ELEMENTARY
CORE Academy
UTAH STATE OFFICE OF EDUCATION & UTAH STATE UNIVERSITY

Participant Handbook

ELEMENTARY CORE ACADEMY • ELEMENTARY CORE ACADEMY • ELEM

6TH GRADE

2007

UTAH STATE OFFICE OF EDUCATION

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ISBN: 1-890563-61-7

Acknowledgements

These materials have been produced by and for the teachers of the State of Utah. Appreciation is expressed to the numerous individuals who provided input and effort into the creation of this curriculum. Delivery of the Elementary CORE Academy, including the development and delivery of content, coordination of sessions, distribution of materials, and participant interaction, has been a collaborative effort of many educational groups across Utah. The following organizations, Utah teachers, and educational leaders contributed ideas and activities as part of this professional development project:

Organizations:

Utah State Office of Education (USOE)
Utah State University (USU)
State Science Education Coordination Committee (SSECC)
State Mathematics Education Coordination Committee (SMECC)
Special Education Services Unit (USOE)

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UTAH STATE OFFICE OF EDUCATION

Leadership…Service…Accountability

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Dear CORE Academy Teachers:

Thank you for your investment in children and in building your own expertise as you participate in the Elementary CORE Academy. I hope your involvement helps you to sustain a laser-like focus on student achievement.

Teachers in Utah are superb. By participating in the Academy, you join a host of teachers throughout the state who understand that teaching targeted on the core curricula, across a spectrum of subjects, will produce results of excellence. The research is quite clear–the closer the match of explicit instruction to core standards, the better the outcome on core assessments.

I personally appreciate your excellence and your desire to create wonderful classrooms of learning for students. Thank you for your dedication. I feel honored to associate with you and pledge my support to lead education in ways that benefit all of our children.

Sincerely,

Patti Harrington

Patti Harrington, Ed.D.
State Superintendent of Public Instruction

Funding Sources

Appreciation is expressed for the tremendous educational input and monetary commitment of several organizations for the successful delivery of the Elementary CORE Academy. This year's Elementary CORE Academy was developed and funded through a variety of sources. The Utah State Office of Education (USOE), in collaboration with Utah State University (USU) and local school districts of Utah, have supported kindergarten through sixth grade teachers with professional development experiences that will enhance the educational experience for Utah children.

Major funding for the Academy comes from the following sources:

Federal/State Funds:

Utah State Office of Education
- Staff Development Funds
- Special Education Services Unit

ESEA Title II

Utah Math Science Partnership

District Funds:

Various sources including Quality Teacher Block, Federal ESEA Title II, and District Professional Development Funds

School Funds:

Trust land, ESEA Title II, and other school funds
Utah State Office of Education Special Education Services

The state and district funds are allocations from the state legislature. ESEA is part of the "No Child Left Behind" funding that comes to Utah.

Additionally, numerous school districts, individual schools, and principals in Utah have sponsored teachers to attend the Academy. Other educational groups have assisted in the development and delivery of resources in the Academy.

Most important is the thousands of teachers who take time from their summer to attend these professional development workshops. It is these teachers who make this program possible.

Goals of the Elementary CORE Academy

Overall

The purpose of the Elementary CORE Academy is to create high quality teacher instruction and improve student achievement through the delivery of professional development opportunities and experiences for teachers across Utah.

The Academy will provide elementary teachers in Utah with:

1. Models of exemplary and innovative instructional strategies, tools, and resources to meet the Core Curriculum standards, objectives, and indicators.
2. Practical models and diverse methods of meeting the learning needs of all children, with instruction implementation aligned to the Core Curriculum.
3. Meaningful opportunities for collaboration, self-reflection, and peer discussion specific to innovative and effective instructional techniques, materials, teaching strategies, and professional practices in order to improve classroom instruction.

Learning a limited set of facts will no longer prepare a student for real experiences encountered in today's world. It is imperative that educators have continued opportunities to obtain instructional skills and strategies that provide methods of meeting the needs of all students. Participants of the Academy experience will be better equipped to meet the challenges faced in today's classrooms.

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Core Curriculum

Utah Elementary Math Core Curriculum

Introduction

Most children enter school confident in their own abilities; they are curious and eager to learn more. They make sense of the world by reasoning and problem solving. Young students are building beliefs about what mathematics is, about what it means to know and do mathematics, and about themselves as mathematical learners. Students use mathematical tools, such as manipulative materials and technology, to develop conceptual understanding and solve problems as they do mathematics. Students, as mathematicians, learn best through participatory experiences throughout the instruction of the mathematics curriculum.

Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, the term *mathematical proficiency* has been chosen to capture what it means to learn mathematics successfully. Mathematical proficiency has five strands: computing (carrying out mathematical procedures flexibly, accurately, efficiently, and appropriately), understanding (comprehending mathematical concepts, operations, and relations), applying (ability to formulate, represent, and solve mathematical problems), reasoning (logically explaining and justifying a solution to a problem), and engaging (seeing mathematics as sensible, useful, and doable, and being able to do the work) (NRC, 2001).

The most important observation about the five strands of mathematical proficiency is that they are interwoven and interdependent. This observation has implications for how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve that goal. At any given moment during a mathematics lesson or unit, one or two strands might be emphasized. But all the strands must eventually be addressed so that the links among them are strengthened. The integrated and balanced development of all five strands of mathematical proficiency should guide the teaching and learning of school mathematics. Instruction should not be based on the extreme positions that students learn solely by internalizing what a teacher or book says, or solely by inventing mathematics on their own.

The Elementary Mathematics Core describes what students should know and be able to do at the end of each of the K-6 grade levels. It was developed and revised by a community of Utah mathematics teachers, mathematicians, university mathematics educators, and

State Office of Education specialists. It was critiqued by an advisory committee representing a wide variety of people from the community, as well as an external review committee. The Core reflects the current philosophy of mathematics education that is expressed in national documents developed by the National Council of Teachers of Mathematics, the American Association for the Advancement of Science, and the National Research Council. This Mathematics Core has the endorsement of the Utah Council of Teachers of Mathematics. The Core reflects high standards of achievement in mathematics for all students.

Organization of the Elementary Mathematics Core

The Core is designed to help teachers organize and deliver instruction.

- Each grade level begins with a brief description of areas of instructional emphasis which can serve as organizing structures for curriculum design and instruction.
- The INTENDED LEARNING OUTCOMES (ILOs) describe the skills and attitudes students should acquire as a result of successful mathematics instruction. They are found at the beginning of each grade level and are an integral part of the Core.
- A STANDARD is a broad statement of what students are expected to understand. Several Objectives are listed under each Standard.
- An OBJECTIVE is a more focused description of what students need to know and be able to do at the completion of instruction. If students have mastered the Objectives associated with a given Standard, they have mastered that Standard at that grade level. Several Indicators are described for each Objective.
- INDICATORS are observable or measurable student actions that enable students to master an Objective. Indicators can help guide classroom instruction.
- MATHEMATICAL LANGUAGE AND SYMBOLS STUDENTS SHOULD USE includes language and symbols students should use in oral and written language.
- EXPLORATORY CONCEPTS AND SKILLS are included to establish connections with learning in subsequent grade levels. They are not intended to be assessed at the grade level indicated.

Guidelines Used in Developing the Elementary Mathematics Core

The Core is:

Consistent With the Nature of Learning
In the early grades, children are forming attitudes and habits for learning. It is important that instruction maximizes students' potential and gives them understanding of the intertwined nature of learning. The main intent of mathematics instruction is for students to value and use mathematics as a process to understand the world. The Core is designed to produce an integrated set of Intended Learning Outcomes for students.

Coherent
The Core has been designed so that, wherever possible, the ideas taught within a particular grade level have a logical and natural connection with each other and with those of earlier grades. Efforts have also been made to select topics and skills that integrate well with one another and with other subject areas appropriate to grade level. In addition, there is an upward articulation of mathematical concepts and skills. This spiraling is intended to prepare students to understand and use more complex mathematical concepts and skills as they advance through the learning process.

Developmentally Appropriate
The Core takes into account the psychological and social readiness of students. It builds from concrete experiences to more abstract understandings. The Core focuses on providing experiences with concepts that students can explore and understand in depth to build the foundation for future mathematical learning experiences.

Reflective of Successful Teaching Practices
Learning through play, movement, and adventure is critical to the early development of the mind and body. The Core emphasizes student exploration. The Core is designed to encourage a variety of interactive learning opportunities. Instruction should include recognition of the role of mathematics in the classroom, school, and community.

Comprehensive
By emphasizing <u>depth</u> rather than <u>breadth</u>, the Elementary Mathematics Core seeks to empower students by providing a comprehensive background in mathematics. Teachers are expected to teach all the standards and objectives specified in the Core for their grade level, but may add related concepts and skills.

Feasible

Teachers and others who are familiar with Utah students, classrooms, teachers, and schools have designed the Core. It can be taught with easily obtained resources and materials. A handbook is also available for teachers and has sample lessons on each topic for each grade level. The handbook is a document that will grow as teachers add exemplary lessons aligned with the new Core.

Useful and Relevant

This curriculum relates directly to student needs and interests. The relevance of mathematics to other endeavors enables students to transfer skills gained from mathematics instruction into their other school subjects and into their lives outside the classroom.

Reliant Upon Effective Assessment Practices

Student achievement of the standards and objectives in this Core is best assessed using a variety of assessment instruments. Performance tests are particularly appropriate to evaluate student mastery of mathematical processes and problem-solving skills. Teachers should use a variety of classroom assessment approaches in conjunction with standard assessment instruments to inform instruction. Sample test items, keyed to each Core Standard, may be located on the "Utah Mathematics Home Page" at http://www.usoe.k12.ut.us/curr/math. Observation of students engaged in instructional activities is highly recommended as a way to assess students' skills as well as attitudes toward learning. The nature of the questions posed by students provides important evidence of their understanding of mathematics.

Based Upon the National Council of Teachers of Mathematics Curriculum Focal Points

In 2006, the National Council of Teachers of Mathematics (NCTM) published ***Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*** (NCTM, 2006). This document is available online at http://www.nctm.org/focalpoints. This document describes three focal points for each grade level. NCTM's focal points are areas of emphasis recommended for the curriculum of each grade level. The focal points within a grade are ***not the entire curriculum*** for that particular grade; however, Utah's Core Curriculum was designed to include these areas of focus.

Intended Learning Outcomes for Third through Sixth Grade Mathematics

The main intent of mathematics instruction is for students to value and use mathematics and reasoning skills to investigate and understand the world.

The Intended Learning Outcomes (ILOs) describe the skills and attitudes students should acquire as a result of successful mathematics instruction. They are an essential part of the Mathematics Core Curriculum and provide teachers with a standard for student learning in mathematics.

ILOs for mathematics:

1. **Develop a positive learning attitude toward mathematics.**
2. **Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.**
3. **Reason logically, using inductive and deductive strategies and justify conclusions.**
4. **Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.**
5. **Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.**
6. **Represent mathematical ideas in a variety of ways.**

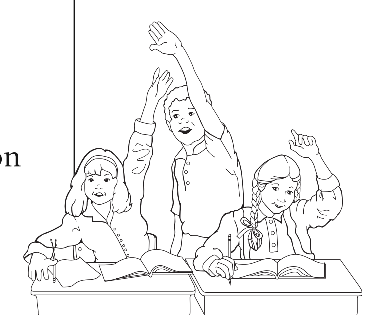
Significant mathematics understanding occurs when teachers incorporate ILOs in planning mathematics instruction. The following are ideas to consider when planning instruction for students to acquire the ILOs:

1. **Develop a positive learning attitude toward mathematics.**

   When students are confident in their mathematical abilities, they demonstrate persistence in completing tasks. They pose mathematical questions about objects, events, and processes while displaying a sense of curiosity about numbers and patterns. It is important to build on students' innate problem-solving inclinations and to preserve and encourage a disposition that values mathematics.

2. **Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.**

   Problem solving is the cornerstone of mathematics. Mathematical knowledge is generated through problem solving



as students explore mathematics. To become effective problem solvers, students need many opportunities to formulate questions and model problem situations in a variety of ways. They should generalize mathematical relationships and solve problems in both mathematical and everyday contexts.

3. **Reason logically, using inductive and deductive strategies and justify conclusions.**

   Mathematical reasoning develops in classrooms where students are encouraged to put forth their own ideas for examination. Students develop their reasoning skills by making and testing mathematical conjectures, drawing logical conclusions, and justifying their thinking in developmentally appropriate ways. Students use models, known facts, and relationships to explain reasoning. As they advance through the grades, students' arguments become more sophisticated.

4. **Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.**

   The ability to express mathematical ideas coherently to peers, teachers, and others through oral and written language is an important skill in mathematics. Students develop this skill and deepen their understanding of mathematics when they use accurate mathematical language to talk and write about what they are doing. When students talk and write about mathematics, they clarify their ideas and learn how to make convincing arguments and represent mathematical ideas verbally, pictorially, and symbolically.

5. **Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.**

   Students develop a perspective of the mathematics field as an integrated whole by understanding connections within mathematics. Students should be encouraged to explore the connections that exist with other disciplines and between mathematics and their own experiences.

6. **Represent mathematical ideas in a variety of ways.**

   Mathematics involves using various types of representations including concrete, pictorial, and symbolic models. In particular, identifying and locating numbers on the number line has a central role in uniting all numbers to promote understanding of equivalent representations and ordering. Students also use a variety of mathematical representations to expand their capacity to think logically about mathematics.

Sixth Grade Mathematics Standards

By the end of grade six, students have mastered the four arithmetic operations with whole numbers, positive rational numbers, positive decimals, and positive and negative integers; they accurately compute and solve problems. They find prime factorizations, least common multiples, and greatest common factors. They create, evaluate, and simplify expressions, and solve equations involving two operations and a single variable. They solve problems involving an unknown angle in a triangle or quadrilateral, and use properties of complementary and supplementary angles. Students know about π as the ratio between the circumference and the diameter of a circle and solve problems using the formulas for the circumference and area of a circle. Students analyze, draw conclusions, and make predictions based upon data and apply basic concepts of probability.

Standard I:
Students will expand number sense to include operations with rational numbers.

Standard I: Students will expand number sense to include operations with rational numbers.

Objective 1: Represent rational numbers in a variety of ways.

a. Recognize a rational number as a ratio of two integers, a to b, where b is not equal to zero.

b. Change whole numbers with exponents to standard form (e.g., $2^4 = 16$) and recognize that any non-zero whole number to the zero power equals 1 (e.g., $9^0 = 1$).

c. Write a whole number in expanded form using exponents (e.g., $876{,}539 = 8 \text{ x } 10^5 + 7 \text{ x } 10^4 + 6 \text{ x } 10^3 + 5 \text{ x } 10^2 + 3 \text{ x } 10^1 + 9 \text{ x } 10^0$).

d. Express numbers in scientific notation using positive powers of ten.

Objective 2: Explain relationships and equivalencies among rational numbers.

a. Place rational numbers on the number line.

b. Compare and order rational numbers, including positive and negative mixed fractions and decimals, using a variety of methods and symbols, including the number line and finding common denominators.

c. Find equivalent forms for common fractions, decimals, percents, and ratios, including repeating or terminating decimals.

d. Relate percents less than 1% or greater than 100% to equivalent fractions, decimals, whole numbers, and mixed numbers.

e. Recognize that the sum of an integer and its additive inverse is zero.

Objective 3: Use number theory concepts to find prime factorizations, least common multiples, and greatest common factors.

a. Determine whether whole numbers to 100 are prime, composite, or neither.

b. Find the prime factorization of composite numbers to 100.

c. Find the greatest common factor and least common multiple for two numbers using a variety of methods (e.g., list of multiples, prime factorization).

Objective 4: Model and illustrate meanings of operations and describe how they relate.

a. Relate fractions to multiplication and division and use this relationship to explain procedures for multiplying and dividing fractions.

b. Recognize that ratios derive from pairs of rows in the multiplication table and connect with equivalent fractions.

c. Give mixed number and decimal solutions to division problems with whole numbers.

Objective 5: Solve problems involving multiple steps.

a. Select appropriate methods to solve a multi-step problem involving multiplication and division of fractions and decimals.

b. Use estimation to determine whether results obtained using a calculator are reasonable.

c. Use estimation or calculation to compute results, depending on the context and numbers involved in the problem.

d. Solve problems involving ratios and proportions.

Objective 6: Demonstrate proficiency with the four operations, with positive rational numbers, and with addition and subtraction of integers.

a. Multiply and divide a multi-digit number by a two-digit number, including decimals.

b. Add, subtract, multiply, and divide fractions and mixed numbers.

c. Add and subtract integers.

Mathematical language and symbols students should use:
prime, composite, exponent, least common multiple, least common denominator, greatest common factor, decimals, percents, divisible, divisibility, equivalent fractions, integer, dividend, quotient, divisor, factor, simplest terms, mixed numeral, improper fraction

Exploratory Concepts and Skills

- Explore the addition and subtraction of positive and negative fractions.
- Investigate the concepts of ratio and proportion.
- Investigate the distributive property of multiplication over addition of double-digit multipliers.

Standard II: Students will use patterns, relations, and algebraic expressions to represent and analyze mathematical problems and number relationships.

Objective 1: Analyze algebraic expressions, tables, and graphs to determine patterns, relations, and rules.

a. Describe simple relationships by creating and analyzing tables, equations, and expressions.

b. Draw a graph and write an equation from a table of values.

c. Draw a graph and create a table of values from an equation.

Objective 2: Write, interpret, and use mathematical expressions, equations, and formulas to represent and solve problems that correspond to given situations.

a. Solve single variable linear equations using a variety of strategies.

b. Recognize that expressions in different forms can be equivalent and rewrite an expression to represent a quantity in a different way.

c. Evaluate and simplify expressions and formulas, substituting given values for the variables (e.g., 2x + 4; x = 2; therefore, 2 (2) + 4 = 8).

Standard II:
Students will use patterns, relations, and algebraic expressions to represent and analyze mathematical problems and number relationships.

Mathematical language and symbols students should use:
order of operations, sequence, function, pattern, algebraic expression, approximately equal, ≈, notation for exponents: 4^3 or 4^3, a number in front of a variable indicates multiplication (e.g., 3y means 3 times the quantity y), formula, generalization

Exploratory Concepts and Skills
- Use physical models to investigate and describe how a change in one variable affects a second variable.
- Use models to develop understanding of slope as constant rate of change.
- Model situations with proportional relationships and solve problems.

Standard III: Students will use spatial and logical reasoning to recognize, describe, and analyze geometric shapes and principles.

Objective 1: Identify and analyze attributes and properties of geometric shapes to solve problems.

a. Identify the midpoint of a line segment and the center and circumference of a circle.

b. Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.

c. Develop and use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle in a triangle or quadrilateral.

Objective 2: Visualize and identify geometric shapes after applying transformations on a coordinate plane.

a. Rotate a polygon about the origin by a multiple of 90° and identify the location of the new vertices.

b. Translate a polygon either horizontally or vertically on a coordinate grid and identify the location of the new vertices.

c. Reflect a polygon across either the x- or y-axis and identify the location of the new vertices.

Mathematical language and symbols students should use:
midpoint, circumference, complementary and supplementary angles, rotate, translate, reflect, transformation

Exploratory Concepts and Skills
- Use manipulatives and technology to model geometric shapes.
- Investigate tessellations.
- Explore the angles formed by intersecting lines.
- Identify and draw shapes and figures from different views/perspectives.

Standard III:
Students will use spatial and logical reasoning to recognize, describe, and analyze geometric shapes and principles.

Standard IV: Students will understand and apply measurement tools and techniques and find the circumference and area of a circle.

Standard IV: Students will understand and apply measurement tools and techniques and find the circumference and area of a circle.

Objective 1: Describe and find the circumference and area of a circle.

a. Explore the relationship between the radius and diameter of a circle to the circle's circumference to develop the formula for circumference.

b. Find the circumference of a circle using a formula.

c. Describe pi as the ratio of the circumference to the diameter of a circle.

d. Decompose a circle into a number of wedges and rearrange the wedges into a shape that approximates a parallelogram to develop the formula for the area of a circle.

e. Find the area of a circle using a formula.

Objective 2: Identify and describe measurable attributes of objects and units of measurement, and solve problems involving measurement.

a. Recognize that measurements are approximations and describe how the size of the unit used in measuring affects the precision.

b. Convert units of measurement within the metric system and convert units of measurement within the customary system.

c. Compare a meter to a yard, a liter to a quart, and a kilometer to a mile.

d. Determine when it is appropriate to estimate or use precise measurement when solving problems.

e. Derive and use the formula to determine the surface area and volume of a cylinder.

Mathematical language and symbols students should use:
cylinder, radius, diameter, circumference, area, surface area, volume, π

Exploratory Concepts and Skills

- Investigate volumes and surface areas of a variety of three-dimensional objects.

Standard V: Students will analyze, draw conclusions, and make predictions based upon data and apply basic concepts of probability.

Objective 1: Design investigations to reach conclusions using statistical methods to make inferences based on data.

a. Design investigations to answer questions.

b. Extend data display and comparisons to include scatter plots and circle graphs.

c. Compare two similar sets of data on the same graph and compare two graphs representing the same set of data.

d. Recognize that changing the scale influences the appearance of a display of data.

e. Propose and justify inferences and predictions based on data.

Objective 2: Apply basic concepts of probability and justify outcomes.

a. Write the results of a probability experiment as a fraction between zero and one, or an equivalent percent.

b. Compare experimental results with theoretical results (e.g., experimental: 7 out of 10 tails; whereas, theoretical 5 out of 10 tails).

c. Compare individual, small group, and large group results of a probability experiment in order to more accurately estimate the actual probabilities.

Mathematical language and symbols students should use:
data display, scatter plot, circle graph, scale, predict, justify, probability, experimental results, theoretical results

Exploratory Concepts and Skills
- Investigate the notion of fairness in games.

Standard V:
Students will analyze, draw conclusions, and make predictions based upon data and apply basic concepts of probability.

Facilitated

Activities

New Math Core Curriculum
Elementary CORE Academy 2007

Since the 2003 adoption of Utah's Elementary Mathematics Core Curriculum, ideas such as coherence, focus, high expectations, computational fluency, representation, and important mathematics have become regular elements in discussions about improving school mathematics. As the next step in devising resources to support the development of a coherent curriculum, the National Council of Teachers of Mathematics (NCTM) released *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence.*

With NCTM's release of the Curriculum Focal Points and discussion regarding high expectations, it became important for Utah to revise the Elementary Mathematics Core Curriculum. The placement of concepts within the Curriculum Focal Points guided the placement of concepts within Utah's Core.

The Core has also been designed so that, wherever possible, the ideas taught within a particular grade level have a logical and natural connection with each other and with those of earlier grades. Efforts have also been made to select topics and skills that integrate well with one another and with other subject areas appropriate to grade level. In addition, there is an upward articulation of mathematical concepts and skills. This spiraling is intended to prepare students to understand and use more complex mathematical concepts and skills as they advance through the learning process.

The Core takes into account the psychological and social readiness of students. It builds from concrete experiences to more abstract understandings. The Core focuses on experiences with concepts that students can explore and understand in depth to build the foundation for future mathematical learning experiences.

The Elementary Mathematics Core describes what students should know and be able to do at the end of each of the K-6 grade levels. It was developed and revised by a community of Utah mathematics teachers, mathematicians, university mathematics educators, and State Office of Education specialists. It was critiqued by an advisory committee representing a wide variety of people from the community, as well as an external review committee. The Core reflects the current philosophy of mathematics education that is expressed in national documents developed by the National Council of Teachers of Mathematics, the American Association for the Advancement of Science, and the National Research Council. This Mathematics Core has the endorsement of the Utah Council of Teachers of Mathematics. The Core reflects high standards of achievement in mathematics for all students.

E-D-P Model
Elementary CORE Academy 2007

Each day good educators observe and interact with students to determine what course of action should be taken to achieve the best educational results for each learner. These observations, in many instances, are made with limited formal data. The E-D-P Model assists educators in the collection and use of information justifying implementation of practices. Many educators struggle with the ability to articulate and align teaching actions with student learning needs. The E-D-P Model is a method of aiding this articulation.

When assessing, it is important to know that correct answers do not necessarily mean students understand a concept. Conversely, incorrect responses may not indicate that a student hasn't learned a concept. It is important for educators to look for hidden understandings and possible misconceptions. Ongoing assessments, observations, and interviews may be necessary. When using this process, instructors should select assignments/tasks where students have opportunities to explain their understanding. Developing a tool to aid teachers in the collection of information and to assist them in determining student understanding has been the driving force in creating the E-D-P Model.

Our discussion begins with a description of the E-D-P Model. This model is based on a medical metaphor of **E**valuation-**D**iagnosis-**P**rescription (E-D-P). It is important to understand the difference between three main types of assessment: diagnostic (usually occurring prior to instruction), formative (concurrently occurs with instruction), and summative (occurs at the conclusion of an instructional period). The E-D-P Model targets diagnostic and formative assessments. By conducting ongoing assessments and using this formative information, educators can effectively impact student learning and plan instruction to meet individual learning needs (McNamee & Chen, 2005).

Evaluation

In classrooms across the country one may observe teachers interacting with students in a variety of ways. The **E**valuation portion of the E-D-P Model provides teachers with a way to identify student learning as it relates to the standard and objective of instruction. As a teacher sees a particular student response she is able to identify understandings and misunderstandings.

EXAMPLE: Marcia responded with the answer of 12 when she was asked to add 14 and 8. Using Marcia's work, an instructor sees that Marcia needs instruction on renaming. Other conclusions for the same response may also be apparent. The **E**valuation phase can then transition to the **D**iagnosis.

Diagnosis

As the student response is investigated the instructor may need to ask questions or inquire regarding the reasoning used to formulate the response. This is similar to a physician, where if a pain in the abdomen is described, the doctor poses questions to the patient or performs a physical exam to determine the source of pain. Educators can employ a similar method as they determine the cause of the incorrect responses given by a student. The diagnosis may consume large amounts of time or be rapidly identified based on student work.

Prescription

Once a learning need is **D**iagnosed/identified, renaming in the case of our example, the teacher can then determine what **P**rescriptive action should be taken. In the medical profession, the instructor or doctor has multiple medicines or treatments that can be prescribed. These multiple medicines affect individuals in different ways based on body chemistry and make up. This is also true with education in relation to learning styles. In education, teachers should have multiple activities, learning situations, or practice methods that can be prescribed to help students understand. In our example the teacher could prescribe numerous interventions to help our student understand the renaming concept. (e.g., place value practice, peer discussion groups focused on a single problem, one-on-one discussion about place value, manipulative extensions, etc.)

As teachers formalize the work that is done in a classroom they will be able to define the learning that occurs in a classroom and what learning should take place in the future. There can be a fine line between instruction and assessment when educators use quality formative assessment tasks to guide instruction and learning (Leahy, et al., 2005). The E-D-P Model encourages teachers to evaluate student work, diagnose learning needs, and determine the best prescription for continued growth in knowledge. Some teachers complete these three stages daily in classrooms around the nation without defining the process. This model provides educators a method to formalize current practice and aid them in the implementation process.

Citations

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McNamee, G.D., Chen, J.Q. (November 2005). Dissolving the Line Between Assessment and Teaching. *Educational Leadership, 63:3, p.72-76.*

Medical Metaphor T-Chart	
Physician	Educator
Why would a physician complete an Evaluation?	Why would an educator complete an Evaluation?
What would a physician use to make make a medical diagnosis?	What would an educator use to make a learning diagnosis?
When evaluation and diagnosis are complete what kind of prescription would be given?	When evaluation and diagnosis are complete what kind of prescription would be given?

ELEMENTARY CORE Academy

E-D-P Assessment Form

Evaluation: ______________________

Name ______________________

Date ______________________

Task/Objective ______________________

( ) Individual ( ) Partner ( ) Group

Diagnosis:		
	Strengths	Weakness
1)		
2)		
3)		
4)		
5)		
6)		

Prescription:

ELEMENTARY CORE Academy

E-D-P Assessment Form

Evaluation: ______________________

Name ______________________

Date ______________________

Task/Objective ______________________

( ) Individual ( ) Partner ( ) Group

Diagnosis:		
	Strengths	Weakness
1)		
2)		
3)		
4)		
5)		
6)		

Prescription:

ELEMENTARY CORE Academy

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1)		
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Prescription:

ELEMENTARY CORE Academy

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( ) Individual ( ) Partner ( ) Group

Diagnosis:		
	Strengths	Weakness
1)		
2)		
3)		
4)		
5)		
6)		

Prescription:

E-D-P Assessment Form

Evaluation: ______________________________

Students:	Diagnosis:							Prescription:				
Task:	Communication	Representation	Computation					Task #4	Comp. #6	Assignment #1		
1) Kyler	√-	√	√					X				
2) Jose	√	√+	√-							X		
3) Kyler	√+	√+	√+						X			
4) Sammy	√	√	√-							X		
5) Shelby	√-	√-	√-							X		

E-D-P Assessment Form	
Diagnosis:	Prescription:

*Copy to a label and place on student work.

E-D-P Assessment Form

Evaluation: ______________________________

Students:	Diagnosis:							Prescription:				
Task:	Communication	Representation	Computation					Task #4	Comp. #6	Assignment #1		
1) Kyler	√-	√	√					X				
2) Jose	√	√+	√-							X		
3) Kyler	√+	√+	√+						X			
4) Sammy	√	√	√-							X		
5) Shelby	√-	√-	√-							X		

E-D-P Assessment Form	
Diagnosis:	Prescription:

*Copy to a label and place on student work.

Core Standard & Objective:	Assessment FOR Learning (Evaluate)			Assessment of Needs (Diagnosis)								
	Type of Assessment			Needs Missing Foundational Skills			Needs Basic Grade Level Core Skills			Needs Extension of Core Skills		
	Pre/Post Dates: /			Plan for Instruction (Prescription) Examples: Expicit Instruction using Graphic Organizers, Skill Specific Activities, Guided Practice, etc.								
Student Name	Limited Knowledge	Partial Knowledge	Mastery									

Mathematical Proficiency
Elementary CORE Academy 2007

How do educators know when a student "Gets It?" Elementary teachers interact with students daily using a variety of individual views regarding mathematical understanding. Success in mathematics is created through a student's composite view and aptitude in five areas of mathematics. In the book, *Helping Children Learn Mathematics,* we are introduced to this composite view of mathematics learning. The term mathematical proficiency is used to describe what it means when a person successfully learns mathematics.

Mathematical proficiency includes five strands:

1) **Understanding:** Comprehending mathematical concepts, operations and relations-knowing what mathematical symbols, diagrams, and procedures mean.
2) **Computing:** Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately.
3) **Applying:** Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately.
4) **Reasoning:** Using logic to explain and justify a solution to a problem or to extend from something known to something not yet known.
5) **Engaging: Seeing mathematics as sensible, useful, and doable-if you work at it-and being willing to do the work.**

It is critical to understand that each of these strands is interwoven and interdependent. Various views of success in mathematics emphasize one aspect of mathematical proficiency with the expectation that the other areas of mathematical knowledge will follow. Success in mathematics comes through achieving mathematical proficiency, which includes each of the five strands.

We see parents, students, and educators focus on only one strand of proficiency, which results in memorized facts that do not necessarily lead to mathematical success. This narrow treatment of math does not provide the strong basis of mathematical learning that students need.

As students learn all the aspects of mathematical proficiency, learning will become stronger, more durable, more adaptable, more useful, and more relevant. It is difficult to master any one of these strands in isolation and is therefore essential to teach the strands in an interconnected method. Developing the strands together builds a student's knowledge of any one strand through connected knowledge points that are memorable.

Citation

National Research Council. (2002). Helping Children Learn Mathematics. Mathematics Learning Study Committee, J. Kilpatrick and J. Swafford, Editors. Center for Education, Division of Behavioral and Social Sciences and Education. Washington, D.C.: National Academy Press.

Building Academic Vocabulary
Elementary CORE Academy 2007

Teaching students vocabulary that will be encountered during the study of content provides a solid background for a positive interaction with that content. Building academic vocabulary is much more than simply placing words upon a word wall or providing a matching exercise with a definition and new terms.

Initially the selection of the terms to be provided to students takes effort and time. Educators should identify key words that are important to the understanding of specific content areas, and are included in the Core Curriculum. The background work of identifying the terms is critical to providing an accurate direction for the subsequent instruction. However, the key to the success of building academic vocabulary ultimately rests upon the quality of the instruction provided by the teacher. Marzano and Pickering provide the following six-step Process for teaching new terms.

The Six-Step Process for Teaching Academic Vocabulary:

1) **Provide a description, explanation, or example of the new term.**
2) **Ask students to restate the description, explanation, or example in their own words.**
3) **Ask students to construct a picture, symbol, or graphic representing the term or phrase**.
4) **Engage students periodically in activities that help them add to their knowledge of the terms in their notebooks**.
5) **Periodically ask students to discuss the terms with one another.**
6) **Involve students periodically in games that allow them to play with the terms.**

With guidance and monitoring students have the ability to generate their own description and representations of vocabulary terms provided. The ownership of this process is valuable in that students see the term as a new tool that aids their learning. An integral step in the process of learning new vocabulary is the student notebook. As students add new terms to their notebook they also refine and update descriptions, which deepens and clarifies their understanding of the content and the terms.

Creating a deeper understanding of vocabulary terms will provide students with multiple points of learning as they encounter new content. These points of learning will broaden the knowledge base and allow students to develop an awareness of the language of learning.

Citation

Marzano, R.J., Pickering, D.J., (2005). *Building Academic Vocabulary Teachers's Manual* ASCD, Alexandria, VA.

Math I-5&6

Activities

Modeling Operations

Multiplication of Large Numbers by place value

Standard I:
Students will expand number sense to include operations with rational numbers.

Objective 5:
Solve problems involving multiple steps.

Objective 6:
Demonstrate proficiency with the four operations, with positive rational numbers, and with addition and subtraction of integers.

Content Connections:
Language Arts VIII-6; Writing for different audiences.

Math Standard I

Objective 5&6

Connections

Background Information

By the age of seven, children gain the ability to think in a concrete realm. They can separate fantasy from reality. This ability allows them to read and reason mathematically. Elementary school students have a disposition to discover, to see why the world works and how they function within it. Problem solving is that discovery process. It enhances student ability to work with difficult ideas before ever seeing algorithms. Students need contextual situations and models in problem solving. A model or manipulative is a thinking tool. Counters, number lines, money etc. are used to help understand what is happening in the problem and to keep track of numbers while problem solving (Van De Walle, 2004). This is the gateway to understanding the algorithm process.

In multiplication, it is important that students understand groups of things as single entities while understanding that a group contains a given number of objects. Experiences with making and counting groups, especially in contextual situations, are extremely useful (Van De Walle, 2004).

In problem solving, students have freedom of choice in choosing the technique(s) used to solve the problem. However, they must explain their process in pictures and writing to show how they found the solution and why it makes sense.

Research Basis

Fusion, K. C., (2003). Toward Computational Fluency in Multidigit Multiplication and Division, Teaching Children Mathematics.

Problem solving situations allow children to directly model solutions to a problem. Students who are able to model mathematical situations from a problem solving basis, simultaneously develop their mathematical fluency and their ability to problem solve.

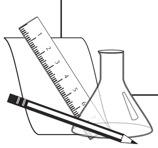
Arbaugh, F., Barker, D.D., Lannin, J.K., Townsend, B.E. (2006). Making the Most of Student Errors.

Student errors provide insight into student thought and process. If educators evaluate student errors, they can guide students toward autonomy in problem solving and sense-making of the mathematical process. This process of student reflection provides the student with strategies for recognizing and reconciling errors.

Invitation to Learn

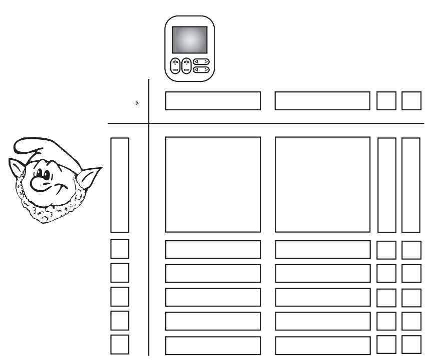
Materials

- ❏ *Base Ten Grid*
- ❏ Santa's Workshop
- ❏ Overhead base ten blocks
- ❏ Base ten blocks for two students.
- ❏ Multiplication T's.



In Santa's workshop, he has fifteen elves working in electronics. If each elf makes twenty-two video games a day, how many video games will the workshop have at the end of the day?

- Give students the *Santa's Workshop* story problem and show it on the overhead. Pass out the base ten blocks (or a hundred grid line if base ten blocks are unavailable) for use in student groups.
- Read the problem as a class. Ask a few questions such as: "What are we trying to answer?" and "How do you think we can do it?" Write their theories on the board. Give students a brief time slot to try to solve each problem.
- Next, take out your overhead base ten blocks. Tell the students that it is sometimes hard to keep track of the information. So you are going to use a manipulative to help keep track of the number of games and the number of elves. The width of the base ten blocks represents the number of elves; the length of the base ten blocks represents how many video games they could complete. Show students how you can fill in the length and width of the array to create a rectangle. This array is a physical representation of the answer.



Side: Number of Elves

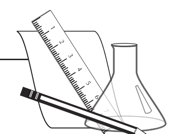
Top:Video Game

- As you fill in the array, lead a discussion on place value within multiplication. List the different place value portions of the answer. For example: If ten elves make ten video games each in a morning, there would be a hundred video games. If they make another ten video games each in the afternoon, there would be a second hundred video games. If five elves make ten video games each in the morning, there would be fifty more video games. If the same five elves make ten more video games each in the afternoon, then there would be fifty more video games. If the ten elves work overtime making two more video games each, they would have made twenty more video games. And, if the last five elves each make two more video games, they make ten more video games. After the base ten blocks are all laid out, use the place value of the base ten blocks to count the value. 100+100+50+50+20+20+10 = 350. That is how many games were made in a day.
- Ask the students, "What did we just determine?" and "How did the manipulatives help us find the answer?"
- Compare the answer to the original theories that students presented. Did the other theories provide the answer? Were there things that were the same in the student's ideas and the class discussion?

Instructional Procedures

This lesson activity, including the Invitation to Learn, should take four to ten days. Don't try to introduce every idea at once. The ideas build upon one another. Present a few ideas per day, making sure that students are invited to share their problem solving ideas as the problems are solved.

1. Hand out the *Think* story problem page. This page includes two story problems similar to the *Invitation to Learn*. Tell the students that the first problem will be solved as a class--but that the students will lead the teacher. Read the story problem, hand out the base ten blocks and ask for student input. "What are you trying to answer?" "What should we do first?" "How many cooks are there?" "How many potatoes were peeled by each cook?" etc. Use the base ten manipulatives to create a physical representation of the answer. Create the array. Then look at the individual place value segments for the answer.
2. Next, have students work in pairs to solve the last problem on the *Think* story problem page. Give them approximately



Materials

- ❑ *Base Ten Grid*
- ❑ Think
- ❑ *Mind over Matter: Mental Math*
- ❑ Four Square Method
- ❑ Partial Products without Four Square
- ❑ Relating Partial Products
- ❑ Fast Freddie
- ❑ Sharing The Strategy with Parents
- ❑ Lattice Multiplication
- ❑ Overhead base ten blocks
- ❑ Base ten blocks for two students.
- ❑ Multiplication T's.

fifteen to twenty minutes to work in groups. Move about the classroom, supporting individual groups in their problem solving process. At the end of the group section, ask different groups to come up and share how they solved the story problem.

3. Tell the students they are going to multiply using mental math. Hand out *The Mind over Matter: Mental Math* page. Work the first problem together. Then have students solve problems in groups using the base ten grid. This exercise is important to help students understand and use the proper place value in the multiplication process. The most important part of this exercise will be the written comparisons of the problems. Students need to be able to explain the relationship between the single place value problems and the multiple place value problem found on the same page. As students are working, walk around, visit each group, and listen to each discussion. Have each group tell you what they are seeing. Remind them to write down what they see happening.
4. Next, give the students the *Four Square Method* page. Tell students that they are going to use their estimation and their mental math knowledge to help solve problems. Show students how to break each number into expanded place value. Then mentally multiply the place value numbers together. Add the partial sums for the final answer. This exercise may take a few days to finish. Have students work on two to three problems a day until they are finished. Have students work out problems, then work in pairs to find the answers.
5. The four square method of partial products is a great way to graphically organize place value multiplication of numbers. However, students need to see a relationship between the place value computation they have been working on and the standard algorithm for multiplying. Hand out the *Partial Products without Four Square* page. This page will have problems that look more like the standard algorithm. The problems they will work with will be written according to the standard algorithm. However, have students solve the problem in a partial product fashion. Students need to see that they are still multiplying in groups of tens and ones.

Example:

```
     26
X    16
     36    -- 6x6
    120    --6x20
```

```
 60    --10x6
200    --10x20
416
```

6. Hand out the *Relating Partial Products Algorithm* page. The last part of this lesson directs students to see the place value, working within the standard algorithm. The problems in this exercise will be the same as the problems given in the *Partial Products* page, so that the students can make a comparison. Write the same problem on the overhead twice. Solve the first problem using the partial products method and the second using the standard algorithm. As you are solving, make sure you talk to your students. Talk about how the strategies relate to one another. Have students try the rest of the problems in groups or on their own. As individual students work on the problems, make sure you discuss the place value nature of the computation. Show how you are keeping track of it within the problem. Have individual students work on problems; discuss what they see and how they are keeping track of the answer. The first four problems on this page will be exactly the same as the first four problems on other *Partial Products* page. Once students complete the first four problems, have them compare the answers to the *Partial Products* page. Have them answer questions about the page, and remark about what they see.

Example:

```
   26
 X 16
  156
  260
  416
```

Assessment Suggestions

- Have students assess a problem from *Fast Freddie*. Tell the students that *Fast Freddie* loves recess. He is a good student most of the time. But when the recess bell rings, he sometimes gets careless in his work. Show a problem that *Fast Freddie* tried to complete as the recess bell rang. Have students write about his process and answer. Was he correct in his thinking?

Example *Fast Freddie Problem*:

"Ring!" The recess bell rang and *Fast Freddie* was worried about being late to recess. He had been working on a story problem as the bell rang. The story problem read this way: Twelve students each brought eleven cans for the Thanksgiving food drive. How many cans did the class collect together? Freddie's answer was: 12 + 11 = 23. Did his answer make sense according to the question asked in the problem?

- Have students come up with a story problem to fit this answer. I have 240 apples. What are some possible ways to get 240 apples?

Curriculum Extensions/Adaptations/ Integration

- For advanced learners: have students write their own multiplication story problems, have them swap problems and solve.
- Always allow slower learners to use the models to facilitate thinking within the problems. Have counters (base ten blocks) readily available.
- Lattice method of multiplication

Family Connections

- Have students search for things in their home that can come in groups. How many pairs of socks were washed in one load of laundry? How many cans of soup can be stacked on one shelf?

- Since this is a new strategy, have students take the strategy home and share the new idea with parents using the *Sharing the Strategy with Parents* worksheet.

Name ________________________________

Santa's Workshop

In Santa's workshop, he has fifteen elves working in electronics. If each elf makes twenty-two video games in a day, how many video games will the workshop have at the end of the day?

Name ________________________________

"Think"

Problem #1

A hotel hired thirteen cooks to help make Thanksgiving dinner for their guests. Each cook peeled seventeen potatoes. How many potatoes did the hotel use for the feast?

Problem #2

Twelve trees were sent to the Festival of Trees. Each tree had fifteen candy canes. How many candy canes were there?

Name ________________________________

"Think"

Problem #1

A hotel hired thirteen cooks to help make Thanksgiving dinner for their guests. Each cook peeled seventeen potatoes. How many potatoes did the hotel use for the feast?

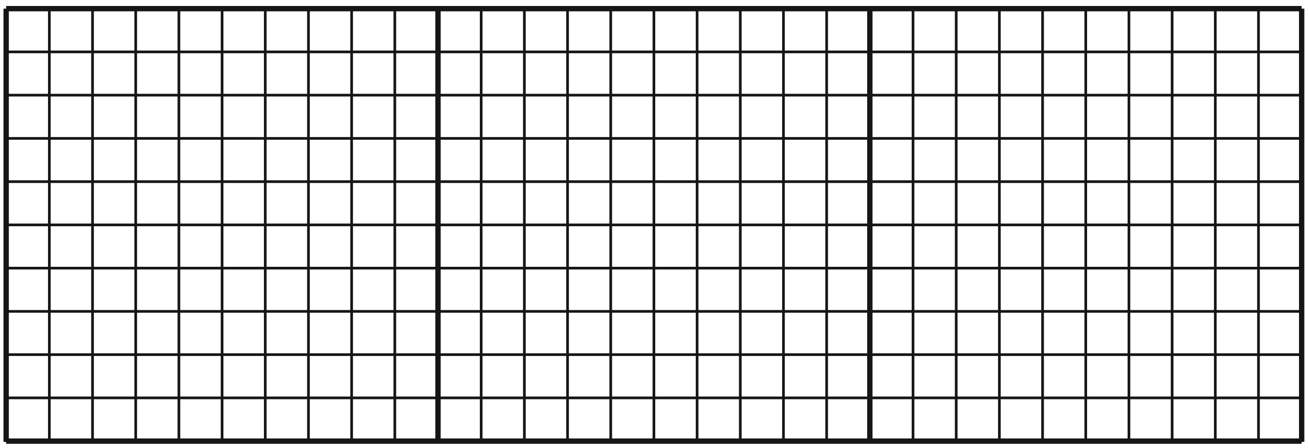
Problem #2

Twelve trees were sent to the Festival of Trees. Each tree had fifteen candy canes. How many candy canes were there?

Mind over Matter: Mental Math

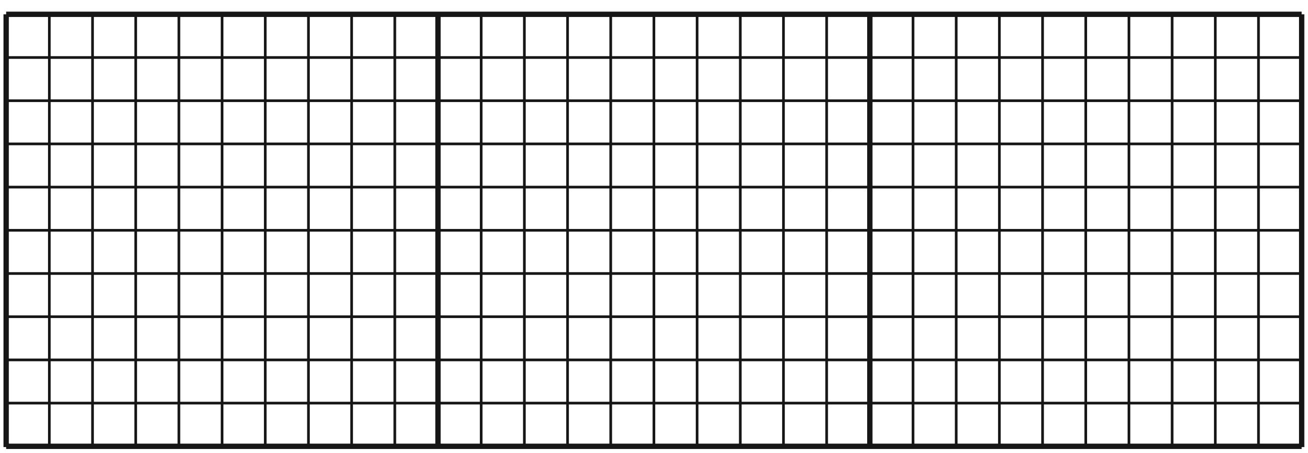
Section I

2*1____
2*10____
2*100____



Draw each Problem:
3*5______
30*5______
300*5______

Draw each Problem:



What Patterns do you see?

____

____

Section II

2*2________
2*10________
20*2________
20*10________

Draw each Problem:

Section III

12*22________

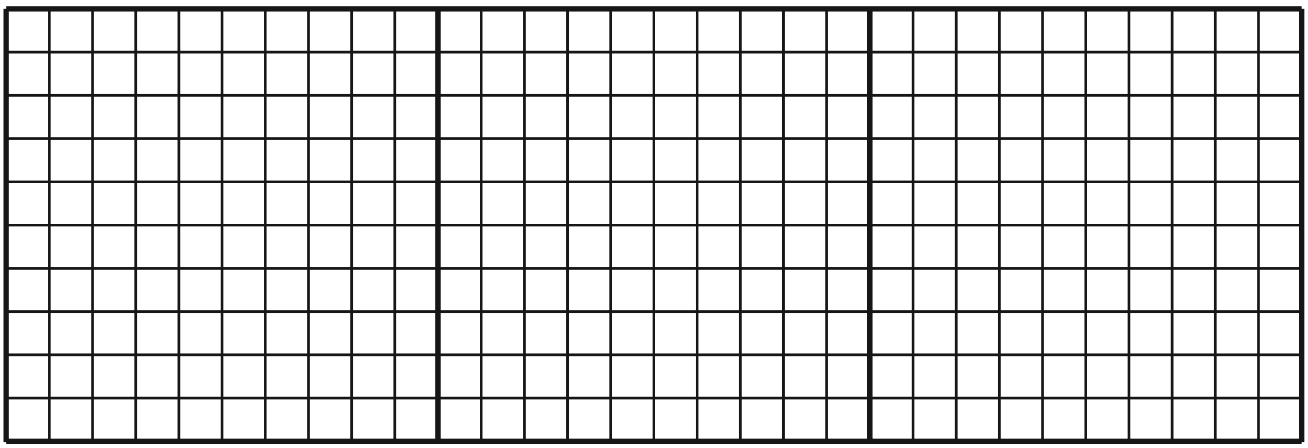
Draw each problem:

What is similar about the problems in section two and section three?

____
____

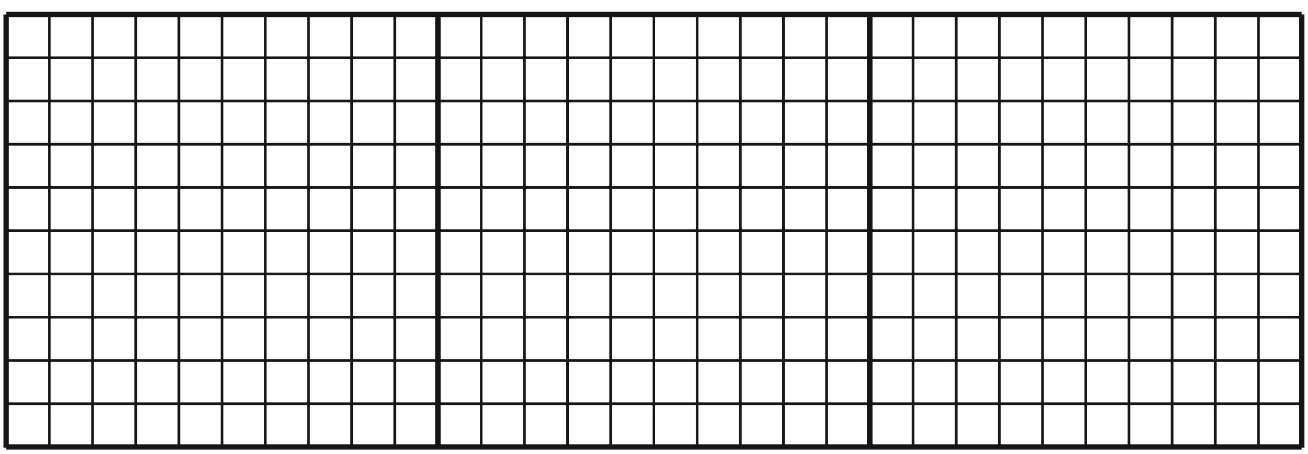
Section IV

4*3___
4*20___
10*3___
10*20___



Draw each problem:
Section V

23*14



Draw the problem:
What is similar with the problems that you see in sections four and five?

Four Square Method

Directions:
1. Estimate the final answer.
2. Break apart the numbers by place value.
3. Multiply by place value inside each box (60*40= 2400)
4. Add all of the box answers for your final answer.

Example: 65 • 42

Step #1. Estimation: 60 • 40 = 2400 (so the answer should be close to 2400)

Step # 2. & 3.

	40 +	2
60 +	2,400	200
5	200	10

Step # 4.

2400
200
120
+ 10

2,730 (Final Answer)

1. 34 • 58	2. 46 • 97
3. 76 • 78	4. 567 • 36
5. 123 • 34	6. 245 • 249
7. 345 • 87	8. 645 • 176

Name ________________________________

Partial Products without Four Square

Directions:
1. Estimate the answer to the nearest ten
2. Multiply in place value
3. Add the place value answers together

Example:	Estimation:	Partial Product:
26	30	26
x 16	x 20	x 16
	600	36
		120
		60
		200
		416

Estimation Partial Product	Estimation Partial Product
12 x 24	56 x 23
45 x 47	254 x 34
167 x 23	236 x 127

Name ________________________________

Relating Partial Products

Directions:
1. Estimate the Answer to the nearest ten
2. Multiply
3. Add the multiplied answers together.

Example:	Estimation:	Standard Way:
26	30	26
x 16	x 20	x 16
	600	156
		260
		416

Estimation Standard Way	Estimation Standard Way
12 x 24	56 x 23
45 x 47	254 x 34

1. Pull out your partial problems page. Examine the first four problems. What do you see that is similar? What do you find different?

____

____

____

____

____

2. Can you explain why we add zeros as place holders in each succeeding line that is multiplied?

3. How does using zeros as place value holders relate to the partial products process?

Estimation Standard Way	Estimation Standard Way
112 x 44	36 x 23
344 x 49	146 x 57

Name ________________________________

Fast Freddie

"Ring!" The recess bell rang and "Fast Freddie" was worried about being late to recess. He had been working on a story problem as the bell rang. The story problem read: Twelve students brought eleven cans each for the Thanksgiving food drive. How many cans did the class collect together? Freddie's answer was: 12 + 11 = 23. Did his answer make sense according to the question asked in the problem?

Name ________________________________

Fast Freddie

"Ring!" The recess bell rang and "Fast Freddie" was worried about being late to recess. He had been working on a story problem as the bell rang. The story problem read: Twelve students brought eleven cans each for the Thanksgiving food drive. How many cans did the class collect together? Freddie's answer was: 12 + 11 = 23. Did his answer make sense according to the question asked in the problem?

Name ______________________________

Sharing the Strategy with Parents

Dear Parents,

As a math class, we have been looking at how place value fits when multiplying large numbers. In this learning process, we have used a few strategies to multiply. We would like to share with you a few multiplication strategies and show you how they fit with the algorithm that most people use.

Problem: 57 x 43	Estimated answer to the nearest ten: x______
Solve the problem with *Partial Products*: 57 x43	Solve the problem using the regular algorithm: 57 x43

Talk as a family:

What do you find similar between the methods?

____

____

____

What do you find different?

____

____

____

Why is it important to estimate the answer?

____

____

____

Division of Numbers Using Partial Quotients

Math Standard I

Objective 5 & 6

Standard I:
Students will expand number sense to include operations with rational numbers.

Objective 5:
Solve problems involving multiple steps.

Objective 6:
Demonstrate proficiency with the four operations, with positive rational numbers, and with addition and subtraction of integers.

Content Connections:
Language Arts VIII-6; Writing in different forms.

Connections

Background Information

From the ages of seven to twelve, children are curious. They have a need to find out how and why the world works. Their concrete ability to think leads them to tinker and discover principles and ideas for themselves. Model problems become a powerful tool for students to reason and problem-solve within a concrete context. The experience of manipulating and moving portions of numbers allows students to see how concepts intertwine with the algorithm process. Division-modeled problems, using base ten blocks or money, allow children to recognize the place value when portioning out numbers. Dividing in place value allows students to solve simpler division problems and compute the quotient at the same time.

There are two different ways to represent division in problem solving. The first is partitive division, or looking for the number of partitions you can make from an original number. The second is measurement or quotitive division. This type of problem examines how much there will be in each individual partition.

Research Basis

Fusion, K. C., (2003). Toward Computational Fluency in Multidigit Multiplication and Division, Teaching Children Mathematics

Problem solving situations allow children to directly model solutions to problems. Students who are able to model mathematical situations from a problem-solving basis simultaneously develop their mathematical fluency and their ability to solve problems.

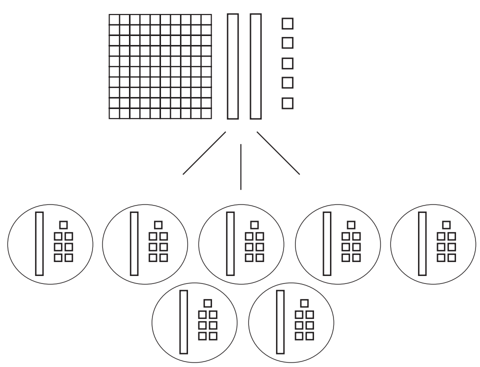
Fitzgerald, W.M., Bouck, M.K., (1992). Models of Instruction.

Inquiry-based mathematics allows students to hypothesize about mathematical ideas, share solutions, and increase understanding as they observe other students' solutions.

Invitation to Learn

There are 125 M&M's in grandma's candy jar. Seven of her grandchildren have come to visit. They all must have an equal share of the candy. Grandma always eats the left over candy. How many M&M's would each grandchild get? How many are left for grandma?

- Start with the Grandma's Candy Jar. Pass out the base ten blocks or the hundred grid line for groups of students to use.
- Read the problem as a class. Ask a few questions like: "What are we trying to answer?" "How do you think we can do it?" Write down the students' suggested methods on the board. Have students take a minute to solve using the methods introduced.
- Next, take out your overhead base ten blocks. Tell the students that because it is sometimes hard to keep track of information, you are going to use the base ten blocks to represent the M&M's in the candy jar. Once you have the entire amount of M&M's represented physically, tell the students that you are going to use your knowledge of place value to share equal amounts of candy with all the grandchildren.



*Use the base ten blocks to show how you can distribute your answer.

- First break apart the hundred's grid so that you have ten groups of ten. Then, share one group of ten M&M's with each grandchild. That leaves fifty-five M&Ms to share. Break the remaining fifty-five into groups. Share them in groups of one, two, or three until all M&Ms have been equally shared. The ones that cannot be equally shared will be saved for grandma. When you have finished sharing, ask the students to tell you, in place value, how many each grandchild shared? One possible solution may be that each grandchild received a grouping of ten, and then they each receive two more. After those were eaten, they could each equally have three more, plus a last candy jar

Materials

- ❏ *Base Ten Grid*
- ❏ *Grandma's Candy Jar*
- ❏ Overhead base ten blocks
- ❏ Base ten blocks

raid amount of two. Six were left in the jar for grandma. So 10 +2 +3 +2 = 17 each grandchild ate seventeen M&M's.

- Ask the students, "How did the manipulatives help us find the answer?"
- Compare the class-directed problem solving to the original theories that the students displayed. Did the other theories provide the answer? Were there things that were the same in the student's ideas and the class problem solving discussion?

Instructional Procedures

Materials

- ❏ *Base Ten Grid*
- ❏ *Think #2*
- ❏ *Lucky Seven Partial Quotients*
- ❏ Lucky Seven Strategy Comparison
- ❏ *Share the strategy with Parents #2*
- ❏ *Fast Freddie #2*
- ❏ Overhead base ten blocks
- ❏ Overhead money
- ❏ Base ten blocks
- ❏ Play money

This lesson activity, including the Invitation to Learn, should take three to eight days. Don't try to introduce every idea at once. The ideas build upon one another. Present a few ideas per day. Make sure students are invited to share their problem-solving ideas as they work through the problems.

1. Hand out the *Think* #2 page. This page includes two story problems, similar to those from the invitation to learn. Tell the students that the first problem will be worked on and discussed as a class. However, the students will lead the teacher. Read the story problem, hand out the base ten blocks, and ask for student input. "What are you trying to answer?" "What should we do first?" "How many books came to the Library?" "How many librarians shelved the books?" etc. Use the base ten manipulatives to create a physical representation of the books. Next, show by place value how many books each librarian put away for the day. Then look at the individual place value segments of the answer.
2. This needs to be a teachers directed activity. Show students how to use the change first as one example then have students work in groups to use money to display different examples. Next, have students work in pairs to solve the last problem on the *Think* #2 page. As a class, solve this problem by partitioning coins. Put "$1.24" on the overhead. Tell the students that the problem calls for large dollar amounts, but you can solve a simpler problem and find the same portions using dollars and cents. Tell the students that they are going to put $1.24 into four portions. One way to solve the problem would be to give each portion twenty-five cents, a nickel, and four pennies. Walking the poodle, Jake earned 25+5+4 = 34 cents. Remind the students that they are not trying to solve for the answer in cents. This answer needs to be given in whole dollars so Jake earned 34 whole dollars. Give students fifteen to twenty

minutes to work in groups. Move around the classroom, supporting individual groups in their problem solving. At the end of the group period, have different student groups come up and share how they solved the story problem. With this problem, make sure that all students have the opportunity to use coins in their problem solving process.

3. Hand out the *Lucky Seven Partial Quotients* worksheet. Tell the students that we are going to solve the division problems by solving simpler place value problems. Just like using the base ten blocks, students will use what they know to come up with the quotient. Have students work in pairs using models to answer the questions on the worksheet. In solving simpler division problems, 10's, 5's and 2's are great dividing numbers. For the first few problems, allow students to use the base ten block or coins to help find the answer.

Example:

```
     18  R3
4 )  75 |10
   - 40 |
     35 |  5
   - 20 |
     15 |
   - 12 |+3
      3 |18
```

10 + 5 + 3 = 18 r 3

4. After students become comfortable with the partial products method, hand out the *Lucky Seven Comparison* page. Have students complete the algorithm both ways. The most important part of this lesson will be the insights gained regarding place value partitioning and the exact number needed for division.

Example:

Lucky Seven:

```
     18  R3
4 )  75 |10
   - 40 |
     35 |  5
   - 20 |
     15 |
   - 12 |+3
      3 |18
```

Standard Way:

```
     18  R3
4 )  75
   - 4
     35
   - 32
      3
```

10 + 5 + 3 = 18 r 3

5. Send home the *Share the Strategy with the Parents* #2 page. Have students share the new strategy with parents. Make sure the students talk about the comparison of the two strategies.

Assessment Suggestions

- Have student assess a problem from *Fast Freddie* #2. Tell the students that "Fast Freddie" loves recess. He is a good student most of the time, but when the recess bell rings he sometimes gets careless in his work. Show a problem that "Fast Freddie" tried to complete as the recess bell rang. Have students write about his process and answer. Was he correct in his thinking?

  Example problem:

  "Ring" Fast Freddie heard the recess bell and decided he needed to work a little faster. He had two division problems left before he could go outside. He started his problem. 98÷4. Freddie decides that four groups of ten should be partitioned out first (using the Lucky Seven method), which gives him 40. He adds 40 to the ninety-eight to try to divide his answer. How is Fast Freddie doing with the partitioning process?
- Have students come up with a problem to this question. Each child earned $1.25 each after a lemonade sale. How did they share the total amount of money earned at the sale? Or for an easier version say six children earned $1.25 each after a lemonade sale. How did they share the total amount of money earned at the sale?

Curriculum Extensions/Adaptations/ Integration

- For advanced learners, have students write their own division story problems. Have them exchange problems with friends and solve.
- Always allow special needs learners to use models to facilitate their thinking. Have counters (base ten blocks) or money readily available.

- Lattice Division

Family Connections

- Since this is a new strategy have students explain to their parents the problem solving process. Use the *Share the Strategy with Parents #2*.

Additional Resources

Van De Walle, J.A. (2004) Elementary and Middle School Mathematics Teaching Developmentally, 5th Edition

Armstrong. T. (2006) The Best Schools

Name ______________________________

Grandma's Candy Jar

There are 125 M&Ms in grandma's candy jar. Seven of her grandchildren have come to visit. They all must have an equal share of the candy. Grandma always eats the left over candy. How many M&M's would each grandchild get? How many are left for grandma?

Name ______________________________

Grandma's Candy Jar

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Name ______________________________

"Think" #2

Problem #1
On Tuesday, the library received a shipment of 234 books. Twelve librarians were available to shelve the books. Each librarian shelved an equal number of books. The rest are waiting to be put away on Wednesday. How many books did each librarian shelve on Tuesday?

Problem #2
Jake saved four times as much money from his paper route than from walking the neighbor's poodle. If Jake has now saved $124.00, how much money did he make walking the neighbor's poodle?

Name ______________________________

"Think" #2

Problem #1
On Tuesday, the library received a shipment of 234 books. Twelve librarians were available to shelve the books. Each librarian shelved an equal number of books. The rest are waiting to be put away on Wednesday. How many books did each librarian shelve on Tuesday?

Problem #2
Jake saved four times as much money from his paper route than from walking the neighbor's poodle. If Jake has now saved $124.00, how much money did he make walking the neighbor's poodle?

Name ______________________________

Lucky Seven Partial Quotients

Directions:

1. Divide each portion using groups of 100's, 10's, 5's and 1's
2. Add the partial answers to find the quotient
3. Write what is left over as a remainder

Example:

```
     18  R3
4 )  75 |10
   - 40 |
   ---- |
     35 |  5
   - 20 |
   ---- |
     15 |
   - 12 |+3
   ---- |---
      3 |18
```

10 + 5 + 3 = 18 r 3

497÷4	238÷3
3,945÷7	960÷30
735÷11	1470÷23
7550÷25	327÷14

Name ________________________________

Lucky Seven Comparison

Directions:
1. Solve the algorithm using the Lucky Seven method.
2. Solve the algorithm using the standard way.
3. Compare the answers.

Lucky Seven	Standard Way
160÷40	160÷40
855÷19	855÷19
5683÷54	5683÷54

1. How are these two strategies similar?

____

____

2. How does the place value of the Lucky Seven method help you?

____

____

Name ______________________________

Sharing the Strategy with Parents #2

Dear parents,

As a math class, we have been looking at how place value fits when you divide large numbers. In this learning process, we have used a few strategies to divide. We would like to share with you a few division strategies and show you how they fit with the algorithm that most people use.

Problem: 1460÷24	
Solve the Problem with the Lucky Seven method: 1460÷24	Solve the problem using the regular algorithm: 1460÷24

Talk as a family:

What do find similar?

____

____

____

What do you find different?

____

____

Name ______________________________

Fast Freddie #2

“Ring” Fast Freddie heard the recess bell and decided he needed to work a little faster. He had two division problems left before he could go outside. He started his problem. 98÷4. Freddie decides that four groups of ten should be partitioned out first (using the Lucky Seven method), which gives him 40. He adds 40 to the ninety-eight to try and divide his answer. How is Fast Freddie doing with the partitioning process?

Name ______________________________

Fast Freddie #2

“Ring” Fast Freddie heard the recess bell and decided he needed to work a little faster. He had two division problems left before he could go outside. He started his problem. 98÷4. Freddie decides that four groups of ten should be partitioned out first (using the Lucky Seven method), which gives him 40. He adds 40 to the ninety-eight to try and divide his answer. How is Fast Freddie doing with the partitioning process?

The Decimal Dimension

Standard I:
Students will understand the basic properties of rocks, the processes involved in the formation of soils, and the needs of plants provided by soil.

Objective 5:
Solve problems involving multiple steps.

Objective 6:
Demonstrate proficiency with the four operations, with positive rational numbers, and with addition and subtraction of integers.

Content Connections:
Language Arts VIII-6; Writing for different audiences.

Math Standard I

Objective 5 & 6

Connections

Background Information

Teaching students to estimate answers provides insight into students understanding of place value. Estimation, using easy to handle parts of the problem prior to decimal computation, is a checking strategy that allows the student to still work in the place value of the given number. Although students are working with parts of whole numbers with multiplication and division, they need to understand that they are working with grouping and partitioning numbers. By computing an estimate, they solve a simpler problem that will be close to the actual decimal answer.

Research Basis

Rubenstein, R.N., (2001). Mental Mathematics beyond the Middle School: Why? What? How?

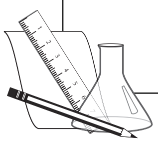
"One simple reason to emphasize mental math is that it is useful for workers, consumers, and citizens. In daily life, adults use estimation more often than exact computation." Estimating is a daily tool in buying groceries, budgeting, and providing for others. It allows us to know "about how much" before we ever know the exact amount of what will be needed (Rubenstein, 2001).

Fitzgerald, W.M., Bouck, M.K., (1992). Insights from Research on Mathematical Problem Solving in the Middle Grades

Students need to have more than just computational knowledge in order to solve problems. In the complexities of finding solutions, students will call upon their concept, linguistic, and algorithmic knowledge. It is important to use real life applications for students to engage in finding solutions.

Materials

- ❑ *Decimal!*



Invitation to Learn

Students must first identify words, pictures and symbols that represent decimal numbers. Students will play Decimal! a game of comparing decimals, cents, and standard form.

Before starting the game students must regard the 100's block of the base ten blocks as one whole. They will regard a ten stick as one tenth and one cube as one hundredth. In money, one dollar will be the whole, ten cents represents one tenth, and one cent will represent one hundredth.

Rules:

1. The object of this game is to be the first person with no cards. **The game is played in pairs**. During a player's turn, they can lay down matches they have in their hand or add to matches that have already been laid down.
2. Each player draws seven cards.
3. The rest of the cards are turned face down. This becomes the draw pile.
4. Turn the first card from the draw pile face up. **Players will either add to the card turned over, or place the matches that they have in their hands down.**
5. Both players look at their hand to see if they have an equivalent representation for the number. For example: From the draw pile, you draw twenty cents. The student who has the card that reads *two tenths*, .2, or the base ten representation, can match the card.
6. All matches are laid out on the table so that other players can add to them.
7. If a player does not have a match, they have to draw from the discard pile.

A player wins the game:

1. **When all the cards from the discard pile are used, the player with the least amount of cards in their hands will win.**
2. **Getting rid of all cards in their hand and yelling DECIMAL! before anyone else**.

Instructional Procedures

This lesson activity, including the invitation to learn, should take three to four days. Don't try to introduce every idea at once. The ideas build upon one other. Present a few ideas per day and make sure students are invited to share their problem solving ideas as the problems are solved.

1. After the students play the Decimal game, tell the students that they can use these manipulatives to multiply and divide decimal problems. Give a group of students money. To another group, give base ten blocks. Have them use what they know to solve the first problem from the "*Think" #3* page.

   Example: Twelve students in Ms. Christensen's class earned 25 cents each for collecting box tops for her class. How much money did they earn all together?

   Walk around the room. Visit each group as they are working on the problem. Discuss ideas and possible solutions. Then have students come up to the overhead and model possible solutions to the question.

2. Have the groups of students trade their manipulatives so that they are working with the manipulatives that they haven't used before. Have students solve the second problem.

   Altogether, the seven students in Mrs. Beckstrand's class earned forty-nine dollars and twenty-one cents. What did each student individually earn if they each have the same amount?

   Have groups of students solve the problem. Again, visit individual groups and discuss their ideas of possible solutions. Then have groups of students model problem solutions for the class.

3. As a class, discuss the problem solving experience. Ask students, "How did we solve the problem?" After they give their answers, point out that, even if they were adding or subtracting, grouping or partitioning, the process was still a form of multiplying or dividing. Ask them, "Was it easy or hard to multiply or divide decimals?" "Why or why not?" "What were some important things to remember as you solved the problem?" "How did you know where to put the decimal point in your answer?"

4. The next day, refer to the story problems that you multiplied or divided with decimals. Talk to the students about solving problems in daily life. "You will not always have base ten blocks or money to help. How can you always make sure your answer has the decimal point in the right place?"

Materials

- ❑ Think #3
- ❑ Estimation Station
- ❑ *Keep It Simple #1*
- ❑ Keep It Simple #2
- ❑ Fast Freddie #3
- ❑ My Favorite Strategy
- ❑ Overhead Base Ten Blocks
- ❑ Base Ten Blocks
- ❑ Overhead Money
- ❑ Money
- ❑ Multiplication T's

5. Have them practice estimating decimal numbers to the nearest whole number. Estimating the answer will help them determine where to place the decimal.

Example:

Number	Estimation
3.4	3
2.351	2
4.56	5
.77	1

Hand out the *Estimation Station* sheet. Have students practice estimating decimal numbers to the nearest whole numbers.

6. Give the students *Keep It Simple #1*. Students will practice estimating the answer to the nearest whole amount. Have the students multiply without the decimal. After they find the numerical answer, they will use estimation to help place the decimal. Remember to take one to two days to complete this assignment. Practice two to three problems each day. If students choose, allow them to use manipulatives when solving these problems.
7. Pass out *Keep It Simple* #2. This page deals with estimation and division problems. Have students complete this page using partial quotients with decimal numbers. Have students divide the numbers using the money system. If students choose, allow them to use dollars and cents as a way of tracking the portioning of parts of whole numbers.

Assessment Suggestions

- *My Favorite Strategy* - Have students use their favorite strategies to help solve multiplication/ division problems. Have them discuss their favorite method and why they like it so much.
- Have students assess a problem from *Fast Freddy #3*. Tell the students that Fast Freddy loves recess. He is a good student most of the time, but when the recess bell rings he sometimes gets careless in his work. Show a problem that Fast Freddy tried to complete as the recess bell rang. Have students write about his process and answer. Was he correct in his thinking?

Example problem:

"Ring." Fast Freddy heard the recess bell and decided he needed to work a little faster. He had one estimation problem left. His problem read 35.16. He estimated that the answer would be close to 36 whole numbers. Was Tommy correct? Explain how you know?

*The problem asked him to round 35.16 to the nearest whole number.

"Ring" Terrible Tommy heard the recess bell and decided he needed to work a little faster. He had two division problems left before he had to go outside. He starts to use the Lucky Seven method to solve his problem 24.24÷12. Tommy decides that 200 can be divided into 24.24. Will Tommy find the correct answer to his problem? Explain how you know?

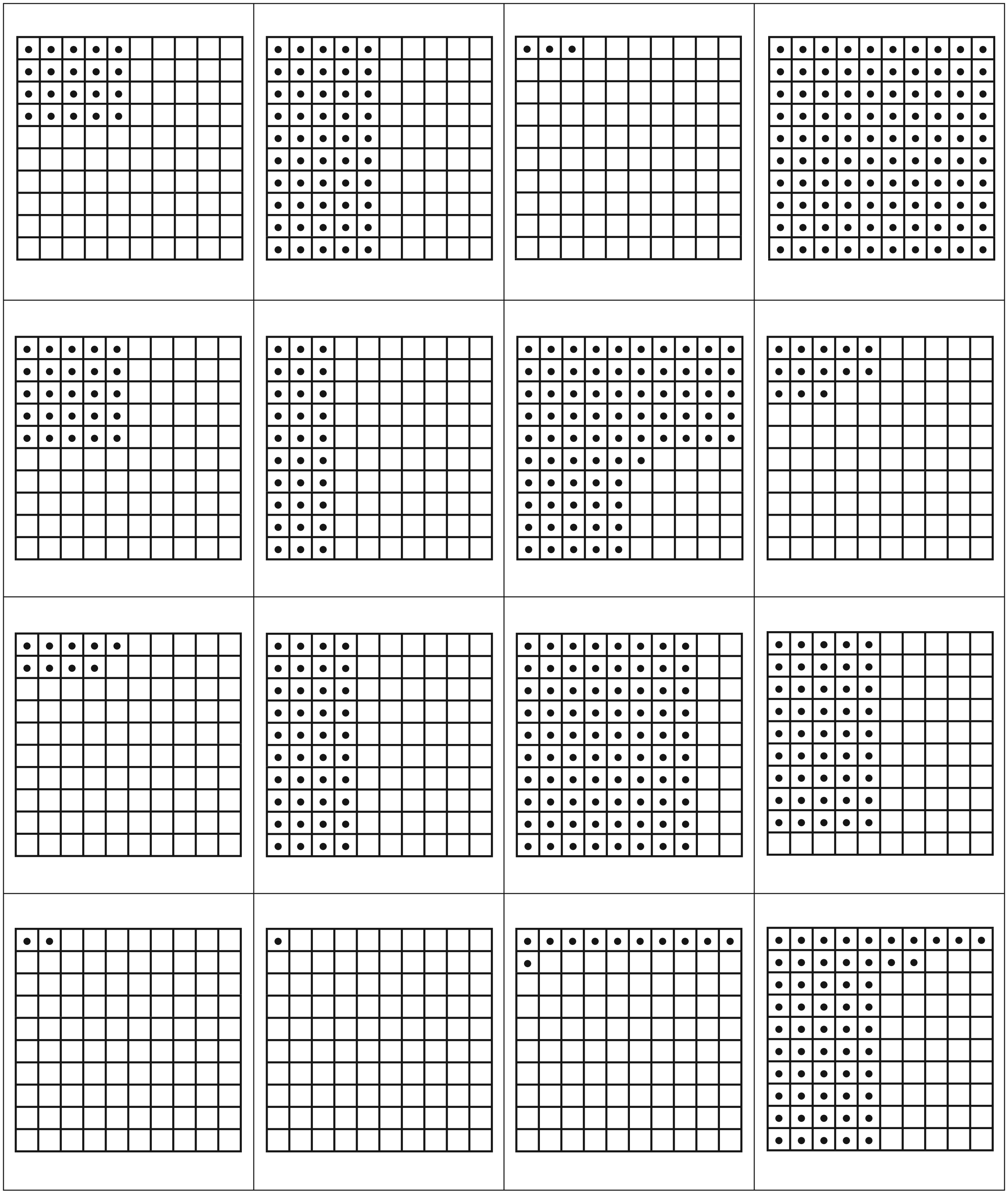
Curriculum Extensions/Adaptations/ Integration

- Always allow special needs learners to use models to facilitate their thinking with the problems. Have counters (base ten blocks) or money readily available.
- Using partial quotients to teach division of repeating decimals is not suggested. It is difficult to represent the partial place value answers in the division process.

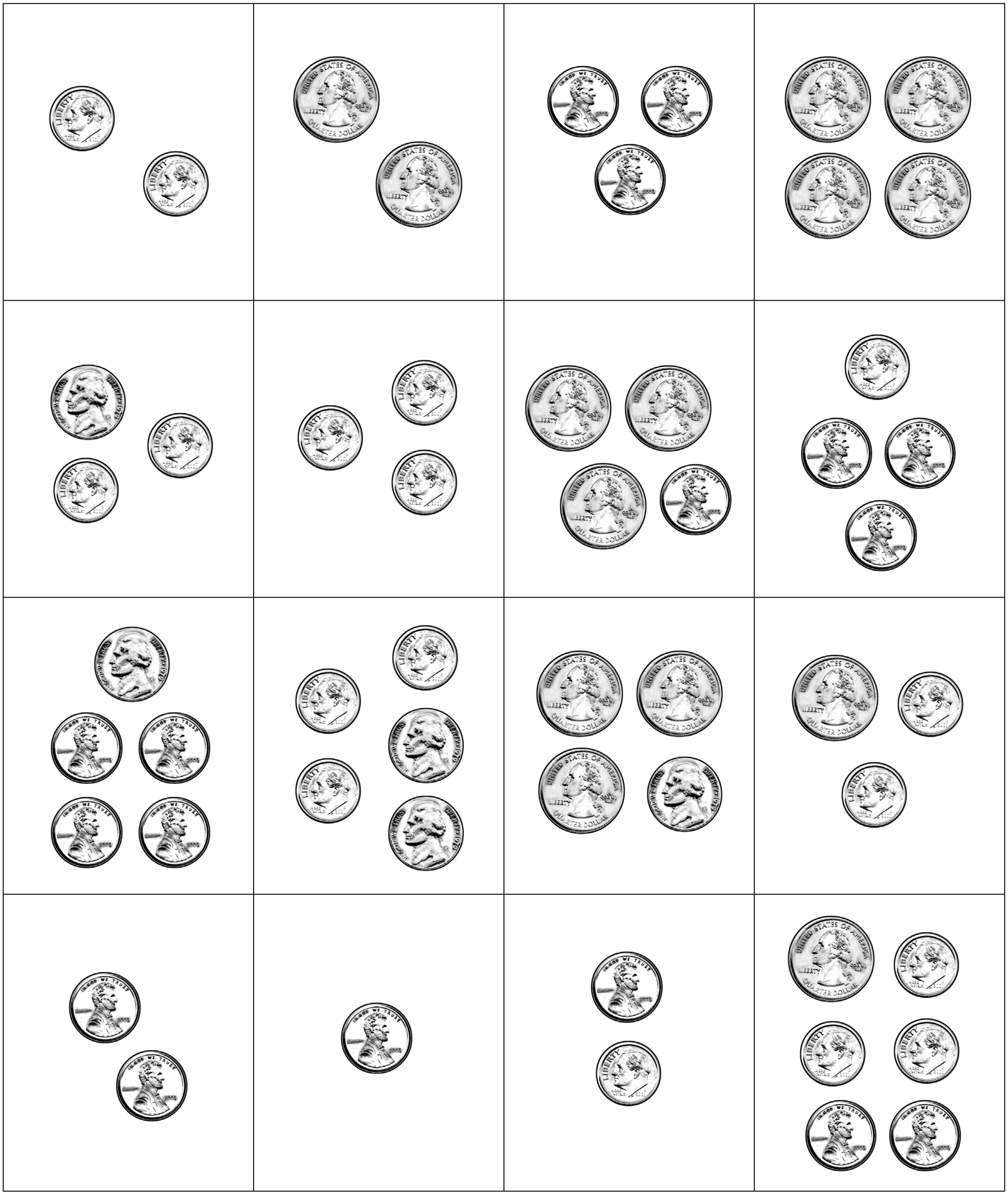
Family Connections

- Have the students write a letter to a parent, brother, or sister explaining the importance of the different multiplication and division strategies you have been using in class. Tell the person you are writing about something new you discovered. Why does this discovery interest you? Which is your favorite strategy? How do you expect to use it?

Decimal!



Decimal!



Decimal!

.2	.5	.03	1
.25	.3	.76	.13
.09	.4	.8	.45
.02	.01	.11	.57

Decimal! Written Form

Two tenths	Five tenths	Three hundredths	One whole
Twenty-five hundredths	Three tenths	Seventy-six hundredths	Thirteen hundredths
Nine hundredths	Four tenths	Eight tenths	Forty-five hundredths
Two hundredths	One hundredth	Eleven hundredths	Fifty seven hundredths

Name ________________________________

"Think" # 3

Twelve students in Ms. Christensen's class earned 25 cents each for collecting box tops for her class. How much money did they earn all together?

Altogether, seven students in Mrs. Beckstrand's class earned forty-nine dollars and twenty-one cents. What did each student individually earn if the amount were the same for each?

Name ______________________________

Estimation Station

Directions:
Round the decimal numbers to the nearest whole number

Number	Estimation	Number	Estimation
2.3		3.45	
23.6		456.4	
13.651		67.564	
56.67		79.151	

Name ________________________________

Keep It Simple #1

Problem/Estimation/Simpler Problem	Problem/Estimation/Simpler Problem
2.34 x 1.5 From the estimation where do you place the decimal point? __________	4.3 x 2.6 From the estimation where do you place the decimal point? __________
12.34 x 4.56 From the estimation where do you place the decimal point? __________	11.23 x 2.78 From the estimation where do you place the decimal point? __________
57.3 x 23.4 From the estimation where do you place the decimal point? ___	124.3 x .12 From the estimation where do you place the decimal point? _________

Name ______________________________

Keep It Simple #2

Directions:
Use the Lucky Seven method, along with dollars and cents, to divide the problems.

Example:

```
        .34
10 | 3.40 | 10
   - 1.00 |
     2.40 | 10
   - 1.00 |
     1.40 | 10
   - 1.00 |
      .40 | + 4
    - .40 | 34
      .00 |
```

Problem	Lucky Seven	Problem	Lucky Seven
8.4÷6		94.85÷5	
11.25÷25		29.4÷21	

How does the Lucky Seven strategy help you know where to put the decimal?

____

____

Name ______________________________

My Favorite Strategy

Problem	Strategy Used
$\begin{array}{r} 3.46 \\ \underline{\times\ 2.7} \end{array}$	

How does the strategy help me find the answer?

____

____

____

____

Problem	Strategy Used
$45.75 \div 25$	

How does the strategy help me find the answer?

____

____

____

____

Name ______________________________

Fast Freddie #3

"Ring" Fast Freddie heard the recess bell and decided he needed to work a little faster. He had one estimation problem left. His problem read 35.16. He estimated that the answer would be close to 36 whole numbers. Was Freddie correct? Explain how you know?

"Ring" Fast Freddie heard the recess bell and decided he needed to work a little faster. He had two division problems left before he had to go outside. He starts to use the Lucky Seven method to solve his problem, 24.24÷12. Freddie decides that 200 can be divided into 24.24. Will he find the correct answer to his problem? Explain how you know?

Name ______________________________

Fast Freddie #3

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Math IV-1

Activities

Measurement

Planetary Circles

Standard IV:
Students will understand and apply measurement tools and techniques and find the circumference and area of a circle.

Objective 1:
Describe and find the circumference and area of a circle.

Intended Learning Outcomes:
5. Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.

Content Connections:
Science III-1; Compare the size of the planets.

Math Standard IV

Objective 1

Connections

Background Information

In this activity, students will find the pattern between the diameter and the circumference of a circle. They will find that the formula for finding circumference is diameter x pi. Before this activity students should know how to identify and measure the radius and diameter of a circle. They should also know that radius is one half of the diameter, and that diameter is twice the radius.

Research Basis

Thorson, A.E. (2002). Mathematics and Science across the Curriculum. ***ERIC Source*** (ERIC # ED463169). Retrieved November 28, 2006, from http://www.eric.ed.gov

This issue for classroom teachers provides a collection of articles focusing on mathematics and science curriculum. Topics addressed in the essays include experiencing mathematics through nature; connecting science, fiction and real life; exploring science and human health; and learning daily from everyday problems.

Ball, D. (1991). What's all this talk about discourse? *Professional Standards for Teaching Mathematics.* National Council of Teachers of Mathematics, 1991.

Deborah Ball defines "discourse" as described by the NCTM Standards. A discussion taken from her classroom, along with entries from her teaching journal, illustrate how thoughtful discourse can be used to help students learn to discuss and understand mathematic concepts.

Invitation to Learn

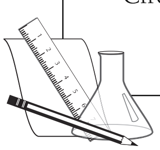
Show a variety of circular objects to your class, and draw some circles on the overhead or board. As a class, review the definitions for radius and diameter. Explain circumference to your students, and show them what it is on a circular object. Using a piece of string and two rulers, demonstrate an easy way for your students to find the diameter of a circle. Put the two rulers on each side of the circular object, making sure that the rulers are parallel to each other, and measure (with the string) in between the two rulers. This will be the circular objects diameter. Then show your students how they would measure a circular object's circumference.

Instructional Procedures

1. Put students into groups and have them go on a search for circles. Have them find ten circular objects and using a string and ruler, measure the diameter and circumference. Have them record the measurements on the worksheet, *Circle Search*.
2. Have students use the data from their search and complete the worksheet.
3. Discuss the worksheet, having students analyze their data for any patterns. They should notice that when the diameter and circumference were multiplied, the product was always close to 3. (The product will vary because measurements are not always exact.)
4. Explain to your students that this number is called pi, and is actually ~3.14. Talk about pi and how the real number extends on forever without ever repeating any patterns of numbers. Cornell University recently found the trillionth digit of pi, but they still haven't found the last digit. The first few digits of pi are 3.14159265. The numbers 3.14 are commonly used to represent the value of pi.
5. As a class, come up with a formula for finding circumference, diameter x pi = circumference.
6. In their math journals, have students write about what they learned on their circle search. Have them describe their thinking as they noticed a pattern on the worksheet and created a formula for finding circumference.
7. Read the book, *Space Circles*. As you read about each planet, have students record the radius and diameter of the planet on the worksheet, *Circles in Space*.
8. After the book is finished, have students complete the worksheet by finding the circumference of each planet.
9. In their science journals, have students write down what they learned about the planets by listening to *Space Circles*, and by completing the worksheet.

Materials

- ❑ string
- ❑ rulers
- ❑ *Circle Search*
- ❑ *Space Circles – Learning About Radius and Diameter*
- ❑ *Circles in Space*
- ❑ Radius, Diameter and Circumference



Assessment Suggestions

- *Circle Search*
- *Circles in Space*
- Informal assessment includes class discussion and journals.
- *Radius, Diameter and Circumference*

Curriculum Extensions/Adaptations/ Integration

- National Pi Day is March 14 (3.14). Celebrate it with circle activities, circular treats, etc.
- Social Studies IV- 1 & 2; Ancient Civilizations. Research pi, and give a report on its origins.

Family Connections

- Have students teach their family how to measure a circular object's diameter by putting the object in between two rulers. Show family how to find the circumference by multiplying the diameter by pi.
- Have students create a game that deals with diameter, circumference and pi. Have them explain diameter, circumference and pi to their family, and then play the game.

Additional Resources

Books

Space Circle - Learning About Radius and Diameter, by Kerri O'Donnell; ISBN 0-8239-8878-3

Circle Search

Measure the circumference (c) and diameter (d) of ten circular objects. Record the measurements in the table. When you get back to class, complete the table by finding the sum, difference, product and quotient of the circumference and diameter for each object.

Object	c	d	c + d	c – d	c x d	c ÷ d
1.						
2.						
3.						
4.						
5.						
6.						
7.						
8.						
9.						
10.						

Examine each column to see if there is a pattern. Describe any patterns you find.

Do you think these patterns exist with all circular objects? Explain your thinking.

Circles in Space

Listen to the book, *Space Circles*. As each planet is discussed, write down the radius and diameter. After finishing the book, calculate the circumference of each planet.

Name of Planet	Radius (r)	Diameter (d)	Circumference (c)
Mercury			
Venus			
Earth			
Mars			
Jupiter			
Saturn			
Uranus			
Neptune			
Pluto			

1. Which planet has the largest circumference? ______________________________
2. Which planet has the smallest circumference? ______________________________
3. What is the difference between the largest circumference and the smallest circumference?

____

4. Do any of the planets have a circumference that is about the same?

____

Radius, Diameter and Circumference

Use the given measures to complete the table. Use 3.14 for ***Pi***.

Radius	Diameter	Circumference
1. 4 in.	8 in.	
2.	16 cm	50.24 cm
3. 2.5 mm		15.7 mm
4.	2 m	
5.		18.84 in.
6. 2 ft.		
7.		31.4 cm
8.	20 mm	
9. 6 ft.		
10.	9 m	
11.		43.96 cm
12. 20 mm		

Circles, Circles, Area

Standard IV:
Students will understand and apply measurement tools and techniques and find the circumference and area of a circle.

Objective 1:
Describe and find the circumference and area of a circle.

Intended Learning Outcomes:
6. Represent mathematical ideas in a variety of ways.

Math Standard IV

Objective 1

Connections

Background Information

This activity helps students explore the formula for the area of a circle, (r^2 x pi). By working with circles, students will develop an understanding of the relationships among the area of a circle, the radius, and pi. Before beginning this activity, students should understand the meaning of the diameter and radius of a circle.

Research Basis

Hinzman, K.P. (1997). Use of Manipulatives in Mathematics at the Middle School Level and Their Effects on Students' Grades and Attitudes. *ERIC Source* (ERIC # ED411150). Retrieved December 10, 2006, from http://www.eric.ed.gov

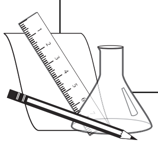
This paper reports on a study that examines mathematical scores when hands on manipulatives and group activities were used in the classroom. Results indicate that student performance was enhanced by the use of manipulative materials; and students' attitudes toward mathematics were significantly more positive than those in previous years when manipulatives were not used.

Reid, J. (1992). The Effects of Cooperative Learning with Intergroup Competition on the Math Achievement of Seventh Grade Students. *ERIC Source* (ERIC # ED355106). Retrieved November 28, 2006, from http://www.eric.ed.gov

This paper reports a study designed to determine the effect of cooperative learning strategies on mathematics achievement of 7th graders. Students were divided into two groups. One group participated in cooperative learning strategies, and the other group received individual/competitive instruction. Pre-tests indicated that no differences existed in the groups prior to instruction, but that the cooperative learning groups performed significantly higher on the post–test. The paper concluded that cooperative group learning strategies are more effective in promoting mathematics achievement.

Materials

- ❏ *A Circles Square*
- ❏ cm graph paper
- ❏ circular objects
- ❏ *A Circles Area*
- ❏ Going in Circles



Invitation to Learn

Pose this problem to your students:

A pizzeria decides to sell three sizes of its new pizza. A small pizza is 10 inches in diameter, a medium pizza is 14 inches in diameter, and a large is 18 inches in diameter. The owner of the pizzeria decides that the price of the small pizza will be $5.00, the medium pizza will be $8.00, and the large pizza will be $12.00. Do you think a large pizza is always the best deal? How would you estimate which pizza size is the best value?

Have the students brainstorm ideas for figuring out which pizza is the best value. Do not lead them in any direction, just listen to their ideas. You will come back to this problem at the end of the Day Two activity.

Instructional Procedures

Day One

1. Ask students how they find the area of triangles and parallelograms. Remind them that they came up with the formula for the area of triangles and rectangles by comparing them to rectangles. For triangles they multiply base x height and then divide it by 2, because a triangle is 1/2 the area of a rectangle. For parallelograms, they can cut it up to make it a rectangle, so base x height is also the formula for area. Explain to them that by using squares, they can learn more about the area of circles.
2. Pass out the worksheet, *A Circles Square.*
3. Explain to the students what a radius square represents.
4. Have students figure out how many radius squares it takes to cover one of the circles. Have them cut out radius squares so they can see how many radius squares it takes to cover that circle.
5. After completing the worksheet, have students write their findings in their math journals. What did they find out about the radius squares? How many radius squares did it take to cover the area of the circle? Did it take the same number of radius squares for all three of the circles?
6. Have a class discussion about what they learned from this activity. Listen to the students comments and ask questions to

deepen their thinking. Do not explain how to find the area of a circle, this will happen in the next part of the activity.

Day Two

7. Hand out centimeter graph paper to groups of students.
8. Have students find two circular objects and trace them on graph paper. Encourage students to center the objects at an intersection of grid lines so that the pair of perpendicular grid lines will divide the traced figures into four equal quadrants. This position will make it easier for students to count the number of square units in the area of each circle. (You may want to show how to line up a circular object on overhead graph paper.)
9. Pass out *A Circles Area*, and have them work with their groups to find the radius and area for two circular objects. Encourage students to be as accurate as possible as they count squares to determine the area of the circles.
10. Have the groups share their data from one of their circular objects with the class to complete their charts for radius and area.
11. Point out that the chart is designed to help them explore a formula for the area of a circle. Have them look at the column that asks them for r^2. Explain that area is a two-dimensional measurement in square units. Remind them that they need two linear dimensions – base x height – for their previous calculation of area (for a rectangle, parallelogram, triangle, and square). The area of a circle is also a two-dimensional measurement, but instead of using base x height, they use radius x radius, or r^2.
12. Have students complete the column for radius squared.
13. Bring the class together again and have them look at the A/r^2 column. Explain to the students that for this column, the area is divided by radius squared.
14. Have students complete the column for A/r^2. The results should be around 3.14, or pi. However, because of inaccuracies in measurement of radius and area, the values may very significantly. Taking an average of all the values in the column might help generate a value close to pi.
15. Have a group report their results for the A/r^2 column. Ask students why they think the data varied for each circular object. Have the students come up with a class average for the column.

16. Have a class discussion on the relationships among the radius, area, and pi. Have students complete their worksheet by writing a formula for the area of a circle. Talk about the formula they created.
17. Have students write the formula in their math journals. Also have them explain what they learned about the area of a circle from doing Day One and Day Two of this activity.
18. Go back to the pizza problem you posed in the Invitation to Learn. Have students figure out which pizza size would be the best deal. Talk about the results and how they figured out which size was the best deal. Have them write their results in their math journals.
19. Have them complete the assessment, *Going in Circles*.

Assessment Suggestions

- Informal assessment includes class discussion, math journals and observation of group work.
- *A Circles Square*
- *A Circles Area*
- *Going in Circles*

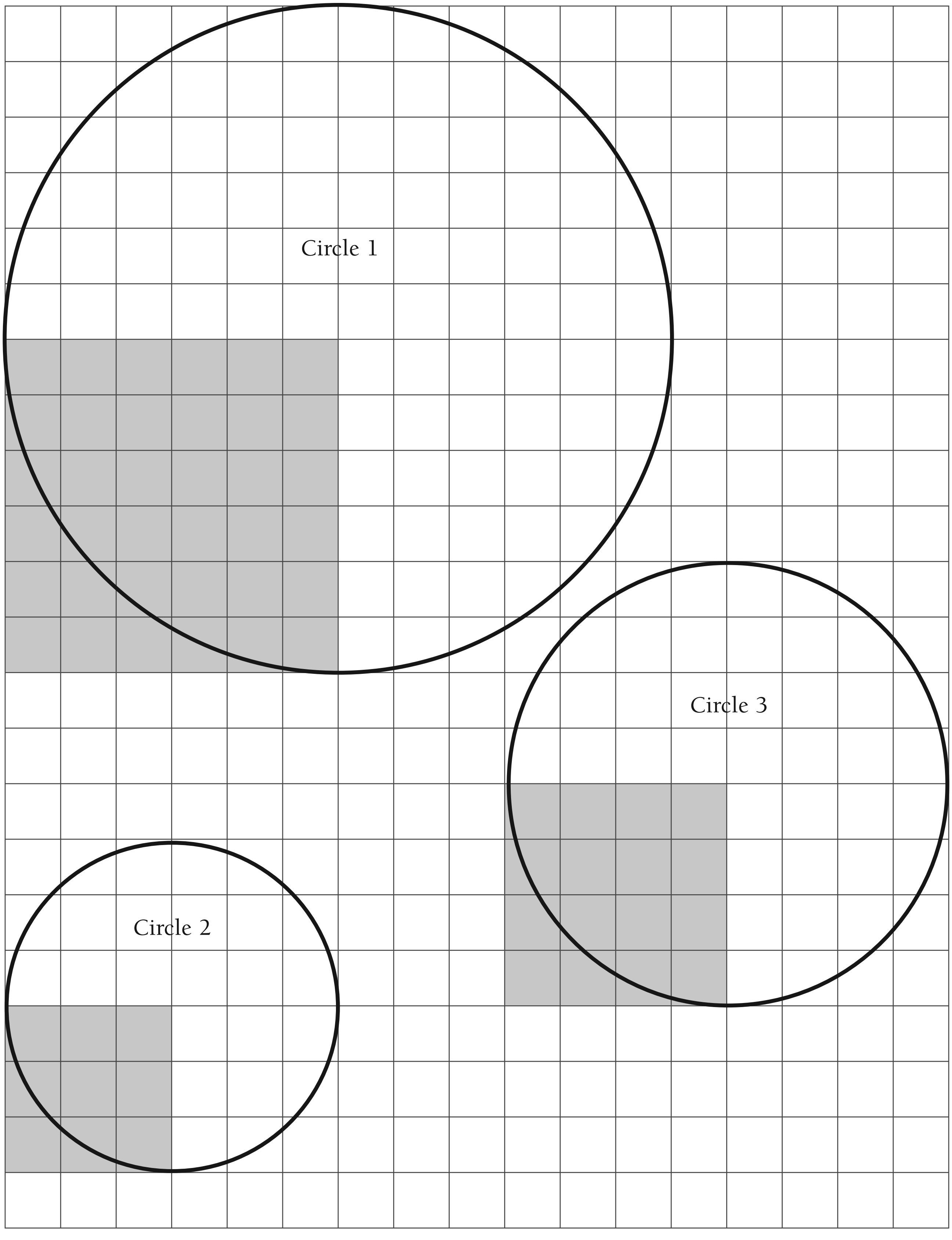
Curriculum Extensions/Adaptations/ Integration

- Have your class go to the gym or playground (anywhere where there is a large circle painted on the ground). Have the students line up around the circle. Have them pair up with a student opposite them (across an imaginary diameter line). Have student A run the diameter of the circle, and student B run the circumference. Have them run until they end up at the same place (or close to it). Student A will have to run the diameter three times while student B makes one revolution. Have each pair of students race, and then talk about pi and the ratio between the circumference and diameter of a circle.

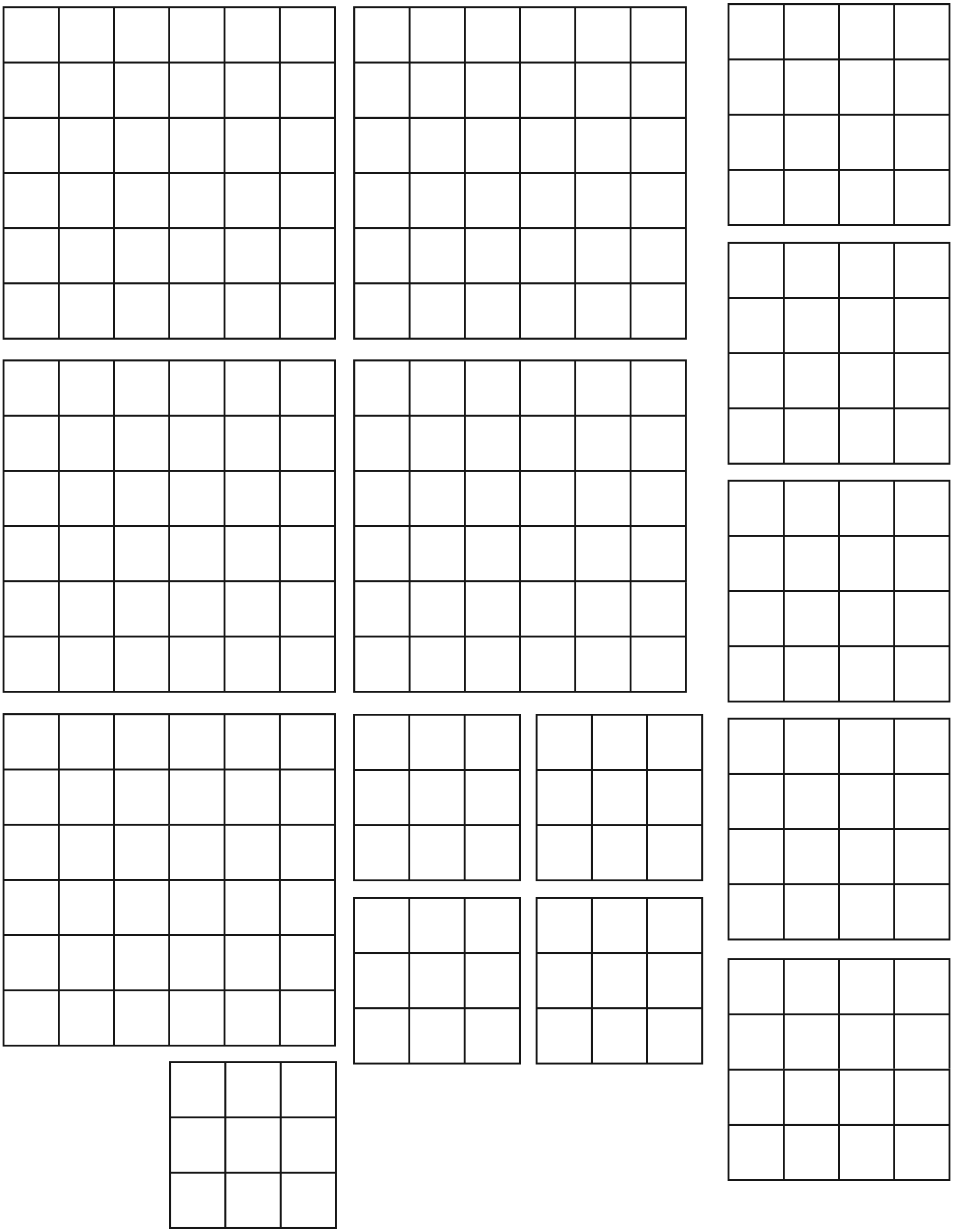
Family Connections

- The next time their family wants to order a pizza, have students find out which pizza place has the best deal for a large pizza. Have them turn in their findings for extra credit (or something like it).

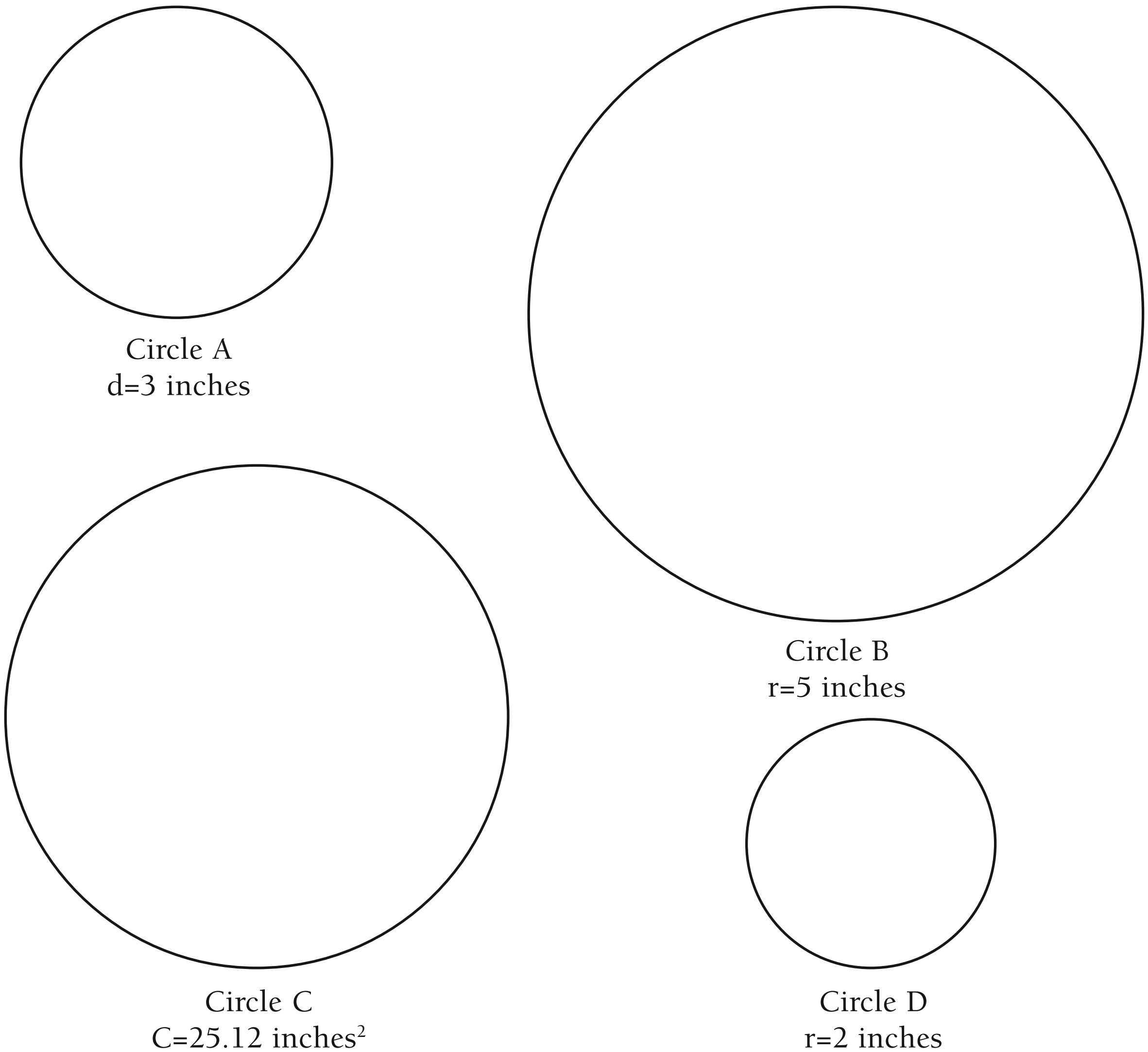
A Circles Square



A Circles Square 2



Going in Circles



1. Which circle has the same area and circumference? ____________________

2. Which circle's circumference is bigger than its area? ____________________

3. Which circle's circumerence is half of its area? ____________________

4. Which circle has an area of 78.5 in^2? ____________________

A Circles Area

Trace two circular objects on centimeter graph paper. Try to center the drawing at an intersection of grid lines. Use the centimeter units to make your measurements.

1. What is the diameter of the first circle? ________ What is the radius? ________

2. What is the diameter of the second circle? ________What is the radius? ________

3. How are the diameter and the radius related? ________________________________

____

4. What is the area of each circle? First: ________ Second: ________
(To find the area, count the number of full squares first and record that number. Then try to piece together partial squares together to make "full" squares. Add the number of these squares to your first number.)

5. Record your data in the chart for the radius and area.

Object	Radius (r)	Area	r^2	A/r^2
1.				
2.				
3.				
4.				
5.				
6.				

6. Compute the average of the values in the last column. Your average should be close to 3.14 or pi. Use that value to write a formula for the area of a circle in terms of its radius.

The Isle of Immeter

Standard IV:
Students will understand and apply measurement tools and techniques and find the circumference and area of a circle.

Objective 1:
Describe and find the circumference and area of a circle.

Intended Learning Outcomes:
6. Represent mathematical ideas in a variety of ways.

Math Standard IV

Objective 1

Connections

Background Information

This activity shows a different way to find the area of a circle, A = 1/2 Circumference x radius. Students will decompose a circle into a number of wedges and rearrange the wedges into a shape that approximates a parallelogram (or rectangle) to develop the formula for the area of a circle. Students should have an understanding of a circles radius and circumference before starting this activity.

Research Basis

Von Drasek, L. (2006). Teaching with Children's Books: The "Wow" Factor. *ERIC Source* (ERIC # EJ729683). Retrieved March 14, 2007, from http://www.eric.ed.gov

Teaching math through children's books motivates children to learn math in exciting new ways; encourages students to think and reason mathematically and builds students' appreciation for math and literature.

Ward, R. (2005). Using Children's Literature to Inspire K-8 Preservice Teachers' Future Mathematics Pedagogy. *ERIC Source* (ERIC # EJ738003). Retrieved March 14, 2007, from http://www.eric.ed.gov

A growing body of research in the fields of mathematics education and literacy supports the inclusion of children's literature with the teaching and learning of mathematics. The author presents a variety of activities and ideas that are sound strategies for effectively integrating children's literature with the teaching of mathematics.

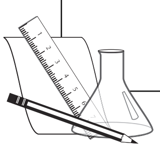
Invitation to Learn

In their math journals, have students create a K-W-L chart on area and perimeter. For the "K" section, have them write down all they know about the perimeter of figures (squares, rectangles, parallelograms, triangles, circles) and what they know about the area of figures (same figures as previously mentioned). For the "W" section of their chart, have them write what they want to know about perimeter and area of figures. Have them leave the "L" section blank. After the activity they will write what they learned about perimeter and area. Have a class discussion on what they put on the "K" and "W" sections of their K-W-L charts.

Instructional Procedures

Materials

- ❏ *Sir Cumference and the Isle of Immeter*
- ❏ overhead tiles
- ❏ *Eighths*
- ❏ *Sixteenths*
- ❏ *Circles Overhead*
- ❏ Perimeter and Area



1. Begin reading, *Sir Cumference and the Isle of Immeter*.
2. After the first page, stop. On the overhead, put up tiles like the square Sir Cumference made. Explain what the book means by inners and edges.
3. Read the next page, on the overhead put the shape that Lady Di made. Talk about the inners and edges.
4. Give a few more examples of squares and rectangles so students understand how to find inners and edges.
5. Read page 5, and explain what Per said about finding the inners and edges with squares and rectangles.
6. Read until the bottom of page 8, and then put tiles on the overhead representing the first doorway. Explain how Radius found the inners so quickly (multiplied the length by the width).
7. After reading page 10, ask students what they think the clue means. "Count half as many inside as out. This unlocks the towers without a doubt."
8. Read until the bottom of page 16.
9. Hand out the copies of the circles divided into eighths to groups of students. Have them cut the wedges and form it into a rectangle. Discuss how it is a "lumpy, bumpy rectangle".
10. Hand out the copies of the circles divided into sixteenths. Have the groups cut the wedges apart and form it into a rectangle. Have the groups try and figure out what the long side of the rectangle represents (one half of the Circumference) and what the short side of the rectangle represents (the radius). Discuss their ideas as a class.
11. Read page 17 and discuss how Per multiplied 1/2 the Circumference by the radius (or length x width), to figure out the area of a circle.
12. Read page 18 and 19, and then make sure students understand what Per is doing.

13. Show the *Circles Overhead* of the three circles, and have the groups figure out the area by multiplying 1/2 Circumference by radius.
14. Read until the end of the book. Review the area and perimeter (inners and edges) of a figure and discuss any questions the students have about the book.
15. Explain to students that we normally don't use the formula 1/2 Circumference x radius to get the area of a circle. Show them the steps of how A = (1/2 C) x r can be changed to the standard formula of

    A = pi x r^2.

    - Area of a rectangle = length x width or 1/2 Cr
    - But to find circumference, you need another formula. Circumference = pi x diameter.
    - Since one diameter equals two radii (d=2r), 2r can be substituted for d, so C = 2r.
    - Put all this information together to make one formula.

      Area of a circle = 1/2 x circumference x radius

      1/2 x 2r x r

      1 x   x r^2 = r^2

      A = r^2

15. In their math journals, have students write what they learned about the perimeter and area of figures in their "L" section of their K-W-L chart. Have a class discussion on what they added to their chart.
16. Have students complete the worksheet, *Area and Perimeter*.

Assessment Suggestions

- Informal assessment includes class discussion, *Circles Overhead*, and the K-W-L chart in their math journals.
- *Area and Perimeter*

Curriculum Extensions/Adaptations/ Integration

- Have students create their own picture book explaining a mathematical concept.

- Have students create a quiz on perimeter and area that they can trade with a classmate.

Family Connections

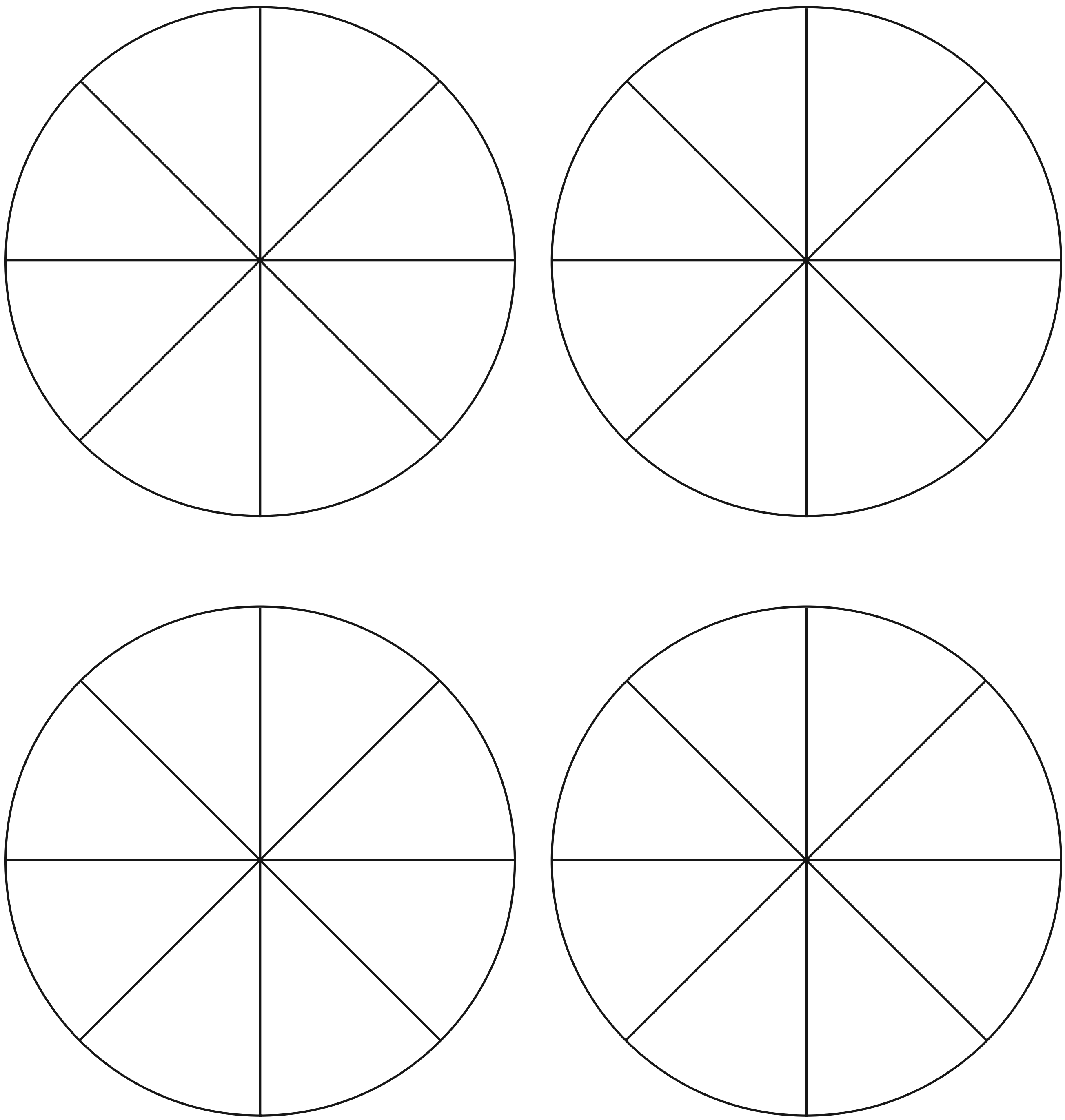
- With your family, measure three circular objects in your home. Show your family how to find the area of the circles by multiplying 1/2 the Circumference by the radius.
- Read *Sir Cumference and the Isle of Immeter* with your family. Answer any questions they may have about perimeter, circumference, radius or area.
- Play a game of inners and edges with your family.

Additional Resources

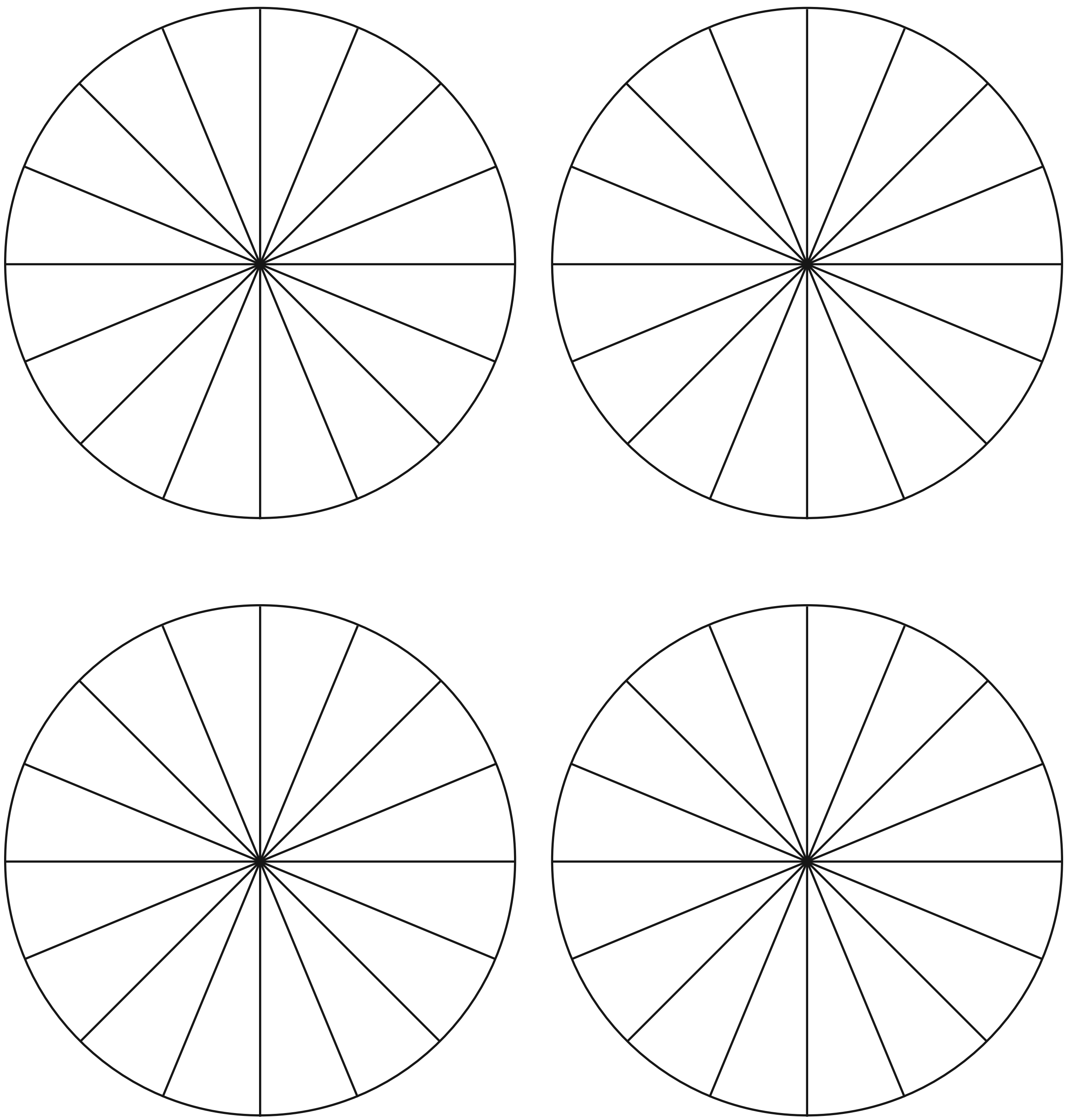
Books

Sir Cumference and the Isle of Immeter by Cindy Neuschwander; ISBN 1-57091-681-0

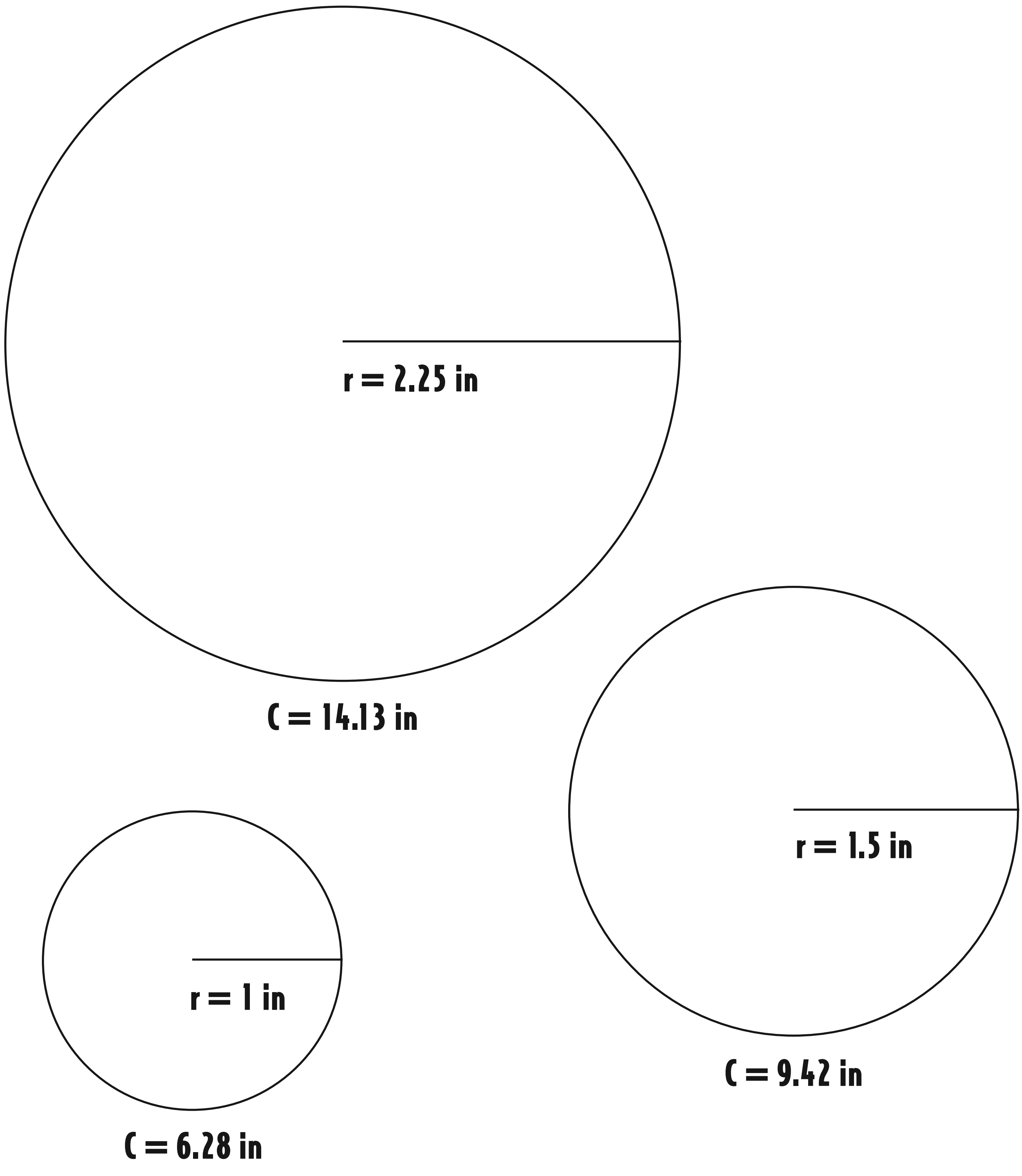
Eighths



Sixteenths



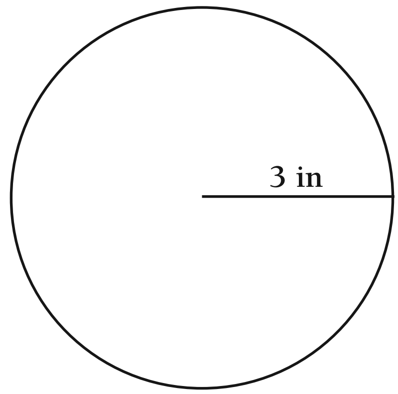
Circles Overhead



Perimeter and Area

Use the clues to determine which figure is being described.

Figure A



CLUES:

- **Its area is less than the area of Figure A.**
- **Its perimeter or circumference is greater than the circumference of Figure A.**

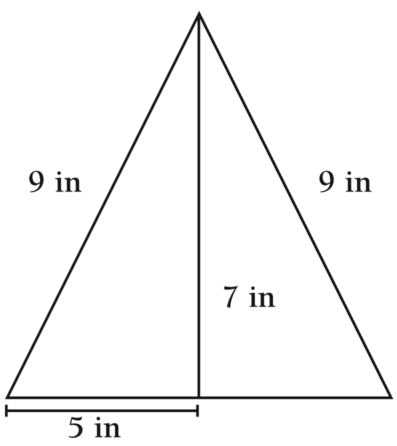


Figure L

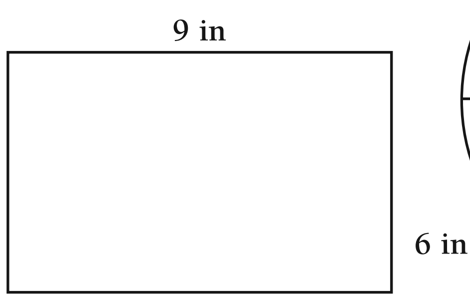


Figure M

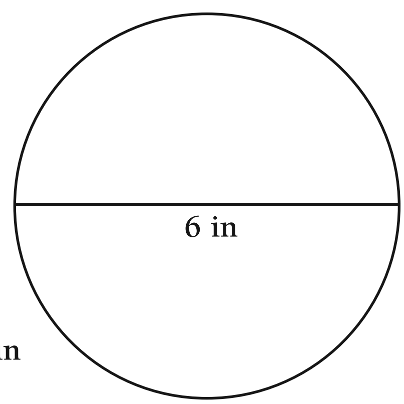


Figure N

1. **Which figure is being described?** ________________________________
2. **How do you know?** ____

____

____

Math V-2

Activities

Probability

How Likely is it?

Standard V:
Students will analyze, draw conclusions, and make predictions based upon data and apply basic concepts of probability.

Objective 2:
Apply basic concepts of probability and justify outcomes.

Intended Learning Outcomes:
1. Develop a positive learning attitude toward mathematics.
2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.

Content Connections:
Math I-2; Convert fractions, decimals, and percents
Science I, II, II, IV

Math Standard V

Objective 2

Connections

Background Information

There is a misconception that the chance of an event occurring is on a scale from 1 to 100. Probability is expressed as a ratio (7 out of 10), a decimal (0.7), a fraction (7/10), or a percent (70%). The first number (numerator) represents the chances of the event happening. The second number (denominator) is the number of attempts made. Which means it is really a fraction from 0 to 1. How likely or unlikely is an event to happen? The "Invitation to Learn" will give you an idea of what students understand about probability.

Research Basis

Chapin, S.H., & Johnson, A. (2000). *Math Matters*. Sausalito: Math Solutions Publications.

Chapter 13 of *Math Matters* it talks about probability, types of probability, and ways to look at probability. The authors let teachers know that students need to experiment with probability to develop an understanding.

Ma, L., (1999). *Knowing and Teaching Elementary Mathematics. Mahwah, New Jersey: Lawrence Erlbaum Associates, Publishers.*

This research emphasizes the importance of teachers understanding the content they are teaching before students can learn. We expect students to understand what they learn, so should we, as teachers understand what we teach. It compares the understanding of fundamental mathematics with the U.S. teachers and China's teachers.

Question of the Day

(Answer the following question in your journal. After answering the question go onto the Probability Meter.)

What is probability and how is it measured?

Probability Meter

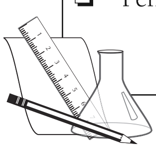
1. Measure 2 feet of addingmachine paper.
2. Fold 2 times and be ready to participate. (see diagram)

Invitation to Learn

Provide a copy of the activity worksheet *Question of the Day/ Probability Meter* for each group or have one on an overhead. After each student has had a chance to write in a journal, or on a piece of paper, the 'Question of the Day', "What is probability and how it is measured?" discuss and share. Students will also follow the directions provided to start creating their probability meters.

Materials

- ❏ Math Journals
- ❏ *Question of the Day/ Probability Meter?*
- ❏ Adding machine tape
- ❏ Clear gallon bag
- ❏ 10 Yellow and 20 orange centimeter cubes/counters
- ❏ Pencil



Instructional Procedures

1. Prepare a probability meter on the board using directions from *Question of the Day/Probability Meter*. The ELL students and others that may struggle can follow along with you now if they were unable to complete this on their own.
2. Put 10 yellow counters in a clear gallon bag. Ask, "What are the chances of pulling a orange marker out of the bag?" 0 or impossible. Do a few tests so the students see what happens. Write a "0" and "impossible" on the left end of your meter (paper). Ask students to brainstorm any synonyms. Next ask, "What are the chances of pulling a yellow counter out of the bag?" It is certain or 1. Do 2 or 3 tests (events) to show what happens. Write a "1" and "certain" on the right end of your meter (paper).

fold	fold	fold	

3. Next, place 10 orange counters in the gallon bag with the 10 yellow counters. Ask, "What are the chances of pulling a orange counter out of the bag?" Have the students brainstorm what the chances are could be. The teacher may suggest, only if students have not come up with anything, "It may or may not happen, a 50-50 likelihood." Ask where this chance would go on their probability meter and why. Do a few tests (events) to see what happens. On the center fold write "1/2", and "equally likely to happen and not happen."
4. Take out 5 of the orange counters, now there are 5 orange and 10 yellow, 15 total. Ask, "What are the chances of pulling a orange counter out of the bag?" Let the students brainstorm first. If no students respond then the teacher could suggest, "Is it impossible, 50-50 (1/2), certain or Unlikely?" On the fold between the 0 and 1/2, write "¼" and "unlikely".
5. Put 15 orange counters in the bag, making 20 orange and 10 yellow, with a total of 30 counters. Ask the same question,

"What are the chances of pulling a orange marker out?" Possible answers could be 50-50 or likely (3/4) an orange will come out. On the fold between the 1/2 and 1, write ¾ and likely.

6. Discuss a few situations so students have a chance to practice measuring and explaining probability. As you go through these situations bring in percentages (0%, 25%, 50%, 75%, 100%) and where they would go on the meter. Use the following examples:

   It will rain tomorrow. (50:50, 50% chance)
   The sun will rise in the morning. (Certain, 100%)

   Two students will be absent tomorrow.
   You will have two birthdays this year. (0, 0%)

   You will get tails if you toss a coin
   The earth revolves around the sun.

   In a new box of crayons at least one will be blue. You will be in school tomorrow.

   When you grow up you will be 9 feet tall.

6. Review what probability is and how it is measured. Probability—the chance or amount an event will happen out of the tries of the event. Probability is measured by using words, fractions, ratios, and percents, all between 0 and 1. Students can now add to or alter their journal entries from invitation to learn. Ask students to write a couple of examples in their journals.

Assessment Suggestions

- Using each measurement on their meters, they will write a situation to show they understand each measurement. e.g., Certain – the sun will go down in the west. From there they will put them up on a class probability meter.
- Design an experiment (in partners or a small group) for another group to try and measure on their probability meters.

Curriculum Extensions/Adaptations/ Integration

- Working the whole activity with a partner.
- Create other fractions on the meter such as, 1/3, 2/3, etc.
- Use this in science with happenings in astronomy. e.g., The sun is the center of our solar system, the order of the planets, etc.

Family Connections

- Have students share the probability meter with their families and work together to come up with 4-5 situations to bring back to class the next day.

Additional Resources

Books

Do You Wanna Bet?, by Jean Cushman; ISBN 0-395-56516-2

Navigation Through Data Analysis and Probability, NCTM, Navigations Series; ISBN 0-87353-523-5

Math Matters, by Suzanne H. Chapin; ISBN 0941356268

Web sites

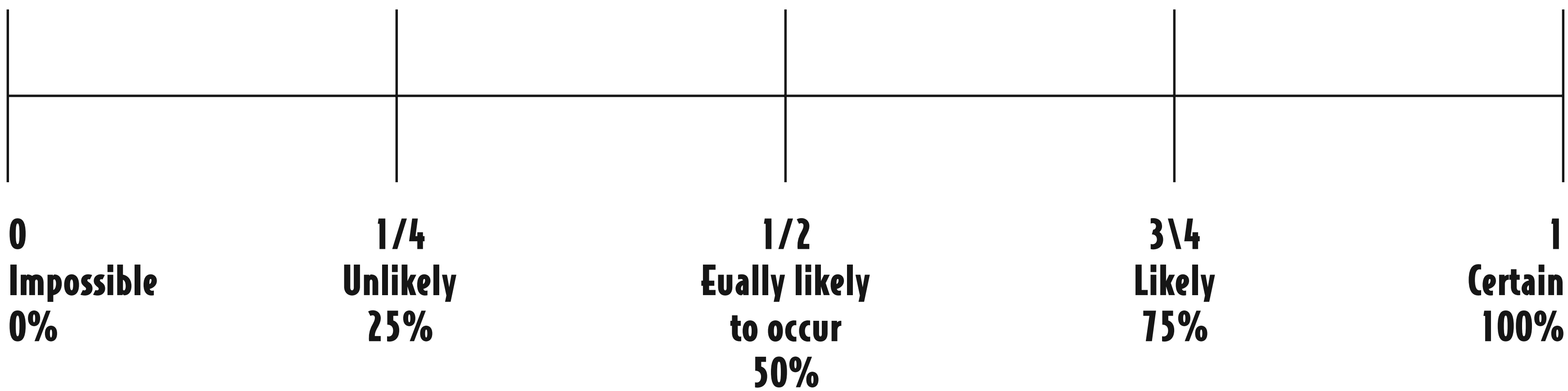
http://illuminations.nctm.org/Activities.aspx?grade=all&standard=all

http://exploringdata.cqu.edu.au/probabil.htm

http://www.learnalberta.ca/content/me5l/html/Math5.html?launch=true

http://www.m-ms.com/

Probablity Meter



E.T. Probability (Experimental/Theoretical)

Math Standard V

Objective 2

Standard V:

Students will analyze, draw conclusions, and make predictions based upon data and apply basic concepts of probability.

Objective 2:

1. Develop a positive learning attitude toward mathematics.
3. Reason logically, using inductive and deductive strategies and justify conclusions.

Content Connections:

Math I-1; Prime factorization, Social Studies I-2; Ancient Egypt
Language Arts VIII-1; Writing

Connections

Background Information

Probability is experimental and theoretical (anticipated). Experimental probability describes the actual event, "Will you absolutely roll a 5 in 6 rolls?" You may or may not, etc. When we are determining the probability of something we are figuring out the theoretical (anticipated) probability. e.g., "I have a 1 in 6 chances of rolling a 5 on a dice. If possible students should have the chance to experiment with probability, then move into the theoretical (anticipated). They need to know and understand the concepts of, experimental and theoretical (anticipated) probability.

Research Basis

Bright, G.W., Frierson, Jr., D., Tarr, J.E., & Thomas, C. (2003). *Navigating through Probability in Grades 6-8*. Reston: The National Council of Teachers of Mathematics, Inc.

This book addresses many aspects of probability. It mentions that learning how to use, and using tree diagrams helps in understanding probability. Tree diagrams also help build conceptual understanding. Many ideas and ways to teach probability and applications are provided.

James, Alisa, (2005), Journaling as an Assessment Option, *ERIC Source,* November 25, 2006, from http://www.eric.ed.gov

This research states that journaling is a tool that can assess student learning in affective and cognitive domains. It allows students a nonthreatening environment to communicate their knowledge.

Coin Flip

- Choose a partner
- Get a coin or two-colored counter
- Make a tally chart in your math journal
- Together flip the coin 20 times and record the results in your journal or on a piece of paper.
- Be ready to discuss and share your results with class.
- Example below

Heads	Tails

Invitation to Learn

With a partner, flip a coin 20 times and make a tally chart of the number of heads and tails. Record this in your math journal or on a piece of paper and be ready to share your findings.

Materials

- ❑ *Coin Flip*
- ❑ Coins or two-colored counters
- ❑ Math journals

Instructional Procedures

1. Discuss the results from the *Coin Flip* activity. (Similarities and differences) Discuss what the chances (head or tail) are for each flip.
2. Talk about experimental probability (which is what they just did). Discuss what the students think they should have flipped if they flipped 20 times. (10:10) Show on the board the theoretical probability of flipping a coin (tree diagram).
3. Show the diagram of *Pigs in a Pipe* on the overhead. Explain that there will be 80 balls going into this machine. They will go different directions and we will need to find out how many (of the 80) end up in each dumpster. They will split evenly, with the same amount going down each pipe. Work this through together, students on their paper and you using the overhead. Suggestion for teachers: use 2 different colors of overhead markers on this activity, one color for the fractions and one color for the numbers going into the pipe. The goal is to find out what percent of the balls end up in each of the dumpsters.
4. Play *The Stick Game*. Pass out three sticks per small group. The students can get the sticks ready for the game by coloring them. Two of the sticks should be red on one side and plain on the other. One stick should be blue on one side and plain on the other. Use the backline for the instructions on coloring the sticks and for playing and scoring the game.
5. Record then discuss what happens during the game in student's math journals or on a piece of paper.
6. After the majority have finished, discuss what happened and then as a class write the theoretical (anticipated) probability on the board, the answer. Discuss how the experimental and theoretical can be similar. When doing the tree diagram on the board, use 3 different colors of markers (chalk), one color to represent each stick.

Materials

- ❑ *Pigs in a Pipe*
- ❑ *Pigs in a Pipe* (key)
- ❑ Math journals
- ❑ 3 popsicle sticks per group
- ❑ Crayons
- ❑ The Stick Game

Assessment Suggestions

Materials

- ❑ *Secret Rooms*
- ❑ *Family reunion*
- ❑ The Game Show

- Students will be given another situation, *Secret Rooms,* in which people will be going into a pyramid (You can decide the number of people). Then in their journals they write about the situation, how they figured it out, and draw their tree diagram.
- Given a word situation, such as, *Family Reunion*, the students will be able to draw a probability tree diagram to show their answer.

Curriculum Extensions/Adaptations/ Integration

- Advanced learners could be given a tree diagram and have to come up with the situation, numbers and provide the tree showing who or what ended where. e.g., similar to Secret Rooms, Family Reunion, etc.
- ELL and others will work with a partner.
- http://www.rainforestmaths.com Site where students can work together on chance and probability.

Family Connections

- Have the students take the worksheet *The Game Show* home to do with their family.
- If they have Internet available go to http://www.rainforestmaths.com/ and then into 6th grade, then into chances and probability.

Additional Resources

Books

Everyday Mathematics, University of Chicago; ISBN 1-57039-510-1

Elementary and Middle School Mathematics, by John A. Van De Walle

Web sites

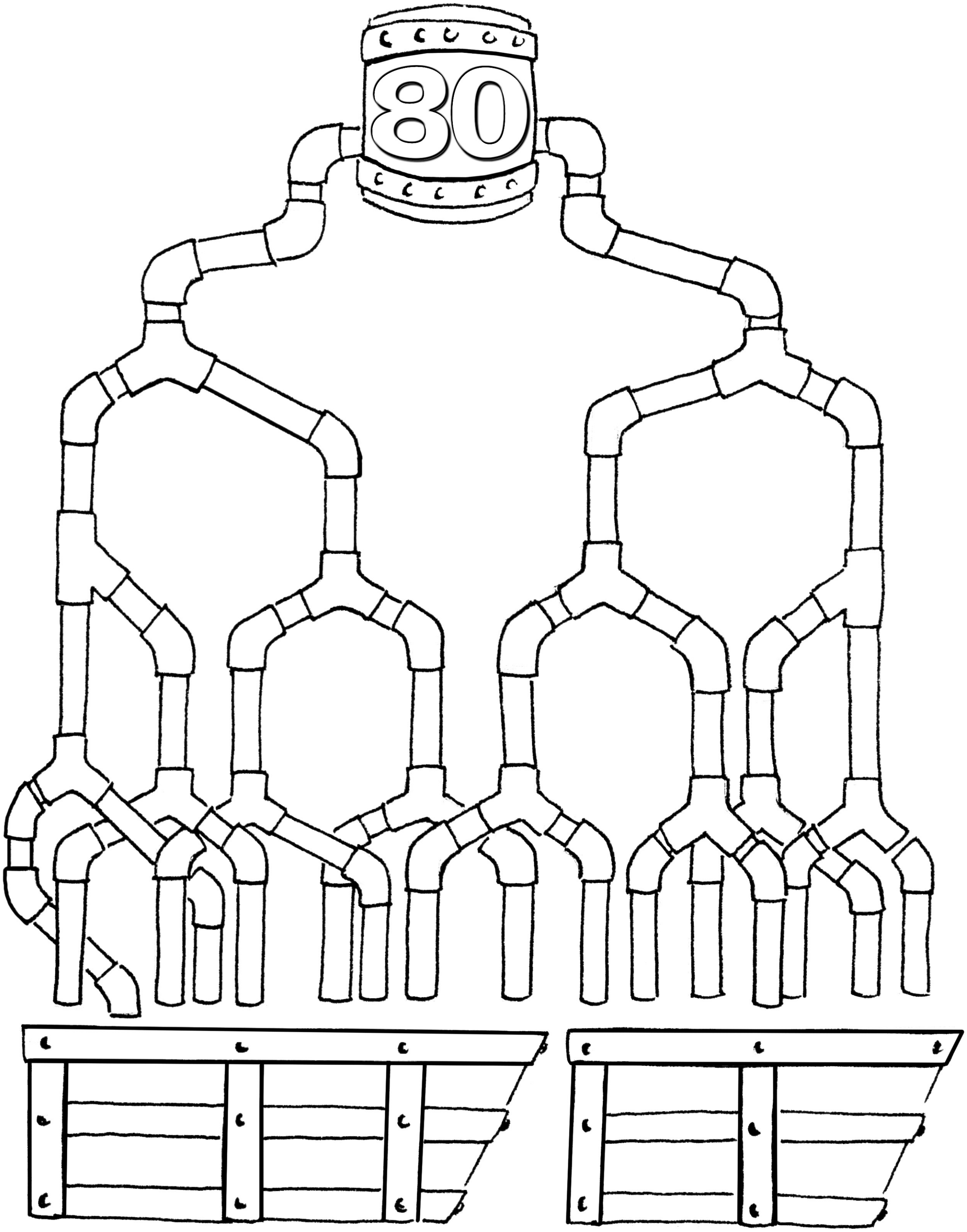
http://fhss.byu.edu/anthro/mopc/pages/Education/EarthActivities/agames.htm

http://www.rainforestmaths.com/

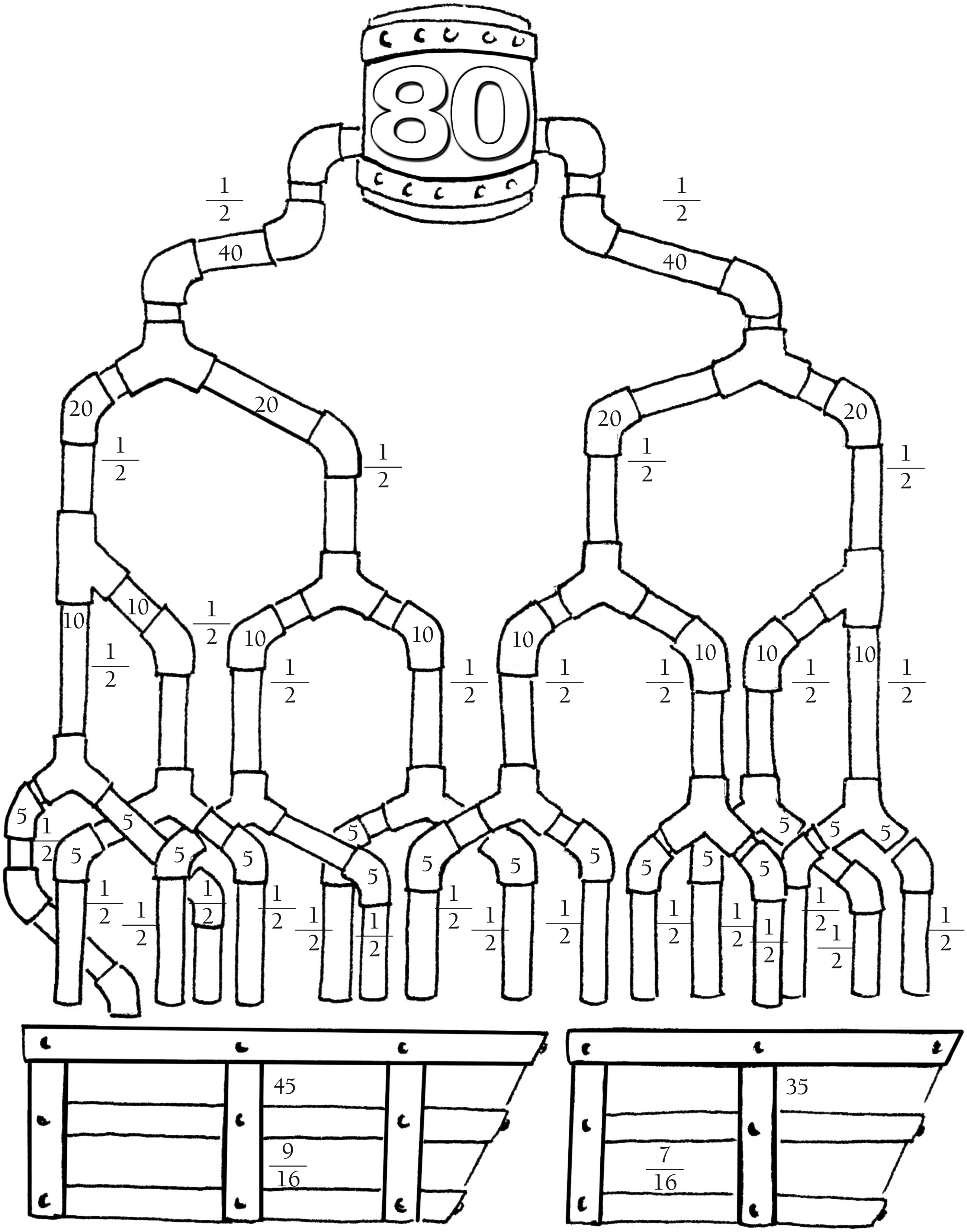
http://regentsprep.org/Regents/math/tree/PracTre.htm

http://mathforum.org/library/drmath/view/56541.html

Pigs in a Pipe



Pigs in a Pipe Answer Key



The Stick Game

Materials:
3 flat sticks (like popsicle sticks)
Crayons (red and blue)

Preparation:
Color two sticks red on one side and leave the other side plain. Color one stick blue on one side and leave the other side plain.

How To Play And Score:
Hold all three sticks in one hand. Hold your hand above the desk and drop the sticks. Below is how you score each drop. Record your score in your journal or on a piece of paper. Add your points as you go so you know when someone reaches 50 points.

All plain sides land face up	4 points
All colored sides land face up	4 points
Two red and one plain land face up	6 points
Two plain and one colored land face up	6 points
One plain, one red, and one blue land face up	0 points

Play until someone reaches 50 points.

The Stick Game

Materials:
3 flat sticks (like popsicle sticks)
Crayons (red and blue)

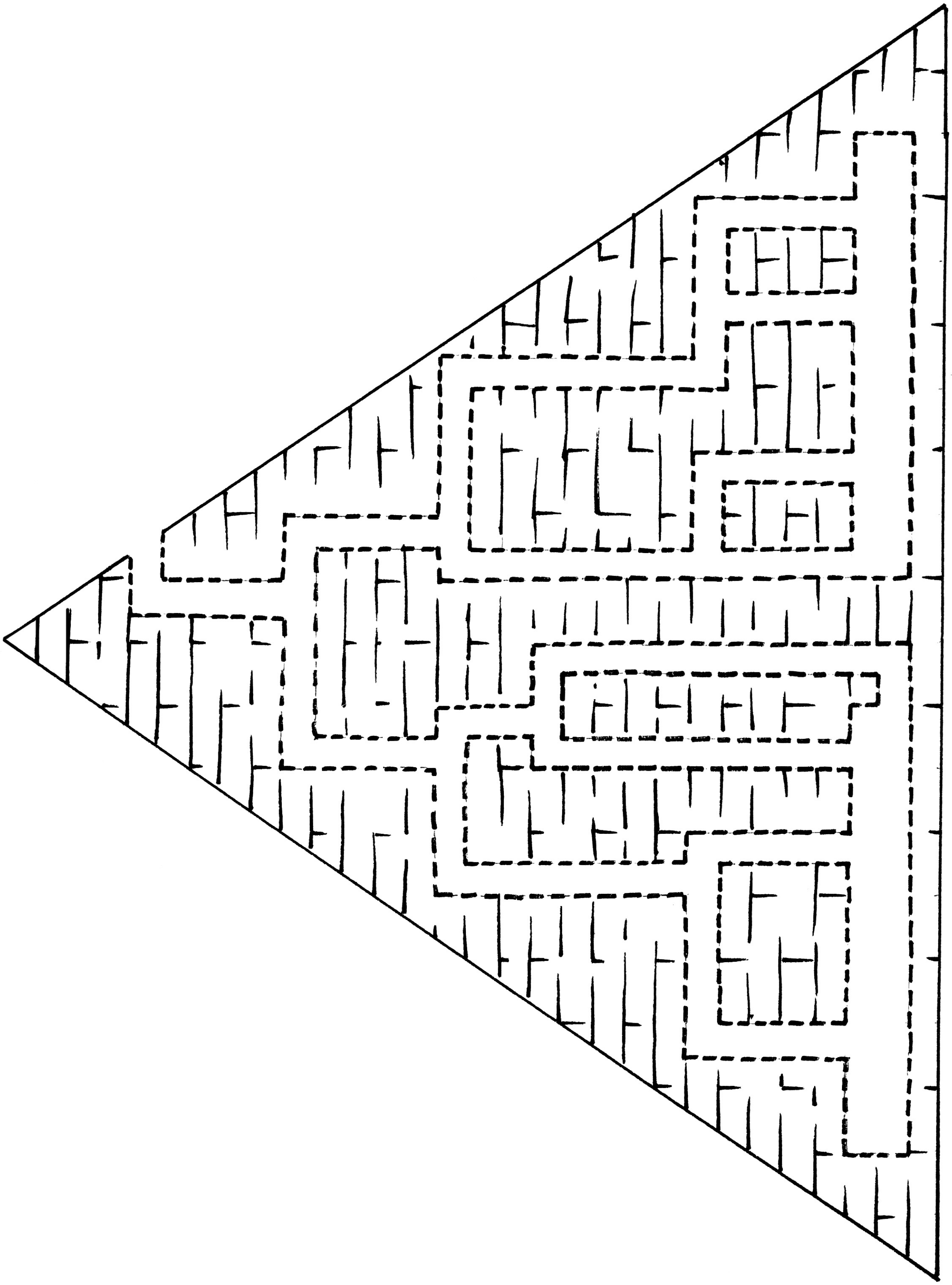
Preparation:
Color two sticks red on one side and leave the other side plain. Color one stick blue on one side and leave the other side plain.

How To Play And Score:
Hold all three sticks in one hand. Hold your hand above the desk and drop the sticks. Below is how you score each drop. Record your score in your journal or on a piece of paper. Add your points as you go so you know when someone reaches 50 points.

All plain sides land face up	4 points
All colored sides land face up	4 points
Two red and one plain land face up	6 points
Two plain and one colored land face up	6 points
One plain, one red, and one blue land face up	0 points

Play until someone reaches 50 points.

Secret Rooms



Family Reunion

There are 5 trails leading to Grandma and Grandpa's camp from your camp. There are 3 trails leading from Grandma and Grandpa's to Uncle John's camp. How many different routes are there from your camp to Uncle John's camp? Draw a tree diagram below to show your answer. Next to your tree diagram put the answer to how many routes.

Family Reunion

There are 5 trails leading to Grandma and Grandpa's camp from your camp. There are 3 trails leading from Grandma and Grandpa's to Uncle John's camp. How many different routes are there from your camp to Uncle John's camp? Draw a tree diagram below to show your answer. Next to your tree diagram put the answer to how many routes.

The Game Show

You are a contestant in a game show. One of the games asks you to pick a curtain and then pick a door behind the curtain. There are 4 curtains and 5 doors behind each curtain. How many choices are possible for the player?

Draw a tree diagram below showing the possible choices. Then write how many choices are possible.

The Game Show

You are a contestant in a game show. One of the games asks you to pick a curtain and then pick a door behind the curtain. There are 4 curtains and 5 doors behind each curtain. How many choices are possible for the player?

Draw a tree diagram below showing the possible choices. Then write how many choices are possible.

P.I.'s (aka – Probability Investigators)

Standard V:
Students will analyze, draw conclusions, and make predictions based upon data and apply basic concepts of probability.

Objective 2:
Apply basic concepts of probability and justify outcomes.

Intended Learning Outcomes:
5. Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.

Content Connections:
Language Arts VIII-1

Math Standard V

Objective 2

Connections

Background Information

Students need to understand that probability is all around them, it's used in everyday life. What is the probability you will get the prize you want out of the box of cereal you bought? How many boxes will it take? What are the chances it will rain, snow, or be sunny? There are chances each day that you don't realize you are doing. What is the probability you will get the parking spot, bike, a certain roll of the dice, or you will even get the lunch you want? When students understand probability and how it works they will see how it affects their lives.

Research Basis

Van De Walle, J.A., (2004). *Elementary and Middle School Mathematics*. Boston, Mass: Pearson Education Inc.

Van De Walle talks about how statistics bombards the public in all areas of our society from advertising to health risks, from opinion polls to students' progress in schools. Students face the same types of situations; from lunch to which assignment to do first, from which friend to go with to winning a game.

Ask Dr. Math, (1944-2006), Probability in the Real World, The Math Forum @ Drexel, November 20, 2006, http://mathforum.org/dr.math/faq/faq.prob.world.html,

This article describes how students need a basic understanding of probability. Students need this to understand things from batting averages to the weather report, to being struck by lightning.

O'Connell, T., Dyment, J., (2006), Reflections on Using Journals, ERIC Source, December 9, 2006, from http://www.eric.ed.gov

This research document noted that even through higher education teachers are having their students do journaling daily. The faculty that participated found that students' perceptions of journal writing changed along with the faculty. Journaling helped them process the subject on which they were writing. It concluded that journal writing should be used as an instructional technique in all areas of the curriculum.

Invitation to Learn

Put the question up on the board: How many ways do people use probability in the real world? Make sure you have them list as many as they can in their math journal or on a piece of paper, example-weather person.

Instructional Procedures

1. Explain the scenario to the students then read aloud *Holes* by Louis Sachar – from page 229, "This is pretty much. . ." through page 230, ". . . and for Hector to hire a team of private investigators."
2. They are the investigators and need to help Hector. Investigators/Detectives cannot make mistakes or it could affect the final results.
3. Put students in small groups and explain the following situation.
4. There are 5 women who are claiming to be Hector's mom. The news has spread that Hector is worth a lot of money, now everyone wants to be his mother. How will the detectives prove the true identity to the media, themselves, and Hector? They decide to use the coding of blood samples.
5. They coded the blood samples of the women with color counters and put them in 5 sacks. One sack contains an exact match to Hector's blood. Show sample of Hectors blood.
6. Pull one tile out at a time, record the result, then replace the tile. Shake the sack before drawing the next tile. By doing this your team of detectives will be able to reveal the contents of the sack.
7. Give each group one copy of *The Situation* sheet. This will give the students something to refer to when doing this activity.
8. Your group will need to present the results and rationale after gathering the data, during a class discussion.

(Idea from NCTM)

Assessment Suggestions

- Students explain in their journals how they came to the conclusion of who is Hector's mother.

Curriculum Extensions/Adaptations/ Integration

- Advanced learners could find another book and try the same idea.
- Students with special needs such as (ELL), or learning disabilities can work with partners.

Family Connections

- Work with their families to add to their list of jobs or situations that use probability

Additional Resources

Books

Holes, by Louis Sachar; ISBN 0-439-12845-5

Articles

R2 Math News, Granite School District, January 2004

Web sites

http://mathforum.org/dr.math

Where in the World is Mrs. Zeroni?
Teacher's Copy

You will need the following sacks prepared for Maria, Nancy, Patty, Grace, and Yolanda. The following is a possible answer key for each paper bag. (Do not do a bag for Hector, write his blood sample on the board or show a sample.)

Blood Sample Code	Yellow	Blue	Red
Hector	5	2	2
Maria	5	2	2
Nancy	6	3	1
Patty	6	3	0
Grace	3	3	4
Yolanda	7	4	0

The Situation
Where in the World is Mrs. Zeroni?

Samples of blood can give us a lot of information about people. Information is coded from blood samples so that comparisons can be made and family relationships can be established.

Hector Zeroni is looking for his mother. She accidentally left him behind one day. Hector has decided to hire investigators/detectives to locate his mother and bring her to him.

Detectives don't like to make mistakes. Right now they have five women claiming to be Hector's mother. It seems news has gotten around that Hector, and his partner Stanley Yelnats, is worth a lot of money, now everyone wants to be his mother. How will the detectives prove the identity of Hector's mother to themselves, the media, and most important, Hector.

The detectives have coded the blood samples using color counters in five paper sacks. One of these sacks contains an exact match to Hector's blood. Pulling one tile out at a time, recording the results, then replacing the counter in the sack and shaking the sack before drawing the next counter can only reveal the contents of the sack.

Your group needs to come up with a plan and gather data to find the answer to this question, **Where in the world is Mrs. Zeroni?** Some things your group needs to decide are:
The way you will go about gathering the data.
The amount of data you will gather.
The way you organize your data.
Ways to present your results to others.

Name ________________________________

Student Recording Sheet
Where in the World is Mrs. Zeroni?

	Yellow	Blue	Red
Hector	5	2	2

	Yellow	Blue	Red
Maria			
Nancy			
Patty			
Grace			
Yolanda			

Name ________________________________

Student Recording Sheet
Where in the World is Mrs. Zeroni?

	Yellow	Blue	Red
Hector	5	2	2

	Yellow	Blue	Red
Maria			
Nancy			
Patty			
Grace			
Yolanda			

Math II-1

Activities

Patterns & Functions

Perceiving Patterns

Standard II:

Students will use patterns, relations, and algebraic expressions to represent and analyze mathematical problems and number relationships.

Objective 1:

Analyze algebraic expressions, tables, and graphs to determine patterns, relations, and rules.

Intended Learning Outcomes:

3. Reason logically, using inductive and deductive strategies and justify conclusions.
6. Represent mathematical ideas in a variety of ways.

Content Connections:

Social Studies 4-1 Ancient Greek Culture

Math Standard II

Objective 1

Connections

Background Information

Patterns are all around us: In nature, technology, financial trends, and especially in math. The ability to see patterns and predict what will come next in patterns will help students in all aspects of their life. Students have been creating, understanding, and discovering patterns throughout their schooling. The toddler separates yellow and green blocks; the act of separating is a pattern. The third grader learns their multiplication facts, which is a series of patterns (repeated addition).

Likewise, the sixth grader needs to be able to recognize, create, and predict patterns. These patterns should be seen with manipulatives, pictures, graphs, tables, and numbers. Patterns can be simple one-step rules (add 1) or multiple-step operations (times 2 minus 1). Pattern recognition is one of the first steps in algebraic thinking.

Once a pattern is identified, students need to be able to communicate a rule associated with the pattern. For example, if the numerical pattern is 3, 5, 7, 9 . . ., the rule is to add 2. If the pattern is 3, 11, 43, 171 . . ., the rule is to multiply by 4 and subtract 1. This skill needs practice and exposure to different pattern representations to be mastered. To assist students with correct identification of the pattern, they can create a T-chart or table. The left side is the order the number is in, while the right side is the actual number. Keep in mind that this is not a function table, so the left column doesn't necessarily help with the pattern prediction. If the pattern is 2, 5, 14, 41 . . ., the table would look like this:

Number order	Number in Pattern
1	2
2	5
3	14
4	41
5	?

The students use trial and error to find the rule, which will help them find what is next in the pattern. The following activities will help students become proficient pattern predictors!

Research Basis

Furner, J. M., Yahya, N., & Duffy, M. L. (2005). 20 ways to teach mathematics: Strategies to reach all students. *Intervention in School and Clinic, 41*(1), 16-23.

Many excellent mathematical teaching strategies are presented in this article. The use of manipulatives, interdisciplinary connections, student drawings, heterogeneous grouping, and the consideration of multiple intelligences are some of the strategies that will be used in these activities.

Le, V., Stecher, B.M., Lockwood, J.R., Hamilton, L.S., Robyn, A., Williams, V.L., Ryan, G., Kerr, K.A., Martinez, J.F., & Klein, S.P. (2006). Improving mathematics and science education: A longitudinal investigation of the relationship between reform-oriented instruction and student achievement. *Rand Source* (MG-480-NSF). Retrieved January 2007 from rand.org

This three-year study explored the connection between student achievement and the use of manipulatives (reform-oriented instruction) in math and science. Students who had more hands-on lessons performed better on standardized tests than those who did not. Problem-solving skills were especially improved by use of manipulatives.

Invitation to Learn

1. Hand out the *What's Next in the Pattern?* worksheet to each student. You may have them work alone, in partners, or in small groups.
2. Students will see the pattern and draw the appropriate amount of items for each situation. This starter will be a good gauge to see how well students are able to see patterns. The first problem is a simple single-step problem; they will get progressively harder. As students are working, monitor for understanding and

Materials

- ❑ *What's Next in the Pattern?*

help them as needed. If you wish, you may allow students to write the number rather than draw the items.

3. You may choose for students to write down the pattern rule (e.g., add 4).
4. You may extend this activity by asking students what the eighth, tenth, twelfth number in the pattern would be.
5. The last two is for the students to create their own patterns with their own ideas.
6. You may have students share their examples to the rest of the group to see if they can stump their classmates.

Instructional Procedures

Herculean Task

1. Read or paraphrase the following story (or make your own up!):

   Ah, Heracles (known as Hercules in Roman mythology) is in for it again! One of the lesser-known tasks that he had to endure was the strength-sapping Boulder Task. Each day, Heracles must lift an increasing amount of boulders from the base of Mt. Olympus to the river Styx. As he began transporting the massive rocks, Heracles calculated that the most he could carry was exactly 27 boulders in one day. Unaware of how many days he had to do this task, he became concerned that his time would run out!

2. Summarize to students what the objective is: To figure out how many days Heracles can carry the boulders before he reaches or exceeds 27 boulders.
3. Hand out student materials. Go over the *Boulder Task* worksheet. Tell students that on the first day, Heracles had to carry 3 boulders. Students will put 3 "boulders" in the first box or egg carton holder. On the second day, he carried 5 boulders; 5 boulders will go in the day 2 box, and so on. Write the following on the board:

Materials

- ❏ Centimeter cubes, beans, beads, marbles, candy, copies of *Boulders* black line master, or any other manipulative for each student to represent the boulders in the activity
- ❏ Egg cartons
- ❏ *Boulder Task*

DAY	BOULDERS
1	3
2	5
3	7
4	9
5	11
6	13
7	15
8	17
9	19
10	21

4. As students put the boulders in the boxes, they may figure out the pattern (add 2).
5. How many days will pass before Heracles has to carry 27 or more boulders to the river Styx? (13 days). Fortunately for our muscular friend, he only had to carry the boulders for 12 days!
6. Next, introduce a couple more patterns that go with the same story. Here are some examples:

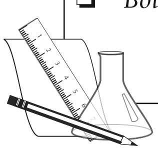
DAY	BOULDERS
1	2
2	5
3	8
4	11
5	14
6	17
7	20
8	23
9	26
10	29

DAY	BOULDERS
1	1
2	5
3	9
4	13
5	17
6	21
7	25
8	29
9	33
10	37

Bold Boulder Patterns

Materials

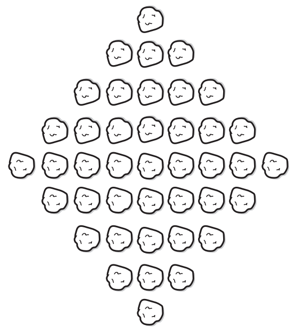
❏ *Bold Boulder Patterns*



1. Read or paraphrase the following story (or make your own up!):

   While Heracles was forced to move boulders from Mt. Olympus, he placed them in the shape of a diamond (Okay, so he was bored!). When he had moved 25 boulders, this is what it looked like (draw this on the board):

   Heracles was really proud of himself, and marveled at all of the different patterns he saw in this boulder diamond.



2. At this point, have students share any patterns they see right off. Write them on the board. Ask questions to guide the students' thinking:
   - What numerical pattern can you describe from the array of boulders? (An obvious pattern is 1 + 3 + 5 + 7 + 5 + 3 +1.)
   - What other patterns do you see? (Point out the 3 x 3 square in the middle.)
   - What numerical pattern can you see now? [9 + 4 + 4 + 4 + 4 or 9 + (4 x 4)]
3. Hand out the *Bold Boulder Patterns* worksheet. Tell the students that their job is to find as many ways as they can to partition the boulders into different patterns. Listen to their discussions and observe the different patterns they come up with. Emphasize the translation of the visual patterns into numerical patterns. It's amazing to see how many different patterns they can come up with!

Assessment Suggestions

- The *What's Next in the Pattern?* starter is an excellent pre-assessment to gauge student ability of seeing patterns.
- For an assessment of the Herculean Task activity, students will come up with their own patterns in an attempt to stump their neighbors. They may continue to use the manipulatives or start using a chart and numbers, which ever they prefer. This may be drawn and handed in, or performed in front of the teacher.

Curriculum Extensions/Adaptations/ Integration

- To extend the Herculean Task activity, you can craft patterns in which numbers go below 1 or get very large, including exponents.

- As an extension of the Bold Boulder Patterns activity, use an array of forty-one and/or sixty-one (see the other pages of the worksheet).
- To adapt the Herculean Task activity, start with one-step patterns until students have a solid grasp on the concept.
- For integration of the Herculean Task activity, you may have students come up with other stories from Egyptian, Greek, or Roman mythology similar to the story presented.
- Tessellations are patterns of shapes that cover a plane without overlapping or gaps. Explore with students what shapes can be used to create tessellations. Start with one and work into multiple shapes.

Family Connections

- Have students create patterns and take them home in an attempt to stump other family members.
- Students will look for patterns in their everyday life. They may observe bowling pins, measuring cups and spoons, etc.
- Students will watch television for 30 minutes, recording how long the program segments are and how long the commercial breaks are. When they bring back their results, have them compare with the rest of the class. This would be a great time to create a graph to show the class results. What patterns do you see?

Additional Resources

Book

Navigating Through Algebra in Grades 3-5, by Gilbert J. Cuevas and Karol Yeatts; ISBN 0-87353-500-6

Web sites

http://www.learner.org/teacherslab/math/patterns/more.html (Logic, number, and word patterns)

http://mythweb.com (Greek mythology)

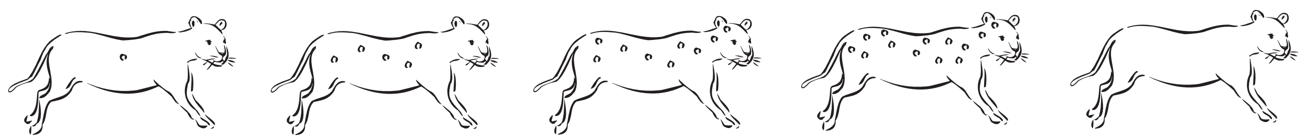
http://nlvm.usu.edu/en/nav/topic_t_1.html (National Library of Virtual Manipulatives)

Name ______________________________

What's Next in the Pattern?

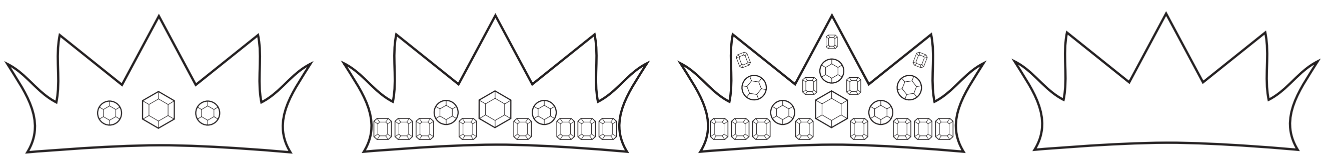
Directions: For each situation, draw or write the correct amount if the pattern continued.

1. Each leopard is born with a different amount of spots. If the pattern continued how many spots will the fifth leopard have? Draw them on.



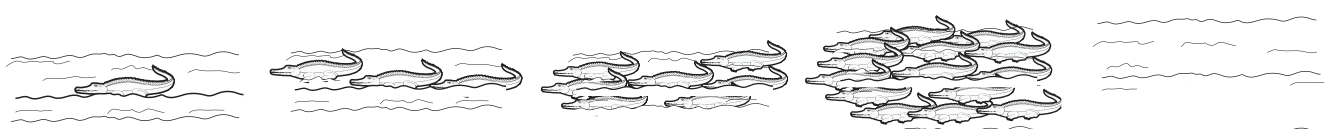
2. Each chocolate chip cookie has more chips than the one before. If the pattern continues, how many chocolate chips will the fourth cookie have?

3. Each King Henry gts more jewels on his crown than his predecessor. If the pattern continues, how many jewels will Henry IV have?



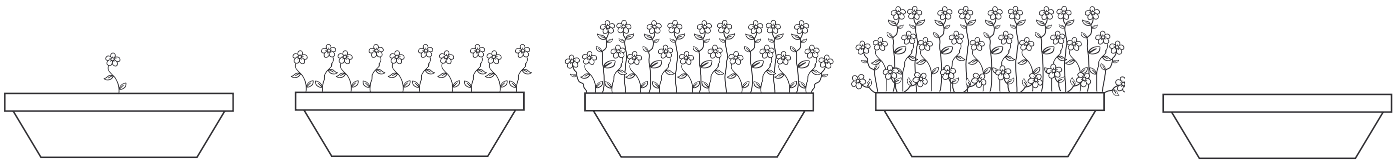
Henry I   Henry II   Henry III   Henry IV

4. Each week, more and more crocodiles pass by on the Nile river. If the pattern continues, how many crocodiles will pass by on the week five?



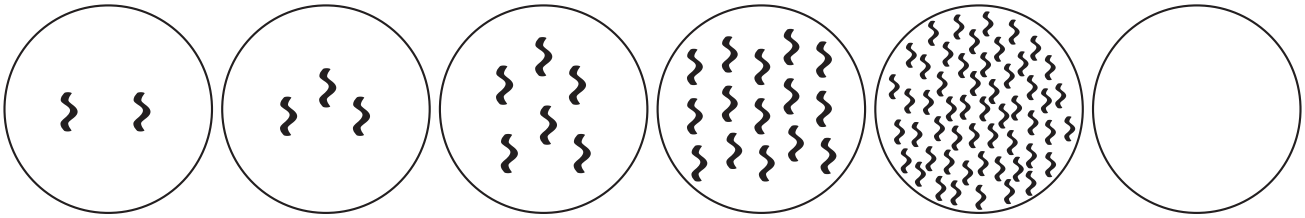
Week 1   Week 2   Week 3   Week 4   Week 5

5. As they are watered properly, these flowers bloom at the same rate. How many flowers will there be on the fifth week? ____________



Week 1   Week 2   Week 3   Week 4   Week 5

6. Each hour the microorganisms in this petri dish multiply. At this rate, how many microorganisms will there be at 5:00?



12:00 1:00 2:00 3:00 4:00 5:00

7. For numbers 7 and 8, identify the next number in the pattern.
   5, 15, 25, 35, ______

8. 1, 4, 13, 40, 121, ______

9. For 9 and 10, create your own pattern.

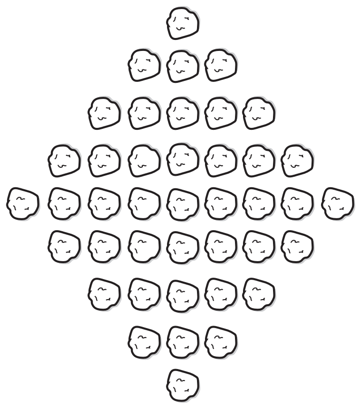
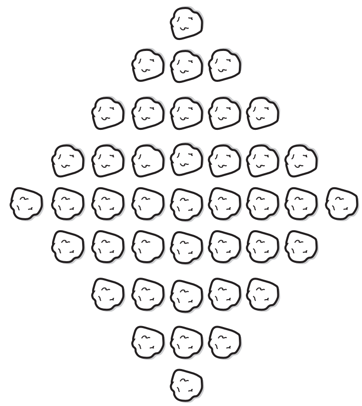
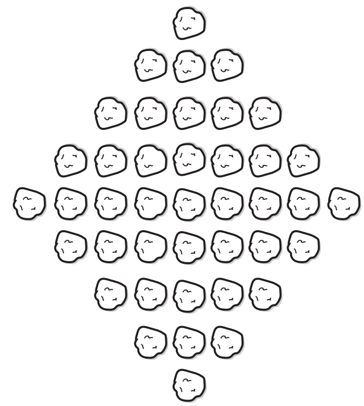
10.

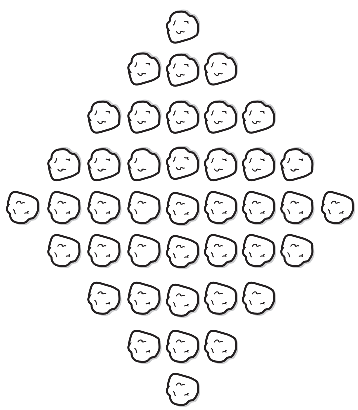
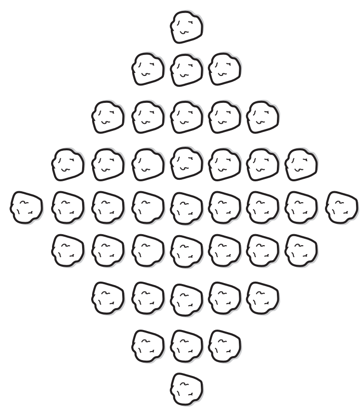
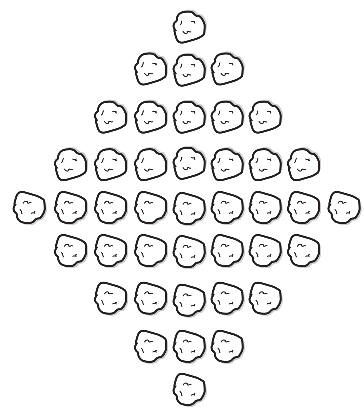
Boulder Task

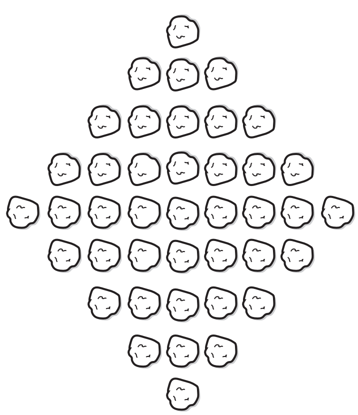
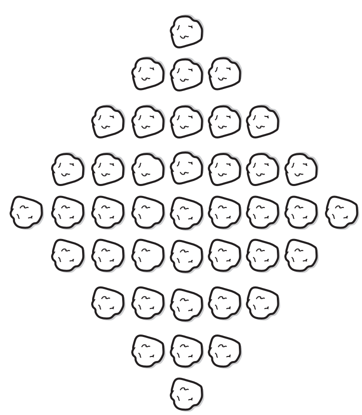
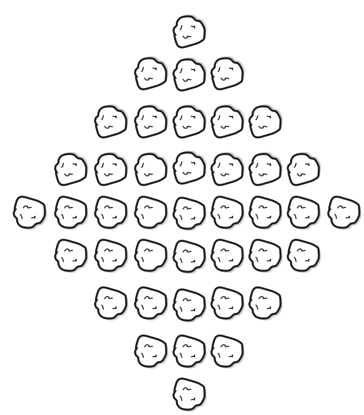
Day 1	Day 2	Day 3	Day 4	Day 5
Day 6	Day 7	Day 8	Day 9	Day 10
Day 11	Day 12	Day 13	Day 14	Day 15

Bold Boulder Patterns

While Heracles was forced to move boulders from Mt. Olympus, he placed them in the shape of a diamond (okay, so he was bored). Heracles was really proud of himself, and marveled at all of the different patterns he saw in this boulder diamond. Your task is to find as many ways as you can to partition the array of diamonds below. Record each way as a numerical sentence.

Bold Boulder Patterns

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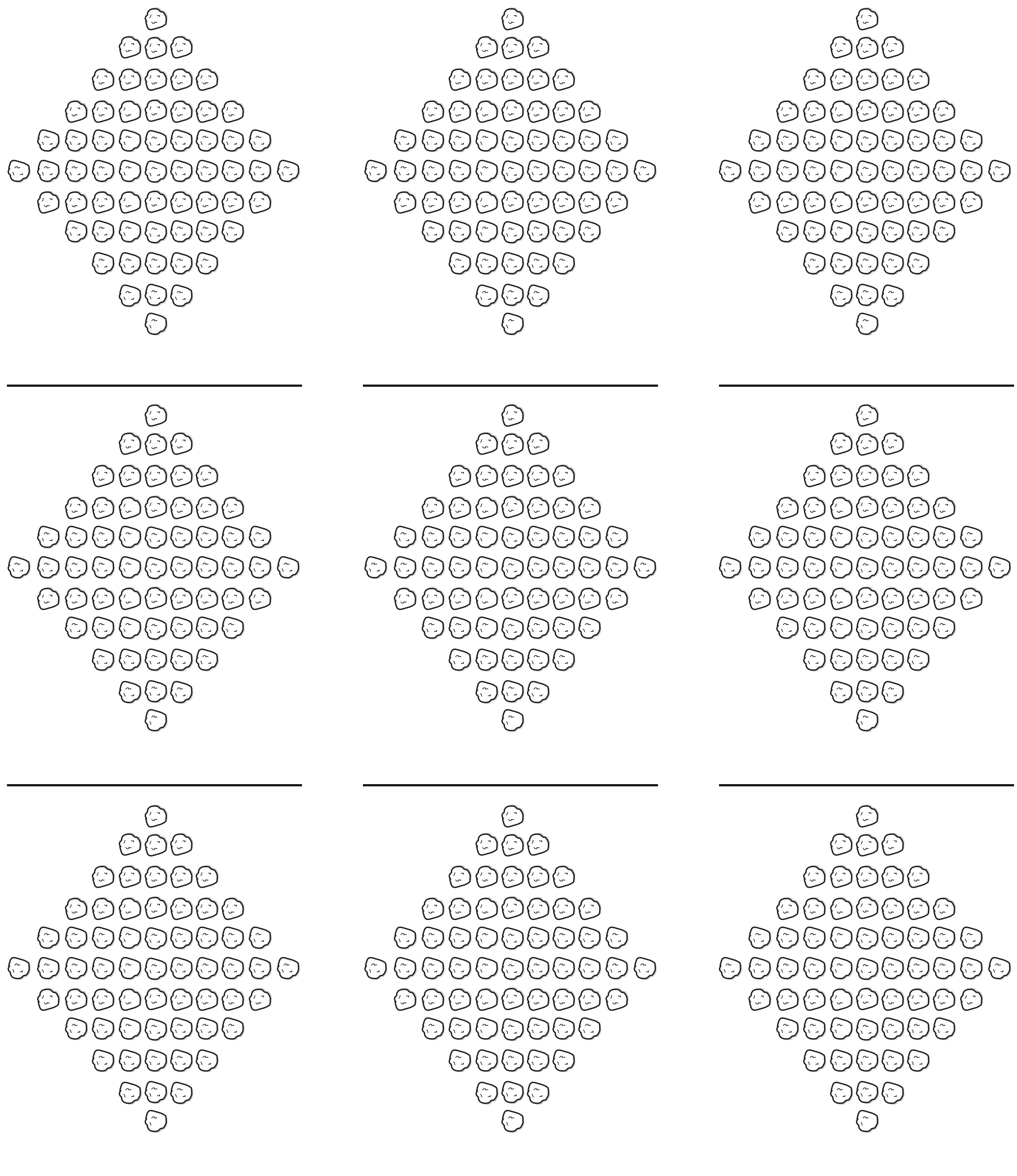
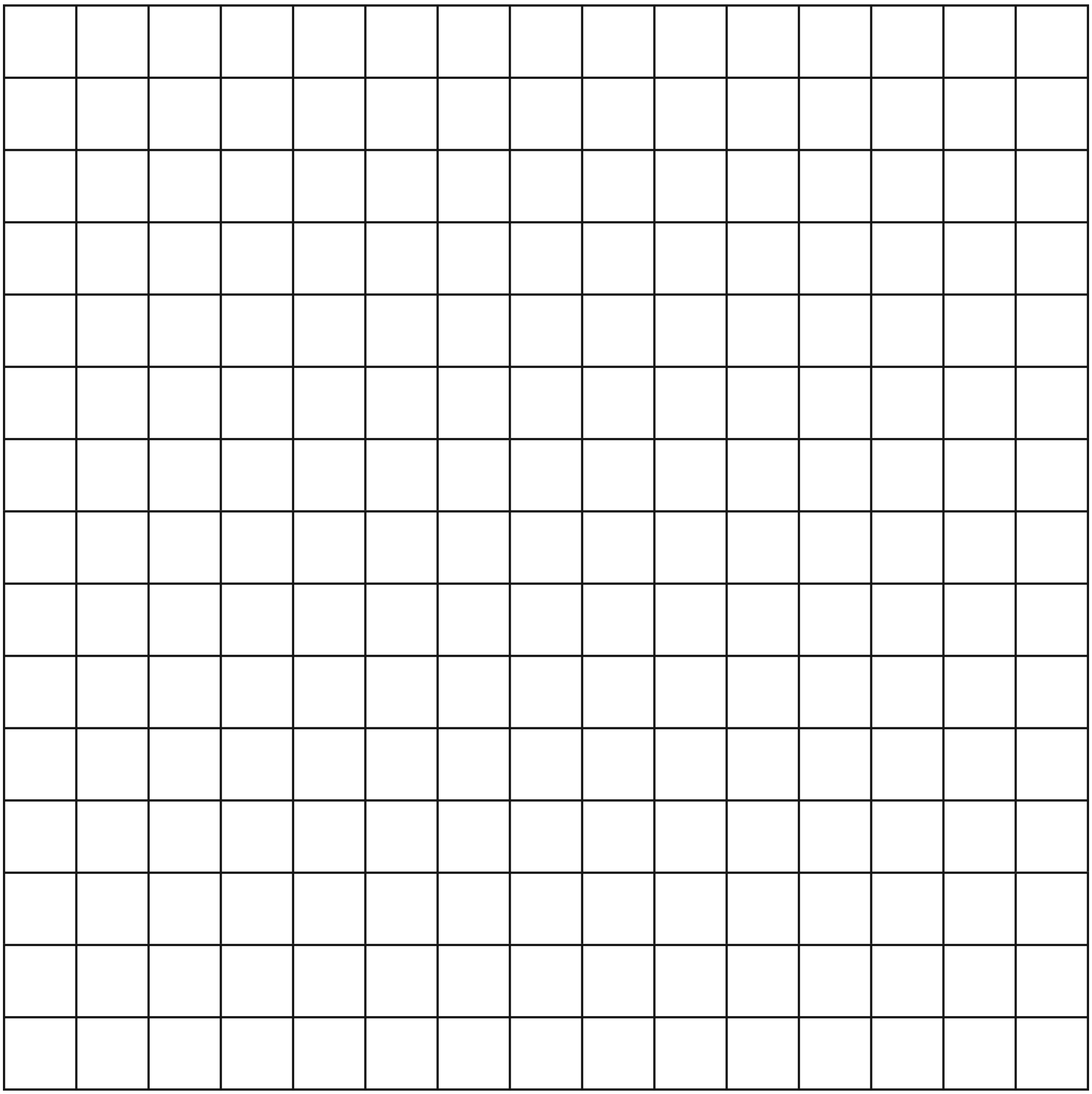


Coordinate Grid



Function-al Machines & Spaghetti Graphs

Standard II:
Students will use patterns, relations, and algebraic expressions to represent and analyze mathematical problems and number relationships.

Objective 1:
Analyze algebraic expressions, tables, and graphs to determine patterns, relations, and rules.

Intended Learning Outcomes:
3. Reason logically, using inductive and deductive strategies and justify conclusions.
6. Represent mathematical ideas in a variety of ways.

Content Connections:
Math III-2; Coordinate Geometry

Math Standard II

Objective 1

Connections

Background Information

There is a powerful pattern identified when any number is put into an equation and consistently follows the rule. This is called a *function*. When the rule is identified, each number does not have to be solved, but one could simply skip to the input number desired, insert it into the "rule" or equation, and the answer (output) will be given. Functions are easily shown in tables, such as the example below:

Input (X)	Output (Y)
1	3
2	4
3	5
4	6
5	7

It is easy to see that the output increases by one each time. The relationship between the input and output is the key, however. The "rule" is to add 2. If 2 is added to the input number 1, the answer is 3. Therefore, an equation can be formed. An *equation* is a mathematical sentence that contains an equal sign. The equation for the above example is x + 2 = y. Students should become proficient at spotting the pattern, recognizing the rule, and creating an equation from that rule. The rule or equation should be a one- or two-step problem, or it becomes really difficult to solve.

Another skill that students need to master is the ability to change an equation back into a function table. If the equation is x + 2 =y, the student can choose *any* number to represent the input (x). They will then "plug in" that number to get the output (y). So, if 8 were chosen for the input, then 10 would be the output. Keep in mind that any variable can and should be used, not just x and y each time.

Moving into the final skill, students need to first be able to graph a function table. This is a simple plotting exercise (taught in 5th grade and in 6th grade Standard III Objective 2). If x is 1 and y is 3, the coordinates will be (1,3). At least 2 (preferably 3) coordinates must be plotted, then connected to create a line (for these types of equations, a straight line will be created). The goal is for students to be able to graph an equation. In summary, here are the steps:

1. Change the equation to a function table
2. Graph the function table
3. Connect the plots to form a line

Research Basis

Cwikla, J. (2004). Less experienced mathematics teachers report what is wrong with their professional support system. *Teachers & Teaching, 10*(2), 181-197.

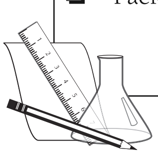
When less-experienced mathematics teachers interviewed, they expressed disappointment that many of their more experienced colleagues lacked content knowledge. Overall, they were not satisfied with the mentoring or collaboration offered by fellow teachers because they often knew more content than their more experienced peers.

Holly, K. R. (1997). Patterns and functions. *Teaching Children Mathematics, 3*, 312-313.

This article gives many ideas and activities for teaching patterns and functions in elementary grades K-6. Venn diagrams, function machines, and building cubes are some ideas presented.

Materials

- ❏ Cookie ingredients
- ❏ Package of store cookies



Invitation to Learn

Show students ingredients for cookies. Ask students if they would like to eat each ingredient. They may want the sugar, but not the salt, etc. Explain to students that these ingredients go through a “magical” change from their separate ingredients until they are spit out of a factory machine. The magic, of course, is the mixing of the ingredients and chemical change when they are cooked together. Tell students that today they will be putting numbers through a machine, which will “magically” change the number. The magic, of course, is the function rule. You may give the students a cookie, notifying them that this cookie may stimulate their brain and make them even better mathematicians.

Instructional Procedures

Function-al Machine

1. Teach students the basics of functions: give examples of function tables, discuss how to discover the rule, and how to change that rule into an equation.
2. It may take a few examples for students to catch on, but they will begin to see this as a fun game. Allow students to come up with their own functions with rules. Let a few of them try to stump the class.
3. Students will pair up. Student A will think of a function table, rule, or equation and secretly write it down on a piece of scratch paper. The rule or equation should be a one- or two-step problem.
4. Student B will write an input number (x) on the *Functional Machine* worksheet and Student A will write the output (y). The second student will then guess the function. If correct, they switch. If incorrect, Student B guesses again.
5. After 3 guesses, if Student B has not guessed correctly, Student A will unveil their table, rule, or equation and explain it to Student B.
6. Students switch roles.

NOTE: You may want to allow students to choose to use a table, rule, or equation, but eventually move students to using equations.

Materials

❏ *Functional Machine*

The Ins and Outs of Functions

1. Using the instructions in Background Information, ensure that students understand the basics of functions.
2. Students will be in heterogeneous groups of 3 or 4. Each group will be given 20 *Ins and Outs of Functions* cards. The first card is the simplest, with each card becoming more difficult. They will work as a group to answer each question on the card:
   1. What's the rule?
   2. What are three more examples?
   3. What's the equation?

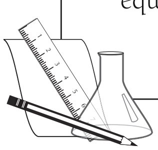
Materials

❏ *Ins and Outs of Functions* (cards)

3. When they have finished a card, they may check with you to see if their answers are correct. If they are incorrect, they need to go back to their group to understand where they went wrong. If a group is consistently getting answers wrong, determine what they are missing and reteach the group.
4. When they are working, walk around and make sure that everyone is participating. They may split up the cards, but they should also help each other.
5. The first group to finish all 20 cards is declared the winner and will become "experts" that will go around to help the other groups (not give answers, but assist). You could even give them stickers to put on to show they are the experts.

Materials

- ❏ Play Dough or other clay
- ❏ Spaghetti
- ❏ Graph paper
- ❏ Spaghetti Graphs equations



Spaghetti Graphs

1. FIRST THINGS FIRST: The prerequisite to this is that students need to have the ability to change an equation to a function table, like this:

   $2x + 3 = y$

   If $x = 1$, then $y = 5$

   If $x = 2$, then $y = 7$

   If $x = 3$, then $y = 9$

X	Y
1	5
2	7
3	9

Then, they need to plot a function table.

Step 1: Students will put an equation into a function table (at least 3 sets to plot).

$7 + x = y$

X	Y
1	8
2	9
3	10

Step 2: Students will graph the above table using play dough "dots."

Step 3: They will put the dried spaghetti in all 3 dots. This will ensure that the points are straight.

Step 4: They will repeat the process with the rest of the equations in the set.

Step 5: If done correctly, the 3 lines will intersect with at least one other line on the same graph.

Equation sets to use:

<u>SET A:</u>

X + 1=Y

X – 2=Y

2X + 3=Y

<u>SET B:</u>

X + 3=Y

3X – 2=Y

X ÷ 2=Y

<u>SET C:</u>

X - 1=Y

X ÷ 3 + 1=Y

5X - 3=Y

<u>SET D:</u>

X•X=Y

X=Y

3X ÷ 2=Y

<u>SET E:</u>

3X + 7=Y

X – 7=Y

6X ÷ 3=Y

Assessment Suggestions:

- Give students 6 equations. If they can accurately plot the equations on a graph, they are proficient. This may also be done with function tables and many other math concepts.

0-2 correct: Intervention-These students need direct reteaching instruction

3-4 correct: Practice-These students need extra practice

5-6 correct: Proficient-These students have mastered the content. Give them an enrichment/extension activity to do

- In a gym, have students create a coordinate grid, using masking tape as the x and y axes. Students will be the points, and they may use a broomstick, etc. to create a line by connecting the "points."

Curriculum Extensions/Adaptations/ Integration

- The equations used for the spaghetti graphs were positive slopes (lines that go from right to left). Your advanced learners can be exposed to negative slopes, which are lines that go from left to right. If the equation has both a negative number before X and the second number, it will be a negative slope. For example, -5X – 3=Y is a negative slope.
- For an introduction to coordinate grids, a great picture book is *The Fly on the Ceiling* about Rene` Descartes creating the Cartesian coordinate system.
- Include ideas for integration for other curricular areas (use appropriate subject area headings).

Family Connections

- Using spaghetti, clay, and graph paper, students will show a parent or older sibling how to graph equations.
- Students will take home the *Ins and Outs of Functions* cards, or create their own function tables. They will show family members how to figure out the rule and create an equation.

Additional Resources

Book

The Fly on the Ceiling, by Julie Glass; ISBN 0679886079

Web sites

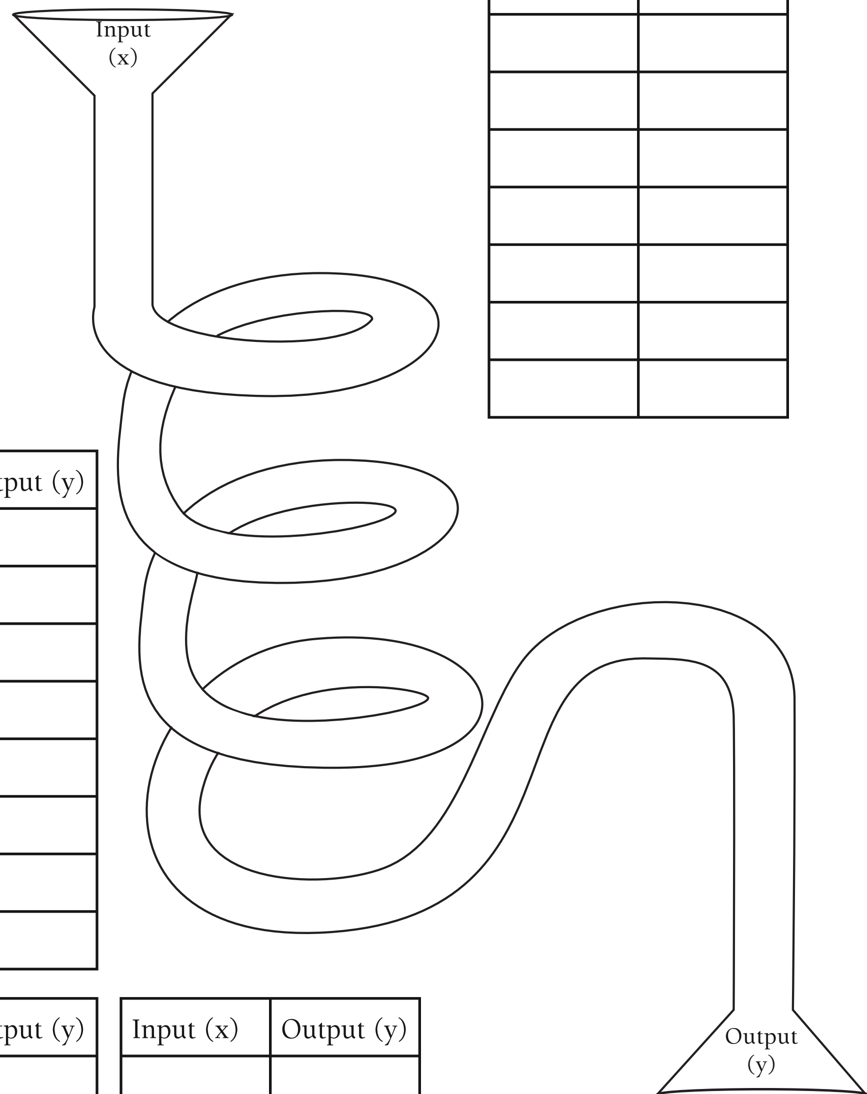
www.math.com (this free site has information and well-explained tutorials on all math subjects)

Name ______________________________

Functional Machine

Input (x)	Output (y)

Input (x)	Output (y)



Input (x)	Output (y)

Input (x)	Output (y)

Input (x)	Output (y)

Input (x)	Output (y)

Input (x)	Output (y)

Ins and Outs of Functions Cards

#1

In (x)	Out (y)
1	4
2	8
3	12
4	16
5	20

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#2

In (x)	Out (y)
1	11
2	12
3	13
4	14
5	15

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#3

x	y
1	50
2	100
3	150
4	200
5	250

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#4

x	y
1	0
2	1
3	2
4	3
5	4
6	5

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#5

x	y
10	5
12	6
14	7
16	8
18	9

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#6

m	a
1	3
2	9
3	15
4	21
5	27

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#7

j	k
1	9
3	13
5	17
7	21
10	27

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#8

x	y
1	22
2	44
3	66
4	88
5	110

1. What's the rule?
2. What are three more examples?
3. What's the equation?

Ins and Outs of Functions Cards

#9

c	d
1	37
2	67
3	97
4	127
9	277

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#10

k	p
1	10
2	18
3	26
4	34
5	42

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#11

r	e
1	9
2	12
3	15
4	18
5	21

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#12

x	y
1	-1
2	0
3	1
4	2
5	3

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#13

a	r
1	9
2	20
3	31
4	42
5	53

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#14

y	x
1	11
2	23
3	35
4	47
8	95

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#15

a	b
2	5
4	10
6	15
8	20
12	30
300	750

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#16

x	y
1.2	2.4
1.4	2.8
1.5	3.0
1.8	3.6
2.0	4.0

1. What's the rule?
2. What are three more examples?
3. What's the equation?

Ins and Outs of Functions Cards

#17

x	y
6	7
21	11
30	14
36	16
60	24

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#18

m	n
8	-2
12	-1
16	0
24	2
36	5

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#19

x	y
0	0
3	27
5	125
6	216
1	1

1. What's the rule?
2. What are three more examples?
3. What's the equation?

#20

f	a
1/6	1/216
1/4	1/64
1/3	1/27
1/2	1/8
1	3

1. What's the rule?
2. What are three more examples?
3. What's the equation?

Math II-2

Activities

Algebraic Situations

Pyramid Equality

Standard II:
Students will use patterns, relations, and algebraic expressions to represent and analyze mathematical problems and number relationships.

Objective 2:
Write, interpret, and use mathematical expressions, equations, and formulas to represent and solve problems that correspond to given situations.

Intended Learning Outcomes:
3. Reason logically, using inductive and deductive strategies and justify conclusions.
4. Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.

Content Connections:
Social Studies IV-1; Ancient Egyptian culture

Math Standard II

Objective 2

Connections

Background Information

The key to solving algebraic equations is to understand equality. Many students think of an equal sign as a symbol *to solve something*. For example, 6 + 13 = ?. Similarly, 6 x 13 + 15 = ?. Students think of equality as calculating a set of numbers to get an answer. This can lead to misconceptions.

Equality is a statement that indicates two quantities are equal. It can be thought of as a *balance*. To solve an equation means to maintain the equality between the two sides of the equal sign.

To help students develop an understanding of equality, they should have already learned how to substitute values from tables into an equation to calculate the missing variable (Standard II, Objective 1). In this activity, students will use a pictorial situation to develop a symbolic method for solving a linear equation, or an equation with a constant rate of change that will produce a straight line on a graph.

This lesson draws on knowledge of burial rituals in Ancient Egypt. This lesson will probably come after a study of Ancient Egypt and will therefore be a review, but it will not hinder the students' learning if this lesson is done beforehand.

Research Basis

Falkner, K.P., Levi, L., & Carpenter, T.P. (1999, December). Children's understanding of equality: a foundation for algebra. *Teaching Children Mathematics, 6*, 232-236.

Equality is a crucial idea for developing algebraic reasoning in young children. Children need to understand that equality expresses the idea that two mathematical expressions hold the same value. This article explores misconceptions of the equal sign and relates the experiences of a teacher's classroom lessons on equality.

Freiman, V., & Lee, L. (2004). *Tracking primary students' understanding of the equality sign. Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education,* 2, 415-422.

The NCTM standards consider equality as a concept that must be taught and understood starting in the younger grades. This article highlights a research study in Quebec that proves misconceptions of the equal sign may be prevented with early introduction of equality.

Invitation to Learn

Write the following starter problem on the board:

The equation 35 = 20 + 15 states that the quantities 35 and 20 + 15 are equal. What do you have to do to keep both sides equal if you:

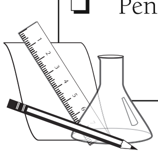
- subtract 15 from the left hand side of the equation?
- add 10 to the right hand side of the original equation?
- divide the left-hand side of the original equation by 5?
- multiply the right-hand side of the original equation by 4?

Try your methods on another example of equality. With your math partner, summarize what you know about maintaining equality between two quantities.

Discuss strategies as a class. Students should understand that anything done on one side of an equation must be done on the other to maintain equality.

Materials

- ❏ *Tomb Treasures*
- ❏ *Pyramid Equality*
- ❏ *Pyramid Equality* (Key)
- ❏ *Treasure and Pyramid Cut-Outs*
- ❏ *Treasure and Pyramid Cut-Outs* (magnets)
- ❏ Math journals
- ❏ Pencils



Instructional Procedures

1. Create a transparency of *Tomb Treasures* or make enough copies to share with your students. Read the scenario as the students follow along. Make sure students understand the treasures and pyramids problem.
2. Have students work alone or in pairs to figure out the answer, 2 treasures in each pyramid. When all students are finished, summarize as a class. Use the *Treasure and Pyramid Cut-Outs* magnets to help.
3. Pass out the worksheet entitled *Pyramid Equality* to each student. Have students work in partners or groups of three to determine the number of treasures in each pyramid. They need to make sure that all work is explained, whether in words or

pictures. They must also respond to the two questions at the bottom of the page.

4. Pass out the pre-cut *Treasure & Pyramid Cut-outs* for the students to use to figure out each problem, or ask the students to cut out the set for use. As they work, walk around the room and ask the following questions to guide their thinking: What does equality mean? How can we maintain equality? How do you know your answer is correct?
5. When all students are finished, summarize as a class. Have different pairs or groups of three discuss their work for a problem. They should use the *Treasure and Pyramid Cut-outs* magnets on the board to illustrate their thinking. Always check to ensure that the students are always maintaining equality!
6. Have students write down common strategies in their math journals. Ask them to circle the strategy they like best.
7. Use the *Tomb Treasures* problem to help students to make the transition from pyramids to variables. Ask, If we let x represent the number of treasures in a pyramid and 1 represent one treasure, how can we rewrite the equality-using x's and numbers?
8. Have students share their ideas to rewrite the problem as an equation, $8 = 2x + 4$. Then, in pairs or small groups, have students rewrite each set of treasures and pyramids on the *Pyramid Equality* sheet as equations.
9. Instruct the students to create their own pyramid equalities by using their *Treasure and Pyramid Cut-Outs*, drawing their equations, and then writing out the equations. Their pyramids must not contain decimals of treasures, so insist that students find whole number answers and check their work. They should create 3-5 equalities, which may be shared with a partner.
10. Students are now ready to learn how to solve algebraic equations.

Assessment Suggestions

- *Pyramid Equality*
- At the conclusion of the lesson, students should create 3-5 equalities of their own using the *Treasure and Pyramid Cut-outs.*
- Informal class discussion and math journals

Curriculum Extensions/Adaptations/ Integration

- Children with special needs may benefit from working with a partner during step 9 of the instructional procedures.
- Advanced students may research the mummification and burial rituals of Ancient Egypt and prepare some additional information to share with the class.

Family Connections

- Challenge the students to balance their family's weight to within ten pounds. Who will you put on each side of the imaginary equality sign? Students can be creative with household items or even pets to make up for any additional needed weight.

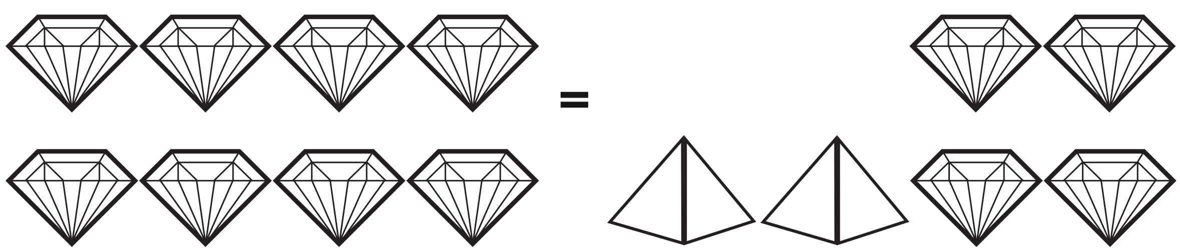
Tomb Treasures

Upon a pharaoh's death in Ancient Egypt, the body was mummified and then buried in an elaborate coffin, or sarcophagus, that was then placed in a pyramid for burial. Along with the mummified Pharaoh rested his jewels and treasures, which he was able to take with him to the afterlife. The pyramid was supposed to protect the pharaoh's body from natural elements (such as weather) and tomb robbers. Unfortunately, tomb robbers still managed to steal the treasures, including the coffins and mummies themselves.

Sefu, an Egyptian archaeologist, is sorting through treasures from robbed tombs. He is trying to decipher how many treasures are still inside the pyramids. He has the following information based on equality:

- Each pyramid contains the same number of small treasures.
- Each small treasure represents one.
- The number of small treasures on both sides of the equality sign is the same, but some of the treasures are still inside the pyramids.

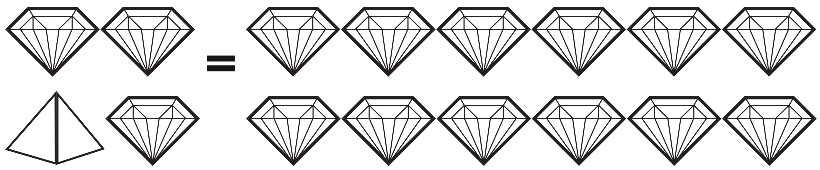
Sefu draws the following picture. Each pyramid contains the same number of small treasures, valued at one unit each.

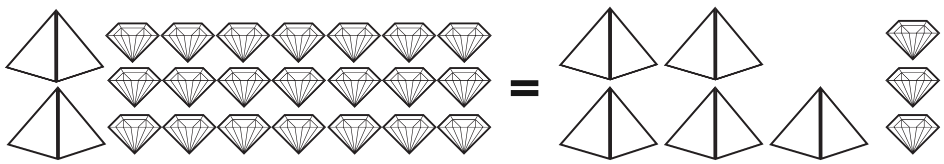


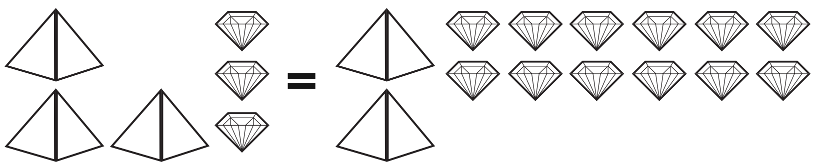
How many treasures are in each pyramid? Explain your reasoning.

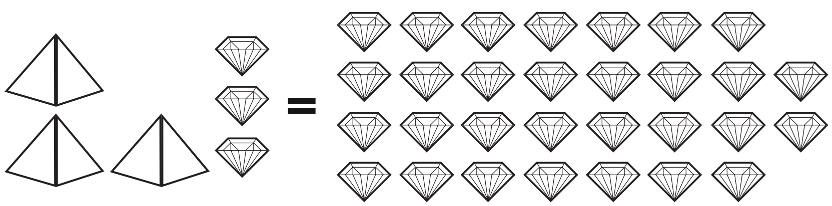
Pyramid Equality

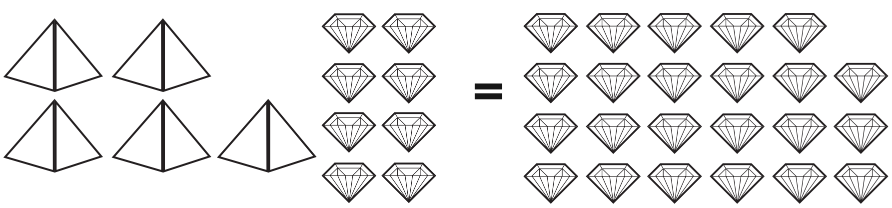
The Egyptian archaeologist, Sefu, has the following information about the treasures from the robbed tombs. For each situation, find the number of treasures in the pyramid. Write down your steps on this paper or in your math journal so that you remember your strategy.

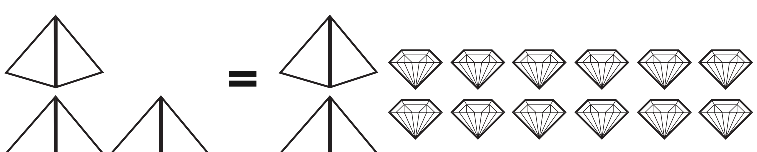
1. 

2. 

3. 

4. 

5. 

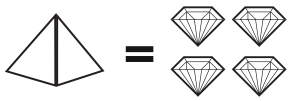
6. 

7. Describe how you can check your answer. How do you know you found the correct number of treasures in each pyramid?

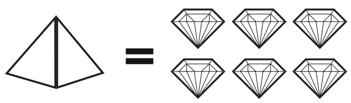
8. Describe how you maintained equality at each step of your solutions.

Pyramid Equality Key

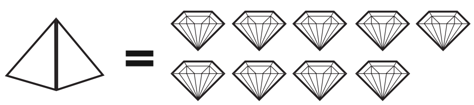
The Egyptian archaeologist, Sefu, has the following information about the treasures from the robbed tombs. For each situation, find the number of treasures in the pyramid. Write down your steps on this paper or in your math journal so that you remember your strategy.

1. 

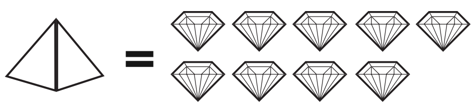
   Check: 2(4) + 4 = 12. 12 = 12.

2. 

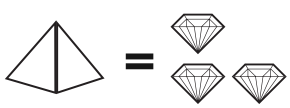
   Check: 2(6) + 21 = 5(6) + 3. 33 = 33.

3. 

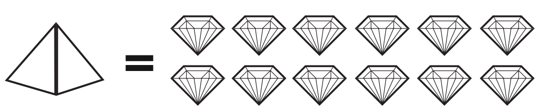
   Check: 3(9) + 3 = 2(9) + 12. 30 = 30.

4. 

   Check: 3(9) + 3 = 27 + 3 = 30.

5. 

   Check: 5(3) + 8 = 15 + 8 = 23.

6. 

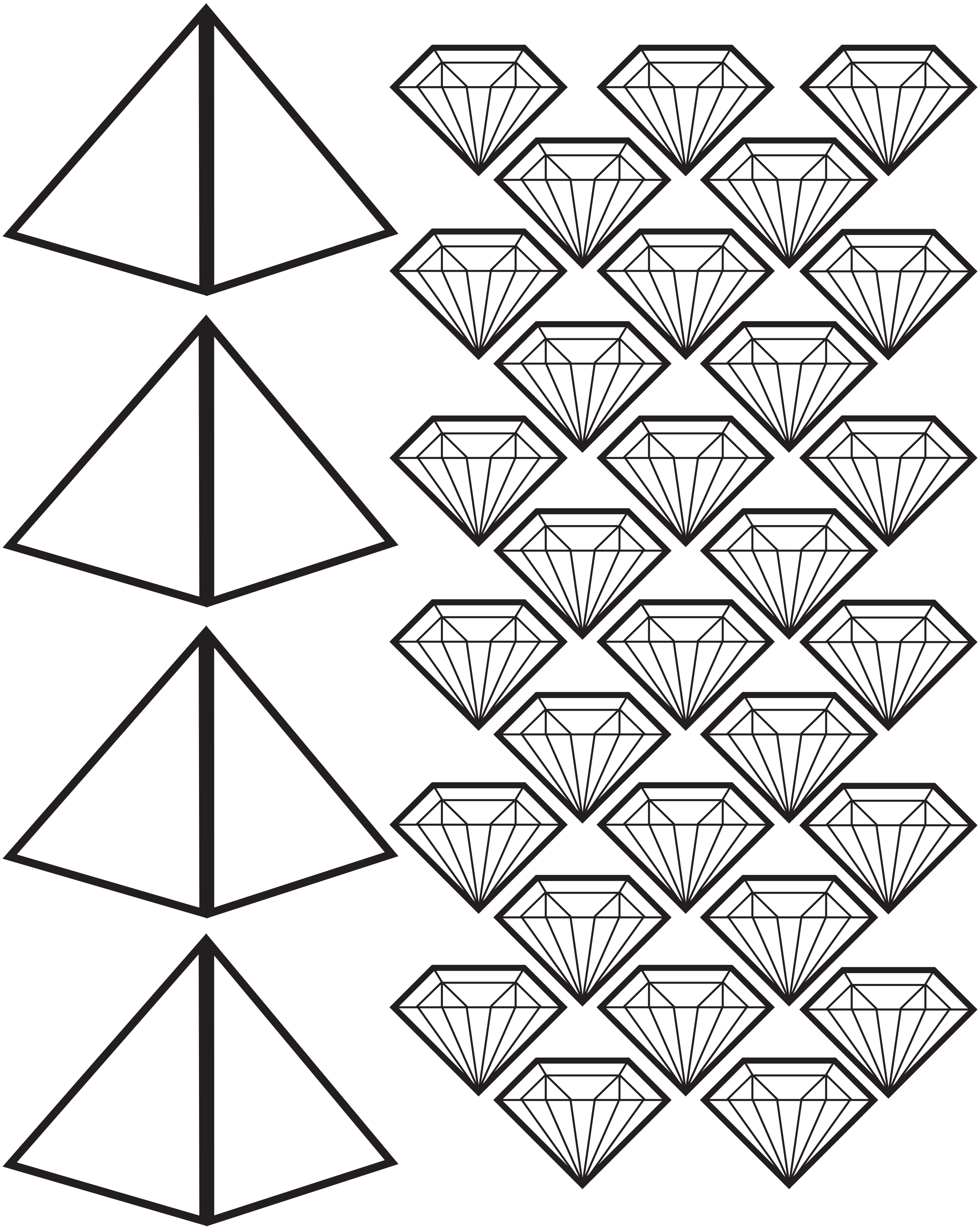
   Check: 3(12) = 2(12) + 12. 36 = 36.

7. Describe how you can check your answer. How do you know you found the correct number of treasures in each pyramid? Students should describe plugging their answer back into the problem to see if it works, as demonstrated above.

8. Describe how you maintained equality at each step of your solutions.

   What students do on one side of the equation must be done on the other!

Treasure and Pyramid Cut-outs



The New Texas Two-Step

Standard II:
Students will use patterns, relations, and algebraic expressions to represent and analyze mathematical problems and number relationships.

Objective 2:
Write, interpret, and use mathematical expressions, equations, and formulas to represent and solve problems that correspond to given situations.

Intended Learning Outcomes:
2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.
3. Reason logically, using inductive and deductive strategies and justify conclusions.
4. Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.

Content Connections:
Math I-4; Model and illustrate meanings of operations

Math Standard II

Objective 2

Connections

Background Information

Solving two-step algebraic equations is a concept used throughout all of algebra. In order to be successful, students must understand equality and variables (which may be taught using the *Pyramid Equality* lesson), as well as the order of operations. They should be able to add and subtract integers, as well as understand the concept of zero pairs of tiles. This occurs when a negative x tile and a positive x tile are together, which create a sum of zero. For example, $-2 + 2 = 0$. Similarly, they should understand that addition and subtraction are the inverse operations of each other, just as multiplication and division are the inverse operations of one another.

Students must also understand exponents. The expression x^2 means x squared, or x times x.

Algebra tiles provide a useful way to introduce algebra operations to students of all ages. Students use the tiles as numbers to replace the variables, which provides a visual image of the equations. This will make the transition to paper and pencil much easier to understand. Using the manipulatives will also aid in retention of the concepts.

Research Basis

Leitze, A.R., & Kitt, N. A. (2000 September). Using homemade algebra tiles to develop algebra and prealgebra concepts. *Mathematics Teacher, 93,* 462-466, 520.

Algebra for all is possible by using algebra tiles as concrete models in the classroom. This article describes how to use homemade tiles to reach a broader group of students for successful algebra thinking. Provides concepts appropriate for this approach.

Leinenbach, M., & Raymond, A.M. (1996). A two-year collaborative action research study on the effects of a "hands-on" approach to learning algebra. *ERIC Source* (ERIC ED398081). Retrieved November 30, 2006, from http://www.eric.ed.gov

A "hands-on" approach to algebra enhances students' confidence, interest in, and ability to solve and retain understanding of algebraic equations. This article describes a two-year research project focused on two phases and data collection.

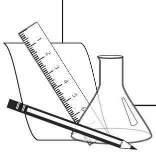
Invitation to Learn

Write the following question on the board: If tickets to a high school football game cost $4 per person, explain in words, numbers, or pictures how you can calculate how much money it will cost your family to go to the game?

Give students a few minutes to work alone, and then pair them up to share their strategies. Discuss as a class. (Students should realize that the number of people per family would cause the cost of the tickets to vary.

Materials

- ❏ Algebra tiles
- ❏ Transparency Algebra Tiles
- ❏ Math journals
- ❏ Colored pencils
- ❏ White piece of paper
- ❏ *Can You Use Algebra Tiles?*
- ❏ *Can You Use Algebra Tiles?* (Key)
- ❏ *Let's Do the Two-Step!*
- ❏ *Let's Do the Two-Step!* (Key)



Instructional Procedures

Day 1: Introduction to Algebra Tiles

1. Prior to this lesson, give each student two sheets of *Algebra Tiles Cut-Outs* on green and red cardstock if you are not using the commercial tiles. The students should cut them out at home, put them in a resealable plastic bag, and return them to school ready to be used. You may also have the students prepare the tiles in school prior to the lesson.
2. Using the transparency Algebra Tiles only, show the students the tiles and name them. The small squares are *units*, or *ones*; one of them stands for 1, and 5 of them stands for 5. The long rectangles are each an *x*, and therefore represent a variable. The large squares represent x^2, x in other words, a variable multiplied by itself.
3. Explain that green tiles indicate addition and red tiles indicate subtraction. In other words, green tiles are positive and red tiles are negative, but it is not necessary for students to understand the term "negative" as they will not be used in the lesson.

4. Make a collection of pieces using all three types of tiles. For example, use one x^2 tile, three x tiles, and 5 ones. Assemble the pieces on the overhead projector, then write down its name: $x^2 + 3x + 5$. This collection represents an algebraic expression. (See figure one.)
5. Have students create their own expression of tiles using a maximum of 10 tiles. They should then draw their expressions in their math journals and write down the name of the expression using numbers and symbols. Remind the students of the commutative property; it does not matter what order the addition occurs: $x^2 + 3x + 5$ is the same as $5 + x^2 + 3x$.
6. To introduce the idea of addition, have the students combine their expressions with a partner. The students should draw their new expression and show the addition using numbers and symbols. For example, the above expression of $x^2 + 3x + 5$ combined with $2x^2 + 8$ would equal $3x^2 + 3x + 13$. (See figure two.)
7. To illustrate the idea of subtraction, have the students experiment with removing tiles. Using their expression from step 5, instruct them to remove three of any type of tiles. Ask, how can you write this new equation and answer? Students should draw their expression and represent it with numbers and symbols. Ask students to remove 2 more tiles from their new expression. Again, they should draw their new expression and represent it with numbers and symbols.
8. Repeat the process of pairing up, adding, and subtracting if desired. The students do not need to use the red tiles for subtracting at this point.
9. After this brief introduction to the tiles, the students are now ready to solve linear equations.

Solving Two-Step Equations with Algebra Tiles

10. Have students take out a white piece of paper and draw a vertical line down the center. Ask them to model the equation $x + 7 = 10$ by placing one positive green x tile and seven positive green unit tiles on the left side of the line, and ten positive green unit tiles on the right side of the line. (See figure 3.)
11. Explain to the students that in order to maintain equality of the "sides," each action must be performed on both sides! Their goal is to isolate the variable, that is, to make sure the variable stands alone on one side of the equality sign.
12. Ask the students what needs to occur for the variable, or the positive green x tile, to stand-alone. After responses, remove

seven tiles from each side of the equation. Now the x tile and 3 units remain, therefore signifying that x = 3. (See figure 4.)

13. Repeat the process with a new equation, 5x – 1 = 9. Have students set up the equation on their own (5 positive green x tiles and 1 negative red unit on the left; 9 positive units on the right). Now how can they get the variable to standby itself? They will need to use the zero pair to remove the one from the left then add that positive unit to the right. In other words, adding a positive green unit tile on the left will remove itself and the 1, and another positive green unit tile also needs to be added to the right to maintain equality. This creates 5x = 10. Now the problem becomes much like the pyramid problem – the students must evenly distribute the units on the right among the x tiles on the left to determine the value of x, which is 2. (See figure 5.)
14. Pass out a *Can You Use Algebra Tiles?* Worksheet for each student. Have them work on the problems in groups of two or three, making sure to draw their tiles for each equation.
15. Correct and summarize strategies. Continue with the tiles, and move on to the paper and pencil method when ready. Students should remember the concept of equality: what you do on one side <u>must</u> be done on the other.

Day 2: Solving Two-Step Equations with Paper and Pencil

16. Review the *Can You Use Algebra Tiles?* Worksheet from Day 1. Look at problem one: 2x – 4 = 10. Ask, how can I solve this problem without using the tiles? Remember, I must maintain equality!
17. Wait for responses. Then teach the two-step paper and pencil method while demonstrating with the above problem.

    Step One: Add or subtract the inverse operation on each side of the equation.

    2x – 4 + 4 = 10 + 4. (After simplifying, 2x = 14.)

    Step Two: Take the inverse (divide) from both sides of the equation. This will affect the number directly beside the variable.

    2x / 2 = 14/2. (After simplifying, x = 7.)
18. Try problem two from *Can You Use Algebra Tiles* as a class. Allow students ownership in finding the answer.
19. Work in groups of two or three to solve the rest of the problems using the paper and pencil method.

Assessment Suggestions

- *Can You Use Algebra Tiles?* Worksheet. You may also revisit this worksheet after teaching the paper and pencil method of solving two-stop equations and have students solve each equation without using tiles.
- *Let's Do the Two-Step!* Worksheet to be completed after step 18 in the instructional procedures.
- Have students explain the steps to solving equations with algebra tiles and with paper and pencil in their math journals.

Curriculum Extensions/Adaptations/ Integration

- Provide extra algebra tiles instruction to students with special needs.
- Allow advanced learners to bypass the algebra tiles when ready and move to paper and pencil.
- Try the activity named Algebraic Equations Gizmo on explorelearning.com to translate English sentences into equations and equations into English sentences.

Family Connections

- Use the activity Algebraic Equations Gizmo on explorelearning.com to translate English sentences into equations and equations into English sentences.
- Try an input-output game. Students create an equation and a list of possible variables and solutions. Now cover up your equation and see if your family can figure it out based on the other information!

Additional Resources

Books

Algebra Tiles for the Overhead Projector, by Hilde Howden; ISBN 0-914040-42-1

Web sites

http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=123

Algebra Tiles Lesson 2

Figure 1:

$x^2 + 3x + 5$

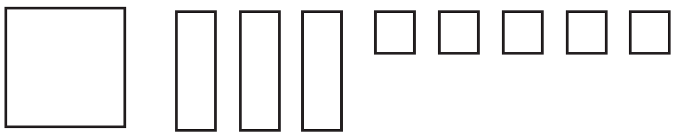


Figure 2:

$3x^2 + 3x + 13$

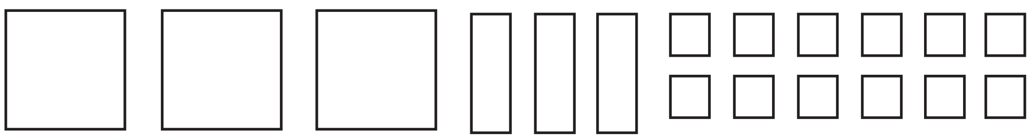


Figure 3:

$x + 7 = 10$

Figure 4:

$x + 7 - 7$ $10 - 7$ $x = 3$

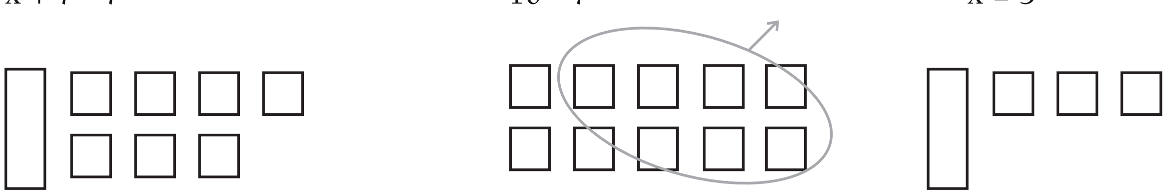


Figure 5:

$5x - 1 = 9$ $5x - 1 + 1$ $9 + 1$

There are 5 groups of 2 to fit in each , so $x = 2$

Can You Use Algebra Tiles?

Solve each problem using algebra tiles, making sure to draw and explain your steps and always maintain equality.

1. $2x + 4 = 10$

2. $3y - 2 = 4$

3. $2x + 3 = 7$

4. $4x - 1 = 3$

5. $2x - 3 = 5$

6. $3x + 1 = 10$

7. $3x - 4 = 5$

8. $5x - 2 = 8$

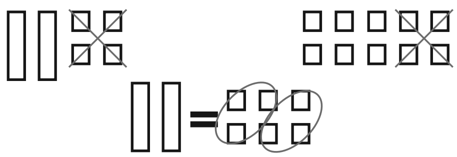
9. $4x + 3 = 19$

Can You Use Algebra Tiles? (Key)

Solve each problem using algebra tiles, making sure to draw and explain your steps and always maintain equality.

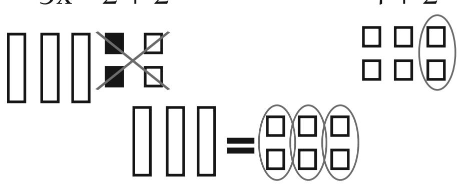
1. $2x + 4 = 10$ $\quad$ $x = 3$

   $2x + 4 - 4$ $\quad$ $10 - 4$



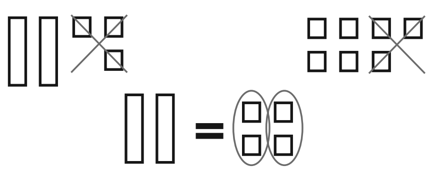
2. $3x - 2 = 4$ $\quad$ $x = 2$

   $3x - 2 + 2$ $\quad$ $4 + 2$



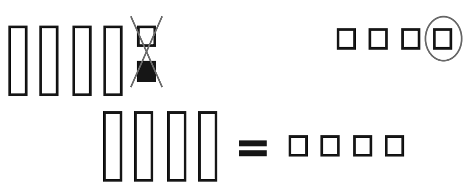
3. $2x + 3 = 7$ $\quad$ $x = 2$

   $2x + 3 - 3$ $\quad$ $7 - 3$



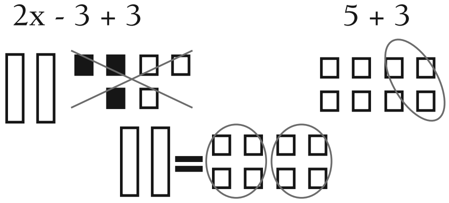
4. $4x - 1 = 3$ $\quad$ $x = 1$

   $4x - 1 + 1$ $\quad$ $3 + 1$



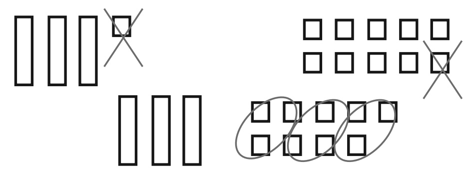
5. $2x - 3 = 5$ $\quad$ $x = 4$

   $2x - 3 + 3$ $\quad$ $5 + 3$



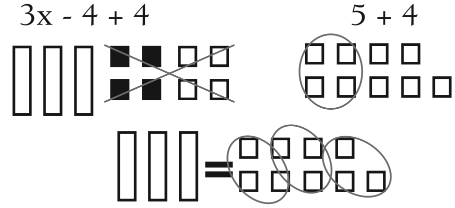
6. $3x + 1 = 10$ $\quad$ $x = 3$

   $3x + 1 - 1$ $\quad$ $10 - 1$



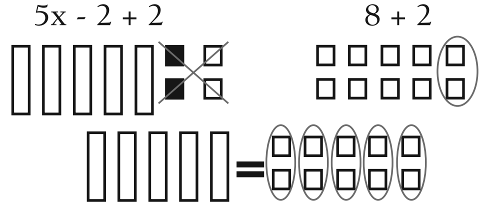
7. $3x - 4 = 5$ $\quad$ $x = 3$

   $3x - 4 + 4$ $\quad$ $5 + 4$



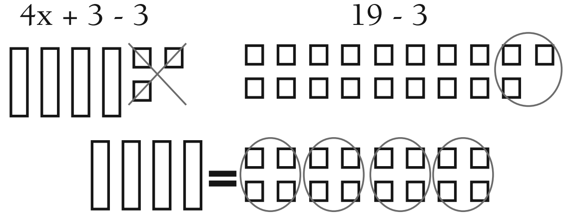
8. $5x - 2 = 8$ $\quad$ $x = 2$

   $5x - 2 + 2$ $\quad$ $8 + 2$



9. $4x + 3 = 19$ $\quad$ $x = 4$

   $4x + 3 - 3$ $\quad$ $19 - 3$



Let's Do the Two-Step!

Solve each algebra problem using the two-step paper and pencil method. Please show your work and your answer clearly.

1. $2 + 6x = 8$

2. $3 + 2x = 7$

3. $2 + 4x = 34$

4. $7x + 1 = 29$

5. $5x + 10 = 35$

6. $8 + 7x = 50$

7. $10 + 4x = 30$

8. $7x + 2 = 65$

9. $6 + 6x = 78$

10. $2x - 5 = 7$

Let's Do the Two-Step! (Key)

1. $2 + 6x = 8$ answer: $x = 1$

2. $3 + 2x = 7$ answer: $x = 2$

3. $2 + 4x = 34$ answer: $x = 8$

4. $7x + 1 = 29$ answer: $x = 4$

5. $5x + 10 = 35$ answer: $x = 5$

6. $8 + 7x = 50$ answer: $x = 6$

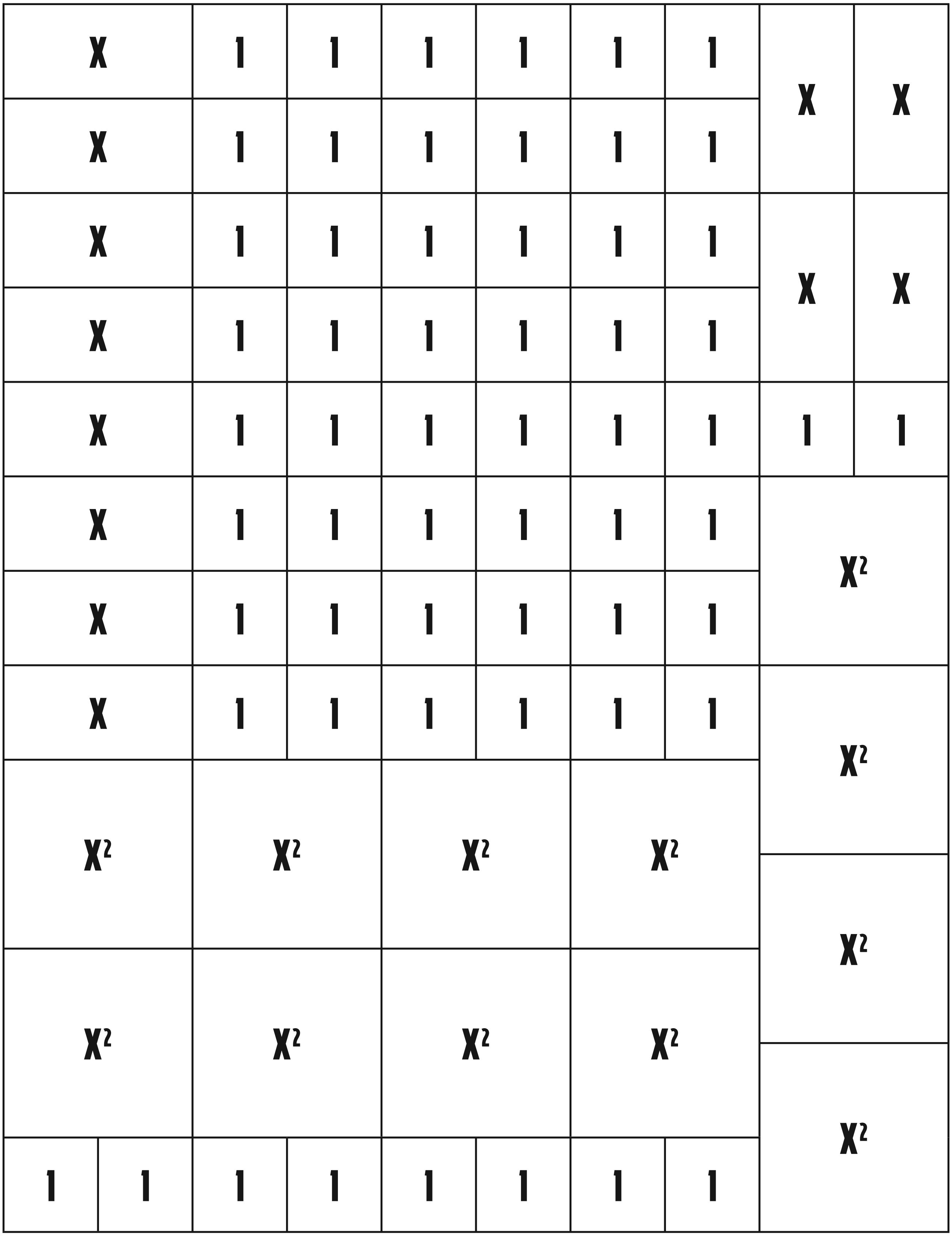
7. $10 + 4x = 30$ answer: $x = 5$

8. $7x + 2 = 65$ answer: $x = 9$

9. $6 + 6x = 78$ answer: $x = 12$

10. $2x - 5 = 7$ answer: $x = 6$

Algebra Tiles Cut-outs



Algebra Applies to the Real World? No Way!

Math Standard II

Objective 2

Connections

Standard II:

Students will use patterns, relations, and algebraic expressions to represent and analyze mathematical problems and number relationships.

Objective 2:

Write, interpret, and use mathematical expressions, equations, and formulas to represent and solve problems that correspond to given situations.

Intended Learning Outcomes:

1. Develop a positive learning attitude toward mathematics.
2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.
5. Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.

Content Connections:

Language Arts I-1; Develop language through listening and speaking

Background Information

This is a follow-up activity designed to extend your students' knowledge of solving two-step equations through a review and real-life situations. Students should have an understanding of equality and solving equations before attempting this lesson.

This lesson employs the skills of Bloom's Taxonomy, which include three overlapping domains: the cognitive, psychomotor, and affective. Bloom's Taxonomy aids these domains through steps of educational objectives: knowledge, understanding, application, analysis, synthesis, and evaluation, all of which are used in the situation cards portion of this lesson. Bloom's is recommended for all curriculum areas to enhance the thinking abilities of your students.

Research Basis

Martinie, S. (2003, October). Families ask: cooperative groups. *Mathematics Teaching in the Middle School, 9,* 106-107.

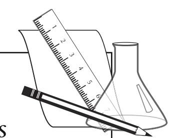
More than 900 studies endorse the use of cooperative learning, which improves student achievement, social skills, and motivation and enthusiasm for math. Students learn and retain information better in cooperative groups. Students are held responsible for their own learning and build confidence and value in their own thinking.

Panitz, T. (2000). Using cooperative learning 100% of the time in mathematics classes establishes a student-centered, interactive learning environment. *ERIC Source* (ERIC ED448063). Retrieved December 3, 2006, from http://www.eric.ed.gov

Cooperative learning activities help to identify student misconceptions and enable the teacher to focus on specific concepts. Verbal, visual, and kinesthetic student learning styles are addressed. The benefits of cooperative learning are indicated, including enhanced critical thinking skills, better student-teacher relationships, and an enjoyment of math classes.

Invitation to Learn

Post this question on the board: Does algebra relate to "real life?" Instruct students to jot down their responses in their math journals. This question will be discussed at the end of the lesson.



Materials

- ❏ *Algepairs Cards*
- ❏ *Situation Cards*
- ❏ *Situation Cards* (Key)
- ❏ Math journals
- ❏ Basic calculators (optional)

Instructional Procedures

1. Have all students stand and pass out one *Algepairs Card* to each. Starting anywhere in the room, have a student read his/her card. The student who can complete that card should read their card, and so on. Students should sit down when finished and try to complete the problems as the cards are read to stay engaged in the game.
2. Redistribute cards and play again, or collect cards.
3. Put students into groups of three and pass out one *Situation Card* and calculator (optional) to each group. Cards are on different colors to indicate whether they are easy (red), mid-level (green), or challenging (blue). You may use these to vary difficulty or to help specific students. Students must still show all of their work if they use a calculator.
4. Have students work cooperatively to understand the problem, write an equation, and find the solution. They should each be able to explain the solution to someone in another group. For example, if a student has *Situation Card 7*, she should understand that since baby-sitting pays $7.50 an hour and she baby-sat for 4 hours that means she made $7.50 each of those hours. The unknown is how much she made. 7.50 x 4 = $30 on Saturday night.
5. If students finish early, ask them questions to extend their thinking, or pass out another situation card not being solved by another group.
6. When all groups are finished, regroup students in threes with each person from a different original group. Have them take turns sharing their situation and equation then allowing the

other students in the group to solve the problem using the equation. Allow enough time for all students to share and solve.

7. Again regroup students into threes and instruct them to create a situation of their own. They must have a variable and their equation must be two steps.
8. Use the finished situations and equations for sharing and/or assessment.
9. Discuss the Invitation to Learn using ideas from the situation cards and earlier discussion. Have students share any new ideas in their math journals.

Assessment Suggestions

- Students should create their own algebraic situation, then write an equation and solve.
- Informal assessment during *Algepairs* game and math journaling.

Curriculum Extensions/Adaptations/Integration

- Pass out more than one *Algepairs Card* to your advanced learners.
- Vary the levels of *Situation Cards* for advanced and special needs students.
- Have students write an essay to convince their peers that algebra relates to real life.
- Read *Divide and Ride,* by Stuart Murphy, and have the students create equations for the situations described.
- Go to futureschannel.com and click on Algebra in the Real World to download videos displaying algebra in real world situations. Could use this as a kick-off or follow-up to the activity.

Family Connections

- Decide on a travel spot for a real or imaginary family vacation. Determine at least 5 expenses (i.e. airline tickets, rental car, food, activity prices, etc.), create equations for each, and solve. Based on your data, how much will the trip cost?

- Create a student edition of *Algepairs Cards* to play at home.

Additional Resources

Books

Divide and Ride, by Stuart Murphy; ISBN 0064467104

Web sites

http://www.coun.uvic.ca/learn/program/hndouts/bloom.html

www.futureschannel.com

Algepairs Cards

19 When x=19 x-13=	**13** When x=13 2x-12=	**15** When x=15 2x-10=
6 When x=6 2x-9=	**14** When x=14 2x-6=	**20** When x=20 x-2=
3 When x=3 5x+1=	**22** When x=22 x+8=	**18** When x=18 x-8=
16 When x=16 2x-7=	**30** When x=30 .5x-3=	**10** When x=10 3x-2=
25 When x=25 x-12=	**12** When x=12 x+3=	**28** When x=28 x-2=

Algepairs Cards

26 When x=26 .5x+4=	**29** When x=29 x-8=	**9** When x=9 2x-14=
17 When x=17 x-15=	**21** When x=21 x+3=	**4** When x=4 4x-11=
2 When x=2 10x+3=	**24** When x=24 .5x-4=	**5** When x=5 3x-8=
23 When x=23 x-12=	**8** When x=8 3x+3=	**7** When x=7 2x-13=
11 When x=11 3x-4=	**27** When x=27 2x-45=	**1** When x=1 9x+10=

Situation Cards

Situation Card #1 Brianna receives $7.50 per hour when she baby-sits the 3 Steigerwald children. If she baby-sits for 4 hours on Saturday night, write an equation to determine how much she will receive. What will she earn if she baby-sits for five and a half hours?	**Situation Card #2** Kolleen and Dallin went ice-skating at Seven Peaks with Kolby and Stephanie on Saturday night. They had to pay for tickets, skate rental, and food. You know that the food and skate rental cost $12 total. You also know they spent $32 that evening. Write an equation to determine the cost of each ticket. How much did each couple pay?
Situation Card #3 Tomorrow is your birthday. You want to bring a Snickers bar to each member of your class, including your teacher. If each candy bar is $.50 and your mom gave you $12.50 to spend, can you afford enough candy bars for your entire class? Write an equation and solve to find out.	**Situation Card #4** Angie and her five siblings want to go to opening night of an upcoming movie. In order to avoid long lines, Angie decides to purchase their tickets on Fandango.com. Tickets are $8 each with a $1 handling fee for each ticket. Her dad gave her $50 to spend. How many of Angie's siblings can she afford to take to the movie? Write an equation and solve to find out.
Situation Card #5 The current temperature is 30 degrees Fahrenheit and is expected to rise 2 degrees per hour for the next several hours. Write an equation that represents the relationship between temperature and time. After how many hours will the temperature be 55 degrees?	**Situation Card #6** Elmbrook Middle School is planning a field trip for the 6th grade and needs to determine how many busses need to be reserved for that day. Your school has 175 6th grade students and 5 teachers. Each bus can hold a maximum of 84 people. Write an equation to determine how many busses the school needs to reserve. If you divide the students and teacher evenly, how many people should go on each bus?

Situation Cards

Situation Card #7

Mr. Risch's class is sponsoring a walkathon to raise money for science supplies. Three students found sponsors who are willing to pledge the following amounts.

- Ashley's sponsors will pay $10 regardless of how far she walks.
- Caroline's sponsors will pay $3 per mile.
- Donnie's sponsors will make a $5 donation, plus $1 per mile.

After creating equations for each pledge plan, decide which plan will bring in the most money if all of the students in the class are planning on walking 8 miles.

Caroline decides to give a t-shirt to each of her sponsors. She is going to use some of the money she collects from her sponsors to cover the $5 cost of each shirt. If the class decides to use her pledge plan, write an equation to represent the amount of money Caroline will make from each sponsor after paying for the cost of the t-shirts.

Situation Card #8

Emily's mom is hiring a magician for her twelfth birthday party. She obtained several prices for the cost of three magicians.

The Great Cardini charges $75 an hour.

Dante Fantasio charges $100 plus $20 an hour.

Amazing Max charges $150 plus $30 an hour.

Write three equations to determine the cost of each magician for two and a half hours. Based on this information, which magician should Emily's mom hire?

Situation Card #9

Natalie wants to purchase an iPod. An electronics store offers two installment plans for buying the $250 version.

Plan A: A fixed weekly payment of $10.50.

Plan B: A $120 initial payment, followed by $5 per week.

After 12 weeks, how much money will Natalie owe on each plan? Write an equation and solve to find out.

Which plan requires the least number of weeks to pay off the iPod?

Situation Cards Key

Note: Students may represent a variable with any letter.

1. 7.50h = p where h = hours worked and p = payment.

   7.50 x 4 = $30 for 4 hours.

   7.50 x 5.5 = $41.25 for 5.5 hours.

2. 4p + 12 = 32 where p = price of tickets.

   p = $5 per ticket.

   5 x 2 = $10 per couple.

3. This answer will vary based on class size. The following uses a class size of 28.

   .5s = 12.50 where s = number of students.

   28 x .5 = 12.50

   Students should reason that since half of 28 is 14, they would be $2.50 short and therefore unable to buy each child a candy bar.

4. 8p + p = 50 OR 9p = 50 OR 8p + 1p = 50 where p = price of ticket.

   For 6 siblings, you would spend $54, which is $4 too expensive.

   For 5 siblings, you will spend $48, which leaves $4 left over.

   Angie can take 4 siblings with you, or let her 5 siblings go to the movie while she stays home. Even better, her dad may give her an extra $4!

5. 30 + 2h = t where h = hours and t = temperature.

   30 + 2h = 55. h = 12.5 hours

6. 84b = 180 where b = number of busses.

   180/84 = 2.14 busses, so you will need 3 busses for the field trip.

   180/3 busses = 60 people on each bus.

7. p = payment earned and m = miles walked

   Ashley's plan p = 10. For 8 miles, p = $10.

   Caroline's plan p = 3m. For 8 miles, p = $24.

   Donnie's plan p = 5 + 1m. For 8 miles, p = $13.

   Caroline's plan will bring in the most money.

   T-shirt equation using Caroline's plan: p = 3m – 5.

8. c = cost and h = hours worked.

c = 75h.	For 2.5 hours, c = $187.50.
c = 100 + 20h	For 2.5 hours, c = $150.00.
c = 150 + 30h	For 2.5 hours, c = $225.00.

   Emily's mom should hire Dante Fantasio for $150.00.

9. Plan A: 250 = 10.50w where w = weeks.

   Plan B: 250 = 120 + 5x.

   After 12 weeks, Natalie will have paid $126 on plan A and $180 on plan B, which means she still owes $124 on plan A and $70 on plan B.

   After working the two-step equation, it will take 23.8 (or about 24) weeks to pay off plan A and 26 weeks to pay off plan B. Plan A takes the least amount of weeks to pay off.

Math I-1&3

Activities

Numbers

Prime Factorization – From Fingerprints to Factorprints

Standard I:
Students will expand number sense to include operations with rational numbers.

Objective 3:
Use number theory concepts to find prime factorizations, least common multiples, and greatest common factors.

Intended Learning Outcomes:
3. Reason logically, using inductive and deductive strategies and justify conclusions.
6. Represent mathematical ideas in a variety of ways.

Content Connections:
Math II-1; analyze algebraic expressions, tables, and graphs

Math Standard I

Objective 3

Connections

Background Information

The number one is a unique number because it only has itself as a factor. A prime number is a counting number larger than one that has exactly two factors. The two factors are one and the number itself. A composite number is a counting number that has more than two factors. Each composite number is divisible by three or more whole numbers.

Each composite number can be renamed as a product of prime numbers. This is known as prime factorization. Understanding prime factorization helps students understand the composition and decomposition of numbers.

Prime factorization is a strategy students may employ to find the Greatest Common Factor (GCF) of two or more numbers. Students may also use prime factorization to find the Least Common Multiple (LCM) of two or more numbers. It may be interesting to note that the product of the LCM and the GCF of two numbers is equal to the product of the two numbers themselves.

Research Basis

Gerlic, I., & Jausovec, N. Multimedia: Differences in cognitive processes observed with EEG. *Educational technology research and development*, September 1999, Vol. 47, Number 3, p5-14.

This study investigated the cognitive processes involved in learning information presented in three different methods: with text; with text, sound, and picture; and with text, sound, and video. Students' brain activity was measured using an EEG in each format. Less mental activity was found using the text only presentation. The results showed higher mental activity with the video and picture presentations, confirming the assumption that these methods induced visualization strategies on the part of the learners.

Zazkis, R., & Liljedahl, P. Understanding primes: The role of representation. *Journal for research in mathematics education*, May 2004, Vol. 35 Issue 3, p164-186.

The authors of this article investigated how preservice elementary teachers understood the concept of prime numbers. They attempted to describe the factors that influenced their understanding. The authors suggested that an obstacle to a full conceptual understanding is a lack of a representation for a prime number. The importance of representations in understanding math concepts is examined.

Invitation to Learn

Materials

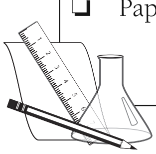
- ❏ Ink pads (1 per group)
- ❏ 1 1/2” x 2” Post-it® Notes
- ❏ Wet wipes
- ❏ Poster of main fingerprint patterns

Pretend you are a detective. What is one piece of evidence that would help you to identify suspects from a crime scene? Fingerprints would be one type of evidence. Every person has a one-of-a-kind fingerprint. Have students make a fingerprint of their right index finger on a Post-it® note. Have students place their Post-it® note on the line plot, matching their fingerprint with one of the nine main patterns pictured on a teacher-made categorical line plot poster. Even though there are nine fingerprint patterns, allow students time to notice that each individual fingerprint is still one-of-a-kind.

Write the following analogy on the board: human is to fingerprint as number is to “factorprint.” Tell students that just as each human has a one-of-a-kind fingerprint, we will learn that each number has a one-of-a-kind “factorprint.”

Materials

- ❏ Overhead color tiles
- ❏ Overhead markers
- ❏ Centimeter graph paper
- ❏ Colored pencils
- ❏ *Prime Factorization*
- ❏ Centimeter cubes
- ❏ *Prime Factorization – Centimeter Cubes*
- ❏ *GCF Mat* (laminated)
- ❏ *LCM Mat* (laminated)
- ❏ Dry-erase markers
- ❏ Paper towels



Instructional Procedures

(The activities listed below are intended to be taught sequentially. They will take several lessons/days to complete with students.)

1. Explain to students that you will be creating a pattern with color tiles. They will copy the pattern by coloring the same pattern on their graph paper. Students are to observe, reflect, and predict the pattern after they see the first few representations
2. Have students copy the following pattern demonstrated on the overhead with color tiles:
   - To begin the first row, color one square black near the top left-hand corner. Label underneath “1.”

- Skip two squares horizontally and color one square red. Label underneath "2."
- Skip two squares horizontally and color one square green. Label underneath "3."
- Skip two squares horizontally and color two vertical squares red. Label underneath "4."
- Skip two squares horizontally and color one square yellow. Label underneath "5."
- Skip two squares horizontally and color one square red and one square green, placed vertically. Label underneath "6."
- Skip two squares horizontally and color one square blue. Label underneath "7."

3. Have students predict what will be next in the pattern. Have students justify their prediction.
4. Continue the pattern, stopping to predict and justify answers at each number, as follows:
   - Skip two squares horizontally and color three vertical squares red. Label underneath "8."
   - Skip two squares horizontally and color two vertical squares green. Label underneath "9."
   - Skip two squares horizontally and color one square red and one square yellow, placed vertically. Label underneath "10."
5. By this time, students may have discovered that "4" was created by multiplying the value of the red square by itself or "2 x 2." They may have found that "6" was created by multiplying the value of the red square by the value of the green square or "2 x 3." Go back over the first 10 numbers and label the expressions for the composite numbers. Label "4" as "2 x 2," "6" as "2 x 3," "8" as "2 x 2 x 2," "9" as "3 x 3," and "10" as "2 x 5." Write PRIME under each prime number.
6. If a horizontal row is full, start a new row about halfway down the page. Continue the pattern, stopping to predict and justify answers at each number, as follows:
   - Skip two squares horizontally and color one square orange. Label underneath "ll" and write "PRIME."
   - Skip two squares horizontally and color two squares red and one square green, placed vertically. Label underneath "12," and label with the expression "2 x 2 x 3."

- Skip two squares horizontally and color one square purple. Label underneath “13” and write “PRIME.”
- Skip two squares horizontally and color one square red and one square blue, placed vertically. Label underneath “14,” and label with the expression “2 x 7.”
- Skip two squares horizontally and color one square green and one square yellow, placed vertically. Label underneath “15,” and label with the expression “3 x 5.”
- Skip two squares horizontally and color four squares red, placed vertically. Label underneath “16,” and label with the expression “2 x 2 x 2 x 2.”

7. Remind students that each composite number is being formed by the multiplication of prime numbers. As you are labeling the expression for “16,” teach students that there is another way to write this expression that would use fewer symbols and would be more efficient. We could use base numbers and exponents. Have students write the expression underneath “16” and “2 x 2 x 2 x 2” as “2^4.” Look back over earlier numbers and write the expressions using base numbers and exponents on each composite number.
8. Continue the pattern, stopping to predict and justify answers at each number as follows:
   - Skip two squares horizontally and color one square pink. Label underneath “17” and write “PRIME.”
   - Skip two squares horizontally and color one square red and two squares green, placed vertically. Label underneath “18,” label with the expression “2 x 3 x 3,” and label with the expression “2^1 x 3^2.”
   - Skip two squares horizontally and color one square brown. Label underneath “19” and write “PRIME.”
   - Skip two squares horizontally and color two squares red and one square yellow, placed vertically. Label underneath “20,” label with the expression “2 x 2 x 5,” and label with the expression “2^2 x 5^1.”
9. Have students complete the patterns to the number 50. This could be done as cooperative teams or as a homework project. Since there is a new color for each prime number, the teacher will need to provide these patterns to create uniformity in correcting:
   - 23 is a black-outlined box with a red dot in the center

- 29 is a black-outlined box with a green dot in the center
- 31 is a black-outlined box with a yellow dot in the center
- 37 is a black-outlined box with a blue dot in the center
- 41 is a black-outlined box with an orange dot in the center
- 43 is a black-outlined box with a purple dot in the center
- 47 is a black-outlined box with a pink dot in the center

10. Have students complete the handout *Prime Factorization*. Label the number one with the word "UNIQUE." Label each prime number with the word "PRIME." For each composite number, write the prime factorization expressions found in the color tile activity.
11. Have students place centimeter cubes on the handout *Prime Factorization – Centimeter Cubes* in the same number and color as the color tile pattern. For example, "1" would have one black cube, "2" would have one red cube, "3" would have one green cube, "4" would have two red cubes, and so on.
12. To find the greatest common factor of two numbers, students must first find what prime factors they have in common. Have students write the numbers 8 and 12 as the two selected numbers on the *GCF Mat* with dry-erase marker. Have students take the cubes from *Prime Factorization – Centimeter Cubes* for 8 and 12 and place them on the *GCF Mat* in the proper squares. Help students to see what factors (represented by the colored centimeter cubes) these two numbers share. The number 8 has three red cubes, while the number 12 has two red cubes and one green cube. These two numbers each have two red cubes, so students would place two red cubes in the GCF column. Thus, the GCF of 8 and 12 is 2x2 or 4.
13. Repeat Step 12 with other pairs of numbers such as 9 and 18, 15 and 20, 8 and 24, 10 and 22, and so on.
14. To find the least common multiple of two numbers, students must first find the prime factors of each number. Have students write the numbers 8 and 12 as the two selected numbers on the *LCM Mat* with dry-erase marker. Have students take the cubes from *Prime Factorization – Centimeter Cubes* for 8 and 12 and place them on the *LCM Mat* in the proper squares. Help students to see what factors (represented by the colored centimeter cubes) these two numbers each has. The number 8 has three red cubes, while the number 12 has two red cubes and one green cube. Place three red cubes in the LCM column to represent 8. Add one green cube (using two of the red cubes

already there) to complete the factors of 12. Thus, there will be three red cubes and one green cube or 2x2x2x3 and the LCM of 8 and 12 is 24.

15. Repeat Step 14 with other pairs of numbers such as 3 and 9, 4 and 5, 4 and 7, and so on.

Assessment Suggestions

- Informal assessment includes observation of students as they complete the color tile activity to the number 50.
- Have a class discussion of answers for the numbers 21 through 50. Model the answers on the overhead projector using color tiles or pictorial representations.
- Correct the handout *Prime Factorization* with the expressions from the color tile activity. Have students save this in a math journal or portfolio for future reference.
- Make a concentration game with 20 index cards. Put composite numbers on ten different cards, and put the prime factorization of the selected composite numbers on the other ten cards.

Curriculum Extensions/Adaptations/ Integration

- Find the prime factorization of a number using the tree method.
- Find the prime factorization of a number using the cake method.
- Find the Greatest Common Factor of two numbers using the prime factorization of the numbers from the color tile activity.
- Find the Least Common Multiple of two numbers using the prime factorization of the numbers from the color tile activity.

Family Connections

- Have students share their graph paper patterns of prime factorization with parents.
- Ask students to explain to parents the difference between unique, prime, and composite numbers.
- Have students explain how a composite number may be renamed as a product of prime numbers to their parents.

- Have parents select a composite number under fifty and have students share a strategy for determining the prime factorization of that number.
- Have students teach parents how to find the GCF and LCM of two numbers using prime factorization.

Additional Resources

Books

Discovering Mathematics with the TI-73: Activities for Grades 5 and 6, by Melissa Nast; ISBN 1-8886309-22-1

Web sites

http://www.fingerprints.tk

http://en.wikipedia.org/wiki/Fingerprint

http://illuminations.nctm.org/ActivityDetail.aspx?ID=64

http://www.learnalberta.ca/content/me5l/html/Math5.html?launch=true

http://illuminations.nctm.org/ActivityDetail.aspx?ID=12

http://nlvm.usu.edu/en/nav/frames_asid_158_g_2_t_1.html

http://nlvm.usu.edu/en/nav/frames_asid_202_g_2_t_1.html

http://amby.com/educate/math/2-1_fact.html

http://purplemath.com/modules/factnumb.htm

Prime Factorization

1	26	51	76
2	27	52	77
3	28	53	78
4	29	54	79
5	30	55	80
6	31	56	81
7	32	57	82
8	33	58	83
9	34	59	84
10	35	60	85
11	36	61	86
12	37	62	87
13	38	63	88
14	39	64	89
15	40	65	90
16	41	66	91
17	42	67	92
18	43	68	93
19	44	69	94
20	45	70	95
21	46	71	96
22	47	72	97
23	48	73	98
24	49	74	99
25	50	75	100

Prime Factorization – Centimeter Cubes

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24

GCF Mat

Number ____________	Number ____________	(GCF)

LCM Mat

Number ______________	Number ______________	(LCM)

Expanded Notation and Scientific Notation – Notable Notation

Math Standard I

Objective 1

Connections

Standard I:
Students will expand number sense to include operations with rational numbers.

Objective 1:
Represent rational numbers in a variety of ways.

Intended Learning Outcomes:

4. Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.
5. Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.
6. Represent mathematical ideas in a variety of ways.

Content Connections:
Ed. Tech. VIII; use appropriate technology resources
Language Arts VII-3; use features of informational text
Language Arts VIII-6; write in different forms
Science III-1; use a model to accurately compare size distance

Background Information

Students must be versatile in different types of notation of numbers. By 6th grade, students need to be familiar with the terms standard notation, expanded notation, and scientific notation. Understanding the latter two types of notation will aid in the composition and decomposition of numbers.

Expanded notation is a method of writing numbers using the distributive property. Expanded notation begins as early as 1st grade. As students progress through school, expanded notation may be represented in different ways. In 4th grade, the number 4,376 may first be expanded to 4,000 + 300 + 70 + 6. By 5th grade, it may then be represented as (4 x 1,000) + (3 x 100) + (7 x 10) + (6 x 1). By 6th grade, students need to be able to write 4,376 as (4 x 10^3) + (3 x 10^2) + (7 x 10^1) + (6 x 10^0).

Scientific notation is a method of writing numbers that are very large or very small with only a few symbols. Numbers in scientific notation are written as a product of two factors. The first factor, also known as a coefficient, is greater than or equal to 1, but less than 10. The second factor is a power of 10. For example, 7 x 10^11 is scientific notation for 700,000,000,000.

Research Basis

Ma, Liping. (1999). Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.

This research investigates the importance of a profound understanding of fundamental mathematics on the part of the teacher. Teachers with this profound understanding incorporate the following four properties in their teaching and learning: connectedness, multiple perspectives, basic ideas, and longitudinal coherence.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics.

Materials

- ❏ *If You Hopped Like a Frog*

Teachers need many different kinds of mathematical knowledge. They must have a deep understanding of concepts, practices, principles, representations, and applications. They need knowledge about math as an entire domain, and they also need a thorough knowledge of the curriculum on their own grade level. Teachers must know how to convey mathematical ideas effectively in a coherent and connected manner.

Invitation to Learn

Read to students the book *If You Hopped Like a Frog*. Point out to students some of the facts using very large numbers. For example, if you grew as fast in your first nine months as you did in the nine months before you were born, you would weigh more than 2,500,000 elephants. Write the number 2,500,000 on the board. Have students practice reading this number. Tell students that this number is written in standard notation. We will be learning about other ways to write very large numbers such as expanded notation and scientific notation.

Materials

- ❏ Calculator
- ❏ *Notation and Powers Table*
- ❏ 3" x 5" index cards
- ❏ Marker
- ❏ Place value house models
- ❏ Paper
- ❏ Pencils
- ❏ *0-9 Digit Cards*
- ❏ Lunch sacks
- ❏ *Power Capture Game*

Instructional Procedures

(The activities listed below are intended to be taught sequentially. They will take several lessons/days to complete with students.)

1. Use a calculator to discover the patterns of the powers of 10. Begin with 10^0 which is equal to 1. Continue with 10^1, 10^2, and so forth. Have students record these in their *Notation and Powers Table* for future reference. As students discover the answers, write each exponential notation on an index card and display in place value house model.
2. Remind students about the number 2,500,000 from the elephant comparison in the book *If You Hopped Like a Frog*. This number is written in standard notation. Another way to write this number is in expanded notation.

3. Show students how to write the large number 2,500,000 in expanded notation. Suggested steps include:
   a. Find the number in the largest decimal place value column. Write down that number and multiply it by the power of ten equivalent to its place value. Students could look at a place value house model that shows periods to find the place value's power of ten, or refer to their *Notation and Powers Table*. In the number 2,500,000, the two is in the one millions place which is 10^6. The first step is (2 x 10^6).
   b. Find the number in the next largest decimal place value column. Write down that number and multiply it by the power of ten equivalent to its place value. In the number 2,500,000, the five is in the hundred thousands place which is 10^5. The expanded notation now should read (2 x 10^6) + (5 x 10^5).
   c. Continue this pattern with all numbers other than zero. Since the rest of the digits in 2,500,000 are zeroes, the final answer should read:

      2,500,000 = (2 x 10^6) + (5 x 10^5)
4. Practice writing other numbers in expanded notation using powers of 10. Use the place value house model and *Notation and Powers Table* for references.
5. Show segment of the movie *Powers of Ten*.
6. To practice the powers of ten place value equivalencies, play the partner game "Power Capture."
   a. Cut apart the *0-9 Digit Cards*. Put the digit cards 0-9 in a lunch sack.
   b. Partner One pulls out a number and writes it in the place value chart in random order on the *Power Capture Game* handout. Partner One returns the number to the sack and continues selecting digits and placing them in random order until a secret seven-digit number is generated.
   c. Partner Two will have three turns to guess a digit in the Partner One's number. For each correct guess, Partner Two scores the number of points for the digit's place in powers of ten. Example: Partner Two guesses there is a 4 in the number and is correct. The 4 is in the hundreds place. Partner Two scores two points because the hundreds place is 10^2. After three guesses, Partner Two's turn is completed.

d. Each partner will keep score for each other on the ***Power Capture Game*** handout.

e. Partner Two then generates a seven-digit number, and Partner One has three guesses to capture his power.

f. The first person to reach a score of 50 power points is the winner.

7. Remind students that we have been looking at numbers in standard notation and expanded notation using powers of ten. Tell students there is another way to write very large numbers called Scientific Notation.

8. Remind students of the number 2,500,000 from the elephant comparison in the book *If You Hopped Like a Frog*. Show students how to write the large number 2,500,000 in scientific notation Suggested steps include:

   a. Find the decimal point in the large number. If there is no decimal point, it is at the end of the number.

   b. Move the decimal point to the left so that you get a number that is greater than or equal to 1, but less than 10. Drop any zeroes that are not needed. This number will be the first factor or the coefficient. In the number 2,500,000, the coefficient is 2.5.

   c. Count the number of places you moved the decimal point to the left. The number of places you moved is the power of 10 to use for the second factor. In the number 2,500,000, the decimal is moved six places to the left. It will be 10^6.

   d. To complete the scientific notation, write the numbers found in steps b and c as a product. The answer for the example given is as follows:

      2,500,000 = 2.5 x 10^6

9. Have students practice writing large numbers from the interesting facts below in scientific notation:

   a. Dogs have about 220,000,000 olfactory receptors to help them smell—roughly 40 times the number humans have.

   b. The population of Tokyo, Japan was approximately 34,450,000 in 2000.

   c. The biggest iceberg ever seen, known as B-15, weighed an estimated 4,000,000,000,000 tons.

   d. One light-year is the distance light travels in one year—about 5,900,000,000,000 miles.

e. Scientists discovered a black hole at the center of M87, a galaxy in the constellation Virgo, rotating at 1,200,000 miles per hour using the Hubble Space Telescope.

f. The planet Mercury travels at 107,000 miles per hour.

g. Microscopic quantities of liquid water were found trapped in salt crystals in a 4,500,000,000-year-old meteorite that fell to Earth at Monahans, Texas in 1998.

h. In the mid 1990's, the world had an estimated 19,200,000 camels, of which nearly half were in Somalia and Sudan.

i. In the early 1990's, Utah's chicken population produced approximately 456,000,000 eggs.

j. During his lifetime, George Eastman (1854-1932), an American inventor of films and cameras, donated $75,000,000 to charities.

Source: http://www.worldalmanacforkids.com

10. Show students how to change numbers in scientific notation to standard notation. Suggested steps include:

a. Look at the exponent in the power of ten of the second factor. In the expression 2.5 x 10^6, the exponent is six.

b. Move the decimal point in the first factor that many places to the right, adding zeroes as needed. If you move the decimal six places to the right in 2.5, you will have to add five zeroes to get the correct answer of 2,500,000.

Assessment Suggestions

- Have students write a number fact book where the numbers are presented in both standard and scientific notation. Students can gather facts from almanacs, encyclopedias or Internet sites such as those listed in additional resources.
- Have students visit one of the Internet sites listed in Additional Resources to learn more about scientific notation. Some of these sites have practice problems, games, and feedback for students.

Curriculum Extensions/Adaptations/ Integration

- Teach students how to write very small numbers such as those encountered in the 6th grade microorganism lessons in scientific notation.
- Show students the complete movie *Powers of Ten* listed in additional resources.
- Teach students about “googol” and “googolplex.”
- Prepare 3 x 5 index cards with different numbers written in scientific notation. Give each student a card. Select 3-5 students at a time to come to the front of the room with their cards and have them stand in order from least to greatest.

Family Connections

- Read completed fact books to family.
- Use the suggested Internet sites in additional resources to find other interesting facts with large numbers. Practice writing these numbers in scientific notation.
- Check out a book from additional resources from the local library to share with family.

Additional Resources

Books

If You Hopped Like a Frog, by David M. Schwartz; ISBN 0-590-09857-8

Powers of Ten, by Phillip Morrison; ISBN 0-7167-1409-4

Big Numbers, by Edward Packard; ISBN 0-7613-1570-5

Zoom, by Istvan Banyai; ISBN 0-670-85804-8

Actual Size, by Steve Jenkins; ISBN 0-618-37594-5

G is for Googol, by David M. Schwartz; ISBN 1-883672-58-9

When There Were Dinosaurs, Using Expanded Notation to Represent Numbers in the Millions, by Orli Zuravicky; ISBN 0-8239-8901-1

Media

Powers of Ten, by Charles and Ray Eames (Pyramid Film and Video, 1-800-421-2304)

Articles

Odyssey, Cobblestone Publishing, Inc.; ISSN 0163-0946

Web sites

http://www.worldalmanacforkids.com

http://www.factmonster.com

http://guinnessworldrecords.com

http://infoplease.com/almanacs.html

http://census/gov/main/www/cen2000.html

https://www.cia.gov/cia/publications/factbook/index.html

http://janus.astro.umd/edu/astro/scinote/

http://www.nyu.edu/pagse/mathmol/textbook/scinot.html

http://www.freemathhelp.com/scientific-notation.html

http://www.gomath.com/exercises/ScientificNotation.php

http://www.321know.com/grade6.htm

http://www.cusd.com/calonline/algebra/module05/module05.htm

http://www.xpmath.com/forums/arcade.php?do=play&gameid=21

http://earth.google.com

Name ________________________________

Name ________________________________

Notation and Powers Table

Exponential Notation	Equivalent Expression	Standard Notation
10^0		
10^1		
10^2		
10^3		
10^4		
10^5		
10^6		
10^7		
10^8		
10^9		
10^{10}		
10^{11}		
10^{12}		

0–9 Digit Cards

0	1	2	3	4
5	6	7	8	9

Name ______________________

Power Capture Game

	Millions 10^6	Hundred Thousands 10^5	Ten Thousands 10^4	Thousands 10^3	Hundreds 10^2	Tens 10^1	Ones 10^0
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							
17							
18							
19							
20							

Parnter's Score

Total Score

Appendix

Mind over Matter: Mental Math

Section I

2*1____
2*10____
2*100____

Draw each Problem:
3*5______
30*5______
300*5______

Draw each Problem:

What Patterns do you see?

Section II

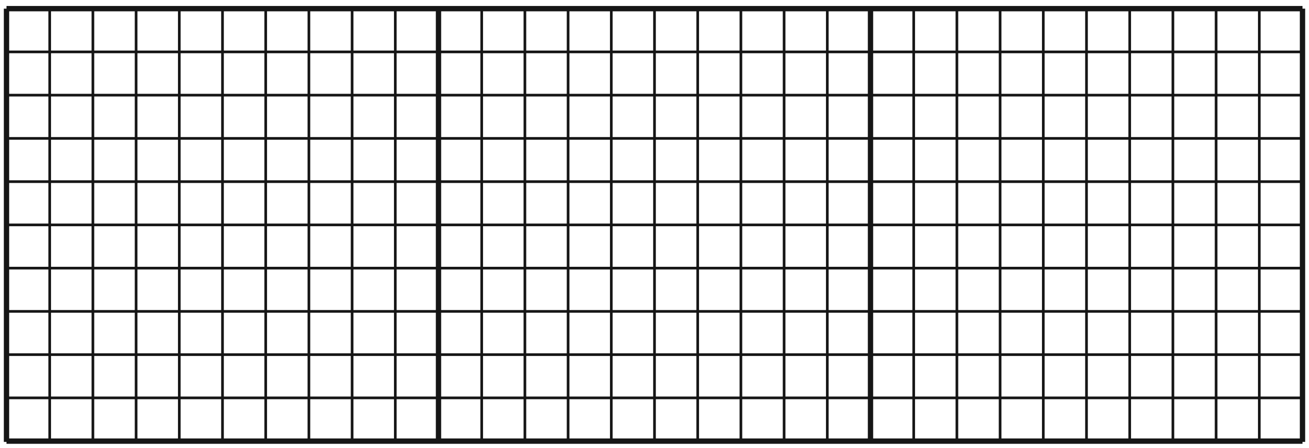
2*2_________

2*10________

20*2________

20*10_______

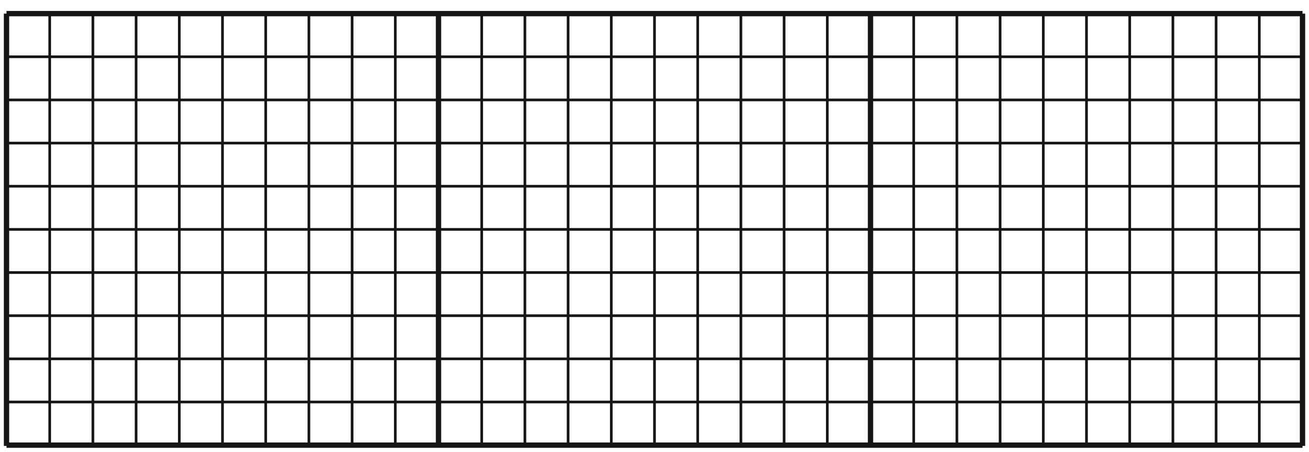
Draw each Problem:



Section III

12*22_________

Draw each problem:



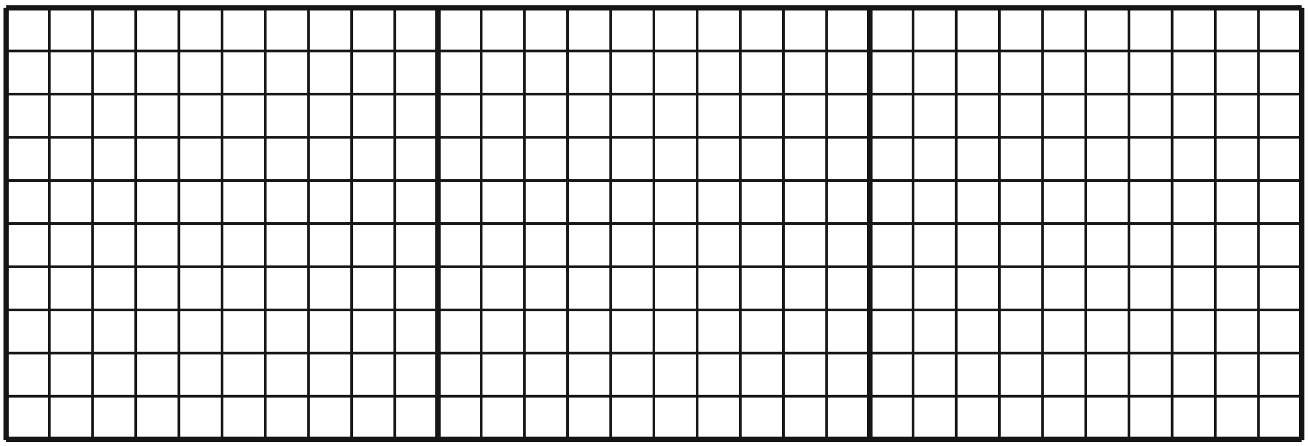
What is similar about the problems in section two and section three?

____

____

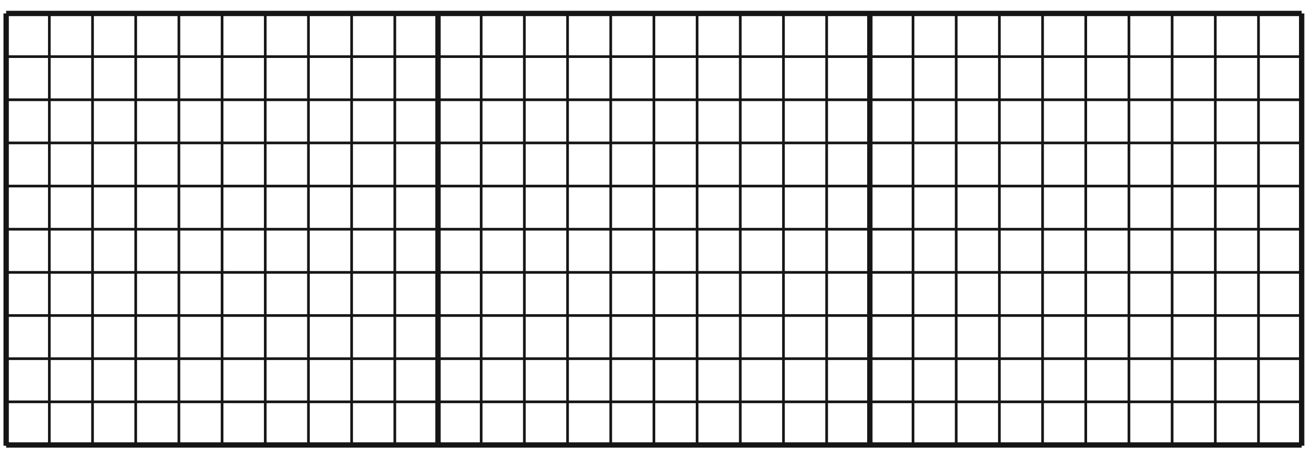
Section IV

4*3___
4*20___
10*3___
10*20___



Draw each problem:
Section V

23*14



Draw the problem:
What is similar with the problems that you see in sections four and five?

Name ______________________________

Partial Products without Four Square

Directions:
1. Estimate the answer to the nearest ten
2. Multiply in place value
3. Add the place value answers together

Example:	Estimation:	Partial Product:
26	30	26
x 16	x 20	x 16
	600	36
		120
		60
		200
		416

Estimation Partial Product	Estimation Partial Product
12 x 24	56 x 23
45 x 47	254 x 34
167 x 23	236 x 127

Name ______________________________

Relating Partial Products

Directions:
1. Estimate the Answer to the nearest ten
2. Multiply
3. Add the multiplied answers together.

Example:	Estimation:	Standard Way:
26	30	26
x 16	x 20	x 16
	600	156
		260
		416

Estimation Standard Way	Estimation Standard Way
12 x 24	56 x 23
45 x 47	254 x 34

1. Pull out your partial problems page. Examine the first four problems. What do you see that is similar? What do you find different?

____

____

____

____

____

2. Can you explain why we add zeros as place holders in each succeeding line that is multiplied?

3. How does using zeros as place value holders relate to the partial products process?

Estimation Standard Way	Estimation Standard Way
112 x 44	36 x 23
344 x 49	146 x 57

Name ______________________________

Lucky Seven Comparison

Directions:
1. Solve the algorithm using the Lucky Seven method.
2. Solve the algorithm using the standard way.
3. Compare the answers.

Lucky Seven	Standard Way
160÷40	160÷40
855÷19	855÷19
5683÷54	5683÷54

1. How are these two strategies similar?

____

____

2. How does the place value of the Lucky Seven method help you?

____

____

Name ______________________________

Estimation Station

Directions:
Round the decimal numbers to the nearest whole number

Number	Estimation	Number	Estimation
2.3		3.45	
23.6		456.4	
13.651		67.564	
56.67		79.151	

Name ______________________________

Keep It Simple #1

Problem/Estimation/Simpler Problem	Problem/Estimation/Simpler Problem
2.34 x 1.5 From the estimation where do you place the decimal point? __________	4.3 x 2.6 From the estimation where do you place the decimal point? __________
12.34 x 4.56 From the estimation where do you place the decimal point? __________	11.23 x 2.78 From the estimation where do you place the decimal point? __________
57.3 x 23.4 From the estimation where do you place the decimal point? ___	124.3 x .12 From the estimation where do you place the decimal point? _________

Name ___________________________

Keep It Simple #2

Directions:
Use the Lucky Seven method, along with dollars and cents, to divide the problems.

Example:

```
       .34
10 ) 3.40 | 10
    - 1.00|
      2.40| 10
    - 1.00|
      1.40| 10
    - 1.00|
       .40| + 4
     - .40| 34
       .00|
```

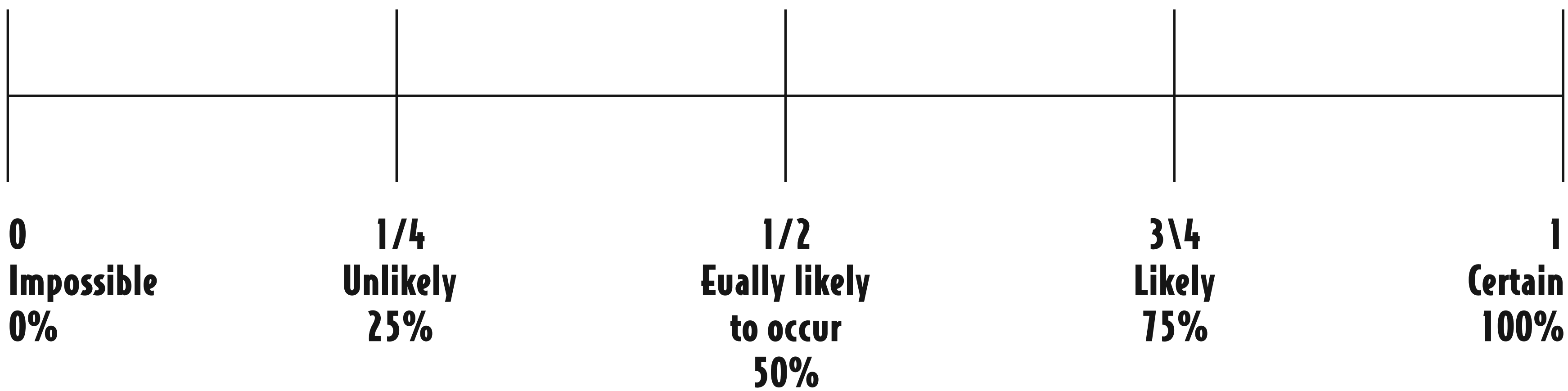
Problem	Lucky Seven	Problem	Lucky Seven
8.4÷6		94.85÷5	
11.25÷25		29.4÷21	

How does the Lucky Seven strategy help you know where to put the decimal?

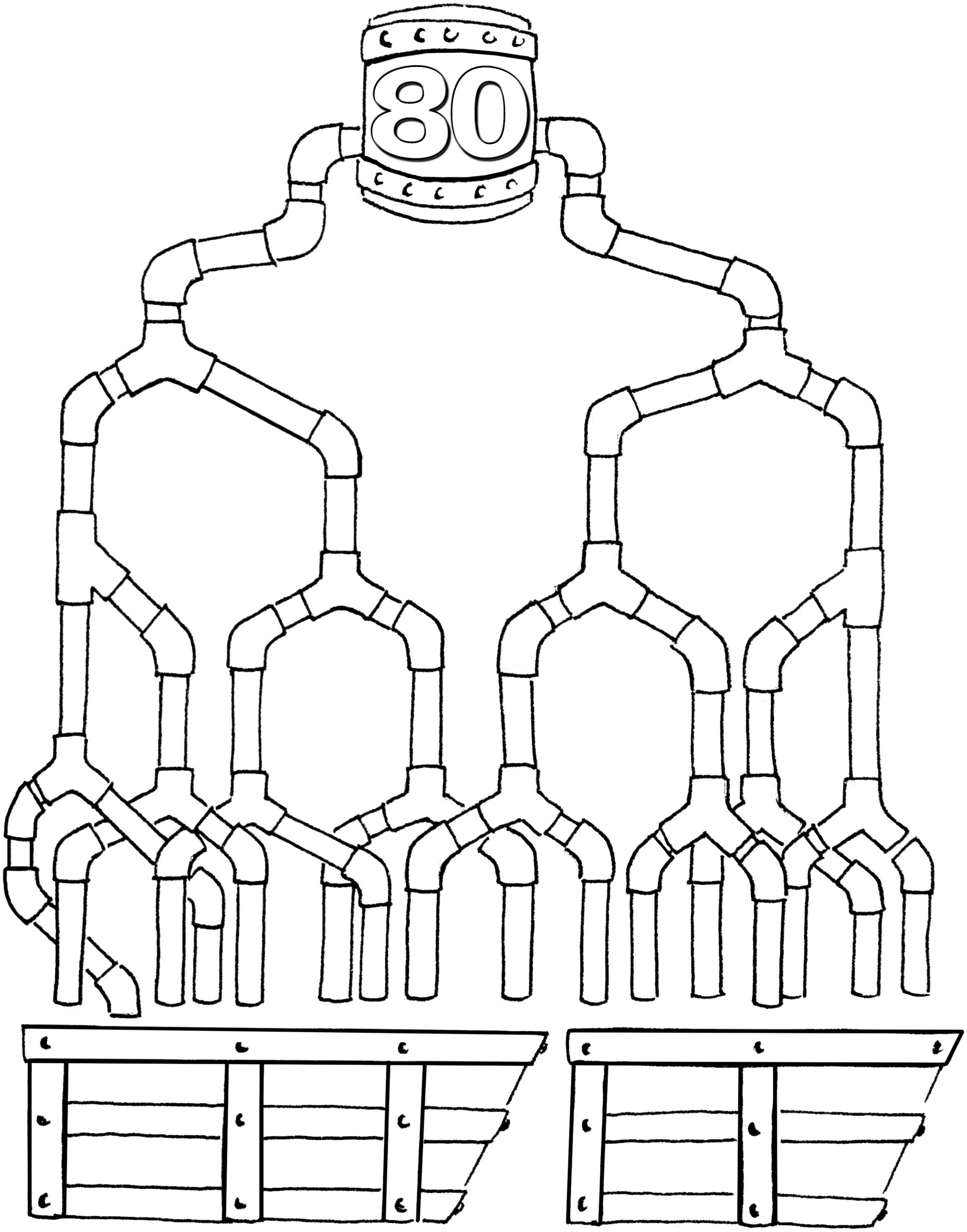
____

____

Probablity Meter



Pigs in a Pipe



The Stick Game

Materials:
3 flat sticks (like popsicle sticks)
Crayons (red and blue)

Preparation:
Color two sticks red on one side and leave the other side plain. Color one stick blue on one side and leave the other side plain.

How To Play And Score:
Hold all three sticks in one hand. Hold your hand above the desk and drop the sticks. Below is how you score each drop. Record your score in your journal or on a piece of paper. Add your points as you go so you know when someone reaches 50 points.

All plain sides land face up	4 points
All colored sides land face up	4 points
Two red and one plain land face up	6 points
Two plain and one colored land face up	6 points
One plain, one red, and one blue land face up	0 points

Play until someone reaches 50 points.

The Stick Game

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3 flat sticks (like popsicle sticks)
Crayons (red and blue)

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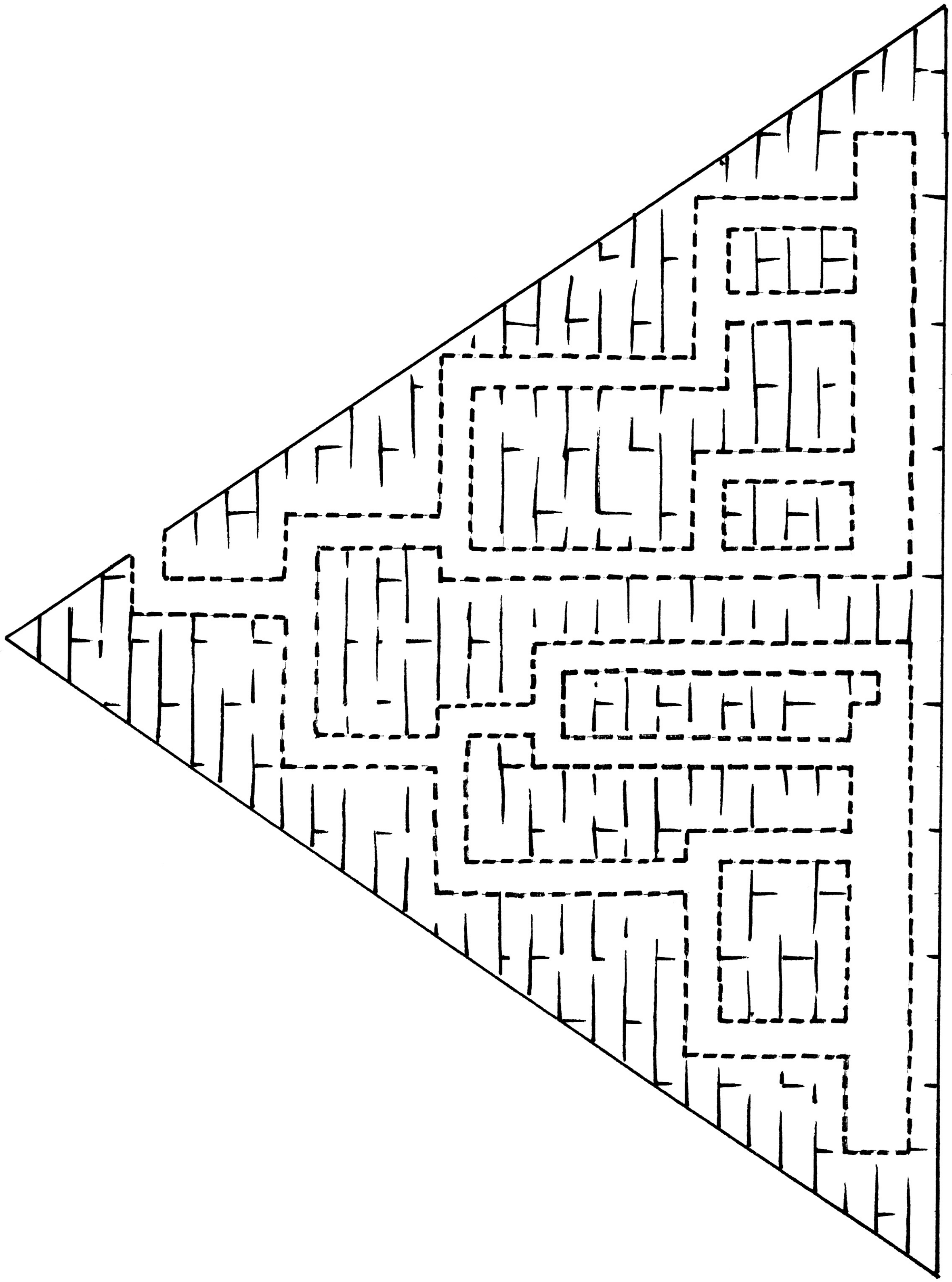
Play until someone reaches 50 points.

Secret Rooms



Family Reunion

There are 5 trails leading to Grandma and Grandpa's camp from your camp. There are 3 trails leading from Grandma and Grandpa's to Uncle John's camp. How many different routes are there from your camp to Uncle John's camp? Draw a tree diagram below to show your answer. Next to your tree diagram put the answer to how many routes.

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The Game Show

You are a contestant in a game show. One of the games asks you to pick a curtain and then pick a door behind the curtain. There are 4 curtains and 5 doors behind each curtain. How many choices are possible for the player?

Draw a tree diagram below showing the possible choices. Then write how many choices are possible.

The Game Show

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Draw a tree diagram below showing the possible choices. Then write how many choices are possible.

The Situation
Where in the World is Mrs. Zeroni?

Samples of blood can give us a lot of information about people. Information is coded from blood samples so that comparisons can be made and family relationships can be established.

Hector Zeroni is looking for his mother. She accidentally left him behind one day. Hector has decided to hire investigators/detectives to locate his mother and bring her to him.

Detectives don't like to make mistakes. Right now they have five women claiming to be Hector's mother. It seems news has gotten around that Hector, and his partner Stanley Yelnats, is worth a lot of money, now everyone wants to be his mother. How will the detectives prove the identity of Hector's mother to themselves, the media, and most important, Hector.

The detectives have coded the blood samples using color counters in five paper sacks. One of these sacks contains an exact match to Hector's blood. Pulling one tile out at a time, recording the results, then replacing the counter in the sack and shaking the sack before drawing the next counter can only reveal the contents of the sack.

Your group needs to come up with a plan and gather data to find the answer to this question, **Where in the world is Mrs. Zeroni?** Some things your group needs to decide are:
The way you will go about gathering the data.
The amount of data you will gather.
The way you organize your data.
Ways to present your results to others.

Name ______________________________

Student Recording Sheet
Where in the World is Mrs. Zeroni?

	Yellow	Blue	Red
Hector	5	2	2

	Yellow	Blue	Red
Maria			
Nancy			
Patty			
Grace			
Yolanda			

Name ______________________________

Student Recording Sheet
Where in the World is Mrs. Zeroni?

	Yellow	Blue	Red
Hector	5	2	2

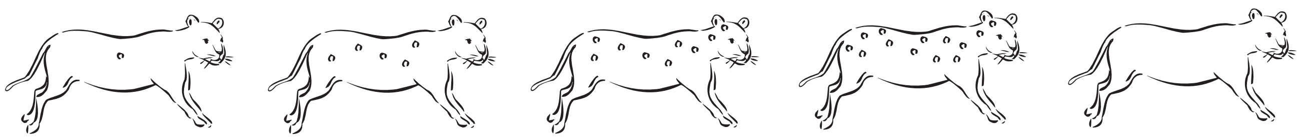
	Yellow	Blue	Red
Maria			
Nancy			
Patty			
Grace			
Yolanda			

Name ______________________________

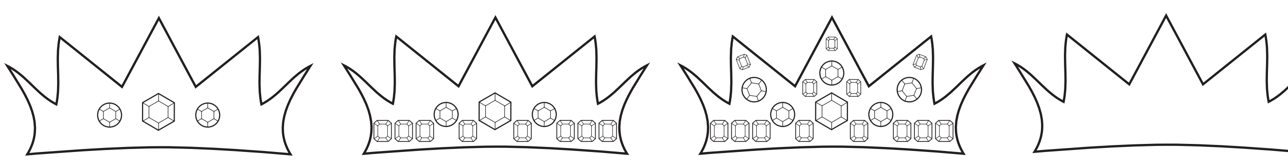
What's Next in the Pattern?

Directions: For each situation, draw or write the correct amount if the pattern continued.

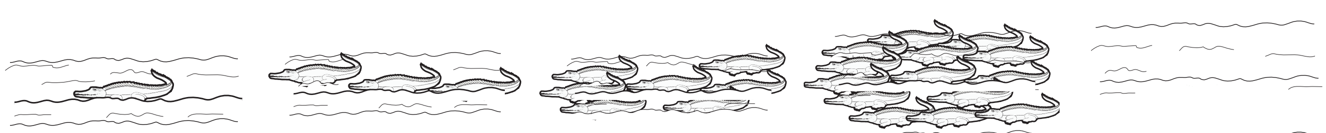
1. Each leopard is born with a different amount of spots. If the pattern continued how many spots will the fifth leopard have? Draw them on.



2. Each chocolate chip cookie has more chips than the one before. If the pattern continues, how many chocolate chips will the fourth cookie have?

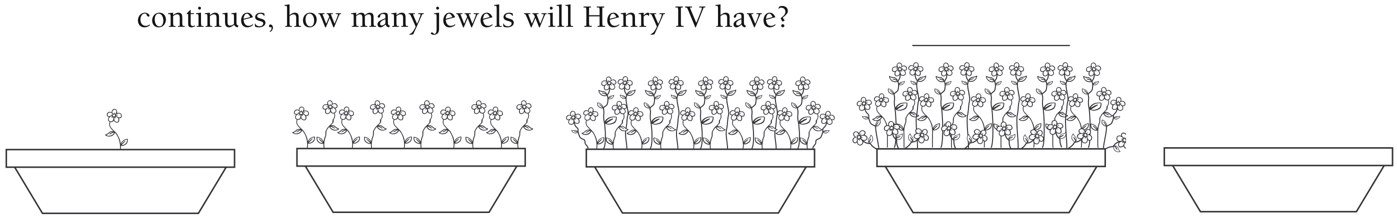


Henry I | Henry II | Henry III | Henry IV



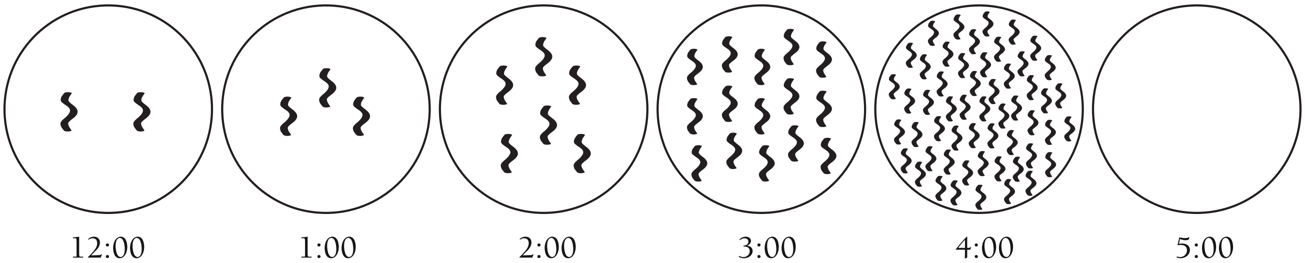
Week 1 | Week 2 | Week 3 | Week 4 | Week 5

3. Each King Henry gts more jewels on his crown than his predecessor. If the pattern continues, how many jewels will Henry IV have?



Week 1 | Week 2 | Week 3 | Week 4 | Week 5

6. Each hour the microorganisms in this petri dish multiply. At this rate, how many microorganisms will there be at 5:00?



7. For numbers 7 and 8, identify the next number in the pattern.
   5, 15, 25, 35, ______

8. 1, 4, 13, 40, 121, ______

9. For 9 and 10, create your own pattern.

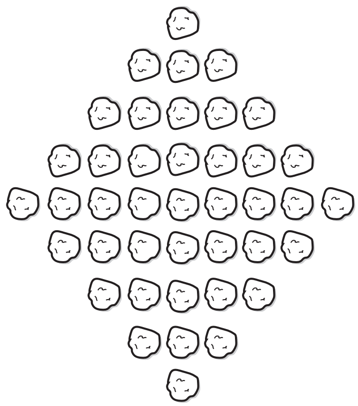
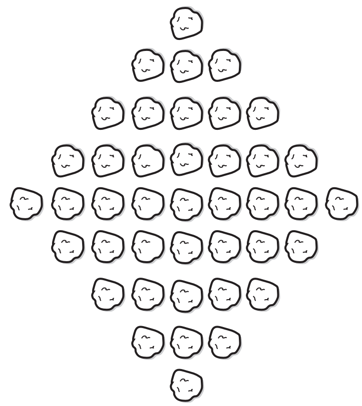
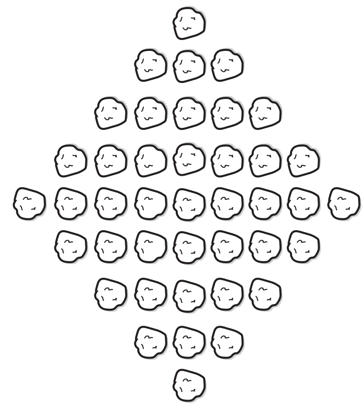
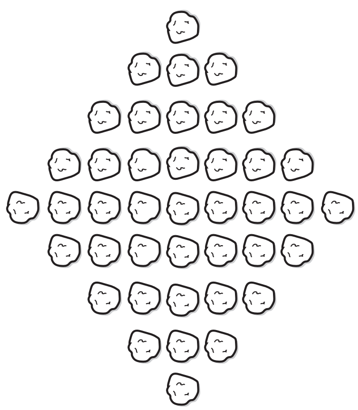
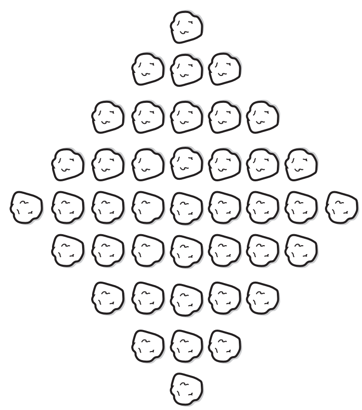
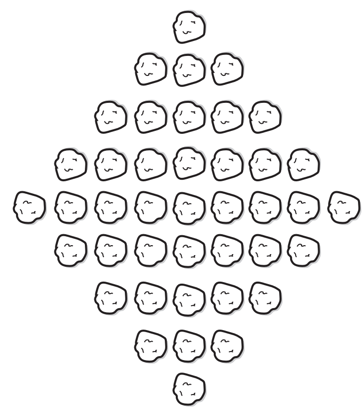
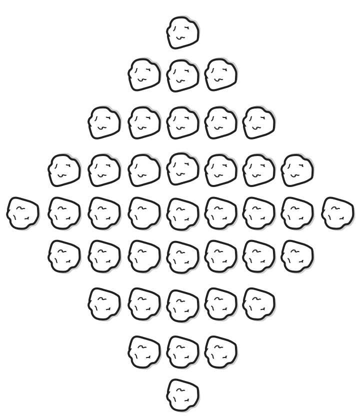
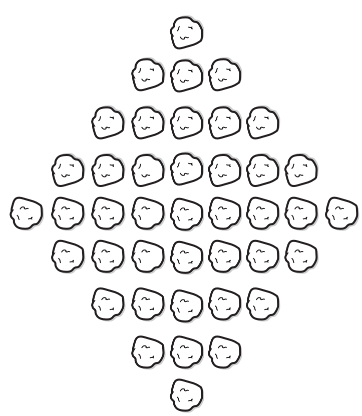
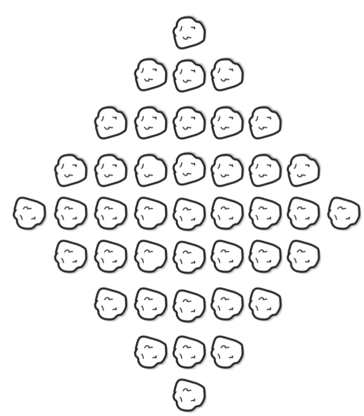
10.

Boulder Task

Day 1	Day 2	Day 3	Day 4	Day 5
Day 6	Day 7	Day 8	Day 9	Day 10
Day 11	Day 12	Day 13	Day 14	Day 15

Bold Boulder Patterns

While Heracles was forced to move boulders from Mt. Olympus, he placed them in the shape of a diamond (okay, so he was bored). Heracles was really proud of himself, and marveled at all of the different patterns he saw in this boulder diamond. Your task is to find as many ways as you can to partition the array of diamonds below. Record each way as a numerical sentence.

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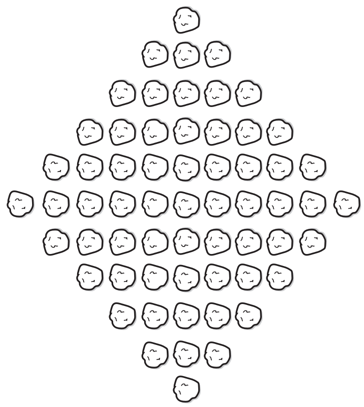
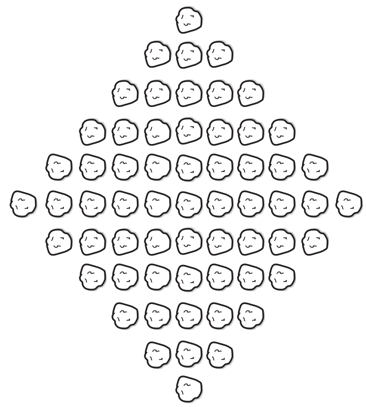
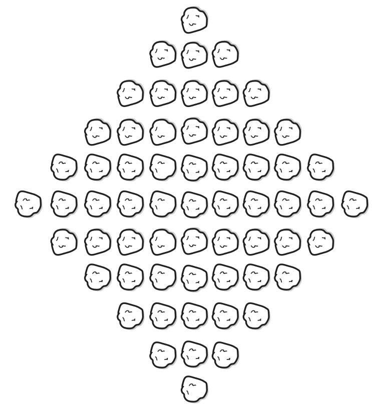
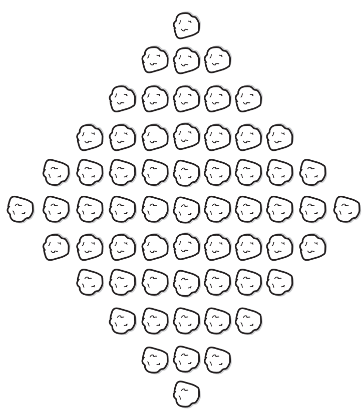
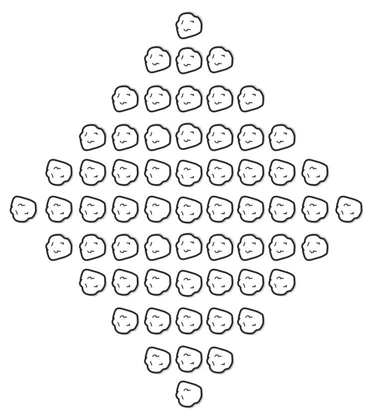
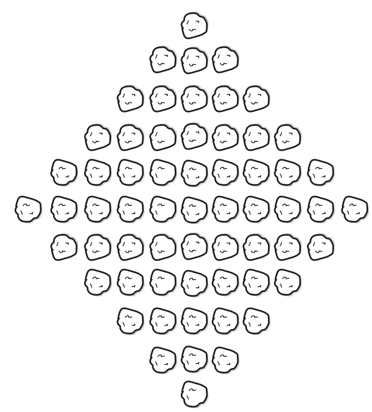


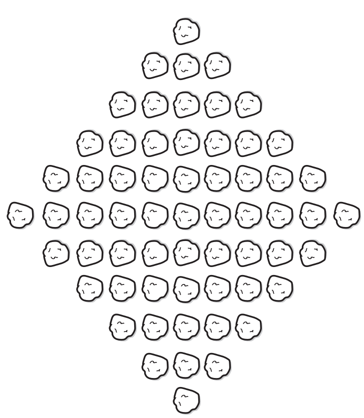


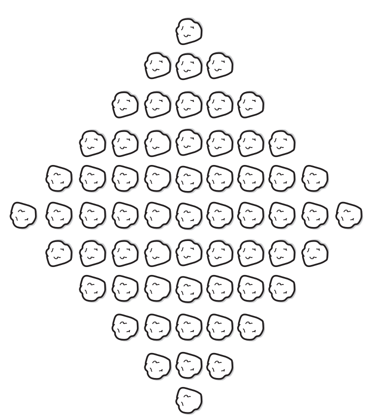


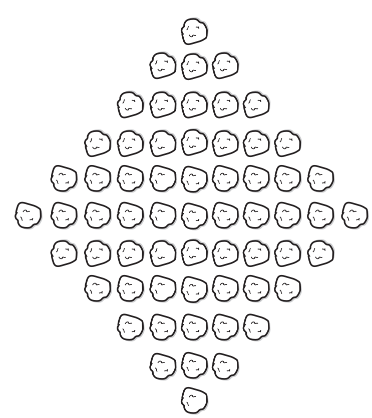




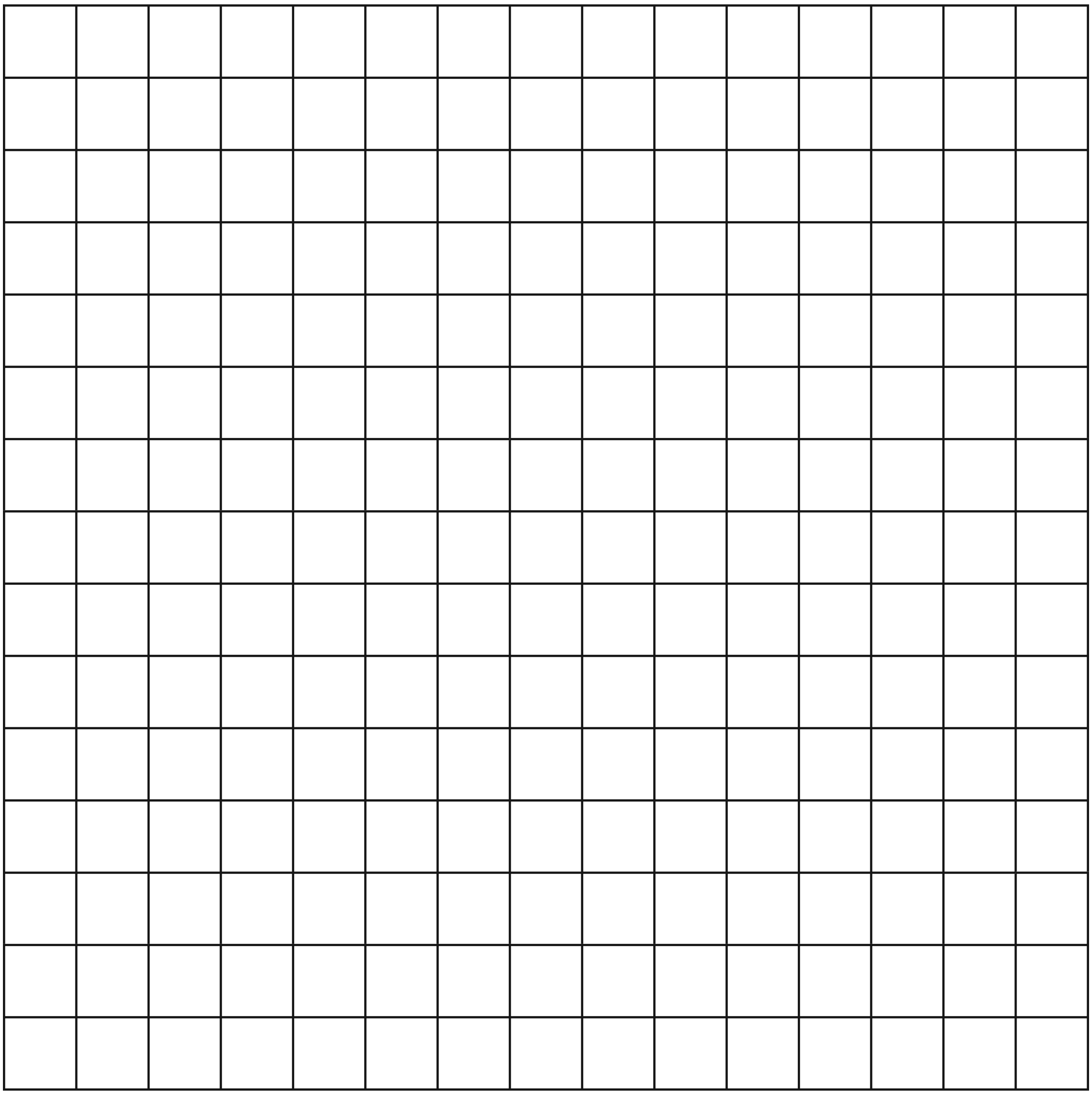








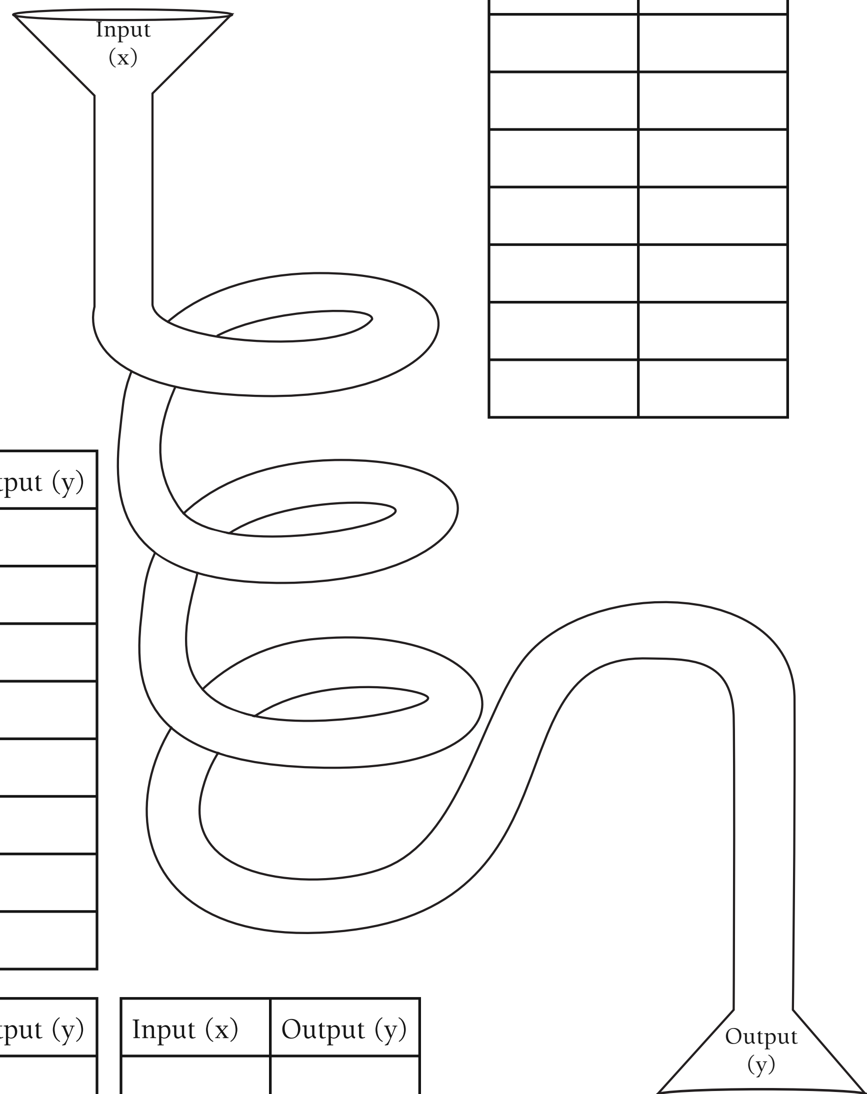
Coordinate Grid



Name ______________________________

Functional Machine

Input (x)	Output (y)



Input (x)	Output (y)

Input (x)	Output (y)

Input (x)	Output (y)

Input (x)	Output (y)

Input (x)	Output (y)

Input (x)	Output (y)

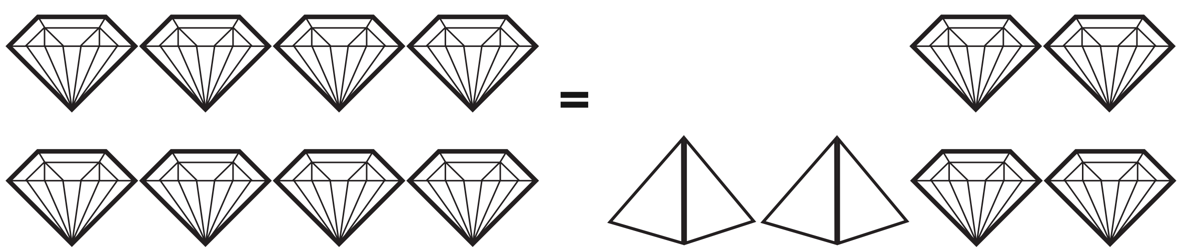
Tomb Treasures

Upon a pharaoh's death in Ancient Egypt, the body was mummified and then buried in an elaborate coffin, or sarcophagus, that was then placed in a pyramid for burial. Along with the mummified Pharaoh rested his jewels and treasures, which he was able to take with him to the afterlife. The pyramid was supposed to protect the pharaoh's body from natural elements (such as weather) and tomb robbers. Unfortunately, tomb robbers still managed to steal the treasures, including the coffins and mummies themselves.

Sefu, an Egyptian archaeologist, is sorting through treasures from robbed tombs. He is trying to decipher how many treasures are still inside the pyramids. He has the following information based on equality:

- Each pyramid contains the same number of small treasures.
- Each small treasure represents one.
- The number of small treasures on both sides of the equality sign is the same, but some of the treasures are still inside the pyramids.

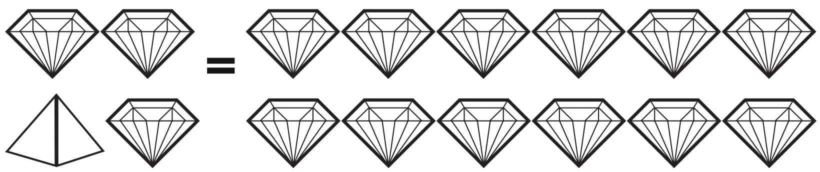
Sefu draws the following picture. Each pyramid contains the same number of small treasures, valued at one unit each.

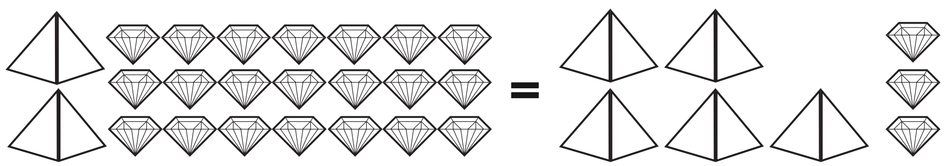


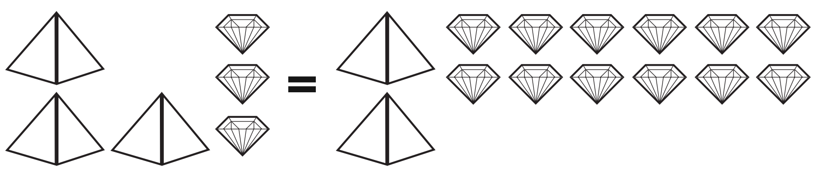
How many treasures are in each pyramid? Explain your reasoning.

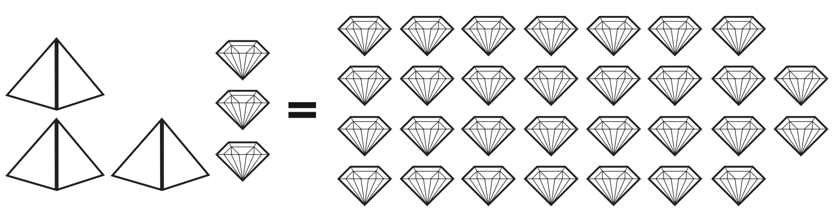
Pyramid Equality

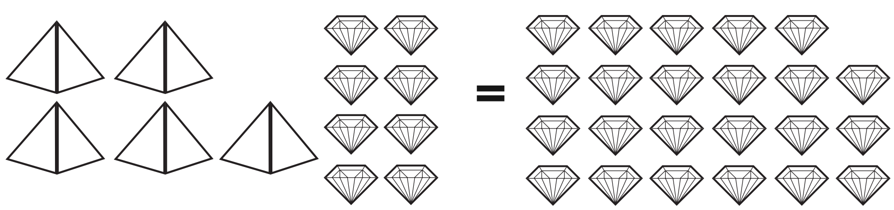
The Egyptian archaeologist, Sefu, has the following information about the treasures from the robbed tombs. For each situation, find the number of treasures in the pyramid. Write down your steps on this paper or in your math journal so that you remember your strategy.

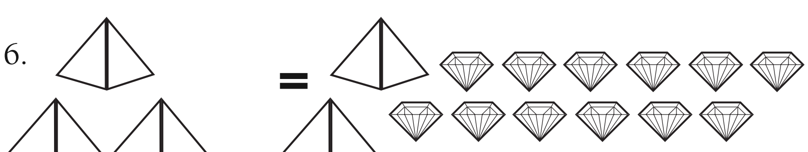
1. 

2. 

3. 

4. 

5. 

6. 

7. Describe how you can check your answer. How do you know you found the correct number of treasures in each pyramid?

8. Describe how you maintained equality at each step of your solutions.

Can You Use Algebra Tiles?

Solve each problem using algebra tiles, making sure to draw and explain your steps and always maintain equality.

1. $2x + 4 = 10$

2. $3y - 2 = 4$

3. $2x + 3 = 7$

4. $4x - 1 = 3$

5. $2x - 3 = 5$

6. $3x + 1 = 10$

7. $3x - 4 = 5$

8. $5x - 2 = 8$

9. $4x + 3 = 19$

Let's Do the Two-Step!

Solve each algebra problem using the two-step paper and pencil method. Please show your work and your answer clearly.

1. $2 + 6x = 8$

2. $3 + 2x = 7$

3. $2 + 4x = 34$

4. $7x + 1 = 29$

5. $5x + 10 = 35$

6. $8 + 7x = 50$

7. $10 + 4x = 30$

8. $7x + 2 = 65$

9. $6 + 6x = 78$

10. $2x - 5 = 7$

Prime Factorization

1	26	51	76
2	27	52	77
3	28	53	78
4	29	54	79
5	30	55	80
6	31	56	81
7	32	57	82
8	33	58	83
9	34	59	84
10	35	60	85
11	36	61	86
12	37	62	87
13	38	63	88
14	39	64	89
15	40	65	90
16	41	66	91
17	42	67	92
18	43	68	93
19	44	69	94
20	45	70	95
21	46	71	96
22	47	72	97
23	48	73	98
24	49	74	99
25	50	75	100

Name ______________________

Notation and Powers Table

Exponential Notation	Equivalent Expression	Standard Notation
10^0		
10^1		
10^2		
10^3		
10^4		
10^5		
10^6		
10^7		
10^8		
10^9		
10^{10}		
10^{11}		
10^{12}		

Name ______________________

Power Capture Game

	Millions 10^6	Hundred Thousands 10^5	Ten Thousands 10^4	Thousands 10^3	Hundreds 10^2	Tens 10^1	Ones 10^0	Parnter's Score
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								

Total Score