ELEMENTARY
CORE Academy
UTAH STATE OFFICE OF EDUCATION & UTAH STATE UNIVERSITY

Participant Handbook

ELEMENTARY CORE ACADEMY • ELEMENTARY CORE ACADEMY •

5TH GRADE

2007

UTAH STATE OFFICE OF EDUCATION

ELEMENTARY CORE ACADEMY

6517 Old Main Hill
Logan, UT 84322-6517

435-797-0939
http://coreacademy.usu.edu

UtahState UNIVERSITY

ISBN: 1-890563-60-9

Acknowledgements

These materials have been produced by and for the teachers of the State of Utah. Appreciation is expressed to the numerous individuals who provided input and effort into the creation of this curriculum. Delivery of the Elementary CORE Academy, including the development and delivery of content, coordination of sessions, distribution of materials, and participant interaction, has been a collaborative effort of many educational groups across Utah. The following organizations, Utah teachers, and educational leaders contributed ideas and activities as part of this professional development project:

Organizations:

Utah State Office of Education (USOE)
Utah State University (USU)
State Science Education Coordination Committee (SSECC)
State Mathematics Education Coordination Committee (SMECC)
Special Education Services Unit (USOE)

Individuals:

Academy Coordination Committee: Max Longhurst, Brett Moulding, Nicole Paulson, Velma Itamura, Janet Gibbs

Academy Director: Max Longhurst

Academy Coordinator: Megan Richards

Academy Facilitators: Rachel Belnap-Alberts, Robin Clement, Kathy Lambert, Kristan Norton

Academy Presenters and Contributors: Laura Ahrnsbrak, Cindy Durante, Pat Merkley, Marsha Newman, Dan Roberts, Cathy Sunderland

Credits for editing, compiling, formatting, and assisting with the materials and delivery of the Elementary CORE Academy are given to Ami Israelsen and Elizabeth Shaw.

UTAH STATE OFFICE OF EDUCATION

Leadership…Service…Accountability

Patti Harrington, Ed.D. State Superintendent of Public Instruction
Voice: (801) 538-7500 Fax: (801) 538-7521 TDD: (801) 538-7876
250 East Cesar E. Chavez Blvd. (500 South) P.O. Box 144200 Salt Lake City, Utah 84114-4200

Dear CORE Academy Teachers:

Thank you for your investment in children and in building your own expertise as you participate in the Elementary CORE Academy. I hope your involvement helps you to sustain a laser-like focus on student achievement.

Teachers in Utah are superb. By participating in the Academy, you join a host of teachers throughout the state who understand that teaching targeted on the core curricula, across a spectrum of subjects, will produce results of excellence. The research is quite clear–the closer the match of explicit instruction to core standards, the better the outcome on core assessments.

I personally appreciate your excellence and your desire to create wonderful classrooms of learning for students. Thank you for your dedication. I feel honored to associate with you and pledge my support to lead education in ways that benefit all of our children.

Sincerely,

Patti Harrington

Patti Harrington, Ed.D.
State Superintendent of Public Instruction

Funding Sources

Appreciation is expressed for the tremendous educational input and monetary commitment of several organizations for the successful delivery of the Elementary CORE Academy. This year's Elementary CORE Academy was developed and funded through a variety of sources. The Utah State Office of Education (USOE), in collaboration with Utah State University (USU) and local school districts of Utah, have supported kindergarten through sixth grade teachers with professional development experiences that will enhance the educational experience for Utah children.

Major funding for the Academy comes from the following sources:

Federal/State Funds:

Utah State Office of Education
- Staff Development Funds
- Special Education Services Unit

ESEA Title II

Utah Math Science Partnership

District Funds:

Various sources including Quality Teacher Block, Federal ESEA Title II, and District Professional Development Funds

School Funds:

Trust land, ESEA Title II, and other school funds
Utah State Office of Education Special Education Services

The state and district funds are allocations from the state legislature. ESEA is part of the "No Child Left Behind" funding that comes to Utah.

Additionally, numerous school districts, individual schools, and principals in Utah have sponsored teachers to attend the Academy. Other educational groups have assisted in the development and delivery of resources in the Academy.

Most important is the thousands of teachers who take time from their summer to attend these professional development workshops. It is these teachers who make this program possible.

Goals of the Elementary CORE Academy

Overall

The purpose of the Elementary CORE Academy is to create high quality teacher instruction and improve student achievement through the delivery of professional development opportunities and experiences for teachers across Utah.

The Academy will provide elementary teachers in Utah with:

1. Models of exemplary and innovative instructional strategies, tools, and resources to meet the Core Curriculum standards, objectives, and indicators.
2. Practical models and diverse methods of meeting the learning needs of all children, with instruction implementation aligned to the Core Curriculum.
3. Meaningful opportunities for collaboration, self-reflection, and peer discussion specific to innovative and effective instructional techniques, materials, teaching strategies, and professional practices in order to improve classroom instruction.

Learning a limited set of facts will no longer prepare a student for real experiences encountered in today's world. It is imperative that educators have continued opportunities to obtain instructional skills and strategies that provide methods of meeting the needs of all students. Participants of the Academy experience will be better equipped to meet the challenges faced in today's classrooms.

Table of Contents

Chapter 1: Fifth Grade Math Core Curriculum

Chapter 2: Facilitated Activities

Chapter 3: Math I-2 Activities - Compare Numbers

Chapter 4: Math Standard I-3 Activities - Modeling Operations

Chapter 5: Math I-2 Activities - Fractions

Chapter 6: Math IV-1&2 Activities - Measurable Attributes

Chapter 7: Math III-2, IV-2 Activities - Quadrants/Measure

Chapter 8: Math V-2 Activities - Probability

Appendix

Fifth Grade
Math
Core Curriculum

Utah Elementary Math Core Curriculum

Introduction

Most children enter school confident in their own abilities; they are curious and eager to learn more. They make sense of the world by reasoning and problem solving. Young students are building beliefs about what mathematics is, about what it means to know and do mathematics, and about themselves as mathematical learners. Students use mathematical tools, such as manipulative materials and technology, to develop conceptual understanding and solve problems as they do mathematics. Students, as mathematicians, learn best through participatory experiences throughout the instruction of the mathematics curriculum.

Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, the term *mathematical proficiency* has been chosen to capture what it means to learn mathematics successfully. Mathematical proficiency has five strands: computing (carrying out mathematical procedures flexibly, accurately, efficiently, and appropriately), understanding (comprehending mathematical concepts, operations, and relations), applying (ability to formulate, represent, and solve mathematical problems), reasoning (logically explaining and justifying a solution to a problem), and engaging (seeing mathematics as sensible, useful, and doable, and being able to do the work) (NRC, 2001).

The most important observation about the five strands of mathematical proficiency is that they are interwoven and interdependent. This observation has implications for how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve that goal. At any given moment during a mathematics lesson or unit, one or two strands might be emphasized. But all the strands must eventually be addressed so that the links among them are strengthened. The integrated and balanced development of all five strands of mathematical proficiency should guide the teaching and learning of school mathematics. Instruction should not be based on the extreme positions that students learn solely by internalizing what a teacher or book says, or solely by inventing mathematics on their own.

The Elementary Mathematics Core describes what students should know and be able to do at the end of each of the K-6 grade levels. It was developed and revised by a community of Utah mathematics

teachers, mathematicians, university mathematics educators, and State Office of Education specialists. It was critiqued by an advisory committee representing a wide variety of people from the community, as well as an external review committee. The Core reflects the current philosophy of mathematics education that is expressed in national documents developed by the National Council of Teachers of Mathematics, the American Association for the Advancement of Science, and the National Research Council. This Mathematics Core has the endorsement of the Utah Council of Teachers of Mathematics. The Core reflects high standards of achievement in mathematics for all students.

Organization of the Elementary Mathematics Core

The Core is designed to help teachers organize and deliver instruction.

- Each grade level begins with a brief description of areas of instructional emphasis which can serve as organizing structures for curriculum design and instruction.
- The INTENDED LEARNING OUTCOMES (ILOs) describe the skills and attitudes students should acquire as a result of successful mathematics instruction. They are found at the beginning of each grade level and are an integral part of the Core.
- A STANDARD is a broad statement of what students are expected to understand. Several Objectives are listed under each Standard.
- An OBJECTIVE is a more focused description of what students need to know and be able to do at the completion of instruction. If students have mastered the Objectives associated with a given Standard, they have mastered that Standard at that grade level. Several Indicators are described for each Objective.
- INDICATORS are observable or measurable student actions that enable students to master an Objective. Indicators can help guide classroom instruction.
- MATHEMATICAL LANGUAGE AND SYMBOLS STUDENTS SHOULD USE includes language and symbols students should use in oral and written language.
- EXPLORATORY CONCEPTS AND SKILLS are included to establish connections with learning in subsequent grade levels. They are not intended to be assessed at the grade level indicated.

Guidelines Used in Developing the Elementary Mathematics Core

The Core is:

Consistent With the Nature of Learning

In the early grades, children are forming attitudes and habits for learning. It is important that instruction maximizes students' potential and gives them understanding of the intertwined nature of learning. The main intent of mathematics instruction is for students to value and use mathematics as a process to understand the world. The Core is designed to produce an integrated set of Intended Learning Outcomes for students.

Coherent

The Core has been designed so that, wherever possible, the ideas taught within a particular grade level have a logical and natural connection with each other and with those of earlier grades. Efforts have also been made to select topics and skills that integrate well with one another and with other subject areas appropriate to grade level. In addition, there is an upward articulation of mathematical concepts and skills. This spiraling is intended to prepare students to understand and use more complex mathematical concepts and skills as they advance through the learning process.

Developmentally Appropriate

The Core takes into account the psychological and social readiness of students. It builds from concrete experiences to more abstract understandings. The Core focuses on providing experiences with concepts that students can explore and understand in depth to build the foundation for future mathematical learning experiences.

Reflective of Successful Teaching Practices

Learning through play, movement, and adventure is critical to the early development of the mind and body. The Core emphasizes student exploration. The Core is designed to encourage a variety of interactive learning opportunities. Instruction should include recognition of the role of mathematics in the classroom, school, and community.

Comprehensive

By emphasizing depth rather than breadth, the Elementary Mathematics Core seeks to empower students by providing a comprehensive background in mathematics. Teachers are expected to teach all the

The Core is:

- Coherent
- Developmentally Appropriate
- Encourages Good Teaching Practices
- Comprehensive
- Feasible
- Useful and Relevant
- Encourages Good Assessment Practices

standards and objectives specified in the Core for their grade level, but may add related concepts and skills.

Feasible

Teachers and others who are familiar with Utah students, classrooms, teachers, and schools have designed the Core. It can be taught with easily obtained resources and materials. A handbook is also available for teachers and has sample lessons on each topic for each grade level. The handbook is a document that will grow as teachers add exemplary lessons aligned with the new Core.

Useful and Relevant

This curriculum relates directly to student needs and interests. The relevance of mathematics to other endeavors enables students to transfer skills gained from mathematics instruction into their other school subjects and into their lives outside the classroom.

Reliant Upon Effective Assessment Practices

Student achievement of the standards and objectives in this Core is best assessed using a variety of assessment instruments. Performance tests are particularly appropriate to evaluate student mastery of mathematical processes and problem-solving skills. Teachers should use a variety of classroom assessment approaches in conjunction with standard assessment instruments to inform instruction. Sample test items, keyed to each Core Standard, may be located on the "Utah Mathematics Home Page" at http://www.usoe.k12.ut.us/curr/math. Observation of students engaged in instructional activities is highly recommended as a way to assess students' skills as well as attitudes toward learning. The nature of the questions posed by students provides important evidence of their understanding of mathematics.

Based Upon the National Council of Teachers of Mathematics Curriculum Focal Points

In 2006, the National Council of Teachers of Mathematics (NCTM) published *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM, 2006). This document is available online at http://www.nctm.org/focalpoints. This document describes three focal points for each grade level. NCTM's focal points are areas of emphasis recommended for the curriculum of each grade level. The focal points within a grade are ***not the entire curriculum*** for that particular grade; however, Utah's Core Curriculum was designed to include these areas of focus.

Intended Learning Outcomes for Third through Sixth Grade Mathematics

The main intent of mathematics instruction is for students to value and use mathematics and reasoning skills to investigate and understand the world.

The Intended Learning Outcomes (ILOs) describe the skills and attitudes students should acquire as a result of successful mathematics instruction. They are an essential part of the Mathematics Core Curriculum and provide teachers with a standard for student learning in mathematics.

ILOs for mathematics:

1. **Develop a positive learning attitude toward mathematics.**
2. **Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.**
3. **Reason logically, using inductive and deductive strategies and justify conclusions.**
4. **Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.**
5. **Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.**
6. **Represent mathematical ideas in a variety of ways.**

Significant mathematics understanding occurs when teachers incorporate ILOs in planning mathematics instruction. The following are ideas to consider when planning instruction for students to acquire the ILOs:

1. **Develop a positive learning attitude toward mathematics.**

   When students are confident in their mathematical abilities, they demonstrate persistence in completing tasks. They pose mathematical questions about objects, events, and processes while displaying a sense of curiosity about numbers and patterns. It is important to build on students' innate problem-solving inclinations and to preserve and encourage a disposition that values mathematics.

2. **Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.**

   Problem solving is the cornerstone of mathematics. Mathematical knowledge is generated through problem solving

as students explore mathematics. To become effective problem solvers, students need many opportunities to formulate questions and model problem situations in a variety of ways. They should generalize mathematical relationships and solve problems in both mathematical and everyday contexts.

3. **Reason logically, using inductive and deductive strategies and justify conclusions.**

   Mathematical reasoning develops in classrooms where students are encouraged to put forth their own ideas for examination. Students develop their reasoning skills by making and testing mathematical conjectures, drawing logical conclusions, and justifying their thinking in developmentally appropriate ways. Students use models, known facts, and relationships to explain reasoning. As they advance through the grades, students' arguments become more sophisticated.

4. **Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.**

   The ability to express mathematical ideas coherently to peers, teachers, and others through oral and written language is an important skill in mathematics. Students develop this skill and deepen their understanding of mathematics when they use accurate mathematical language to talk and write about what they are doing. When students talk and write about mathematics, they clarify their ideas and learn how to make convincing arguments and represent mathematical ideas verbally, pictorially, and symbolically.

5. **Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.**

   Students develop a perspective of the mathematics field as an integrated whole by understanding connections within mathematics. Students should be encouraged to explore the connections that exist with other disciplines and between mathematics and their own experiences.

6. **Represent mathematical ideas in a variety of ways.**

   Mathematics involves using various types of representations including concrete, pictorial, and symbolic models. In particular, identifying and locating numbers on the number line has a central role in uniting all numbers to promote understanding of equivalent representations and ordering. Students also use a variety of mathematical representations to expand their capacity to think logically about mathematics.

Fifth Grade Mathematics Standards

By the end of grade five, students increase their facility with the four basic arithmetic operations applied to whole numbers, fractions, and decimals. They locate integers on a number line and ordered pairs of integers on the coordinate plane. They determine rules for numerical patterns, work with expressions including order of operations, and solve single-operation equations involving a single variable. They classify angles, triangles, and quadrilaterals, and analyze relationships among lines, triangles and quadrilaterals. They recognize and determine surface area and volume of three-dimensional shapes, including right prisms. Students understand the concepts of mean, median, mode, and range of data sets and can calculate them. They use line plots, bar graphs, and line graphs to record and analyze data.

Standard I: Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Standard I: Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 1: Represent whole numbers and decimals from thousandths to one billion, fractions, percents, and integers.

a. Read and write numbers in standard and expanded form.

b. Demonstrate multiple ways to represent whole numbers, decimals, fractions, percents, and integers using models and symbolic representations (e.g., 108 = 2 x 50 + 8; 108 = 10^2 + 8; 90% = 90 out of 100 squares on a hundred chart).

c. Identify, read, and locate fractions, mixed numbers, decimals, and integers on the number line.

d. Represent repeated factors using exponents.

e. Describe situations where integers could be used in the students' environment.

Objective 2: Explain relationships and equivalencies among integers, fractions, decimals, and percents.

a. Compare fractions by finding a common denominator.

b. Order integers, fractions (including mixed numbers), and decimals using a variety of methods, including the number line.

c. Rewrite mixed numbers and improper fractions from one form to the other and represent each using regions, sets of objects, or line segments.

d. Represent commonly used fractions as decimals and percents in a variety of ways (e.g., models, fraction strips, pictures, calculators, algorithms).

e. Model and calculate equivalent forms of a fraction (including simplest form).

f. Rename whole numbers as fractions with different denominators (e.g., 5 = 5/1, 3 = 6/2, 1 = 7/7).

Objective 3: Use number theory concepts to develop and use divisibility tests; classify whole numbers to 50 as prime, composite, or neither; and find common multiples and factors.

a. Identify patterns with skip counting and multiples to develop and use divisibility tests for determining whether a whole number is divisible by 2, 3, 5, 6, 9, and 10.

b. Use strategies for classifying whole numbers to 50 as prime, composite, or neither.

c. Rewrite a composite number between 2 and 50 as a product of only prime numbers.

d. Find common multiples and factors and apply to adding and subtracting fractions.

Objective 4: Model and illustrate meanings of multiplication and division.

a. Represent division-with-remainder using whole numbers, decimals, or fractions.

b. Describe the effect of place value when multiplying and dividing whole numbers and decimals by 10, 100, and 1,000.

c. Model multiplication of fractions and decimals (e.g., tenths multiplied by tenths, a whole number multiplied by tenths, or a whole number with tenths multiplied by tenths) in a variety of ways (e.g., manipulatives, number line and area models, patterns).

Objective 5: Solve problems involving one or two operations.

a. Determine when it is appropriate to use estimation, mental math strategies, paper and pencil, and algorithms.

b. Make reasonable estimations of fraction and decimal sums, differences, and products, including knowing whether results obtained using a calculator are reasonable.

c. Write number sentences that can be used to solve a two-step problem.

d. Interpret division-with-remainder problems as they apply to the environment (e.g., If there are 53 people, how many vans are needed if each van holds 8 people?).

Objective 6: Demonstrate proficiency with multiplication and division of whole numbers and compute problems involving addition, subtraction, and multiplication of decimals and fractions.

a. Multiply multi-digit whole numbers by a two-digit whole number with fluency, using efficient procedures.

b. Divide multi-digit dividends by a one-digit divisor with fluency, using efficient procedures.

c. Add and subtract decimals with fluency, using efficient procedures.

d. Add and subtract fractions with fluency.

e. Multiply fractions.

Mathematical language and symbols students should use

prime, composite, exponent, fractions, numerator, denominator, common denominator, common factor, common multiple, decimals, percents, divisible, divisibility, equivalent fractions, integer, dividend, quotient, divisor, factor, order of operations, simplest terms, various symbols for multiplication and division, mixed numeral, improper fraction

Exploratory Concepts and Skills

- Extend classification of whole numbers from 0-100 as prime, composite, or neither.
- Apply rules of divisibility.
- Explore adding and subtracting integers.
- Divide multi-digit dividends by a two-digit divisor.

Standard II: Students will use patterns and relations to represent and analyze mathematical problems and number relationships using algebraic symbols.

Standard II: Students will use patterns and relations to represent and analyze mathematical problems and number relationships using algebraic symbols.

Objective 1: Identify, analyze and determine a rule for predicting and extending numerical patterns involving operations whole numbers, decimals, and fractions.

a. Analyze and make predictions about numeric patterns, including decimals and fractions.

b. Determine a rule for the pattern using organized lists, tables, objects, and variables.

Objective 2: Use algebraic expressions, inequalities, or equations to represent and solve simple real-world problems. –

a. Use properties and the order of operations involving addition, subtraction, multiplication, division, and the use of parentheses to compute with whole numbers, decimals, and fractions.

b. Use patterns, models, and relationships as contexts for writing and solving simple equations and inequalities with whole number solutions (e.g., 6x = 54; x + 3 = 7).

Mathematical language and symbols students should use

variety of symbols for multiplication and division such as x, i, and * as symbols for multiplication and ÷, e, and a fraction bar (/ or —) as division symbols; variable, order of operations, parentheses, inequality, expression, equation, associative property, commutative property, distributive property

Exploratory Concepts and Skills

- Extend classification of whole numbers from 0-100 as prime, composite, or Solve multi-step equations.
- Construct and analyze tables involving equivalent ratios.

Standard III: Students will use spatial reasoning to recognize, describe, and analyze geometric shapes and principles.

Objective 1: Describe relationships between two- and three-dimensional shapes and analyze attributes and properties of geometric shapes.

a. Draw, label, and describe line segments, rays, lines, parallel lines, and perpendicular lines.

b. Draw, label, and define an angle as two rays sharing a common endpoint (vertex).

c. Classify triangles and quadrilaterals and analyze the relationships among the shapes in each classification (e.g., a square is a rectangle).

d. Relate pyramids and right prisms to the two-dimensional shapes (nets) from which they were created.

e. Identify properties and attributes of solids (i.e., right prisms, pyramids, cylinders, cones) and describe them by the number of edges, faces, and vertices as well as the types of faces.

Objective 2: Specify locations in a coordinate plane.

a. Locate points defined by ordered pairs of integers.

b. Write an ordered pair for a point in a coordinate plane with integer coordinates.

c. Specify possible paths between locations on a coordinate plane and compare distances of the various paths.

Mathematical language and symbols students should use

perpendicular and parallel lines, rays, angles (acute, obtuse, right, straight), triangles (equilateral, isosceles, scalene, right, acute, obtuse), vertex, vertices, edge, face, corresponding angles, similar, polygon, pyramid, right prism

Exploratory Concepts and Skills

- Compare corresponding angles of two triangles and determine whether the triangles are similar.
- Rotate a shape around a fixed point and identify the location of the new vertices.
- Translate a polygon either horizontally or vertically on a coordinate grid and identify the location of the new vertices.
- Reflect a shape across either the x- or y-axis and identify the location of the new vertices.

Standard IV: Students will determine area of polygons and surface area and volume of three-dimensional shapes.

Standard IV: Students will determine area of polygons and surface area and volume of three-dimensional shapes.

Objective 1: Determine the area of polygons and apply to real-world problems.

a. Determine the area of a trapezoid by the composition and decomposition of rectangles, triangles, and parallelograms.

b. Determine the area of irregular and regular polygons by the composition and decomposition of rectangles, triangles, and parallelograms.

c. Compare areas of polygons using different units of measure within the same measurement system (e.g., square feet, square yards).

Objective 2: Recognize, describe, and determine surface area and volume of three-dimensional shapes.

a. Quantify volume by finding the total number of same-sized units of volume needed to fill the space without gaps or overlaps.

b. Recognize that a cube having a 1 unit edge is the standard unit for measuring volume expressed as a cubic unit.

c. Derive and use the formula to determine the volume of a right prism with a triangular or rectangular base.

d. Relate the formulas for the areas of triangles, rectangles, or parallelograms to the surface area of a right prism.

e. Derive and use the formula to determine the surface area of a right prism and express surface area in square units.

Mathematical language and symbols students should use
area, volume, surface area, volume, right prism

Exploratory Concepts and Skills
- Investigate pi as the ratio of the circumference to the diameter of a circle.
- Determine the volume of a right prism with various bases.

Standard V: Students will construct, analyze, and construct reasonable conclusions from data and apply basic concepts of probability.

Objective 1: Formulate and answer questions using statistical methods to compare data, and propose and justify inferences based on data.

a. Construct, analyze, and display data using an appropriate format (e.g., line plots, bar graphs, line graphs).

b. Recognize the differences in representing categorical and numerical data.

c. Identify minimum and maximum values for a set of data.

d. Identify and calculate the mean, median, mode, and range.

Objective 2: Apply basic concepts of probability.

a. Describe the results of experiments involving random outcomes using a variety of notations (e.g., 4 out of 9, 4/9).

b. Recognize that probability is always a value between 0 and 1 (inclusively).

c. Express the likelihood of an outcome in a simple experiment as a value between 0 and 1 (inclusively).

Mathematical language and symbols students should use

data, minimum values, maximum values, mean, median, mode, average, range

Exploratory Concepts and Skills

- Explore the differences in representing categorical and numerical data.

Facilitated

Activities

New Math Core Curriculum
Elementary CORE Academy 2007

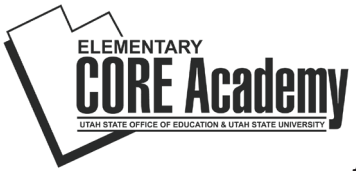
Since the 2003 adoption of Utah's Elementary Mathematics Core Curriculum, ideas such as coherence, focus, high expectations, computational fluency, representation, and important mathematics have become regular elements in discussions about improving school mathematics. As the next step in devising resources to support the development of a coherent curriculum, the National Council of Teachers of Mathematics (NCTM) released *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence.*

With NCTM's release of the Curriculum Focal Points and discussion regarding high expectations, it became important for Utah to revise the Elementary Mathematics Core Curriculum. The placement of concepts within the Curriculum Focal Points guided the placement of concepts within Utah's Core.

The Core has also been designed so that, wherever possible, the ideas taught within a particular grade level have a logical and natural connection with each other and with those of earlier grades. Efforts have also been made to select topics and skills that integrate well with one another and with other subject areas appropriate to grade level. In addition, there is an upward articulation of mathematical concepts and skills. This spiraling is intended to prepare students to understand and use more complex mathematical concepts and skills as they advance through the learning process.

The Core takes into account the psychological and social readiness of students. It builds from concrete experiences to more abstract understandings. The Core focuses on experiences with concepts that students can explore and understand in depth to build the foundation for future mathematical learning experiences.

The Elementary Mathematics Core describes what students should know and be able to do at the end of each of the K-6 grade levels. It was developed and revised by a community of Utah mathematics teachers, mathematicians, university mathematics educators, and State Office of Education specialists. It was critiqued by an advisory committee representing a wide variety of people from the community, as well as an external review committee. The Core reflects the current philosophy of mathematics education that is expressed in national documents developed by the National Council of Teachers of Mathematics, the American Association for the Advancement of Science, and the National Research Council. This Mathematics Core has the endorsement of the Utah Council of Teachers of Mathematics. The Core reflects high standards of achievement in mathematics for all students.



E-D-P Model
Elementary CORE Academy 2007

Each day good educators observe and interact with students to determine what course of action should be taken to achieve the best educational results for each learner. These observations, in many instances, are made with limited formal data. The E-D-P Model assists educators in the collection and use of information justifying implementation of practices. Many educators struggle with the ability to articulate and align teaching actions with student learning needs. The E-D-P Model is a method of aiding this articulation.

When assessing, it is important to know that correct answers do not necessarily mean students understand a concept. Conversely, incorrect responses may not indicate that a student hasn't learned a concept. It is important for educators to look for hidden understandings and possible misconceptions. Ongoing assessments, observations, and interviews may be necessary. When using this process, instructors should select assignments/tasks where students have opportunities to explain their understanding. Developing a tool to aid teachers in the collection of information and to assist them in determining student understanding has been the driving force in creating the E-D-P Model.

Our discussion begins with a description of the E-D-P Model. This model is based on a medical metaphor of **E**valuation-**D**iagnosis-**P**rescription (E-D-P). It is important to understand the difference between three main types of assessment: diagnostic (usually occurring prior to instruction), formative (concurrently occurs with instruction), and summative (occurs at the conclusion of an instructional period). The E-D-P Model targets diagnostic and formative assessments. By conducting ongoing assessments and using this formative information, educators can effectively impact student learning and plan instruction to meet individual learning needs (McNamee & Chen, 2005).

Evaluation

In classrooms across the country one may observe teachers interacting with students in a variety of ways. The **E**valuation portion of the E-D-P Model provides teachers with a way to identify student learning as it relates to the standard and objective of instruction. As a teacher sees a particular student response she is able to identify understandings and misunderstandings.

EXAMPLE: Marcia responded with the answer of 12 when she was asked to add 14 and 8. Using Marcia's work, an instructor sees that Marcia needs instruction on renaming. Other conclusions for the same response may also be apparent. The **E**valuation phase can then transition to the **D**iagnosis.

Diagnosis

As the student response is investigated the instructor may need to ask questions or inquire regarding the reasoning used to formulate the response. This is similar to a physician, where if a pain in the abdomen is described, the doctor poses questions to the patient or performs a physical exam to determine the source of pain. Educators can employ a similar method as they determine the cause of the incorrect responses given by a student. The diagnosis may consume large amounts of time or be rapidly identified based on student work.

Prescription

Once a learning need is **D**iagnosed/identified, renaming in the case of our example, the teacher can then determine what **P**rescriptive action should be taken. In the medical profession, the instructor or doctor has multiple medicines or treatments that can be prescribed. These multiple medicines affect individuals in different ways based on body chemistry and make up. This is also true with education in relation to learning styles. In education, teachers should have multiple activities, learning situations, or practice methods that can be prescribed to help students understand. In our example the teacher could prescribe numerous interventions to help our student understand the renaming concept. (e.g., place value practice, peer discussion groups focused on a single problem, one-on-one discussion about place value, manipulative extensions, etc.)

As teachers formalize the work that is done in a classroom they will be able to define the learning that occurs in a classroom and what learning should take place in the future. There can be a fine line between instruction and assessment when educators use quality formative assessment tasks to guide instruction and learning (Leahy, et al., 2005). The E-D-P Model encourages teachers to evaluate student work, diagnose learning needs, and determine the best prescription for continued growth in knowledge. Some teachers complete these three stages daily in classrooms around the nation without defining the process. This model provides educators a method to formalize current practice and aid them in the implementation process.

Citations

Leahy, S., Lyon, C., Thompson, M., Wiliam, D. (November 2005). Classroom Assessment: Minute by Minute, Day by Day. *Educational Leadership*, 63:3, p.18-24.

McNamee, G.D., Chen, J.Q. (November 2005). Dissolving the Line Between Assessment and Teaching. *Educational Leadership, 63:3, p.72-76.*

Medical Metaphor T-Chart	
Physician	Educator
Why would a physician complete an Evaluation?	Why would an educator complete an Evaluation?
What would a physician use to make make a medical diagnosis?	What would an educator use to make a learning diagnosis?
When evaluation and diagnosis are complete what kind of prescription would be given?	When evaluation and diagnosis are complete what kind of prescription would be given?

ELEMENTARY CORE Academy

E-D-P Assessment Form

Evaluation: ____________________

Name ____________________

Date ____________________

Task/Objective ____________________

( ) Individual ( ) Partner ( ) Group

Diagnosis:		
	Strengths	Weakness
1)		
2)		
3)		
4)		
5)		
6)		

Prescription:

ELEMENTARY CORE Academy

E-D-P Assessment Form

Evaluation: ____________________

Name ____________________

Date ____________________

Task/Objective ____________________

( ) Individual ( ) Partner ( ) Group

Diagnosis:		
	Strengths	Weakness
1)		
2)		
3)		
4)		
5)		
6)		

Prescription:

ELEMENTARY CORE Academy

E-D-P Assessment Form

Evaluation: ____________________

Name ____________________

Date ____________________

Task/Objective ____________________

( ) Individual ( ) Partner ( ) Group

Diagnosis:		
	Strengths	Weakness
1)		
2)		
3)		
4)		
5)		
6)		

Prescription:

ELEMENTARY CORE Academy

E-D-P Assessment Form

Evaluation: ____________________

Name ____________________

Date ____________________

Task/Objective ____________________

( ) Individual ( ) Partner ( ) Group

Diagnosis:		
	Strengths	Weakness
1)		
2)		
3)		
4)		
5)		
6)		

Prescription:

E-D-P Assessment Form

Evaluation: _____

Students:	Diagnosis:							Prescription:				
Task:	Communication	Representation	Computation					Task #4	Comp. #6	Assignment #1		
1) Kyler	√-	√	√					X				
2) Jose	√	√+	√-							X		
3) Kyler	√+	√+	√+						X			
4) Sammy	√	√	√-							X		
5) Shelby	√-	√-	√-							X		

E-D-P Assessment Form	
Diagnosis:	Prescription:

*Copy to a label and place on student work.

E-D-P Assessment Form

Evaluation: _____

Students:	Diagnosis:							Prescription:				
Task:	Communication	Representation	Computation					Task #4	Comp. #6	Assignment #1		
1) Kyler	√-	√	√					X				
2) Jose	√	√+	√-							X		
3) Kyler	√+	√+	√+						X			
4) Sammy	√	√	√-							X		
5) Shelby	√-	√-	√-							X		

E-D-P Assessment Form	
Diagnosis:	Prescription:

*Copy to a label and place on student work.

Core Standard & Objective:	Assessment FOR Learning (Evaluate)			Assessment of Needs (Diagnosis)								
	Type of Assessment			Needs Missing Foundational Skills			Needs Basic Grade Level Core Skills			Needs Extension of Core Skills		
	Pre/Post Dates: /			Plan for Instruction (Prescription) Examples: Expicit Instruction using Graphic Organizers, Skill Specific Activities, Guided Practice, etc.								
Student Name	Limited Knowledge	Partial Knowledge	Mastery									

Mathematical Proficiency
Elementary CORE Academy 2007

How do educators know when a student "Gets It?" Elementary teachers interact with students daily using a variety of individual views regarding mathematical understanding. Success in mathematics is created through a student's composite view and aptitude in five areas of mathematics. In the book, *Helping Children Learn Mathematics,* we are introduced to this composite view of mathematics learning. The term mathematical proficiency is used to describe what it means when a person successfully learns mathematics.

Mathematical proficiency includes five strands:

1) **Understanding:** Comprehending mathematical concepts, operations and relations-knowing what mathematical symbols, diagrams, and procedures mean.
2) **Computing:** Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately.
3) **Applying:** Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately.
4) **Reasoning:** Using logic to explain and justify a solution to a problem or to extend from something known to something not yet known.
5) **Engaging: Seeing mathematics as sensible, useful, and doable-if you work at it-and being willing to do the work.**

It is critical to understand that each of these strands is interwoven and interdependent. Various views of success in mathematics emphasize one aspect of mathematical proficiency with the expectation that the other areas of mathematical knowledge will follow. Success in mathematics comes through achieving mathematical proficiency, which includes each of the five strands.

We see parents, students, and educators focus on only one strand of proficiency, which results in memorized facts that do not necessarily lead to mathematical success. This narrow treatment of math does not provide the strong basis of mathematical learning that students need.

As students learn all the aspects of mathematical proficiency, learning will become stronger, more durable, more adaptable, more useful, and more relevant. It is difficult to master any one of these strands in isolation and is therefore essential to teach the strands in an interconnected method. Developing the strands together builds a student's knowledge of any one strand through connected knowledge points that are memorable.

Citation

National Research Council. (2002). Helping Children Learn Mathematics. Mathematics Learning Study Committee, J. Kilpatrick and J. Swafford, Editors. Center for Education, Division of Behavioral and Social Sciences and Education. Washington, D.C.: National Academy Press.

Building Academic Vocabulary
Elementary CORE Academy 2007

Teaching students vocabulary that will be encountered during the study of content provides a solid background for a positive interaction with that content. Building academic vocabulary is much more than simply placing words upon a word wall or providing a matching exercise with a definition and new terms.

Initially the selection of the terms to be provided to students takes effort and time. Educators should identify key words that are important to the understanding of specific content areas, and are included in the Core Curriculum. The background work of identifying the terms is critical to providing an accurate direction for the subsequent instruction. However, the key to the success of building academic vocabulary ultimately rests upon the quality of the instruction provided by the teacher. Marzano and Pickering provide the following six-step Process for teaching new terms.

The Six-Step Process for Teaching Academic Vocabulary:

1) **Provide a description, explanation, or example of the new term.**
2) **Ask students to restate the description, explanation, or example in their own words.**
3) **Ask students to construct a picture, symbol, or graphic representing the term or phrase**.
4) **Engage students periodically in activities that help them add to their knowledge of the terms in their notebooks**.
5) **Periodically ask students to discuss the terms with one another.**
6) **Involve students periodically in games that allow them to play with the terms.**

With guidance and monitoring students have the ability to generate their own description and representations of vocabulary terms provided. The ownership of this process is valuable in that students see the term as a new tool that aids their learning. An integral step in the process of learning new vocabulary is the student notebook. As students add new terms to their notebook they also refine and update descriptions, which deepens and clarifies their understanding of the content and the terms.

Creating a deeper understanding of vocabulary terms will provide students with multiple points of learning as they encounter new content. These points of learning will broaden the knowledge base and allow students to develop an awareness of the language of learning.

Citation

Marzano, R.J., Pickering, D.J., (2005). *Building Academic Vocabulary Teachers's Manual* ASCD, Alexandria, VA.

Math I-2

Activities

Compare Numbers

How Low Can You Go?

Standard I:

Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 2:

Explain relationships and equivalencies among integers, fractions, decimals, and percents.

Intended Learning Outcomes:

2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.

Content Connections:

Math I-3; common denominator circles

Math Standard I

Objective 2

Connections

Background Information

Before teaching this lesson the students need to have a knowledge of the parts of a fraction (numerator and denominator) and prime and composite numbers. They also need to know that fractions with like denominators can be added and subtracted. Students will also need to understand equivalent fractions.

In this activity students will use fraction circles to build equivalent fractions and manipulate them to discover the least common denominator. Graphic organizers will be used to compare the circle fraction with the numerals involved.

Research Basis

Moore , D.W., (1984). A quantitative and qualitative review of graphic organizer research. *Journal of educational research*. 78, 11-17.

Two research reviews sought to sort out the accumulated evidence of graphic organizer effects on learning. Moore applied meta-analysis procedures to integrate research findings from 16 graphic organizer studies. Strong effects were obtained when students constructed graphic organizers after encountering content.

Loewenberg, D.B. (1992). Magical hopes manipulatives and the reform of math education. *American educator*. Summer, 14-18, 46-47.

This article focuses on manipulatives and how they effect students understanding. It also explains how manipulatives have changed education in a positive way. It discusses real-life examples of how manipulatives have enhanced student understanding.

Invitation to Learn

Materials

- ❑ *Picture Pie*
- ❑ Four inch circles
- ❑ *Fraction Cards*

In this activity the students will be given five four-inch circles that have been cut with a die cut machine. Show the students the book Picture Pie, by Ed Emberley. Ask the students if they recognize any fractions in the pictures. The students will then make a picture by cutting their circles into halves, quarters, and eighths. They will then use these pieces to create a picture (do not use thirds, sixths, or fifths). The teacher will then hand out a fraction card to each of the students. This card will tell the student how many pieces of their circles they can use to make their picture. After the students have had time to make their pictures and have labeled each fraction piece, they will be paired up and they will need to make up an addition mathematical equation using their pictures. The students will write this equation somewhere on their pictures. The pictures will be used for the next activity.

Instructional Procedures

Materials

- ❑ *Factor Tree*
- ❑ Factor Tree (overhead)
- ❑ *How to Find LCD*

1. Distribute two *Factor Tree* handouts per student.
2. Identify the parts of their equations (whole number, numerator, denominator, symbols used).
3. Have each student identify the denominator of their fraction on their picture they created in the previous activity by placing a finger on it.
4. Have the students look at their partner's denominators to see if they have found it.
5. Ask if their denominators are the same? Or different?
6. If there are some that are the same ask the students if the can be added? YES!
7. Most of the student's fractions should have different denominators. These denominators will be placed into the seed on the Factor Tree handouts. The students will then put their denominator into the denominator seed circle on one of their *Factor Tree* handouts.
8. The students will then put their partners denominator on the other *Factor Tree* handout that was given to them.
9. Explain that you will now be splitting the denominator seeds and watching them grow into prime factor fruit. (The teacher can tell the students that farmers pick their fruit when it is prime).
10. Show the students how this is done on the *Factor Tree* transparency.

11. After the students have come up with their factor fruit they will be using the *How to Find LCD* handout to accomplish the next part of the lesson. This handout should be placed into the student's math binder so it can be referred to in the future.
12. Count the number of times each prime number appears in each of the factorizations.
13. For each prime number, take the largest of these counts.
14. Write down that prime number as many times as you counted for it in step #2.
15. The least common denominator is the product of all the prime numbers written down.

Assessment Suggestions

- Observation
- E.D.P. (Evaluate, Diagnosis, Prescribe)
- Journal entry explaining how to find LCD

Curriculum Extensions/Adaptations/ Integration

- Advanced Learners could be given three or more denominators and asked to find the LCD.
- Special needs students could be given a multiplication array to help with facts they do not know.

Family Connections

- Students can go home and show their parents how to get factor fruit from a denominator seed using the *Factor Tree* handout.
- Students can create a story using the vocabulary from the lesson (denominator, numerator, whole number, prime number, etc.).

Additional Resources

Books

Picture Pie, by Ed Emberley; ISBN 0-316-78982-8

Web sites

http://www.glc.k12.ga.us/seqlps/sudisplay.asp?SUID=164

Fraction Cards

2 and 1/4	2 and 1/2	2 and 3/4	2 and 3/8	2 and 5/8
2 and 6/8	2 and 7/8	3 and 1/4	3 and 1/2	3 and 3/4
4 and 1/4	4 and 1/2	4 and 3/4	4 and 3/8	4 and 7/8
3 and 7/8	3 and 3/8	3 and 5/8	3 and 6/8	1 and 7/8

Fraction Cards

2 and 1/4	2 and 1/2	2 and 3/4	2 and 3/8	2 and 5/8
2 and 6/8	2 and 7/8	3 and 1/4	3 and 1/2	3 and 3/4
4 and 1/4	4 and 1/2	4 and 3/4	4 and 3/8	4 and 7/8
3 and 7/8	3 and 3/8	3 and 5/8	3 and 6/8	1 and 7/8

How to find LCD

To find the least common denominator using this method, **factor each of the denominators** into **primes**. Then for each **different** prime number in all of the factorizations, do the following...

1. **Count** the number of times each prime number appears in each of the factorizations.
2. For each prime number, take the **largest** of these counts.
3. Write down that prime number **as many times as you counted** for it in step #2.
4. The least common denominator is the **product** of all the prime numbers written down.

Example: We'll use the same fractions as above: **1/5**, **1/6** and **1/15.**

- **Factor into primes**
  - Prime factorization of **5** is **5** (**5** is a prime number)
  - Prime factorization of **6** is **2 x 3**
  - Prime factorization of **15** is **3 x 5**

Notice that the **different primes** are 2, 3 and 5.

- Now, we do **Step #1 – Count** the number of times **each** prime number appears in **each** of the factorizations...
  - The count of primes in **5** is **one 5**
  - The count of primes in **6** is **one 2** and **one 3**
  - The count of primes in **15** is **one 3** and **one 5**
- **Step #2** - For **each** prime number, take the **largest** of these counts. So we have...
  - The largest count of **2s** is **one**
  - The largest count of **3s** is **one**
  - The largest count of **5s** is **one**
- **Step #3 –** Since we now know the count of each prime number, you simply write down that prime number as **many times as you counted** for it in step #2.

Here are the numbers...

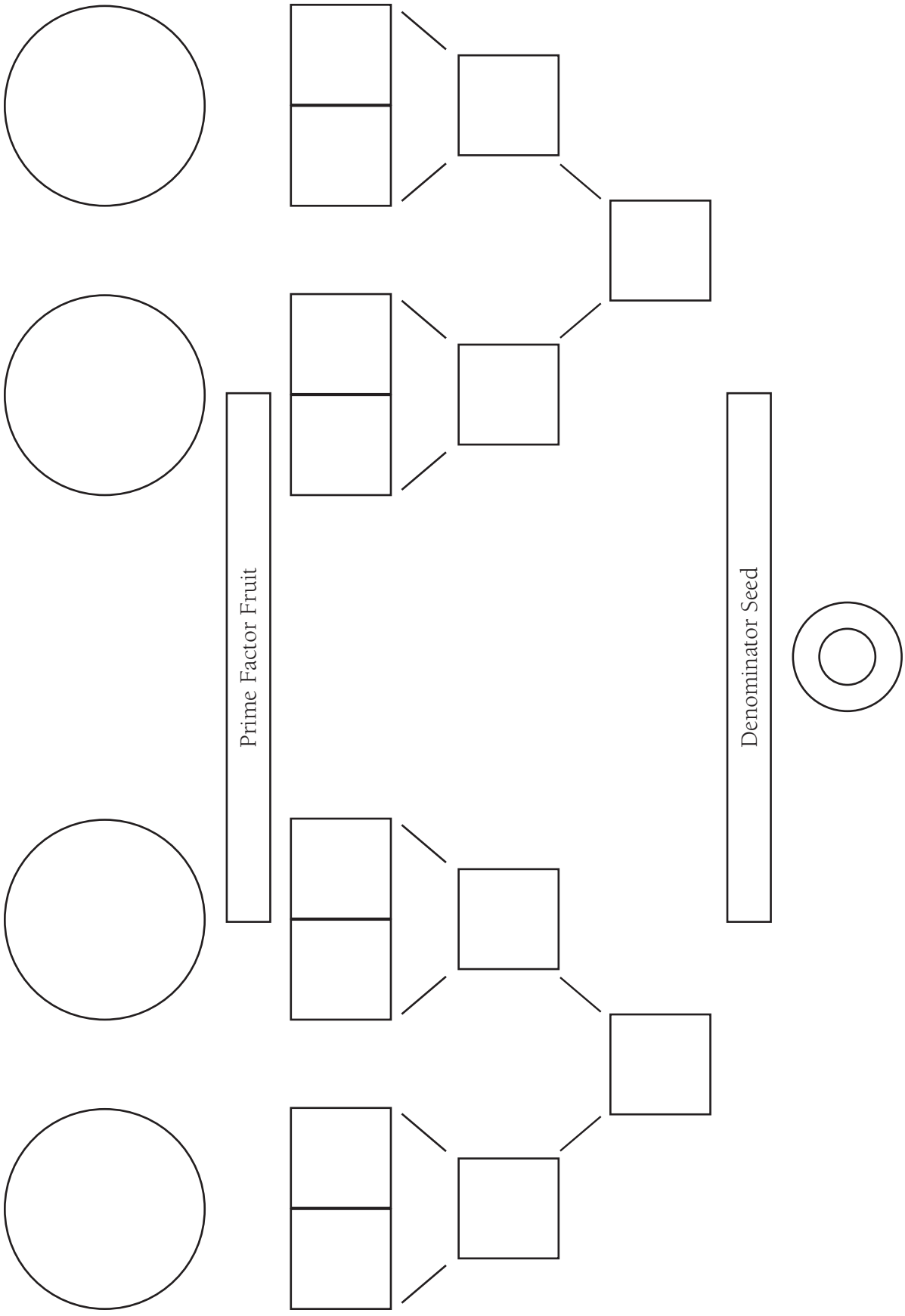
2, 3, 5

- **Step #4 –** The least common denominator is the **product** of all the prime numbers written down.

2 x 3 x 5 = 30

Therefore, the least common denominator of **1/5**, **1/6** and **1/15** is **30**.

Factor Tree



There's Nothing Improper About Them

Math Standard I

Objective 2

Connections

Standard I:

Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 2:

Explain relationships and equivalencies among integers, fractions, decimals, and percents.

Intended Learning Outcomes:

2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.

Content Connections:

Math I-3; Improper Fraction Bingo

Background Information

Before teaching this lesson, mixed and improper fractions should be introduced. The students need to know the definition to numerator, denominator, and whole number.

In this activity students will review what an improper fraction and a mixed number are and how they relate to one another using shapes. Students will then practice converting improper fractions to mixed numbers and vice versa using the well-known game Bingo.

Research Basis

Moore, D.W., (1984). A quantitative and qualitative review of graphic organizer research. *Journal of educational research. 78*, 11-17.

Two research reviews sought to sort out the accumulated evidence of graphic organizer effects on learning. Moore applied meta-analysis procedures to integrate research findings from 16 graphic organizer studies. Strong effects were obtained when students constructed graphic organizers after encountering content.

Ellis, E., (2004). Q&A: *What's the big deal with graphic organizers?* Retrieved December 30, 2006, from http://graphicorganizers.com/about.html.

This article answers some common questions about graphic organizers. It discusses many misconceptions about the use of graphic organizers and describes when and how to use them properly. It also discusses how graphic organizers can be valuable assessment tools.

Materials

- ❑ *Shape Shift Sheet*
- ❑ Assorted Shape Tiles.

Invitation to Learn

This activity is called shape shifting. In this activity the class is split up into pairs. Each pair is randomly given small shapes that can be manipulated and combined to make larger shapes (hand out enough for the pair to be able to make at least two big shapes). Explain that the triangles will need to be combined with the other triangles, squares with the squares, rectangles with rectangles. Do not combine different shapes. The teacher will then ask the question how many small shapes did it take to make your larger shape? Ask each pair to write down their mixed number and draw a picture on the *Shape Shift Sheet*. The groups that do not have whole shapes will need to write them down as fractions. Discuss how one small shape is a fraction of the larger shape.

Materials

- ❏ *Assorted shape tiles*
- ❏ *Improper Instructions #1*
- ❏ *Improper Instructions #2*
- ❏ *Blank Fraction Bingo Card*
- ❏ *Improper Fraction Answers*
- ❏ *Mixed Number Squares*
- ❏ Paper bag
- ❏ Bag of candy
- ❏ Computer

Instructional Procedures

1. Hand out *Improper Instructions #1* and *Improper Instructions #2*, have students use shapes to work through the instructions with the teacher as a review.
2. Pass out *Blank Fraction Bingo Cards* to students.
3. Have students fill in bingo cards with fractions located on overhead *Improper Fraction Answers*.
4. The teacher should have cut up *Mixed Number Squares*.
5. Students will take turns choosing the mixed number out of the bag.
6. Students will place a candy on the block on their card that has the appropriate improper fraction.
7. Students will take turns until a student covers a line (horizontally, vertically, or diagonally). The teacher may provide prizes (optional).
8. Students will use website to check their problems. Students should type in the improper fraction; the web site will change the fraction into a mixed number.

Assessment Suggestions

- Graphic Organizer (Bingo card)
- E.D.P. (Evaluate, Diagnosis, Prescribe)
- Observation of correctly converting fractions

Curriculum Extensions/Adaptations/ Integration

- Advanced learners could write story problems using improper fractions, orlist examples of real life situations where they observe improper fractions/mixed numbers.
- Learners with special needs could draw or use manipulatives to show examples of improper fractions/mixed numbers.

Family Connections

- Students could list examples of improper fractions/mixed numbers that they see at their home (improper fraction hunt).
- Students could ask parents if they use improper fraction/mixed numbers, then write a paragraph explaining how, when, and where their parents use them.

Additional Resources

Web sites

http://www.GraphiOrganizers.com

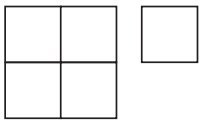
Shape Shift Sheet

Write the mixed number on the line	Write the mixed number on the line
_____	
_____	
_____	

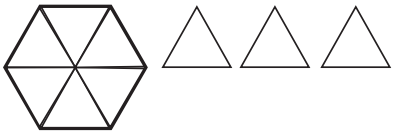
Improper Instructions #1

Improper Fractions are just fractions that have a larger numerator than denminator.
Like this one: **5/4**

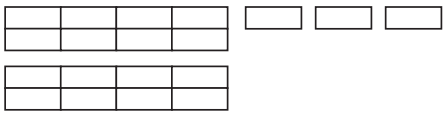
Use your unit blocks to make a model. **5/4 = 1 1/4**
Remember, **5/4** is the same as **1** whole with **1** extra.



Try another one **9/6**
Use your triangle blocks to make a hexagon.
Remember, **9/6** is the same as **1** whole with **3** extra.



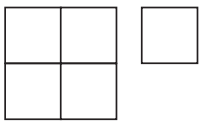
What about this? **19/8**
19/8 is the same as **2 3/8**



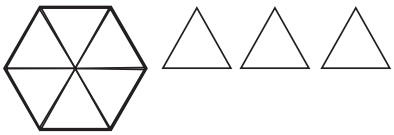
Improper Instructions #1

Improper Fractions are just fractions that have a larger numerator than denminator.
Like this one: **5/4**

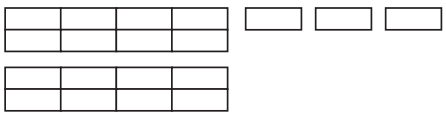
Use your unit blocks to make a model. **5/4 = 1 1/4**
Remember, **5/4** is the same as **1** whole with **1** extra.



Try another one **9/6**
Use your triangle blocks to make a hexagon.
Remember, **9/6** is the same as **1** whole with **3** extra.



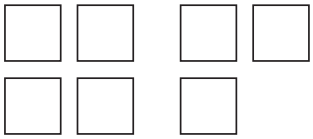
What about this? **19/8**
19/8 is the same as **2 3/8**



Improper Instructions #2

Improper Fractions are just fractions that have a larger numerator than denminator.
How about this **1 3/4**

Use your unit blocks to make a model.
Count the number of squares. **7 squares = 7/4**
Remember **7/4** is the same as **1** whole with **3** extra

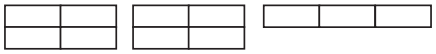


Can you change without the models?

2 3/4

Multiply these. Then add this to the product.
2 x 4 = 8 8 + 3 = 11 ⟵ This is your new numerator
The denominator stays the same. **11/4**

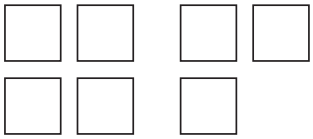
Check with your blocks. **2 3/4 = 11/4**



Improper Instructions #2

Improper Fractions are just fractions that have a larger numerator than denminator.
How about this **1 3/4**

Use your unit blocks to make a model.
Count the number of squares. **7 squares = 7/4**
Remember **7/4** is the same as **1** whole with **3** extra



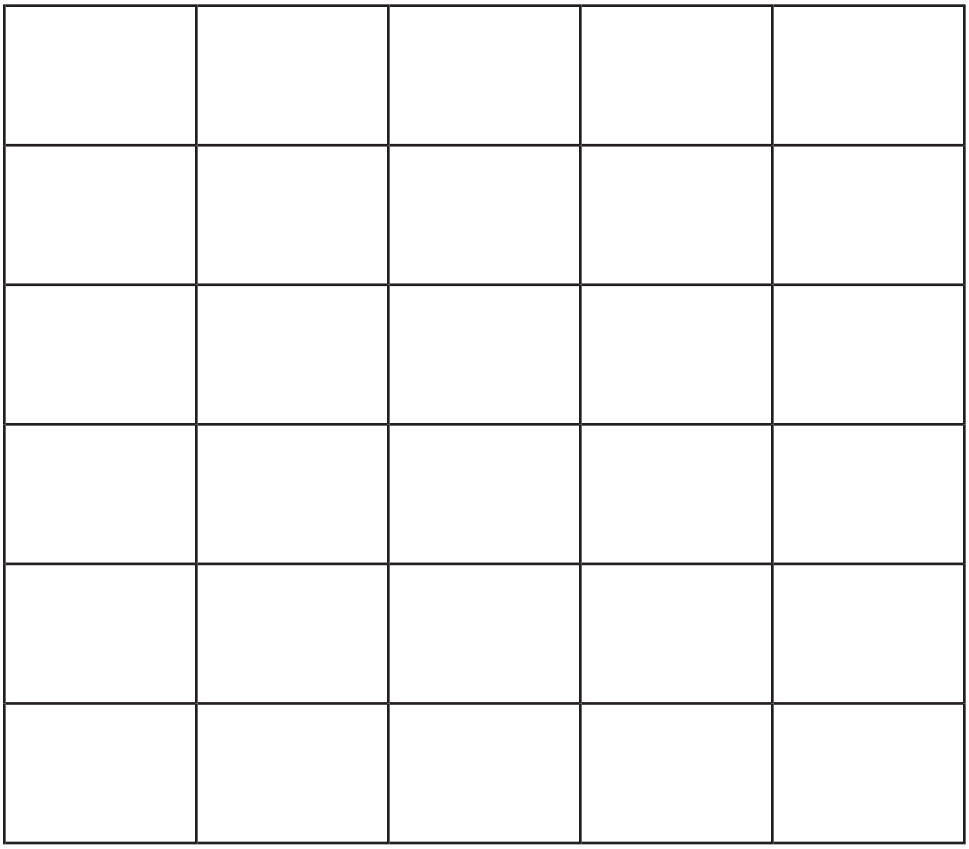
Can you change without the models?

2 3/4

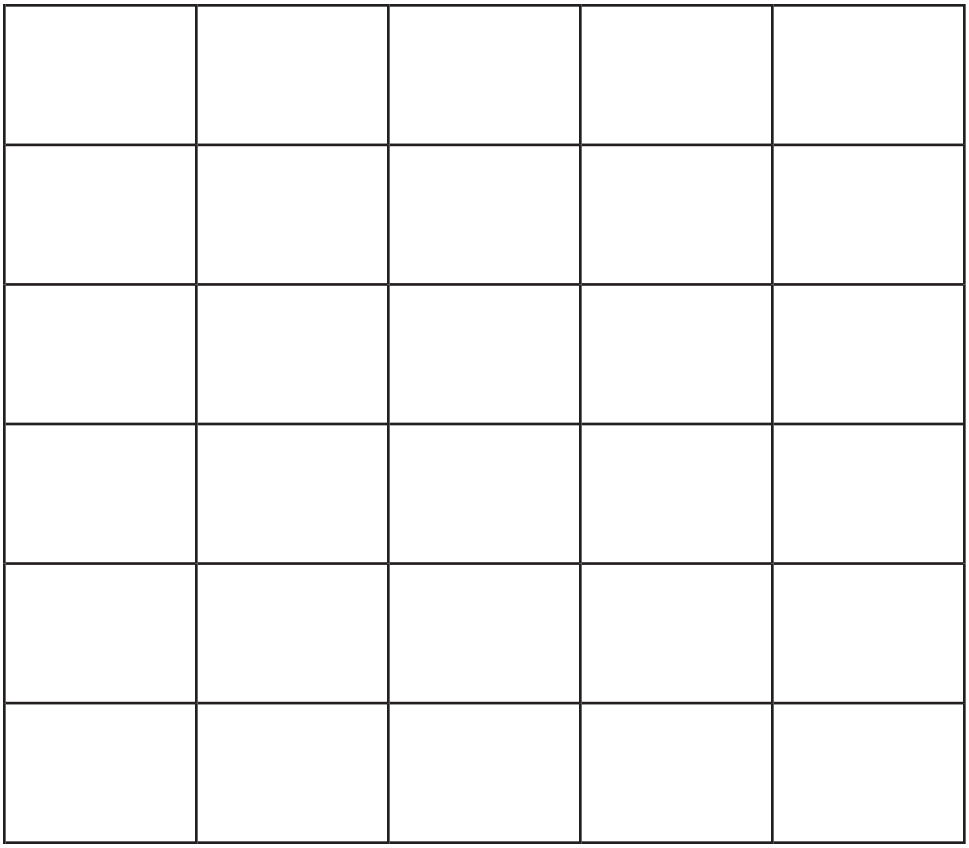
Multiply these. Then add this to the product.
2 x 4 = 8 8 + 3 = 11 ⟵ This is your new numerator
The denominator stays the same. **11/4**

Check with your blocks. **2 3/4 = 11/4**

Fraction Bingo



Fraction Bingo



Improper Fraction Answers

Make transparency or copy to board.

$\frac{17}{6}$	$\frac{12}{7}$	$\frac{19}{4}$	$\frac{10}{3}$	$\frac{16}{6}$	$\frac{11}{4}$	$\frac{21}{7}$	$\frac{18}{5}$
$\frac{10}{6}$	$\frac{15}{3}$	$\frac{14}{3}$	$\frac{15}{2}$	$\frac{12}{7}$	$\frac{23}{6}$	$\frac{8}{6}$	$\frac{4}{3}$
$\frac{25}{4}$	$\frac{11}{6}$	$\frac{22}{4}$	$\frac{36}{7}$	$\frac{54}{7}$	$\frac{31}{9}$	$\frac{37}{6}$	$\frac{29}{7}$
$\frac{41}{8}$	$\frac{6}{6}$	$\frac{34}{5}$	$\frac{26}{8}$	$\frac{13}{5}$	$\frac{25}{4}$	$\frac{8}{3}$	$\frac{51}{8}$
$\frac{17}{4}$	$\frac{12}{7}$	$\frac{43}{5}$	$\frac{33}{6}$	$\frac{44}{9}$	$\frac{19}{4}$	$\frac{58}{9}$	$\frac{61}{6}$
$\frac{39}{9}$	$\frac{15}{7}$	$\frac{59}{7}$	$\frac{52}{5}$	$\frac{71}{8}$	$\frac{53}{4}$	$\frac{51}{6}$	$\frac{32}{9}$
$\frac{48}{6}$	$\frac{21}{6}$	$\frac{15}{4}$	$\frac{25}{3}$	$\frac{32}{7}$	$\frac{14}{6}$	$\frac{17}{6}$	$\frac{13}{6}$

Mixed Number Squares

Cut squares for bag.

$2\frac{5}{6}$	$1\frac{5}{7}$	$4\frac{3}{4}$	$3\frac{1}{3}$	$2\frac{4}{6}$	$2\frac{3}{4}$	3	$3\frac{3}{5}$
$1\frac{4}{6}$	5	$4\frac{2}{3}$	$7\frac{1}{2}$	$1\frac{5}{7}$	$3\frac{5}{6}$	$1\frac{2}{6}$	$1\frac{1}{3}$
$6\frac{1}{4}$	$1\frac{5}{6}$	$5\frac{2}{4}$	$5\frac{1}{7}$	$7\frac{5}{7}$	$3\frac{4}{9}$	$6\frac{1}{6}$	$4\frac{1}{7}$
$5\frac{1}{8}$	1	$6\frac{4}{5}$	$3\frac{2}{8}$	$2\frac{3}{5}$	$6\frac{1}{4}$	$2\frac{2}{3}$	$6\frac{3}{8}$
$4\frac{1}{4}$	$1\frac{5}{7}$	$8\frac{3}{5}$	$5\frac{3}{6}$	$4\frac{8}{9}$	$4\frac{3}{4}$	$6\frac{4}{9}$	$10\frac{1}{6}$
$4\frac{3}{9}$	$2\frac{1}{7}$	$8\frac{3}{7}$	$10\frac{2}{5}$	$8\frac{7}{8}$	$13\frac{1}{4}$	$8\frac{3}{6}$	$3\frac{5}{9}$
8	$3\frac{3}{6}$	$3\frac{3}{4}$	$8\frac{1}{3}$	$4\frac{4}{7}$	$2\frac{2}{6}$	$2\frac{5}{6}$	$2\frac{1}{6}$

Name _____ Date _____

Fraction Quiz

Change the improper fractions to mixed numbers.

$\frac{17}{3}$ = _____ $\frac{31}{6}$ = _____ $\frac{7}{2}$ = _____

$\frac{26}{4}$ = _____ $\frac{42}{8}$ = _____ $\frac{51}{9}$ = _____

$\frac{19}{5}$ = _____ $\frac{16}{3}$ = _____ $\frac{68}{7}$ = _____

Name _____ Date _____

Fraction Quiz

Change the improper fractions to mixed numbers.

$\frac{17}{3}$ = _____ $\frac{31}{6}$ = _____ $\frac{7}{2}$ = _____

$\frac{26}{4}$ = _____ $\frac{42}{8}$ = _____ $\frac{51}{9}$ = _____

$\frac{19}{5}$ = _____ $\frac{16}{3}$ = _____ $\frac{68}{7}$ = _____

Math Standard I-3&4

Activities

Modeling Operations

Playing with Remainders

Standard I:

Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 4:

Model and illustrate meanings multiplication and division.

Intended Learning Outcomes:

2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.
3. Reason logically, using inductive and deductive strategies and justify conclusions.
5. Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.

Content Connections:

Language Arts Standard 1; oral language
Language Arts Standard 2; fluency

Math Standard I

Objective 4

Connections

Background Information

It often takes a leap of understanding for students to apply the procedural algorithm of division with remainders to real-world situations where remainders are encountered. A child who can easily calculate 40 divided by 6 = 6R4 will too often state 6R4 as the answer to the number of cars necessary to transport 40 children to a baseball game if 6 children can fit in each car. The activities in this section will first review the concept of division as proportional reasoning involving equal shares and then they will lead children to discover the three usual ways of dealing with remainders in real life: they are either used to round up to the next whole number, they are dropped and discarded, or they are split evenly among the participants.

Before beginning this lesson, students must be able to express remainders as fractions and decimals.

Materials

- ❑ 12 manipulatives
- ❑ Math journals

Research Basis

Wiebe, A., (1989). Proportionality: A major concept in mathematics—part II: Remainders—what are we to do with them? ***Aims newsletter, volume iii***, No. 7, 6-7.

Dr. Wiebe explores the gap between abstract answers to division problems with remainders and real-life situations where students encounter remainders. Expressing remainders as fractions and decimals are explored and applied, and the choices of rounding up, dropping, and sharing remainders are introduced.

Martinez, J.G.R., (2000). Look smart. *Early years,* January 2000. Retrieved January 12, 2007 from http://www.findarticles.com.

Engaging children in math story problems is easier when the stories have real plots and good endings. By engaging students in the plot, they become interested in solving the math situations, rather than routinely solving a page of "word problems." Additionally, the enthusiasm generated motivates students to write their own stories, developing new problems within the story context, and acting out the story line.

Invitation to Learn

Distribute a set of 12 counting objects to each child. (They may be cubes, blocks, chips, etc.) Tell the students that they each have a set of 12 objects. Then ask the students to divide their sets into four fair shares. Guide them to create four sets with three objects in each set. Discuss the term "fair shares" if it is not part of your usual vocabulary. It means every set has the same number of objects, the dividend is divided equally by the divisor. Then write the following equation on the board and ask the children to copy it into their math journals.

$$4\overline{)12}\ 3$$

Ask what is different from the usual way of writing a division problem. They should notice that the number 1 is written above the divisor. What is significant about the number 1? Take several ideas from students. Lead them to discover that the 1 is implied in every division problem, because the quotient is how many items are in 1 fair share. Then have the children write the following two statements in their journals:

- 1 fair share contains 3 objects.
- 4 fair shares contain 12 objects.

Explore with the children the relationships between the numbers as they discover the proportions: 1/4 = 3/12; 1/3=4/12 and 1X12=3x4. Write all the true statements on the board and have the children list them in their journals.

Next, copy these three equations on the board:

$$4\overline{)12}\ ?\qquad ?\overline{)12}\ 3\qquad 4\overline{)?}\ 3$$

Ask what is the question in the first equation. (The students are asked to form 4 fair shares from 12 objects.) What is the question in the second equation? (The students are asked to find how many fair shares of 3 each can be made with 12 objects.) What is the question in

the third equation? (Students are asked to find how many objects must be used to make 4 fair shares containing 3 objects each. This case involves multiplication rather than division.) Have the children build each situation with their manipulatives, knowing that even though the problem looks the same each time, in the first instance the question is the number of fair shares in each set. In the second instance, the question is the number of sets, and in the third instance, the question is the total number of objects.

Introducing division as proportional reasoning prepares children for equivalency in fractions; proportionality in ratios, proportions and percents; and provides a more concrete understanding of division as the process of creating fair shares.

Instructional Procedures

1. Divide the class into three groups. Each script has enough parts for eight actors. Additional class members could be used to direct, create, and manage props, etc. If you have a really large class, you may wish to double one or more of the scripts and perform the same play(s) twice. The plays also work well in a readers' theater format, shortening preparation time.
2. Practice the plays: "*Round-up!,*" "*Sharing is Very Important!*" and "*You Just Drop It!*"
3. Present each play to the whole class. During and after the presentations, the class completes the graphic organizer *Playing with Remainders.*
4. As a class, discuss the different applications of remainders in the three plays using the graphic organizer to illustrate the different ways each play uses remainders in real life. You may wish to have the children trim the edges of the graphic organizer and glue it into their journals as a reference.
5. As a whole class or in partners complete the worksheet *Remainder Stories.*
6. Note: Another option for these plays is to use them as center activities, with each child participating in each play, using no audience but discussing each play separately as a whole class. This option may increase student engagement.

Materials

- ❏ Math journals
- ❏ *Round-up!*
- ❏ *You Just Drop It!*
- ❏ *Sharing is Very Important!*
- ❏ *Playing with Remainders*
- ❏ *Remainder Stories*
- ❏ Paper
- ❏ Scissors
- ❏ Tape
- ❏ Markers

Assessment Suggestions

- Formative assessment: Check for accuracy as students complete their graphic organizers, participate in the discussion following the presentation of the plays, and solve the word problem worksheet.
- Final assessment: In a word-problem test, students should be able to supply the correct answer and explain in words what they did with their remainders (dropped, shared, or rounded up).

Curriculum Extensions/Adaptations/ Integration

- After reading or acting out these plays, children could write their own stories or plays where the characters must interpret remainders correctly in real-life situations.
- Children new to the United States could be encouraged to set new plays in their homeland countries with names, food, and problem-solving situations common to their life experiences.

Family Connections

- Assign students to create two to five word problems at home using members of their families and either real or made up situations that require the correct use of remainders.
- Which use of a remainder is most common? Give students a few days to collect data at home about which scenario is most common—dropping, rounding or sharing. They might be allowed situations on TV in addition to real-life occurrences. After collecting data, a bar graph could be constructed comparing the three types of remainders' frequency.

Additional Resources

Books

Teaching with the Brain in Mind, by Eric Jensen; ISBN 1-4166-0030-2

Web site

http://www.edhelper.com

"Round-up!"

Scene 1

Narrator: "Our play begins in the family room of a modern home where two 11-year-old children are gathered around the TV."
Mom: (entering from the kitchen) "Tyler! Nikki! I want to talk to you!" (The children stay glued to the TV.) "Nikki! Tyler! You'll want to hear what I have to say!"
Tyler and Nikki together: "Okay, Mom, what's up?"
Mom: "We have our plans for the family reunion. We're going to a dude ranch with all the cousins. You'll spend a week away from the TV—riding horses, rafting a river. You might even get to take part in a cattle round up!"
Tyler: "Cool! When are we going?"
Mom: "Next Friday."
Nikki: "Can I ride with my cousin Brittany?"
Mom: "Everyone's coming to our house to meet. I don't think we'll need to take everyone's cars. Gas is so expensive, we might as well take as few cars as possible."

Scene 2

Narrator: "Now the setting changes to the front yard outside Nikki and Tyler's house. All the relatives are gathered to go to the reunion together. Nikki is standing by her mom, not really listening. Tyler is standing next to his dad."
Tyler's dad: "All right, everyone! Stand together! How many people do we have? Let's see…I think we have 23 people, counting all the children. Each car we are taking has 5 seat belts, so how many cars do we need?"
Tyler: "That's easy, Dad. 23 divided by 5 = 4R3. We need 4R3 cars!"
Narrator: "A strange voice is heard above the crowd. Everyone freezes as it calls in a low, slow, Western drawl…"
Voice: "Round-up!"
Narrator: "Slowly the action returns, but Tyler acts as if he has been struck by lightning."
Tyler: "Dad, no. We don't need 4R3 cars. I have to round up that remainder. We need 5 cars."

Scene 3

Narrator: "We join our cast outside the main lodge at the *No-Remainder Ranch*. Nikki and her mom are standing in front of Wrangler John."
Wrangler John: "Welcome, everyone! Gather round so I can assign you a bunk. Let's see, there are 23 of you, and I can put 4 in a cabin. How many cabins do I need?"
Nikki: "I can do that problem in my head! 23 divided by 4 = 5R3. We need 5R3 cabins!"
Narrator: "Again, a strange voice is heard above the crowd. Everyone freezes as it calls in a low, slow, Western drawl…"
Voice: "Round-up!"
Narrator: "Slowly the action returns, but now Nikki acts as if she has been struck by lightning."
Nikki: "Wait! 5R3 cabins doesn't make any sense. I need to round up the remainder! We need 6 cabins for 23 people. One bunk will just have to be empty."

Scene 4:
Narrator: "Join Nikki and Tyler's family in a clearing next to the bank of a fast-moving river. Family members are putting on life jackets and waiting for instructions from Wrangler John. Nikki and her cousin Brittany are standing together."
Wrangler John: "Be sure your life jacket is on properly. The river is fast and you will encounter some class 4 rapids. We haven't lost anyone yet this year, and we don't expect to. Each raft holds 6 guests, plus a guide who knows the river well. Let's see…we have 23 guests. How many rafts do we need to take?"
Brittany: "That's easy! 23 divided by 6 = 3R5. We need 3R5 rafts!"
Narrator: "Everyone suddenly becomes silent. Tyler and Nikki look around, as if expecting the voice. And, sure enough, seemingly out of nowhere, it calls…"
Voice: "Round-up!"
Narrator: "Brittany rubs her forehead as if she has been hit by lightning. Then she excitedly calls…"
Brittany: "Wait! 3R5 rafts doesn't make any sense! I have to round up! We need 4 rafts."

Scene 5:
Narrator: "It's night time, and the guests of the ***No-Remainder Ranch*** are seated around a campfire. They are listening to Wrangler John tell stories about the mountains around them."
Wrangler John: "Do you want to hear another story?"
Tyler, Nikki, Brittany: "Yes!"
Wrangler John: "Well, okay. This story has been around for a long time, and folks around here believe it to be true. Have you wondered, since you have been here, how the ranch got its name?" (The assembled guests nod their heads, and Wrangler John continues.) "A long time ago, people around here couldn't stay safely through the winter. Gathering enough provisions took too much work, and it made sense to go down to town where it was warmer. So, early every November, after all the harvestin' was done, the animals were driven down to lower ground. There wasn't much of a ranch here, and it wasn't named at all. Those who worked here came back after the animals were secure and gathered the last of their things and then went back to town in their wagons. This happened year after year without incident. That is until 1906. In 1906 snow came earlier than usual, and the cattle drive had to be put together quickly. Five men came back to the ranch after that, just to tidy things up and get the last of their provisions. They had to hurry, because a fierce storm was just a few hours away, and getting stuck at the ranch over the winter would be no picnic."
Narrator: "Wrangler John looked carefully over his audience to see that they were paying attention. No one spoke."
Wrangler John: "The men had one wagon, drawn by two work horses. They divided into teams of two for the last of their chores and then got into the wagon and drove away. What they didn't realize was that in dividing 5 men by 2, they had left one man out. 'Scorch', as thy called him, because he usually burned dinner, had no partner, no job, and had been left to winter alone at the ranch. By the time they realized they'd forgotten 'Scorch', the high country was buried in three feet of snow and it was too late to go back and search for him."
Tyler: "Was he ever seen again? What happened to him?"
Wrangler John: "No. The next spring, when the wranglers returned to the ranch, a careful search was conducted. But no remains were ever discovered. However, a strange legend surrounding

'Scorch's disappearance is told today. It is said that he protects people all over these parts from being left behind. Whenever a group is dividing into sets, and an important remainder might be forgotten, he calls in a low, slow drawl, 'Round-up!' and the group remembers to include the remainder. In fact, it's after one such experience that the name of the ranch was changed to the *No-remainder Ranch*. But it's just a legend. I don't know anyone personally who has heard the voice…"

Narrator: "The crowd grows silent as Brittany, Nikki, and Tyler look at each other in amazement. They know THEY'VE heard the voice. Each time they were about to leave an important remainder behind, the voice instructed them to round up. And, as if to remind them forever, once more they heard the low, slow, Western drawl…"

Voice: "Round-up!"

"Sharing is Very Important"

Scene 1

Narrator 1: "Have you ever had a little brother or sister turn into a nosy tattle-tale? Scott and Travis did. Their little sister Samantha turned five and thought she was the boss of everything! But one day, they decided they were glad to have her around. That day, Little Samantha saved Travis' life."

Narrator 2: "Our story begins in the Hunter family's back yard. Scott is 13, Travis is 10, and Samantha is 5. As usual, Scott and Travis are trying to accomplish something, and Samantha is in their way."

Scott: "Travis, hand me that rope. I want to tie knots in it. We can climb up it to get into our tree fort." (Travis hands Scott the rope.) "If we cut it in two pieces, we can use ½ for the front door and ½ for the back door. Hmm…we have 11 feet of rope. How long does each piece need to be?"

Travis: "That's easy! 11 divided by 2 = 5R1. Each piece needs to be 5R1 feet long."

Samantha: "I'm telling Mom! You're not sharing!"

Travis: "Not sharing what? What are you talking about?"

Scott: "Just ignore her. You said what about the rope?"

Travis: "Each piece needs to be 5R1 feet long."

Narrator 1: "Samantha didn't want to be ignored, so she went to their mother for help. Soon Mom came into the back yard."

Mom: "Boys, Samantha says you aren't sharing. Don't you know that you need to share whenever you can? Sharing is very important."

Narrator 2: "With that bit of advice, Mom went back into the house. And Travis and Scott went back to work."

Travis: (with a long look at Samantha) "Scott, something Mom just said made sense. We can share this remainder. Each piece of rope can be 5 ½ feet long. Thanks, Samantha. You actually helped us with this tree house."

Scene 2

Narrator 1: "Within a few days the tree house was finished, and it was time to have a sleepover in it. Scott and Travis decided there was room for 4 sleeping bags, so each of them invited his best friend. As soon as it was dark, they climbed the ropes and settled in."

Narrator 2: "Of course, no one really sleeps at a sleepover, right? Within minutes, on each boy's sleeping bag heaped a pile of treasure—whole bags of candy, stacks of baseball cards for trading, and Game Boy's and Ipods for later, when the talking wore thin."

Scott: "Justin, are you ready to share your Airheads? I want a blue one."

Justin: "There are 17 in the bag and we have 4 kids. How many does that give each of us?"

Travis: "I'm good at division. 17 divided by 4 = 4R1. Each kid gets 4R1 Airheads."

Narrator 1: "At just that moment, Samantha's head popped up in the entrance to the tree fort."

Samantha: "Hey, you guys forgot to invite me. And you're not sharing! Don't you know sharing is very important?"

Scott: (with a long look at Samantha) "Travis, you're right about Samantha. Sometimes she says just the right thing. We can share that remainder. Each kid gets 4 Airheads, and we can divide the last one into 4 pieces. We'll each get 4 1/4 Airheads."

Travis: "I'm okay about Airheads, but what I'm really eyeing is Hector's Reese's Peanut Butter Cups. Hector, how many Reese's do you have?"

Hector: "There are 10 in the bag. And I know that 10 divided by 4 = 2 R2, so we each get…"
Samantha: "If you don't start sharing, I'm telling Mom again!"
Travis: (looking at Samantha) "Okay, we'll share the remainder. 10 divided by 4= 2R2. But if we share the remainder, we'll each get 2 1/2 Reese's."
Scott: "Now, Samantha, get lost. This is a BOY tree house!"
Scene 3
Narrator 2: "The tree house was a big hit. For most of the summer Scott and Travis had a sleepover in it at least once a week. But in mid-August, Scott's friend, Justin, had another big idea."
Justin: "Hey guys, let's do a survival camp-out on Slickrock Mountain!"
Hector: "What's a survival camp-out?"
Justin: "It's when we each go our own way and we have to stay alone all night, without a tent or anything!"
Travis: "Is it safe?"
Scott: "Sure! We don't go very far from each other—just far enough to not see each other. We'll stay at the old mine camp."
Narrator 1: "The boys got permission from their parents, and decided to meet in exactly one week with all their camping gear. They would get ready in Scott and Travis' back yard."
Scene 4
Narrator 2: "It was still hot at 7 p.m. when the boys gathered for their campout. The mine camp was just a couple of miles from Scott and Travis' house, so they decided to hike in and then separate at bedtime. They piled their stuff on the concrete patio, just to be sure they had thought of everything."
Justin: "Does everyone have a flashlight?"
Boys: "Yeah!"
Hector: "What about mosquito repellent?"
Boys: "Yeah!"
Scott: "What about matches?"
Justin: "Oh, I don't."
Travis: "Neither do I."
Hector: "I don't either. Scott, do you?"
Scott: "Yeah, I have a few books of them. Do you guys want to use some?"
Boys: "Yeah!"
Scott: "Okay, I have 5 books. With 4 boys, we each get…"
Travis: "I know! 5 divided by 4 = 1R1. We each get 1R1 books of matches."
Narrator 1: "All of a sudden Samantha appeared around the corner of the house."
Samantha: "Hey, guys, are you sharing yet? If you don't share, I'm telling Mom. Sharing is very important!"
Travis: (looking at Samantha) "Hmm…can we share this remainder? I guess so. We'll split open the book and each take 5 matches. How's that for sharing?"
Scene 5
Narrator 1: "Travis, Scott, Justin and Hector took off for the old mine camp. There they cooked a fine dinner over a large campfire and then sat late into the evening, roasting marshmallows and counting the constellations. Travis absent-mindedly threw his book of matches into the fire and watched it flare up and then disappear."

Narrator 2: "Then it was time to find a solitary place to camp. The boys decided to each take 100 steps in a different direction, so they wouldn't be too far away. Travis chose to walk 100 steps up the side of Slickrock Mountain, hoping to find a sheltered niche against a fir tree."
Travis: "This is a good spot. I think I'll sleep here."
Narrator 1: "And so he fell asleep with a sweatshirt for a pillow and fir branches for a blanket. He slept soundly all night."
Narrator 2: "But when he woke up, he wasn't sure at all where he was. Everything looked different by daylight. He tried calling his brother and friends, but no one answered. Knowing the rules of survival, he didn't hike away—instead he waited in the same spot for someone to find him."
Travis: "I'll stay right here. I know my family will come looking soon."
Narrator 1: "The day passed without anyone finding Travis. And as night came, it looked like it would snow. Suddenly, Travis was afraid he was in real trouble!"
Travis: "I wish I hadn't thrown all my matches in the fire. I could really use a signal fire about now. I bet if I built a fire, my family would find me soon, and I would stay warm too."
Narrator 2: "Travis reached his hand deep into his left front pocket, wishing he had that book of matches. Almost unbelievingly, his hand found the five remainder matches that Samantha had insisted be shared."
Travis: "Hey! I have 5 matches! It is important to share a remainder! I can build a fire with these and my family will rescue me!"
Narrator 1: "It didn't take long for Travis to build a roaring fire, with smoke and flames reaching high into the sky. It didn't take much longer for Travis' family to find him, high on Slickrock Mountain, and to bring him home. They had been searching all day, but they had been on the opposite side of the mine camp."
Narrator 2: "Travis was very happy to be home. He'd survived all right, because of Samantha's insistence that they share a remainder."
Travis: "Thanks, Samantha."
Samantha: "Sharing is very important!"

"You Just Drop It!"

Scene 1

Narrator: "Marisol and Shailee have lived next door to each other for nine of their eleven years, and except for a few fights every now and then have been best friends the entire time. Marisol puts up with Shailee's moodiness, and Shailee puts up with Marisol's clumsiness. Best friends have to forgive each other—that's why they are best friends. They do have a lot in common: both love sports and good music, and right now both of them want to be veterinarians when they grow up. In fact, they are discussing their future right now."

Marisol: "Shailee, how do you think we are going to be able to afford all the school it takes to be veterinarians?"

Shailee: "I think we should start saving our money now!"

Marisol: "What money? I don't even get an allowance."

Shailee: "Well, let's start a business! If we can start earning money, we'll be able to start saving money."

Marisol: "What could we do? We're a little old to sell lemonade."

Shailee: "Actually, I've been thinking about this for a while. We could set up a roadside stand and sell baked goods, lemonade, and flowers. If we are smart about it, I think we could earn a lot of money."

Scene 2

Narrator: "Marisol's mom had a connection with a flower wholesaler, and Shailee's grandma made the best cookies and brownies in town. It didn't take long for Marisol and Shailee to have a whole kitchen full of flowers and goodies to sell."

Shailee: "Hold these flowers, Marisol, while I tie ribbons around them. I want to put them in bunches of 7. Hmm…we have 37 flowers. How many bunches of 7 can we make with 37 flowers?"

Marisol: "That's easy! We can make 5R2 bunches. Oops, Shailee, I'm sorry! I dropped those two flowers! I'm so clumsy! I accidentally dropped the remainder!"

Shailee: "Don't worry. We couldn't make a bunch with just two flowers; we couldn't use them anyway. It was okay to drop the remainder. We really only could make 5 bunches. Now, hand me those brownies…"

Narrator: "Marisol gave Shailee a tray of brownies."

Shailee: "Okay, we can fit 6 brownies on each plate. How many brownies do we have?"

Marisol: "We have 34 brownies. With 6 on a plate, we can fill 5R4 plates of brownies…Oh, no! I accidentally dropped 4 brownies! They're just crumbs on the floor now! I'm sorry I'm so clumsy."

Shailee: "Marisol, you are clumsy, but you dropped just the remainder, and we couldn't use it anyway. No one would want to pay for a plate that was only 2/3 full. We still have 5 plates of brownies. But will you carefully hand me the chocolate chip cookies? I need to count them."

Marisol: "I'll count them. There are 50 cookies, and they look really yummy! Let's put them in sets of 8. That way we'll have 6R2 plates."

Narrator: "Marisol started handing the tray of cookies to Shailee. But just before Shailee grasped them, Marisol slipped on the brownie crumbs on the floor and two cookies slid off."

Marisol: "Shailee, I just dropped two of the cookies! What will we do now?"

Shailee: "Marisol, don't worry about that! You just dropped the remainder! They were extra anyway. We still have 6 plates of cookies to sell."

Scene 3

Narrator: "Every weekend Shailee and Marisol sold flowers and baked goods at their roadside stand. Soon their business grew so large that they had to hire more employees."
David and Sean: "Marisol and Shailee, thanks for letting us work for you. What do you want us to do?"
Shailee: "David, will you put the flowers in bunches of 7 and tie ribbons around them? Try to choose colors that look good together."
Marisol: "Sean, will you put the brownies and cookies on plates and wrap them? We sell brownies in sets of 6 and cookies in sets of 8."
David: "There are 58 flowers. That means I can make 8 bunches of flowers with a remainder of two. What do I do with the remainder?"
Sean: "There are 40 brownies. That means I can make 6 plates with a remainder of two. And there are 46 cookies. I have enough cookies for 5 plates with a remainder of 6. What do I do with the remainder?"
Narrator: "Shailee and Marisol just looked at each other and laughed. Then they said to David and Sean…"
Marisol and Shailee: "YOU JUST DROP IT!"

Scene 4

Narrator: "The money kept piling up in the bank, and in a little over fourteen years Marisol and Shailee had their very own veterinary clinic. They were still best friends—Shailee was still moody and Marisol was still clumsy. On a June morning as Shailee was standing behind the front desk, she was surprised to see David and Sean show up at their shop. It had been years since the childhood friends had seen one another."
Shailee: "Hi, David and Sean. It's wonderful to see you! Marisol, (she calls into a room behind her), will you bring drinks for everyone?" (She turns back to David and Sean.)
David: "Hi! I'd like you to meet my wife, Brianna, and our Golden Retriever, Lucky. We brought him to you so he could get his shots. You remember Sean, don't you?"
Sean: "Hi! This is my Chihuahua, Bentley. He needs shots too."
Marisol: (entering with 6 cups of water) "Hi! Here, have some water. Oh, I brought 6 cups when I only needed five…"(she slips, spilling one of the cups onto the floor.) "Uh, oh—I dropped it! Oh, well—it was an unimportant remainder anyway. There's nothing wrong with dropping an unimportant remainder!"

Name _____ Date _____

Playing with Remainders

Listen to the play *"Round-up!"* and then answer the following four questions:

1. 23 people went to the family reunion. If 5 people could fit in each car, how many cars were needed to take people to the *No-Remainder Ranch*?

2. At the *No-Remainder Ranch* 4 people were assigned to each cabin. How many cabins were needed for the 23 people?

3. While they were at the ranch, the family members went on a river-rafting trip. If 6 people could fit on each raft, how many rafts were needed for the 23 people?

4. Why did Tyler and Nikki have to round up their remainders each time in this story?

Listen to the play *"You Just Drop It!"* and then answer the following four questions:

1. How many bunches of 7 flowers could Shailee and Marisol make with 37 flowers?

2. How many plates of 6 brownies each could be made with 34 brownies?

3. How many plates of 8 cookies each could be made with 50 cookies?

4. Why did Shailee and Marisol have to drop their remainders each time in this story?

Listen to the play "*Sharing is Very Important?* and then answer the following three questions:

1. Scott and Travis cut an 11-foot rope into two equal pieces. How long was each piece?

2. Four boys are sharing 17 Airheads equally. How many Airheads does each boy get to eat?

3. Why didn't the remainders in this story need to be rounded up or dropped?

Write a good rule for what to do with remainders.

when you need to round up the remainder	when you need to drop the remainder	when you need to share the remainder equally

Name _____ Date _____

Remainder Stories

Answer each question with a complete sentence. Then tell how you used the remainder (whether you rounded up, dropped, or shared the remainder equally). Last, tell why you used the remainder the way you did. Each problem is worth four points (1 point = correct answer; 1 point = complete sentence; 1 point = correct use of remainder; 1 point = explanation for use of remainder).

1. Skyler is helping his mother plan a wedding breakfast for his older sister, Jessica. They are expecting 63 family members to attend, and they are using round tables that seat 8 guests each. How many tables will be needed to seat 63 people?

2. Skyler and his sister, Rylie, are preparing flower bouquets as centerpieces for each table at the wedding breakfast. They hope to have enough to decorate the table that is displaying the wedding cake as well. They have 67 carnations and wish to put 6 carnations in each bouquet. How many bouquets can they make with 67 carnations?

3. Rylie is going to the zoo for her 12th birthday party, and she is taking 9 friends. The zoo has a new baby giraffe, and groups of 3 children are allowed at a time in a special viewing room to see the giraffe and his mother. How many tours will it take for Rylie and her 8 friends to see the giraffe?

4. At Rylie's 12th birthday party, she wants to give each of her friends a jar with a variety of candy from the candy store. She has 5 friends coming, and she has 113 individually wrapped pieces of candy. If she gives each person the same number of pieces of candy, how many pieces will each friend receive in her candy jar?

5. While Rylie is celebrating with her friends, Skyler's mom gave him $10.00 to share equally with his three best friends so they could buy candy too. How much money do Skyler and his friends each get to spend?

6. Jessica is making curtains for her new apartment. She has 15 yards of material to make 2 sets of curtains. How much fabric can she use for each set?

7. Jessica's mother is serving punch at the wedding reception. She has a punch bowl that holds 106 ounces of punch. How many 8-ounce servings can be poured from the punch bowl when it is full?

Divisibility Rules

Standard I:

Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 3:

Use number theory concepts to develop and use divisibility tests; classify whole numbers to 50 as prime, composite, or neither; and find common multiples and factors..

Intended Learning Outcomes:

1. Demonstrate a positive learning attitude toward mathematics.
3. Reason logically, using inductive and deductive strategies and justify conclusions.

Content Connections:

Science 1; Use science process and thinking skills

Math Standard I

Objective 3

Connections

Background Information

The rules of divisibility are simple formulas for understanding how fair shares can be created from large numbers without practicing long or short division. Students usually come to fifth grade with an implicit understanding about why numbers are divisible by 2, 5, and 10, but it is important in fifth grade to make that understanding explicit. Additionally, the formulas for dividing numbers by 3 and 9 must be taught, since they are rarely discovered by children. It is helpful to separate the formulas for 2, 5, and 10 (which depend on the digit in the ones column) from the formulas for 3 and 9 (which depend on the sum of the digits and the formula for 6, which combines the rules for 2 & 3). Note that there are simple formulas for divisibility by 4, and 8, (as well as more complicated formulas for larger numbers) but they are not part of the Utah fifth grade Core Curriculum requirements. Information about these formulas is included in the curriculum extensions section for interested students.

This lesson should be sequenced after division with whole numbers has been reviewed and practiced, division with remainders has been reviewed and practiced, and students are familiar with vocabulary terms dividend, divisor, and quotient. It may also be used to review prime and composite, since every number greater than 2 that is divisible by 2, 3, 5, 6, 9, or 10 is composite; also, when discussing divisibility, students will probably remember that all numbers are divisible by one and themselves.

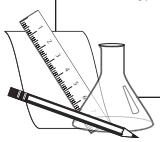
Research Basis

Furner, J.M., Yahya, N., Duffy, M.L. (2005). Teach mathematics: Strategies to reach all students. *Intervention in school and clinic, Vol. 41, No. 1, 16-23.*

In 2000 the National Council of Teachers of Mathematics identified "equity" as the first principle for school mathematics, meaning all children have the right to understand mathematical principles. This article offers 20 teaching strategies to reach the wide variety of learning styles and ability levels in our classrooms as we aim to meet the equity principle. Good lessons may incorporate several of these 20 strategies at one time: we may draw, explain verbally, organize conceptually, demonstrate manipulatively, and practice kinesthetically. Grouping heterogeneously and connecting culturally helps our lessons cross learning barriers and provide opportunities for children to help each other learn.

Materials

- ❑ Index Cards
- ❑ *Divisibility Test*
- ❑ Calculators
- ❑ *Divisibility Rules*
- ❑ Chart paper
- ❑ Markers



Ball, D., (1992). Magical Hopes: Manipulatives and the reform of math education: *American educator,* Summer 1992.

Although this article is 15 years old, its concerns are still valid: are we using manipulatives wisely when we teach mathematics to children? What are the relative merits of different concrete objects? Are lessons using manipulatives sensible to adults because we already understand the concepts they are designed to represent? As teachers it is important for us to understand the purpose behind the manipulatives we use when we design instruction, and it is vital for us to link the activities using manipulatives to the mathematical concepts explicitly for children to make important connections.

Invitation to Learn

Divide the class into teams of three members each. One member is the director, one the recorder, and one the materials coordinator. Each team takes four index cards and writes a different digit from 0-9 on each card. Then, from the four choices of digits, the team makes a list of all the possible four-digit number combinations using each digit once. There will 24 possible number combinations. Next, have the students each take a graphic organizer, *Divisibility Test,* with columns for the numbers they created, plus the columns for 2, 3, 5, 6, 9, and 10 listed across the top. Using calculators if you wish, have the students divide each of their 24 numbers by 2, 3, 5, 6, 9, and 10 to decide if their numbers divide evenly without leaving remainders. If the number divides evenly, have the students write "yes" in the column on the graphic organizer. If the number does not divide evenly, have the students write "no" in the column on the graphic organizer.

After the graphic organizer is complete, have each team record their "yes" examples on chart paper hanging around the room, one piece for

each of the numbers 2, 3, 5, 6, 9, and 10. Once this is done, have each team make a hypothesis about a "rule" for divisibility for each of the numbers 2, 3, 5, 9, and 10. Have them record their hypotheses on the graphic organizer labeled *Divisibility Rules*. It is important that each child have his or her own copy of the two graphic organizers because the next part of the lesson is done as a whole class.

Instructional Procedures

1. After teams have completed their *Divisibility Test* graphic organizer, recorded their numbers on the chart paper, and made hypotheses about divisibility on their *Divisibility Rules* graphic organizer, have them return to their individual seats for a whole-class lesson.
2. Using the chart paper lists as summaries of numbers generated by the class teams, discuss each chart and have the students share their hypotheses of divisibility rules. Guide their discussions to the correct rules for each number, and have them write them on the graphic organizer. Then have them trim the edges of their graphic organizers and glue them into their math journals for later referencing.
3. Ask the students if it is possible to divide their rules into two main categories, using a Venn Diagram to compare and contrast the categories. Lead them to separate the numbers where the ones digit determines the divisibility (2, 5, 10) from the numbers that require adding all the digits (3, 9). Have them complete a Venn Diagram in their math journals while you model one on the board.
4. Play *Divisibility Rocks* using students' journals as reminders of the divisibility rules. Note: if this game is used as one station in a variety of center activities, fewer sets of the game will need to be produced.

How to play *Divisibility Rocks*:

1. Divide the class into groups of two to six students per game. (An ideal size game is three students because each player will always have a job.)
2. Give each group one game set. Each set requires a deck of cards, a bag of rocks, and a *Divisibility Key*.
3. Divide the cards face down evenly among members of a group. Discard any remaining cards. Pile the rocks in the center of the game.

Materials

- ❏ Math journals
- ❏ *Divisibility Rocks Cards*
- ❏ *Divisibility Pebbles*
- ❏ *Divisibility Key*

4. Decide which person will be the first Player. The person to his or her right will hold the *Divisibility Key* and the person to his left will be the Challenger.
5. The first Player turns over his or her top card. The person holding the *Divisibility Key* asks, "is it divisible by 2?" If the Player answers, "yes," then he takes a rock from the pile. The process is repeated with the numbers 3, 5, 6, 9, and 10, with the Player taking a rock for each "yes" answer. (An example is a student would receive three rocks for the number 10 because it is divisible by 2, 5, and 10.)
6. Then the person with the *Divisibility Key* turns to the Challenger and asks, "do you want to challenge him?" If the Challenger believes any answers were incorrect, he or she may answer "yes," telling what numbers are believed to be incorrect.
7. If the Challenger is correct, he gets all the rocks from the Player. If the Challenger is incorrect, he forfeits the next turn.
8. If the Player is wrong and the Challenger refuses to challenge, the person with the *Divisibility Key* corrects the turn and corrects the number of rocks taken.
9. The play then moves clockwise to the left, with the past Player now responsible for the *Divisibility Key*, and the Challenger becoming the next Player.
10. At the end of a round, the person with the most rocks collects the cards used in the round and all the rocks are returned to the center of the game. A new round is played.
11. At the end of a round, if there is a tie, both Players involved in the tie turn over their next card and collect the rocks for that card. Whoever holds the card that earns the most rocks wins the round.
12. A player is out when he is out of cards; the Player with all the cards at the end of the game is the winner.
13. To shorten the game, the teacher may set a time limit; the person with the most cards at the end of the allocated time is the winner.

Assessment Suggestions

- Pre-assessment: Observe the children's hypotheses as they write on their graphic organizers to see if their prior knowledge about divisibility is accurate, especially with numbers 2, 5, and 10.

- Formative assessment: Check for accuracy as students write correct rules on their graphic organizers, complete their Venn diagrams, and verbalize their responses during the Divisibility Rocks game.
- Final assessment: Using the Divisibility Test graphic organizer as a master, list ten numbers with a variety of divisibilities and have the students complete the chart with "yes" or "no" answers.

Curriculum Extensions/Adaptations/ Integration

- Advanced learners may enjoy discovering the rule of divisibility for 4 (last two digits are either 00 or are divisible by 4), and the rule for 8 (last three digits are divisible by 8). Rules for higher numbers are available on the web sites listed in the additional resources.
- Why do the rules work? Advanced learners may enjoy hypothesizing about the rules for 3 and 9—why are adding digits meaningful? Explanations are given on the web sites listed in the additional resources.
- Heterogeneous grouping for the invitation to learn and the card game help struggling learners through cooperative processes.
- The scientific method is used to discover mathematical absolutes. Children may recognize science vocabulary as the rules for divisibility are discovered through the formation of hypotheses, the gathering of data, the formation of conclusions, etc. Explicit teaching of these vocabulary terms strengthens both areas of science and mathematics.

Family Connections

- Can the rules of divisibility apply to real-life situations? Ask the students to find at least one example after school where the rules of divisibility shorten the task of creating equal shares. An example: mom fries scones and makes 15 scones. She knows they can be divided evenly among the five people in her family. Repeat this assignment for a few days, until everyone has had a chance to discover an example.
- Are the rules of divisibility for 3 and 9 unfamiliar enough to mystify people? How many people can you surprise by asking them to tell you a 10-digit number and then you telling them

whether it is divisible by 3 and 9? Record their numbers and their comments and report back to class for a discussion.

Additional Resources

Web sites

http://www.homeschoolmath.net/teaching/md/division_rules.php

http://www.math.about.com/library/bldivide.htm

http://argyll.epsb.ca/jreed/math7/strand1/1104.htm

http://www.mathforum.org/k12/mathtips/ward.html

Name _____ Date _____

Divisibility Test

1. Write the 24 numbers you created in the first column.
2. Decide if your numbers are divisible by 2, 3, 5, 6, 9, or 10. Write yes or no in the correct colums.

Number	2	3	5	6	9	10
1.						
2.						
3.						
4.						
5.						
6.						
7.						
8.						
9.						
10.						
11.						
12.						
13.						
14.						
15.						
16.						
17.						
18.						
19.						
20.						
21.						
22.						
23.						
24.						

Name _____ Date _____

Divisibility Rules

Number Divisible By	My Hypothesis	The Actual Rule
2		
3		
5		
6		
9		
10		

How to Play "Divisibility Rocks"

1. Take a deck of cards, a Divisibility Key, and a bag of rocks.
2. Divide the cards face down evenly among players. Discard any extras.
3. Place the pile of rocks in the center of the playing circle.
4. Decide who is first. The person to his right is in charge of the Divisibility Key.
5. The first player turns over his top card and decides if the number on the card is divisible by 2, 3, 5, 9, and 10. He takes one rock from the center pile for each "yes" answer.
6. If the player to the left disagrees, he or she may "challenge" by saying "Challenge!" Then both players appeal to the person holding the key to see who is right. If the challenger is correct, that person gets the rocks. If the challenger is incorrect, the original player gets to keep the rocks and the challenger loses his or her turn.
7. Play continues clockwise with each person taking a turn, rotating the person who holds the key and the person who is the challenger.
8. When every player has had a turn, the rocks are counted. Whoever has the most rocks gets to keep all the cards from that turn. The rocks are returned to the center pile.
9. If there is a tie, both players involved in the tie turn over their next card and collect the rocks for that card. Whoever holds the card that earns the most rocks wins the round.
10. A player is out when he or she is out of cards; the player with all the cards at the end of the game is the winner.
11. To shorten the game, the teacher may set a time limit; the person with the most cards at the end of the allocated time is the winner.

Divisibility Rocks Cards

24	34
35	36
44	46
48	55

Divisibility Rocks Cards

56	57
60	62
65	72
74	75

Divisibility Rocks Cards

80	84
98	115
117	128
130	140

Divisibility Rocks Cards

150	160
171	175
190	196
200	216

Divisibility Rocks Cards

240	256
260	285
308	309
335	338

Divisibility Rocks Cards

385	408
429	438
447	495
524	567

Divisibility Rocks Cards

625	657
666	669
700	711
715	728

Divisibility Rocks Cards

735	741
770	771
849	888
915	960

Divisibility Rocks Cards

1115	1135
1280	1324
2204	2220
2225	2318

Divisibility Key

Number	2	3	5	6	9	10
24	Y	Y	N	Y	N	N
34	Y	N	N	N	N	N
35	Y	Y	N	Y	Y	N
36	Y	Y	N	Y	Y	N
44	Y	Y	N	N	N	N
46	Y	N	N	N	N	N
48	Y	Y	N	6	N	N
55	N	N	Y	N	N	N
56	Y	N	Y	N	N	N
57	N	Y	N	N	N	N
60	Y	Y	Y	6	N	Y
62	Y	N	N	N	N	N
65	N	N	Y	N	N	N
72	Y	Y	N	Y	Y	N
74	Y	N	N	N	N	N
75	N	Y	Y	N	N	N
80	Y	N	Y	N	N	Y
84	Y	Y	N	Y	N	N
98	Y	N	N	N	N	N
115	N	N	Y	N	N	N
117	N	Y	N	N	Y	N
128	Y	N	N	N	N	N
130	Y	N	Y	N	N	Y
140	Y	N	Y	N	N	Y
150	Y	Y	Y	Y	N	Y
160	Y	N	Y	N	N	Y
171	N	Y	N	N	Y	N
175	N	N	Y	N	N	N
190	Y	N	Y	N	N	Y
196	Y	N	N	N	N	N
200	Y	N	Y	N	N	Y
216	Y	Y	N	Y	Y	N
240	Y	Y	Y	Y	N	Y

256	Y	N	N	N	N	N
260	Y	N	Y	N	N	Y
285	N	Y	Y	N	N	N
308	Y	N	N	N	N	N
309	N	Y	N	N	N	N
335	N	N	Y	N	N	N
338	Y	N	N	N	N	N
385	N	N	Y	N	N	N
408	Y	Y	N	Y	N	N
429	N	Y	N	N	N	N
495	N	Y	Y	N	N	N
524	Y	N	N	N	N	N
567	N	N	N	N	N	N
625	N	N	Y	N	N	N
657	N	Y	N	N	Y	N
666	Y	Y	N	Y	N	N
669	Y	Y	N	Y	N	N
700	Y	N	Y	N	N	Y
711	N	Y	N	N	Y	N
715	N	N	Y	N	N	N
728	Y	N	N	N	N	N
735	N	Y	Y	N	N	N
741	N	Y	N	N	N	N
770	Y	N	Y	N	N	Y
771	N	Y	N	N	N	N
849	N	Y	N	N	N	N
888	Y	Y	N	Y	N	N
915	N	Y	Y	N	N	N
960	Y	Y	Y	Y	N	Y
1115	N	N	Y	N	N	N
1135	N	N	Y	N	N	N
1280	Y	N	Y	N	N	Y
1324	Y	N	N	N	N	N
2204	Y	N	N	N	N	N
2220	Y	Y	Y	Y	N	Y

Math I-2

Activities

Fractions

Equal Shares

Standard I:

Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 2:

Explain relationships and equivalencies among integers, fractions, decimals, and percents.

Intended Learning Outcomes:

3. Reason logically using inductive and deductive strategies and justify conclusions.
4. Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.

Content Connections:

Math II-1; recognize, analyze, and use patterns
Math IV-1a; using customary units of measurement

Math Standard I

Objective 2

Connections

Background Information

This exploration is best done following a class discussion lead by the teacher of what a fraction is and what it really represents. Often time students are intimidated with the concept of fractions. Have them relax and just think of the fraction as another way to write or express a division equation. Mathematicians are known to be very "efficient" folks and seem to always find the most efficient way to write, express, and communicate things quickly. They are always anxious to move on and get the job done. Show the students the ÷ symbol. Do you see the fraction model in this symbol? The line means to "share equally." The denominator is the number of shares. Students relate to sharing with friends, so you might refer to the denominator as, "how many friends you will be sharing with?" The numerator is the portion of the shares to be considered. You can actually cover that number of shares with your hand to give the students an action cue to depend on. The following exploration and experience with the manipulatives is to as much uncover what the students know as much as to allow them to discover!

Research Basis

Zull, J.E. (2004). The art of changing the brain. *Educational leadership*. September 2004

This article explores the fact that learning should feel good. When a student is experiencing, exploring, developing connections, and learning then positive emotions are generated. This biochemical reward of learning is not provided by explanations from the teacher,

Materials

- ❑ Licorice rope

but by the student developing their own idea and ownership of those ideas. It goes on to discuss that the way we feel always influences our brain and strengthens growth and wiring. The article shares some best practices for teachers to optimize learning in the classroom.

De Geest, E., & Watson, A., (2004). Instilling Thinking. *Mathematics Teaching*. June 2004.

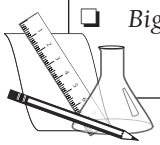
This article shares research done to identify and develop ways of stimulate mathematical thinking. It explores the common practice of giving students in the lowest achieving group repetitive, simplified mathematics. When studies show that more good is done helping learners develop thinking skills and understanding throughout every level of mathematics lessons. This with a teachers high expectations help a student's self-awareness that they are learning and progressing. Students showed significant gains in self-esteem and their ability and willingness to engage with extended, unfamiliar, and complex tasks.

Invitation to Learn

Provide each group of four to five students with a single licorice rope. Ask them to share this one licorice rope with the group "equally." Don't allow them to eat the shares until you have a chance to talk as a class. This activity will only take a few minutes. Children share everyday, all day long, so they will jump right in and get busy sharing. Travel among the groups and listen for snippets or phrases being said during the sharing. Pull the class together and share things you heard and go right into a discussion of "sharing equally." Depending on the responses and your assessment of understanding you might need to "share" more objects on the overhead with the class. Then share the traditional fraction model. Discussing and clarifying as needed. Let them eat!

Materials

- ❑ Manipulative set (one per class)
- ❑ Construction paper-variety of colors
- ❑ Scissors
- ❑ *Can You Make?*
- ❑ *Share Equally*
- ❑ *If This Is...?*
- ❑ *Matching Bars Game*
- ❑ *Big Inch*



Instructional Procedures

1. Provide single manipulative sets on a table or area where students have access: fraction circles, fraction pieces, pattern blocks, fraction bars, 12-centimeter cubes, yard stick, ruler, egg carton, *Cake* worksheet, number line 0-1, and *Clock* worksheet. These are suggestions only. You can pare down the choices or add others depending on the degree of challenge you wish to deal with and availability.
2. Challenge the students to show, model, and name as many equal shares of the tool, object, or manipulative being used.

3. Invite the individual groups to pick the manipulative of their choice.
4. Circulate among the groups and assess knowledge level, vocabulary being used, and progress. Allow about ten minutes for group members to interact on the task.
5. Then suggest to the class the use of a graphic organizer, *Can You Make?*, to help record findings.
6. Some explanation of how the *Can You Make?* graphic organizer is set up and its use may be needed and this usually works itself out if you take a manipulative and start working through an example on the overhead.
7. Students continue working and complete organizer to twelfths. There is value in the sketching of the manipulative pieces and a few groups may be confronted with having to construct sevenths, ninths, and elevenths. Provide construction paper of colors not represented in manipulative pieces.
8. Groups will then present findings to the total class. This will give an opportunity for you to discuss proper vocabulary in depth and clear up misconceptions that might have come up. This is a rich exploration. Students access prior knowledge, organize findings, organize patterns, interpret patterns, identify equivalents, process proportions, use estimation, order relationships of fractions to the whole, and make connections to other concepts in mathematics.
9. Have groups record the patterns that developed as they filled in the graphic organizer. Do this in traditional fraction representation.
10. Discuss and write equivalent fractions on a chart, overhead or chalkboard as they are shared.
11. In their math journal or on the bottom of the *Can You Make?* graphic organizer have them write: What I learned or discovered from this experience?

Assessment Suggestions

- A performance assessment is built into the completion of *Can You Make?* graphic organizer.
- Observation and interview of the experience.
- Journal writing of students reflection on the experience.

Curriculum Extensions/Adaptations/ Integration

- An extension for advanced learners would be the worksheet, *Share Equally* and/or *If This Is...?*
- Adaptations for learners with special needs or as a re-teaching activity for a smaller group is the *Matching Bars Game*.
  1. Place the fraction bar set of 16 pieces face down in the center of the group. Arrange them in equal rows and columns.
  2. To determine which player goes first: each player picks one of the face down bars. The player with the greatest amount shaded goes first. Replace the bars face down.
  3. Now take turns turning over two bars per turn that have the same shaded amount. If the shaded amounts are the same, he keeps the bars and goes again.
  4. If the two bars do not have the same amount shaded, they are turned over again and the next student takes a turn.
  5. Play continues until all the bars have been matched. The student with the most matching bars wins.
- Another adaptation for those needing further practice in linear and length models is the folding activity *Big Inch*.
  1. Pretend that the paper is going to be an inch magnified.
  2. Fold the paper in half end to end.
  3. How many sections do you have?
  4. Draw a line along the fold about three inches long.
  5. Write ½ under that line.
  6. Now fold the paper in half again.
  7. How many sections do you have?
  8. Draw a shorter line on each fold.
  9. Write 1/4 under the first line, 2/4 on the second line, and 3/4 on the last fold line that was created.
  10. Now fold the paper in half again.
  11. How many sections do you have now?
  12. Fill in the numbers on the folds created.
  13. Now fold the paper in half again.
  14. How many sections?
  15. Fill in the numbers on the folds created.
  16. Discuss the experience and allow students to measure with their Big Inch.

- Take the pattern blocks and change the unit whole. For example: two yellow hexagons equal one. What would be the value of the other pieces?
- Have students create a design with pattern blocks. What is the design's value if the unit whole is the green triangle?

Family Connections

Home Fraction Hunt:

- What are the most common fractions found in the home?
- Where are most of the fractions found in your home?

Additional Resources

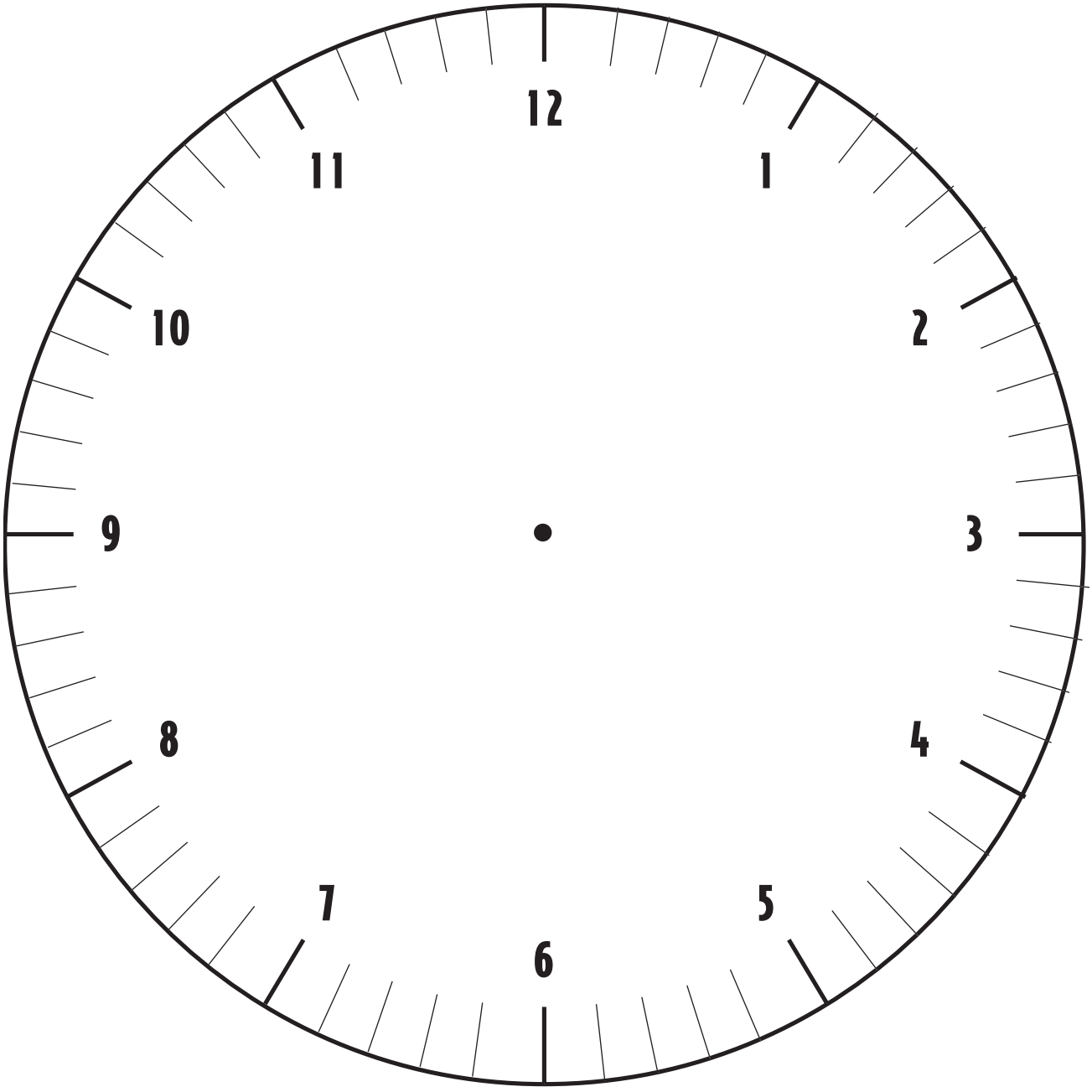
Web sites

Interactive math practice for elementary students www.mathplayground.com/Fraction_Bars.html

Interactive lessons and explorations and information about fraction bars for students, teachers, and people of all ages in mathematics and science. www.fractionbars.com

The Musical Fraction Bars activity connects your knowledge of fractions and length to ... How to play Musical Fraction Bars: Scroll down until you see the ... www.philtulga.com/fractionbars.html

Clock



Cake

Can You Make?

Can You Make?	with wholes	with halves	with thirds	with fourths	with fifths	with sixths
1						
1/2						
1/3						
1/4						
1/5						
1/6						
1/7						
1/8						
1/9						
1/10						
1/11						
1/12						

Can You Make?

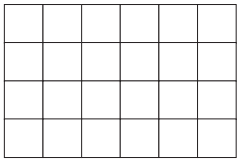
Can You Make?	with sevenths	with eighths	with ninths	with tenths	with elevenths	with twelfths
1						
1/2						
1/3						
1/4						
1/5						
1/6						
1/7						
1/8						
1/9						
1/10						
1/11						
1/12						

Share Equally

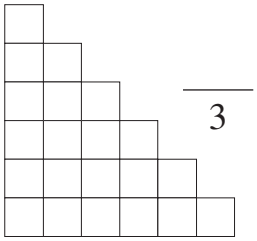
Share each of the items below as directed. Be prepared to explain how you did the sharing.

3

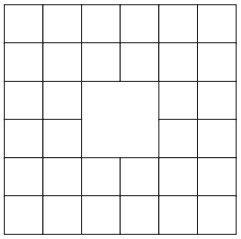
4



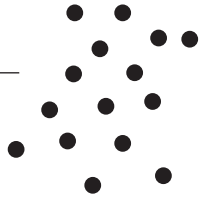
12



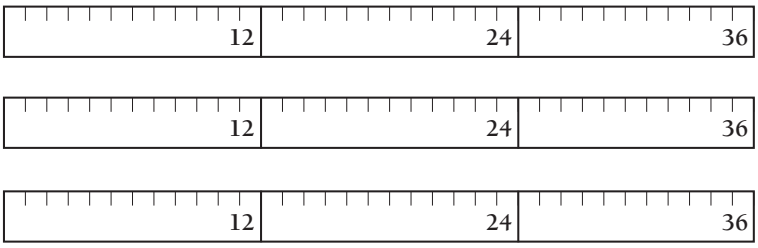
3



3



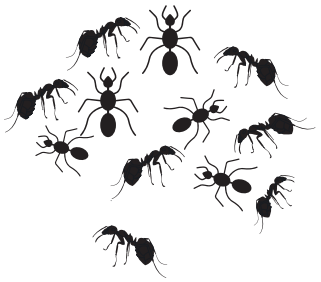
5



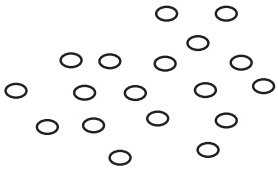
9

8

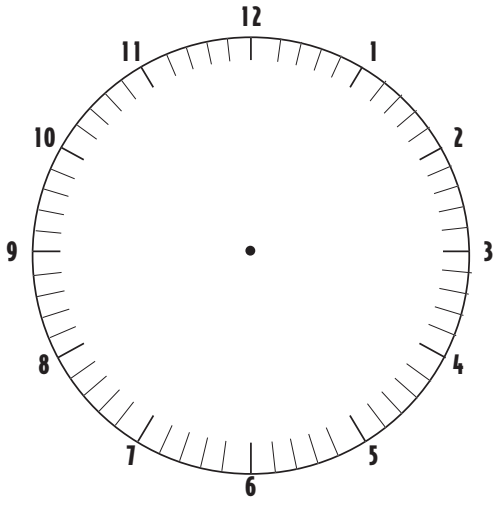
4



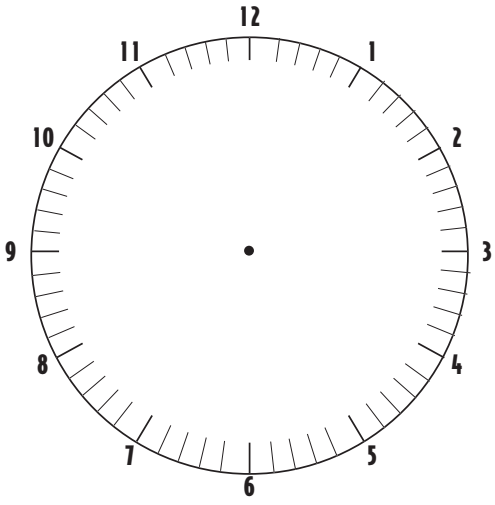
4



6



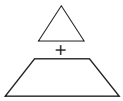
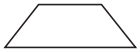
8

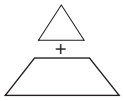
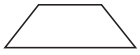


5

"If This Is...?

Challenge yourself to find the answers without using pattern blocks. Then if you need check with pattern blocks. Happy thinking!

If  is					
1/2					
1/4					
1/8					

If  is					
1					
2					
1/2					

Tangram Toil

Math Standard I

Objective 2

Connections

Standard I:

Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 2:

Explain relationships and equivalencies among integers, fractions, decimals, and percents.

Intended Learning Outcomes:

2. Become effective problem solvers by selecting appropriate methods employing a variety of strategies, and exploring alternative approaches to solve problems.
3. Reason logically using inductive and deductive strategies and justify conclusions.
4. Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.

Content Connections:

Math II-1; recognize, analyze, and use patterns

Background Information

Students should be comfortable with "sharing equally." This activity provides students experience with more physical models. The more rich area, set, and length models provided, the more meaning for the students. Tangram Toil makes the transition to algorithms more successful. This will give them an opportunity to apply and practice what they understand about equivalence, fractional relationship to the unit whole, use fraction vocabulary, and create the concrete representation.

Research Basis

Solomon, M., & Hendren, R. (2003). A critical look at brain-based education. *NAESP Middle Matters.* 12(1).

This article addresses new brain research in respect to how children learn. Quantitative thinking requires many different component skills, including decoding of symbols, understanding quantities, counting ability, representing abstract objects, and understanding part-whole relationships. Math teaching strategies need to help students develop representations of number-related concepts, transfer lower-level rote math skills to higher-level problem solving, and generate multiple solutions to problems. This article discusses the importance of simulations, role-play, hands-on activities, collaborative decision-making, group problem solving, and movement for the formation of complex neural connections in the brain.

Green, F.E. (2006). Brain and learning research: Implications for meeting the needs of diverse learners. *Education*. 119(4).

This extensive research article provides implications of meeting the needs of the diverse learners in the classroom. It shares dramatic developments related to brain structure, multiple intelligences, learning styles, emotions and learning, music and cognitive development, and brain-based learning.

Invitation to Learn

Materials

- ❑ Bike picture/cards
- ❑ *Bikes*

As students enter the room they will find various bike picture cards on their desks. Invite students to look over the bike cards, and complete the *BIKE* worksheet provided for them at their desks. Follow with a class discussion of results and ask other probing questions to assess understanding. Pose the opportunity for students to offer questions of fractional relationships with the bikes.

Instructional Procedures

Materials

- ❑ Tangram manipulatives
- ❑ Milky Way miniature candy bars
- ❑ *Milky Way Fraction Hunt*
- ❑ Journal

1. Provide a group of students, pairs of students, or individual students with a tangram manipulative. This will depend on the amount of tangram sets you have.
2. Challenge the students to put the tangram pieces together to form a square. This may take awhile.
3. Once assembled they are to find the fractional value of each of the tangram pieces. Assume that the original square is the whole or one.
4. An additional challenge depending on the students' prior knowledge of decimals would be to refer to the whole as 1.00 or $1.00. Then challenge them to find the decimal value of each tangram piece. This provides an opportunity to practice fraction to decimal relationships.
5. Another challenge would be to change the unit whole. The large right triangle could become the area whole.

Assessment Suggestions

- *Milky Way Fraction Hunt* has a built in self-assessment.
  1. Provide the *Milky Way Fraction Hunt* worksheet on each students' desk.
  2. Place a basket or box of miniature Milky Way bars under the South Pole of a globe in the room prior to students arriving.

Hopefully not too obvious to students' sight as students will decode the clues and follow directions to the candy bars.

Curriculum Extensions/Adaptations/ Integration

- Brain storm and/or provide a list of suggestions of real world fraction opportunities.

The list might resemble: class members, physical classroom, one student's outfit, teachers in building, shoes on students, vowels/ consonants in names, M&M candies in a bag, Skittles in a bag, handful of Fruit Loops cereal, Valentines candies, cars in parking lot, lunch items on plate (carbs/proteins), assignment scores, opinions, and so forth.

As a writing extension have students write their own fraction problem about one of the suggestions in the list. Write the problem on a sheet of paper. Sign your name and add to a class set of problems in book form, exchange with a classmate for them to solve, or use in a center.

Family Connections

- Challenge students to write a fraction hunt (like *Milky Way Fraction Hunt*) of their own to share with the class.

Additional Resources

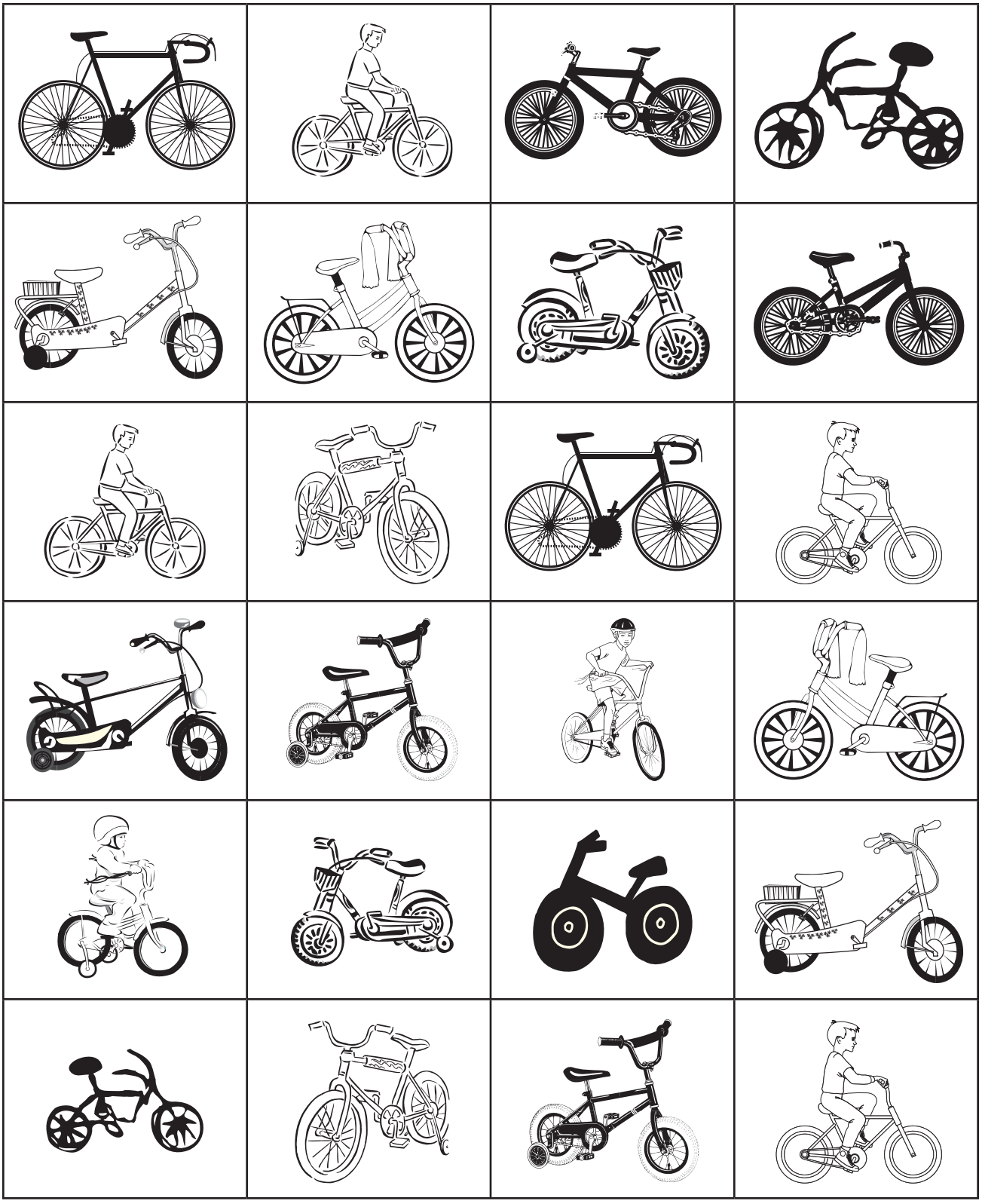
Web sites

Tangrams and Fractions ...

www.pbs.org/teachersource/mathline/concepts/asia/

www.europa.com/~paulg/mathmodels/frac_area.html

BIKE Picture Cards



BIKES

What is the whole unit that you are considering? _____

What fraction of the bikes are mountain bikes? _____

What fraction of the bikes have gears? _____

What fraction of the bikes have training wheels? _____

The boy style frames on the bikes represent _____ of the bikes.

How many of the handle bars have streamers on them. _____

How many of the bikes have fenders? _____

Make **two** true statements about the bike colors using the terms: **half, third, fourth, fifth, sixth, eighth, and/or tenth.**

1._____

_____

_____

2._____

_____

_____

BIKES

What is the whole unit that you are considering? _____

What fraction of the bikes are mountain bikes? _____

What fraction of the bikes have gears? _____

What fraction of the bikes have training wheels? _____

The boy style frames on the bikes represent _____ of the bikes.

How many of the handle bars have streamers on them. _____

How many of the bikes have fenders? _____

Make **two** true statements about the bike colors using the terms: **half, third, fourth, fifth, sixth, eighth, and/or tenth.**

1._____

_____

_____

2._____

_____

_____

Name _____ Date _____

Milky Way Fraction Hunt

Write the appropriate parts of the words on the line to form a new word.

1. The first half of food + the last quarter of door.

_____

2. The last third of hat + the first 2/5 of heavy.

_____

3. The second 1/3 of office + the last 1/4 of door + the first 1/3 of street.

_____

4. The last half of go + the last 1/2 of done.

_____

5. The last 1/8 of elephant + the first 1/5 of order.

_____

6. The first 3/4 of fine + the last 3/4 of dish.

_____

7. The last 1/6 of cement + the first of 3/7 of history.

_____

8. The last half of bath + the finest 1/3 of end + the last 2/7 of require.

_____

9. The first 2/5 of water + the last 3/4 of fits.

_____

10. The last 1/6 of Glenda.

_____

11. The first 1/3 of principal + the first half of zero.

_____

12. The first 1/7 of instant + the first third of fat.

_____

13. The first 2/5 of young + the first 1/10 of understand.

_____

14. The first 1/4 of ugly + the first 1/5 of settlement.

_____

15. The first 1/4 of youthful + the last half of pour.

_____

16. The first 1/4 of hesitate + the last 2/3 of sad.

_____

17. The last 1/3 of rat + the first 2/5 of heart.

_____

18. The first 3/7 of mileage + the last 2/3 of sky.

_____

19. The first 1/5 of white + the last 1/3 of Friday.

_____

20. The last 1/4 of Meri + the first 1/5 of Susan.

_____

21. The first 3/5 of dirty + the last 3/7 of perfect + the first 2/5 of Lynda.

_____

22. The first 3/4 of bent + the last 2/3 of breath.

_____

23. The first 1/3 of Thomas + the first 1/8 of Endicott.

_____

24. The first 3/5 of sound + the last 2/9 of Aylsworth.

_____

25. The first quarter of positive + the first two thirds of Lee.

_____

26. The first 3/5 of quick + the second 1/4 of meat + the last 1/3 of patiently.

_____

27. The first third of get + the second fourth of Jody.

_____

28. The first half of loud + the last half of book.

_____

Write the clues in numerical order:

_____

_____

_____

The ANSWER KEY for the activity is: FOR THE FIRST ONE TO FINISH THIS THERE WAITS A PRIZE IF YOU USE YOUR HEAD THE MILKY WAY IS DIRECTLY BENEATH THE SOUTH POLE QUIETLY GO LOOK

Math IV-1&2

Activities

Measurable Attributes

What's Inside?

Standard IV:

Students will determine area of polygons and surface area and volume of three-dimensional shapes.

Objective 1:

Determine the area of polygons and apply to real-world problems.

Intended Learning Outcomes:

2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.
5. Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.

Content Connections:

Math III-1 Analyze attributes of geometric shapes
Language Arts VIII-2; compose written work

Math Standard IV

Objective 1

Connections

Background Information

This lesson is an opportunity for students to explore area without the requirement of a formula to determine the area of a polygon. These hands-on activities will help students understand the concept of area. Students need to understand that area is the number of square units inside a flat, two-dimensional figure. Math concepts such as area can be abstract and hard to understand. Teaching area from a non-formula basis helps the understanding of the concept become more concrete, making the transition to using a formula easier. Finding the area of an irregular figure requires students to decompose the figure into smaller rectangles or triangles, finding the area of the smaller figures and then adding.

Before beginning this lesson, students should be able to easily identify and name different polygons. It will help if students have worked with polygons in composing other shapes. This lesson serves as a good introduction into area and the determination of such based on a predetermined square unit. By using paper tiles to represent one square unit, students are constructing their own mathematical understanding of area. Practicing with the area of polygons, sets up students for success in developing an understanding of the surface area of a three dimensional solid.

Research Basics

Moyer, P. (2004). Controlling choice: teachers, students and manipulatives in mathematics classrooms. *School Science and Mathematics.* 104(1). 16-32.

This research study of instructional practices of teachers shows those who demonstrate the use of manipulatives as a tool for better understanding of concepts and allow access to manipulatives often are opening doors for students struggling with abstract concepts. Allowing the use of manipulatives encourages student ownership of strategies, ideas, and processes and gives students a strong conceptual base on which they can begin to construct higher mathematical thinking.

Furner,J., Yahya, N., Duffy M.L. (2005). 20 Ways to teach mathematics: strategies to reach all students. *Intervention in School and Clinic.* 41(1). 16-23.

Educators must make every effort to ensure all students have equal access to learning mathematics. Incorporating multiple intelligences enables all learners the opportunities to develop mathematically. Applying skills to a problem-solving task benefits learners in later applying information to real life situations.

Invitation to Learn

Materials

- ❑ Tangram sets
- ❑ Tangram shape laminated cards

Provide small groups or pairs of students with a set of tangrams and a laminated card with a shape on it. Pairs will try to reproduce the shape with no overlapping pieces from the tangram set. This activity leads right into a discussion of what area is and how is it determined.

Instructional Procedures

Materials

- ❑ Math journal
- ❑ Pencil
- ❑ *Pezzettino*
- ❑ Various 1 cm paper tiles
- ❑ *One-on-the-Mountain*
- ❑ Glue sticks
- ❑ Tangram sets
- ❑ Overhead tangram set

1. Read *Pezzettino* to students, emphasizing the illustrations.
2. Discuss and define area with students as the number of square units inside a figure. Share with students that we are finding area without the use of a formula. Also revisit polygons to clarify student understanding.
3. Hand out *One-on-the-mountain* from the story and colored paper tiles. Using the tiles, have students fill in the area of the animal and determine the number of square units used, or area. Students will record area on bottom of sheet and attach to journal.
4. Handout sets of tangrams to individual students.
5. Starting with the small square, trace around shape in journal. Assign the square the area of one square unit. Write one square unit next to the square.
6. Using overhead tangrams, make a square with the two small congruent triangles. Ask students what the area of the square

is? What would the area of one triangle be? Trace around one of the congruent triangles and label the area next to it.

7. Continue on for each different piece within the tangram set. Remind students to label the area for each shape. Allow students time to discover the make-up of each shape.
8. Discuss how area of different shapes can be determined with tangrams. Help students having trouble with the building or visualization of filling the area with other shapes.
9. Students will now be able to make any polygon with the tangrams and determine the area. Each person will design a polygon for a partner to determine the area. Students will trade polygons and determine the area.
10. Allow several pairs to share a polygon, using the overhead tangrams, with the class.
11. Using an overhead tangram set, display a trapezoid. As a class have students determine the area of a trapezoid using the overhead tangram pieces.
12. In Math Journal have students construct and trace their trapezoid. Have students trade journals and determine the area.

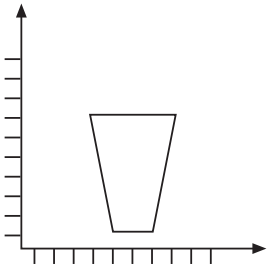
Assessment Suggestions

- Assess ability to determine area by supplying tangram silhouette and assigning a random unit for the square.
- Math journal–examples of polygons with area determined.
- Repeat the same steps with the tangram, changing the square unit to another number such as 3.
- Observation and discussion of the activity.
- Journaling–Ask students to define area and explain one way to determine area.

Curriculum Extensions/Adaptations/ Integration

- Using black line tangram animal figures from the Invitation to Learn, determine the area.
- Special needs students may glue down tangram paper tiles to determine area.
- Use this lesson as a first step in helping students discover their own formula for area.

- Measure the actual area of each tangram piece and chart results of measurements.
- Have students build their own tangram shape, specify a square unit and have a partner determine the area.
- Apply to real life situations by determining the amount of floor covering needed for a room or tile patterns for a floor.
- Language Arts–integrate curriculum by having students write a story, design and illustrate with tile animals, determine and label area of each animal. Write a class story and each student illustrate a portion of the story. Share story with a younger grade level.
- Students can determine area of regular/irregular polygons using coordinate graphing of the polygon vertices and diagonal multiplication.
  1. Place a polygon on coordinate graph and determine ordered pairs of vertices.
  2. List vertex pairs going around the polygon and include starting point at end. Diagonally multiply both sides and total.



	5	1	
4	4	6	30
48	8	6	24
42	7	1	8
5	5	1	7
99			69

  3. Find the difference between the 2 sums and divide by 2.

$$\frac{99-69}{2} = 15 \text{ sq units}$$

Family Connections

- Students will find a polygon shape in the home, trace or plot onto graph paper and determine the area using predetermined square unit or diagonal multiplication.
- Compile a list of uses of area within the home. Share list with the class.
- Send home a set of tangrams for students to share with family.

Additional Resources

Books

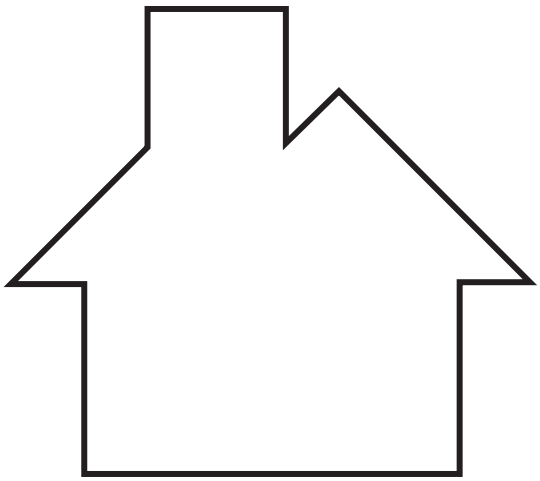
Pezzettino, by Leo Lionni; ISBN 039483156

Tangram Puzzles, by Chris Crawford; ISBN 080697589

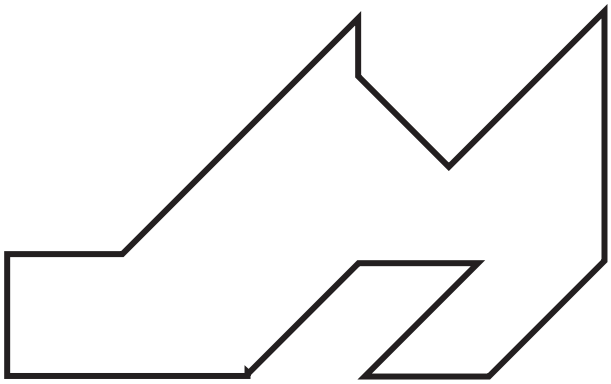
Web sites

http://www.tangrams.ca/

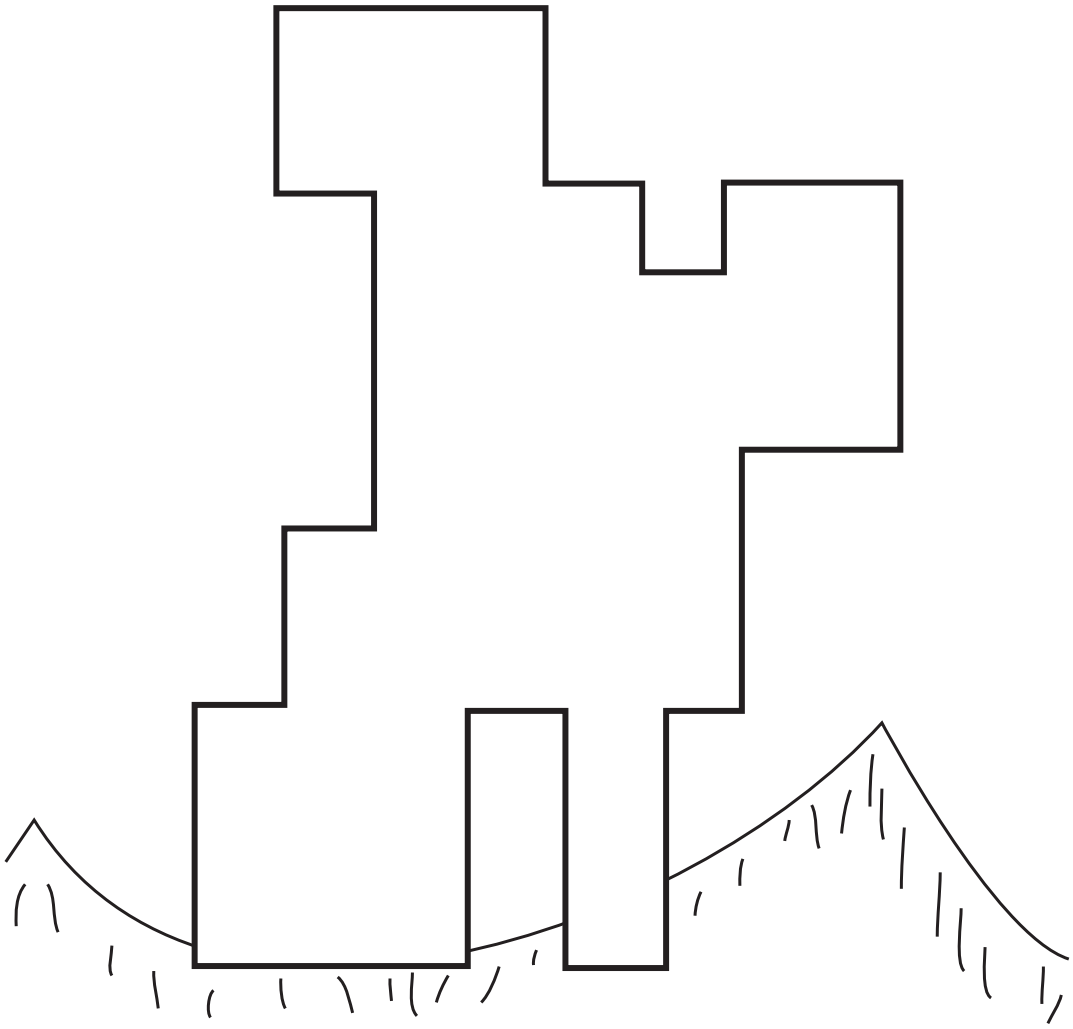
Tangram Puzzle



House



Shoe



One-on-the- Mountain

Fill 'Em Up

Standard IV:

Students will determine area of polygons and surface area and volume of three-dimensional shapes.

Objective 2:

Recognize, describe and determine surface area and volume of three-dimensional shapes.

Intended Learning Outcomes:

2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.
3. Reason logically, using inductive and deductive strategies and justify conclusions.

Content Connections:

Science I-1; All matter has volume
Math III-1; Geometric shapes

Math Standard IV

Objective 2

Connections

Background Information

Volume of a three-dimensional figure is the amount of space inside the figure. While students can be shown and taught the formula to determine a volume measurement, they often need an exploration activity to actually comprehend the more abstract concept. This lesson may serve as an introduction to the concept of volume by providing a hands-on experience to develop an understanding of volume and one way in which it can be measured without the use of a formula.

As students measure the volume of the shapes several relationships should come to light. The volume of the cube is three times the volume of the square pyramid. The volume of a pyramid is 1/3 the volume of a prism with the same base area and height. The volume of a cone is 1/3 the volume of a cylinder with the same base area and height. The sphere is 2/3 the volume of a cylinder.

Using liters to measure the volume transfers to the concept of cubic centimeters since there are 1000 cubic centimeters in a liter and 1000 milliliters in a liter. Students need to become aware that should they overflow the solid or not fill completely, measurements can be inaccurate. If all is done correctly, they should have a close match. After having students measure the volume with milliliters, and providing the formula for area, they can then measure the prism's base and height checking for accuracy.

Research Basis

Ancess, J. (2004). Snapshots of meaning-making classrooms. *Educational Leadership.* 62(1). 36-40.

Teachers have a responsibility to design instruction enabling all students to learn in ways that suit them best. Providing small group activities allows students to share their own and their peers' individual strategies for solving math problems.

Rushton, S., Larkin E. Shaping the learning environment: connecting developmentally appropriate practices to brain research. *Early Childhood Education Journal.* 29(1). 25-33.

Studies reviewed show that pairing brain research with developmentally appropriate practices sets the stage for solid learning. Being aware of both and providing hands-on activities that cater to different learning modalities and stimulate the different regions of the brain makes learning more interesting promoting deeper understanding.

Invitation to Learn

Materials

- ❑ Various containers (size & shape)

Collect and display a variety of shaped containers. Have the students list the containers in order, least to greatest based on their estimate of the volume of each, in their math journal. Ask students to share their ideas, thoughts and methods for determining the container with the greatest volume. Ask students what volume is? What did they look at? Is the height more important than the width or circumference? Where do they see a volume measurement in real life? How do we measure volume? Share with students which containers are larger than others with a quick measurement of the volume of several of the containers.

Instructional Procedures

1. Display a small container of rice, lentils, and water which students might use to measure volume. Question students as to which material would give the best measurement and why?
2. Demonstrate proper measurement, with rice and water, measuring the volume of the small rectangular prism. Compare the measurements and ask which material provides a more accurate measurement. Why?
3. Distribute *How Much Will It Hold?* to each student.

Materials

- ❑ *How Much Will It Hold?*
- ❑ Clear geometric solids
- ❑ Rice
- ❑ Lentils
- ❑ Water
- ❑ Funnels
- ❑ 50ml graduated cylinders
- ❑ Paper towels

4. Working with a partner, students will use the eight geometric solids on their table to estimate which has the least volume and list in order least to greatest.
5. Using a 50ml graduated cylinder and funnel, one student will fill chosen solid and note on chart the volume of the solid. Students will use the same material to measure all of their shapes. Have different pairs use different materials at each table. One pair will use water, one lentils and one group will use rice.
6. Partners will complete measurement a second time to assure accuracy.
7. Repeat the same process with all eight shapes.
8. Students will list solids, greatest volume to least, and compare with estimation.
9. Group students according to the material used to measure volume. Have groups share their findings. Listen as students attempt to explain and question differences in findings.
10. Ask students to compare the relationship between different solids. Do they see any relationships? If students can see a similarity, have them share and discuss their findings. If they cannot see a relationship, aim them toward the cube and square pyramid. Was their measurement of the square pyramid 1/3 of the cube? What are some ways in which they can prove their findings to be true? Can they see any other similarities with other shapes?

Assessment Suggestions

- Performance assessment-completed chart of measurements.
- Journaling-written explanation of relationship of shapes discovered in measuring and how those can be proven. Have students explain possible reasons in differences of volume. Can they suggest ways to increase accuracy?
- Provide another geometric shape for student to measure volume.

Curriculum Extensions/Adaptations/ Integration

- For advanced learners, let them discover the relationship between liters and cubic centimeters using centimeter cubes and water.
- Special needs students can better successfully measure using water from squirt bottles to prevent spilling.
- Science- integrates math to science unit of Matter.

Family Connections

- Students can find a container at home with the volume listed. Compare the volume to another container that is not marked. Is the volume greater? How can they measure the volume? Share with the class their findings.
- Students can answer why a cereal box never seems to be full when opened the first time.

Additional Resources

Books

Math On Call, ISBN 0669457701

How Much Will It Hold??

List least to greatest, which solid has the largest volume.

Estimation

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____

Actual after Measurement

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____

SHAPE	Volume Measurement 1 # ml	Volume Measurement 2 # ml
Large Square prism		
Small Square prism		
Small rectangular prism		
Square pyramid		
Large Rectangular prism		
Cone		
Large cylinder		
Sphere		

Type of material used for measurement:_____

Look at results for possible relationships between different solids. Write and describe your findings below-

Math III-2, IV-2

Activities

Quadrants/Measure

Getting to the Point

Standard III:

Students will use spatial reasoning to recognize, describe, and analyze geometric shapes and principles.

Objective 2:

Specify locations in a coordinate plane.

Intended Learning Outcomes:

3. Reason logically using inductive and deductive strategies and justify conclusions.
5. Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.

Content Connections:

Science IV; fossil formation

Math Standard III

Objective 2

Connections

Background Information

Coordinate graphs are very important since they are the place where algebra and geometry come together.

In the 1600s, Rene Descartes, the French philosopher, mathematician, and scientist, founded analytic geometry and originated the Cartesian coordinates. He is given credit for coming up with the two-axis system we use today. The story goes that he was lying in bed and watching flies crawl over the tiles on the ceiling. He realized that he could describe a fly's position using the intersecting lines of the tiles. The coordinate plane is often called the Cartesian plane after him.

The coordinate plane is divided into four quadrants, which are labeled with Roman numerals. In the fifth-grade math curriculum, the students need to know how to locate and write points defined by ordered pairs of integers in all four quadrants. The center of the coordinate plane is called the origin and has the coordinates of (0, 0). The ordered pairs are referred to as coordinates. We write a point's coordinates inside parentheses, separated by a comma like this: (5, 6). The first number in an ordered pair is called the x-coordinate. The x-coordinate tells us how far from the origin the point is along the x-axis or the horizontal number line. The second number is called the y-coordinate. The y-coordinate tells us how far from the origin the point is along the y-axis or the vertical number line.

Research Basis

Irwin, K.C., (2001). Using everyday knowledge of decimals to enhance understanding. *Journal for research and mathematics education*. 32(4). 399-420.

This study investigated the role of students' everyday knowledge of decimals in supporting the development of their knowledge of decimals. One group worked with problems presented in familiar context, the others were given no contextual connections. The students' ability to make connections between the known and unknown greatly enhanced their understanding of mathematical concepts. Presenting students with real life applications is important when being challenged with new concepts.

Furner, J. M., Yahya, N., and Duffy, M. L., (2005). Teach mathematics: Strategies to reach all students. *Interventions in school and clinic*. 41(1),16-23.

In this article, the authors list 20 different strategies that can help teachers reach all students. These strategies are based on the belief that all students have the right to learn math and feel confident in their ability to do math. It is the responsibility of all teachers to see that mathematics can be learned by every student. The strategies introduced in this article can enable teachers to accomplish this goal.

Invitation to Learn

Materials

- ❑ *The Fly on the Ceiling, a Math Reader*
- ❑ *Graph Paper A*
- ❑ Object that will stick on board

This invitation to learn, allows you to assess the level of students' mastery in locating an object on a coordinate plane using ordered pairs.

- Have a student throw an object that will stick to the board.
- Challenge the students to identify the exact location of the object on the board. Allow time for a short discussion, but try to do as little leading as possible.
- Project transparency of the *Graph Paper A* on board.
- Have a student throw object on to the projected *Graph Paper A*.
- Again challenge the students to identify the specific location of the object.
- If the students suggest numbering the *Graph Paper A,* and using ordered pairs, follow through on their ideas. Let them know that they aren't the first to come up with this idea; Rene Descartes discovered this concept over three hundred years ago. Read the book, *The Fly on the Ceiling, a Math Reader.*
- If they are unable to figure out what to do, proceed directly to reading *The Fly on the Ceiling, a Math Reader* by introducing Rene Descartes, a man who discovered a solution to this problem over three hundred years ago.
- After reading the book, repeat the activity using what they learned from the book.

Instructional Procedures

Getting to the Point

1. Share *The Coordinate Plane* PowerPoint with students introducing them to the coordinate plane with the correct vocabulary. Use the key points to review main ideas.
2. If you're unable to access the PowerPoint, use the *Key Graphing Cling* and introduce the following key points.

Key Points

Plane: a flat surface that goes on forever in every direction

Coordinate plane: made up of an infinite number of points and divided by two number lines

Point of Origin: where the two number lines meet

Axis (plural is "*axes"):*

- *x*-axis: the horizontal line; east of the origin is positive while west is negative.
- *y*-axis: vertical line; north of the origin is positive while south is negative.

Quadrants: the four sections divided by the *x* and *y* axes numbered in order from I-IV starting in the upper right quadrant and going counterclockwise.

Coordinates or ordered pair:

- the two numbers used to locate points on the plane; relative to the point of origin
- always written in parentheses with the *x*-value first *(x,y)*.
- the ordered pair for the point of origin is (0,0).

3. Pass out Dry Erase Mats and using the Key Graphing Cling
   - Give students two different colored strips of paper.
   - Have them make two individual number lines using their mats as a guide with "0" in the center and include both positive and negative numbers.
   - Connect the two strips at "0" using a brad.
   - Rotate the second strip 90 degrees to form the *y-axis* .
   - Overlay these on mats.
   - Begin to label the mats with markers:
     - x- axis
     - y- axis

Materials

- ❑ *The Coordinate Plane*
- ❑ X/Y-axis Dry Erase Mats
- ❑ Key Graphing Cling
- ❑ Marking pens
- ❑ Paper strips
- ❑ Metal brads
- ❑ *Coordinate Cards*

- point of origin
- 4 quadrants (I-IV)

- Using *Coordinate Cards*, have students practice locating and plotting coordinates on their mats.
- Check for accuracy using Key Graphing Cling.
- Working in pairs, students take turns giving and plotting ordered pairs on their Dry Erase Mats.

Points of Interest -Coordinate Activity Stations

These activities are designed for two players each. Pass out *Points of Interest Guidelines* for each team. Have enough materials at each point for at least three to four groups depending on the size of your class. Groups may rotate through each point independently or as directed by teacher. After visiting all *Points of Interest,* have students reflect in journals what they have learned about the coordinate plane and locating points in all four quadrants

Materials

- ❑ *Tic Tac Toe*
- ❑ Pencils
- ❑ Scratch paper
- ❑ *In Search of Buried Treasure*
- ❑ Space Wars
- ❑ Computers
- ❑ Points of Interest Guidelines
- ❑ Coordinate Plane Assessment
- ❑ Math journal

Point 1: "Tic Tac Toe"

1. *Tic Tac Toe* game board.
2. Scratch paper and pencil for each player to record their coordinates.
3. Play rock, paper, and scissors to determine who starts. The winner begins the games, while the other picks X or O symbol.
4. The object of the game is to get four X's or four O's in a row vertically, horizontally, or diagonally.
5. Player one writes down the ordered pairs on scratch paper, then points to that location. It is up to the other player to check for accuracy before a symbol can be placed. If the point is mislabeled, no symbol is made on the game board.
6. Players take turns writing and locating the ordered pairs until one player has four in a row.
7. Students continue playing until they have played a game in all four quadrants.

Point 2: "In Search of Buried Treasure"

1. The object of this game is to practice naming coordinates on a four-quadrant grid.
2. Each player gets one game board, *In Search of Buried Treasure.*
3. Play rock, paper, and scissors to determine who buries the "treasure" first.

4. Player one: Hides the "treasure" in one quadrant by marking it on their coordinate plane (keeps it hidden-a book works well for hiding it).
5. Player two: Guesses the location by writing an ordered pair in the "guess" box on their page while telling Player 1. They then mark it on their coordinate plane.
6. Player one: Marks the same coordinates and then uses the compass to tell Player two in which direction they must go to find the treasure. Caution the students that if Player 1 does not mark their partners point, they may give out the wrong direction.
7. Player two: Writes the direction in their"clue" box.
8. The game continues until the treasure is found.
9. Players switch roles and play again using the second coordinate plane.

Point 3: "Space Wars"

1. Object of the game is to find and destroy each others' hidden spaceships.
2. Players each mark (vertically or horizontally only) their "Fleet" of five ships on their "Air Space" on the coordinate plane. There must be at least one ship in each quadrant.
3. The ships should remain hidden from the opponent's view. A book works well.
4. Taking turns, players call out their "shots" attempting to get "hits" on the opponent's spaceships and destroy them.
5. "Hits" or "misses" should be marked on the other coordinate plane.
6. Use an X for a hit and an O for a miss.
7. A spaceship is destroyed when all points on the craft are hit.
8. A player wins when all five opponent's ships are destroyed.
9. Fleet:

Length	Name
5 points	Death Star
4 points	Warbirds
3 points	Starship
3 points	Fighters
2 points	Starbase

Point 4: Internet Games (optional)

There are many sites on the internet that have interactive games to reinforce the coordinate plane. Here are just a couple.

- Mole Game - http://funbasedlearning.com/algebra/graphing/default.htm

  The students try to catch a mole located within the four quadrants. There are three levels in this game.

  Easy version of Graph Mole - If you are learning how to plot points for the first time, try this fun and easy tutorial and game.

  Medium version of Graph Mole - If you are reviewing how to plot points, play this game.

  Hard version of Graph Mole - Once you have mastered plotting points, try this random question arcade style game.

- Maze game http://www.shodor.org/interactivate/activities/MazeGame/

  This game lets students practice using coordinates by having them move a robot through a mine field to a given target. The students must specify the coordinates of the new location. In order to win, the path must not cross a mine. Challenge the students to place more than five mines. Use the "Help" tab on this site for instructions.

Assessment Suggestions

- In their math journals, have students write what they have learned about the coordinate plane and locating points in all four quadrants.
- *The Coordinate PlaneAssessment* Worksheet

Curriculum Extensions/Adaptations/ Integration

- Art: Have students create simple drawings using coordinates for other students to recreate.
- Place *Points of Interest* -Coordinate Activities in centers for students to continue working with coordinates.

Family Connections

- If students have access to the internet have them play the games found on the internet.
- Play any of the activities from *Points of Interest* with family.

Additional Resources

Books

The Fly on the Ceiling, a Math Reader, by Julie Glass; ISBN 0-679-98607-3

Math Dictionary-The Easy, Simple, Fun Guide to Help Math Phobics Become Math Lovers, by Eula Ewing Monroe; ISBN 978-1-59078-413-6

Mathematicians are People Too, (Vol. 2), by Luretta Rimer; ISBN 0-86651-509-7

Web sites

Background information on the coordinate plane:

http://themathworksheetsite.com/ coordinate plane templates http://www.shodor.org/interactivate/lessons/GraphingCoordinate/

http://mathforum.org/cgraph/cplane/index.html

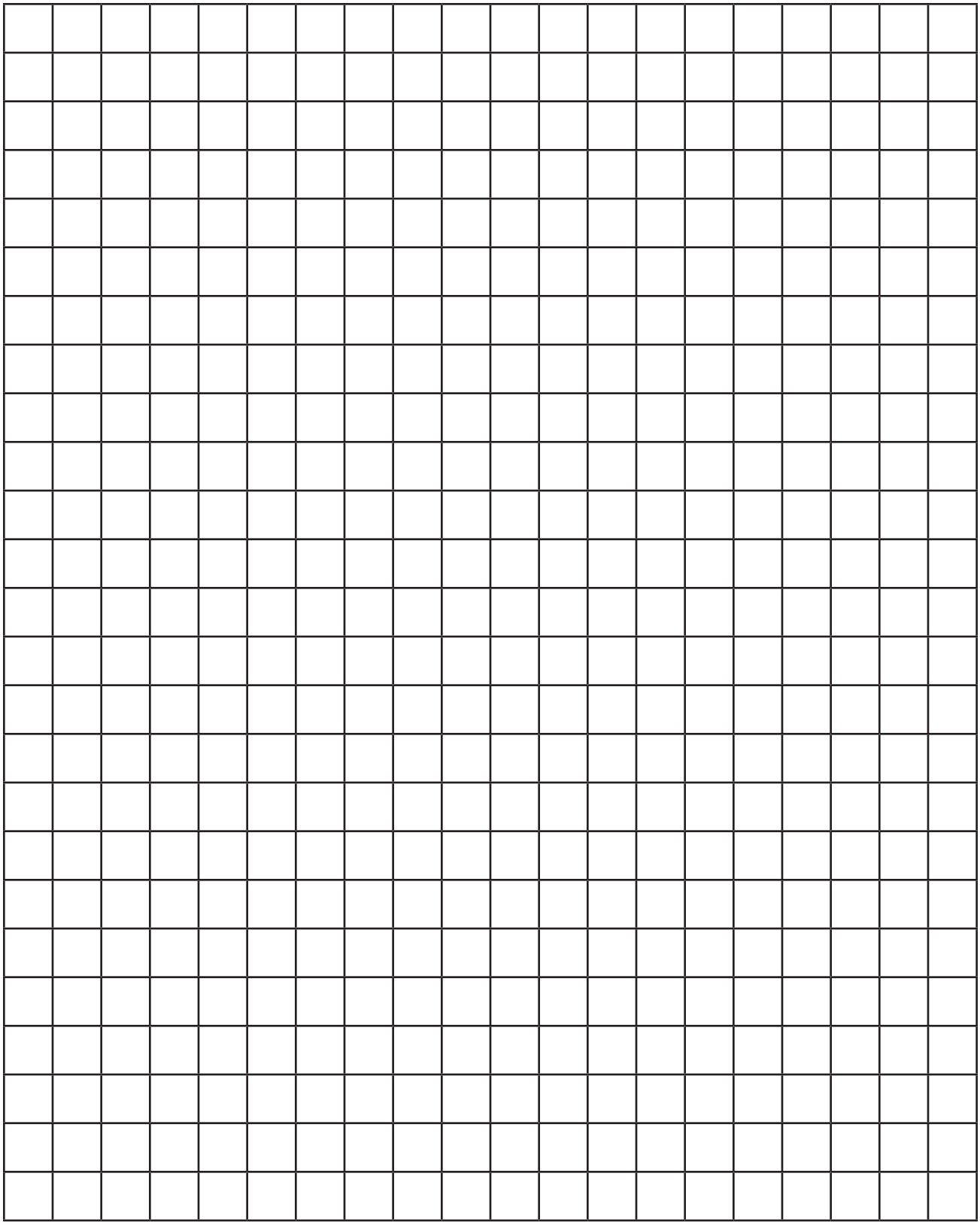
http://www.math.com/school/subject2/lessons/S2U4L1GL.html

http://www.purplemath.com/modules/plane.htm

http://www.learningwave.com/lwonline/algebra_section2/alg_coord.html

http://mathforum.org/cgraph/cplane/index.html

Graph Paper A



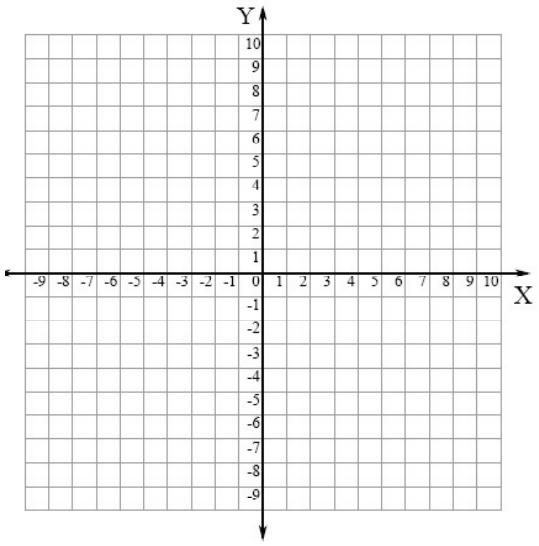
Coordinate Cards

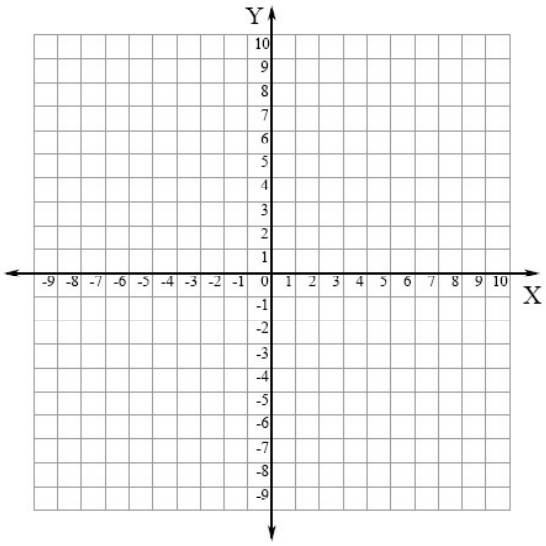
(8,6)	(−10,3)
(−7,−9)	(3,−2)
(4,0)	(−1,5)
(0,−6)	(10,−6)
(9,8)	(−5,8)

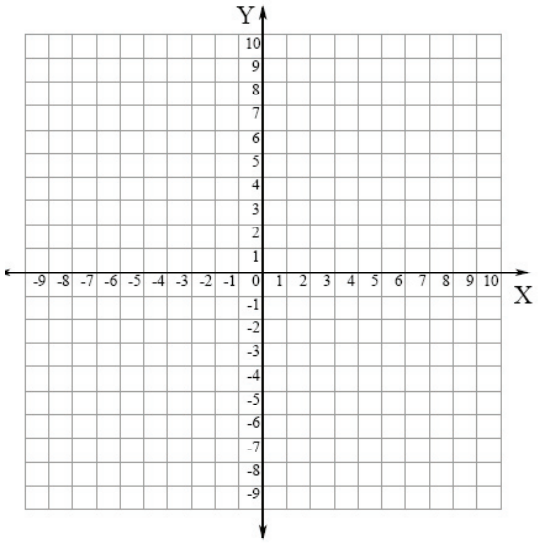
Tic Tac Toe

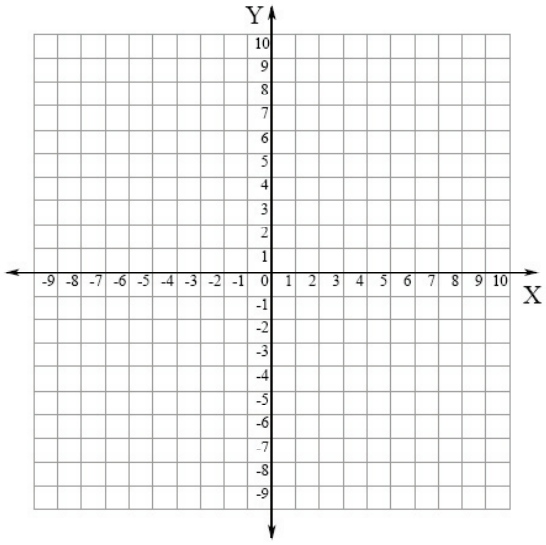
The object of the game is to get four X's or four O's in a row vertically, horizontally, or diagonally.

1. Play rock, paper, and scissors to decide who starts.
2. The winner begins the game, while the other picks X or O.
3. To start, write down the ordered pairs on paper. Point to that location. Other player checks to see if it is correct.
4. If you are right, place your mark. If you are wrong, you lose your turn.
5. Take turns until one player has four in a row.
6. Play four games, one in each quadrant.

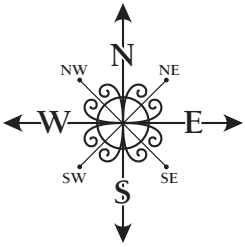


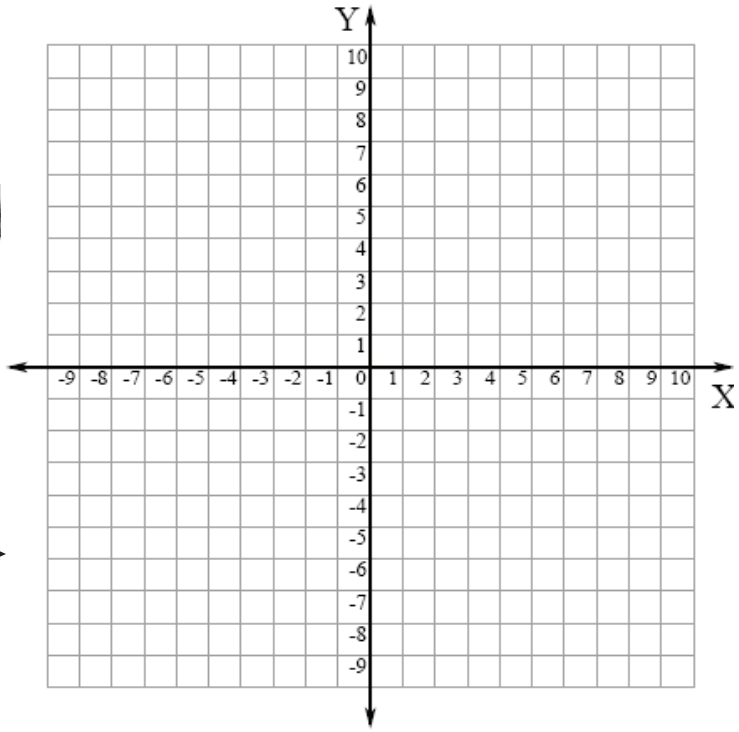






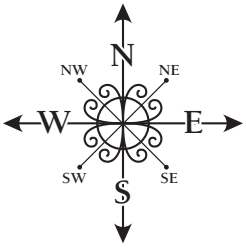
In Search of Buried Treasure

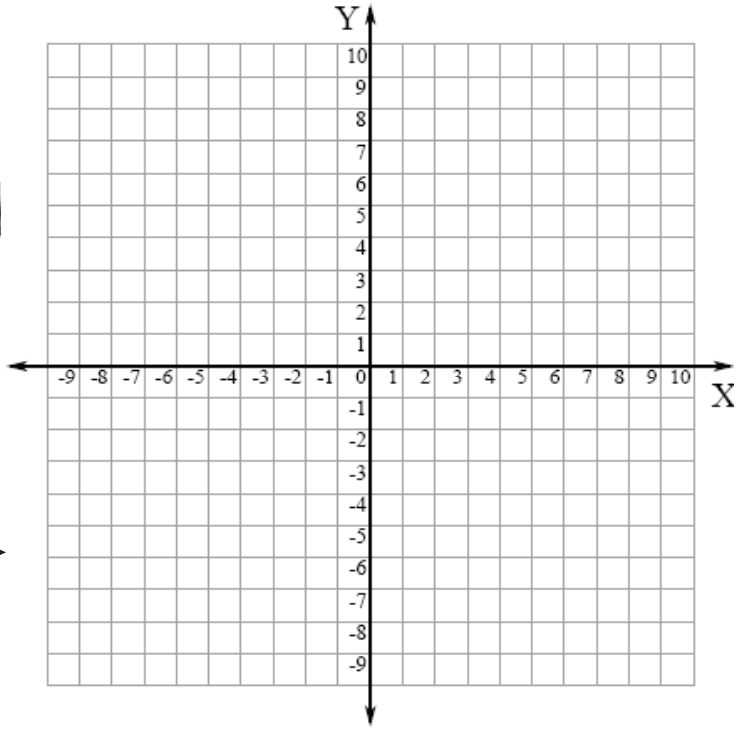




Guess	Clue

In Search of Buried Treasure





Guess	Clue

"In Search of Buried Treasure"

1. The object of this game is to practice naming coordinates on a four-quadrant grid.
2. Each player gets one game board, *In Search of Buried Treasure.*
3. Play rock, paper, and scissors to determine who buries the "treasure" first.
4. Player 1: Hides the "treasure" in one quadrant by marking it on their coordinate plane (keeps it hidden-a book works well for hiding it).
5. Player 2: Guesses the location by writing an ordered pair in the "guess" box on their page while telling player 1. They then mark it on their coordinate plane.
6. Player 1: Marks the same coordinates and then uses the compass to tell player 2 in which direction they must go to find the treasure. Caution the students that if player 1 does not mark their partners point, they may give out the wrong direction.
7. Player 2: Writes the direction in their "clue" box.
8. The game continues until the treasure is found.
9. Players switch roles and play again using the second coordinate plane.

"In Search of Buried Treasure"

1. The object of this game is to practice naming coordinates on a four-quadrant grid.
2. Each player gets one game board, *In Search of Buried Treasure.*
3. Play rock, paper, and scissors to determine who buries the "treasure" first.
4. Player 1: Hides the "treasure" in one quadrant by marking it on their coordinate plane (keeps it hidden-a book works well for hiding it).
5. Player 2: Guesses the location by writing an ordered pair in the "guess" box on their page while telling player 1. They then mark it on their coordinate plane.
6. Player 1: Marks the same coordinates and then uses the compass to tell player 2 in which direction they must go to find the treasure. Caution the students that if player 1 does not mark their partners point, they may give out the wrong direction.
7. Player 2: Writes the direction in their "clue" box.
8. The game continues until the treasure is found.
9. Players switch roles and play again using the second coordinate plane.

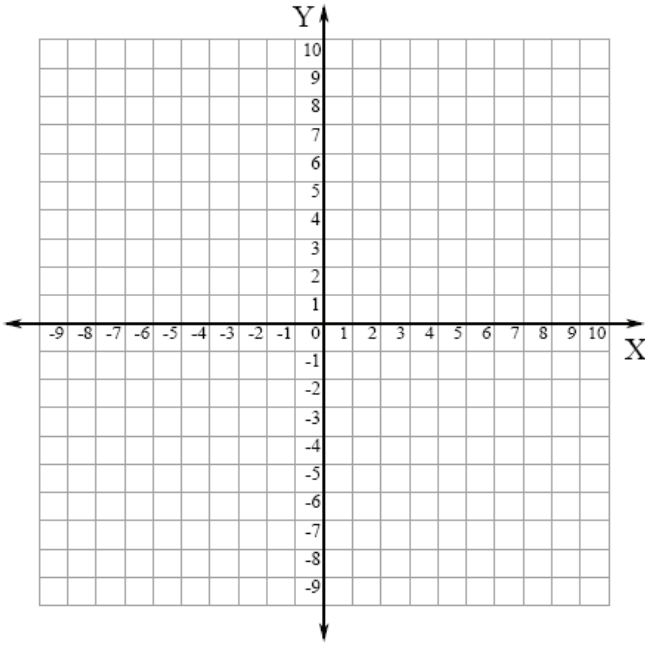
Space Wars

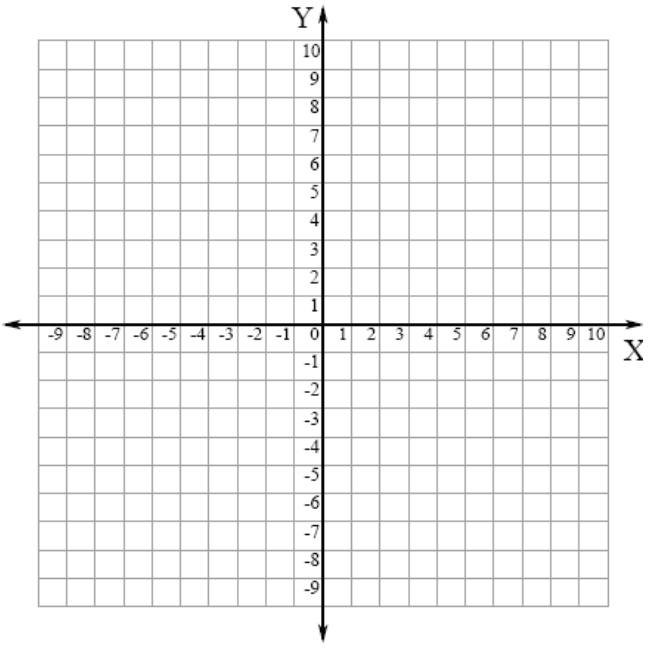
Object: Find and destroy each others' hidden spaceships.
Rules:

1. Players each mark (vertically or horizontally only) their "Fleet" of five ships on their "Air Space" on the coordinate plane. There must be a least one ship in each quadrant.
2. The ships should remain hidden from the opponent's view. A book works well.
3. Taking turns, players call out their "shots" attempting to get "hits" on the opponent's spaceships and destroy them.
4. "Hits" or "misses" should be marked on the other coordinate plane.
5. Use an X for a hit and an O for a miss.
6. A spaceship is destroyed when all points on the craft are hit.
7. A player wins when all 5 opponent's ships are destroyed.

Fleet

Length	Name
5 points	Death Star
4 points	Warbirds
3 points	Starship
3 poin11:37:56 AM	Fighters
2 points	Starbase





Points of Interest Guidelines

Point 1: "Tic Tac Toe"

1. Play rock, paper, and scissors to determine who starts. The winner begins the games, while the other picks X or O symbol.
2. The object of the game is to get four X's or four O's in a row vertically, horizontally, or diagonally.
3. Player 1 writes down the ordered pairs on scratch paper, then points to that location. It is up to the other player to check for accuracy before a symbol can be placed. If the point is mislabeled, no symbol is made on the game board.
4. Players take turns writing and locating the ordered pairs until one player has four in a row.
5. Continue playing until you have played a game in all four quadrants.

Point 2: "In Search of Buried Treasure"

1. Play rock, paper, and scissors to determine who buries the "treasure" first.
2. Player 1: Hides the "treasure" in one quadrant by marking it on their coordinate plane (keeps it hidden-a book works well for hiding it).
3. Player 2: Guesses the location by writing an ordered pair in the "guess" box on their page while telling Player 1. They then mark it on their coordinate plane.
4. Player 1: Marks the same coordinates and then uses the compass to tell Player 2 in which direction they must go to find the treasure. Caution the students that if Player 1 does not mark their partners point, they may give out the wrong direction.
5. Player 2: Writes the direction in their "clue" box.
6. The game continues until the treasure is found.
7. Players switch roles and play again using the second coordinate plane.

Point 3: "Space Wars"

1. Players each mark (vertically or horizontally only) their "Fleet" of five ships on their "Air Space" on the coordinate plane. There must be a least one ship in each quadrant.
2. The ships should remain hidden from the opponent's view. A book works well.
3. Taking turns, players call out their "shots" attempting to get "hits" on the opponent's spaceships and destroy them.
4. "Hits" or "misses" should be marked on the other coordinate plane.
5. Use an X for a hit and an O for a miss.
6. A spaceship is destroyed when all points on the craft are hit.
7. A player wins when all five opponent's ships are destroyed.

Point 4: Internet Games

- *Mole Game* - http://funbasedlearning.com/algebra/graphing/default.htm

  Try to catch the mole located within the four quadrants!!! There are 3 levels in this game. Easy version of Graph Mole - If you are learning how to plot points for the first time, try this fun and easy tutorial and game. Medium version of Graph Mole - If you are reviewing how to plot points, play this game. Hard version of Graph Mole - Once you have mastered plotting points, try this random question arcade style game.

- *Maze Game* - http://www.shodor.org/interactivate/activities/MazeGame/

  Practice using coordinates by moving a robot through a mine field to a given target. You must specify the coordinates of the new location. In order to win, the path must not cross a mine. To make it more difficult place more than 5 mines!!! Use the "Help" tab on this site for further instructions.

The Coordinate Plane Assessment

Write the letter of the point that matches each ordered pair.

_____(0,-5)

_____ (6,5)

_____ (-3,-2)

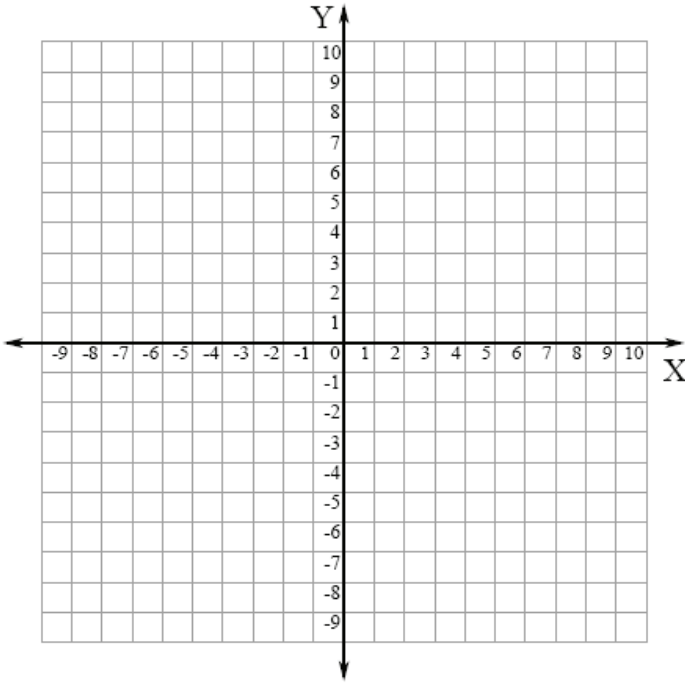
_____ (4,-4)

_____ (-4,-6)

_____ (2,1)

_____ (-5,2)

_____ (-4,3)



Plot each of the following ordered pairs on the coordinate plane.

A (2, 2)

B (3, -2)

C (6, 5)

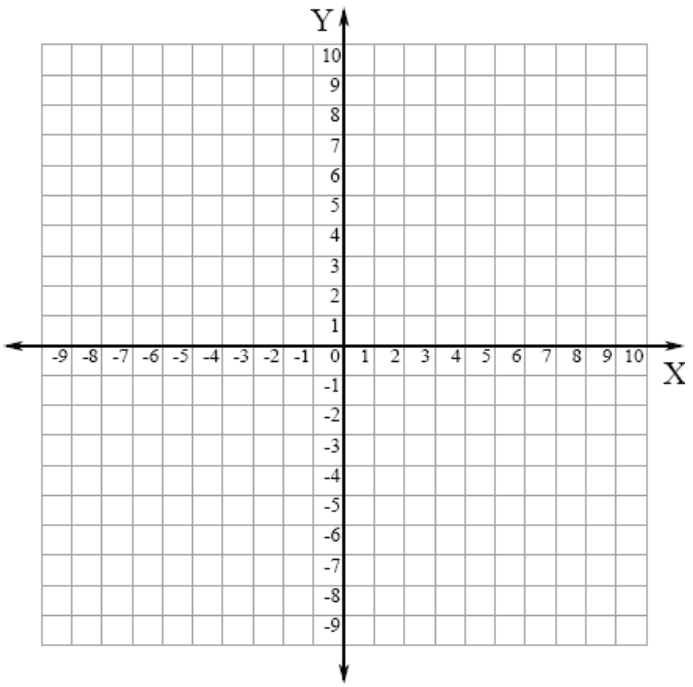
D (-5, -4)

E (-2, -3)

F (-4, 3)

G (0, 3)

H (5, -2)



Insides and Outsides

Math Standard IV

Objective 2

Connections

Standard IV:

Students will understand and apply measurement tools and techniques and determine surface area and volume of three-dimensional shapes.

Objective 2:

Recognize, describe, and determine surface area and volume of three-dimensional shapes.

Intended Learning Outcomes:

3. Reason logically using inductive and deductive strategies and justify conclusions.
5. Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.

Content Connections:

Background Information

In order for students to visualize and determine surface area and volume of three-dimensional shapes, it is important to manipulate actual objects. They must also have experience with the concepts of area, nets and rectangular prisms prior to the introduction of this standard and objective (Standard IV Objective 2). When talking about area, we are referring the measurement of a two-dimensional shape. When talking about surface area, we are referring to the measurement of a three-dimensional shape.

The surface area of a prism is the sum of the areas of all the faces, including the bases. The surface area is measured in square units. Although the students may not be familiar with this concept, they have actually experienced it as they've worked with nets. Surface area takes nets one step further by determining actual measurements. The purpose of this lesson is to help the students make this connection. The mathematical formula for surface area is:

$$SA= 2(l \cdot w) +2(l \cdot h) + 2(w \cdot h)$$

The actual formula for surface area involves using length, width and height, at this level. Having the students find the area of each face and then adding them together as square units to find the total will be less confusing for fifth graders.

The volume of a prism tells how many cubic units it takes to fill the prism. Volume is measured in cubic units. The mathematical formula for volume is:

$$V= l \cdot w \cdot h$$

Research Basis

Sowel, E.J. (1989). Effects of manipulative materials in mathematics instruction. *Journal for research in mathematics education,* 20 (4), 498-505.

This review of research sums up the result of sixty studies addressing the effectiveness of manipulatives on student learning and attitudes in mathematics teaching. Sowell concludes that the more concrete the manipulatives, and the longer the time spent using them, the better instructional outcomes.

Hinzman, K.P. (1997). Use of manipulatives in mathematics at the middle school level and their effects on students' grades and attitudes. *ERIC Source* (ERIC # ED411150). Retrieved December 10, 2006, from http://www.eric.ed.gov

This paper reports on a study that examines mathematical scores when hands on manipulatives and group activities were used in the classroom. Results indicate that student performance was enhanced by the use of manipulative materials; and students' attitudes toward mathematics were significantly more positive than those in previous years when manipulatives were not used.

Invitation to Learn

Materials

- ❑ Cubes
- ❑ *Chart Paper*

- Put students into small groups of two to four, and give each group a container with at least 48 cubes.
- Challenge each group to build as many different regular rectangular prisms that have a volume of 12 cubic units.
- Once a prism has been built, ask them to set it aside to keep as an example so that the same prisms are not repeated. (With 12 cubes, they can build four different rectangular prisms: 1x2x6, 1x3x4, 1x1x12, and 2x2x3.)
- Demonstrate the four different prisms that twelve cubes can make by having a few volunteers stand and describe their prisms – length, height, and width. Because of previous lessons on volume, the students should be able to describe their prisms using length, width and height. A discussion on whether a different orientation makes a difference in the dimensions may be needed.
- Review the definition of volume with your students.
- As a class, find the volume for the four different prisms using length, width, and height.

Length	Width	Height	Volume

- On chart paper, record the students findings . Highlight the fact that because each prism is made of twelve cubes, each has a volume of 12 cubic units.

Instructional Procedures

Materials

- ❑ *X/Y-axis Dry Erase Mats*
- ❑ *Key Graphing Cling*
- ❑ Pre-made rectangular prisms
- ❑ Rectangular prism net patterns
- ❑ Markers
- ❑ Cubes
- ❑ *The Great Cover Up*
- ❑ Graph paper

The Great Cover Up

Use the concept of area with 2-D measurement, using a 3-D prism (1 x 2 x 3), to introduce the concept of surface area or 3-D measurement. Ask the students to predict if the prisms have the same volume, do they have the same surface area. Come back to this question at the end of the *Cut it Out!* activity.

- Show the class the pre-made prism or 3-D object and ask the students how they can use area to determine how much wrapping paper would be needed to cover the entire prism without any overlapping.
  - This measurement is called surface area and can only be found for 3-D objects.
- Discuss their ideas for ways to measure the surface area.
- If they struggle coming up with a solution, bring out nets that were used in previous lessons, to visualize the connection between nets and surface area.
- Review area and its formula by finding the area of each face. This discussion is critical in helping students make the connection between area and surface area.

1. With the help of the students, use the *Key Graphing Cling* to model how to draw the net of the pre-made prism.
   - Use the concept of a room to help them visualize each surface as it is being drawn: floor, ceiling, four walls: two front/back walls, two side walls.
   - Use these same terms to label each part of the net.
   - Discuss how to find the area of each face, leading to the idea that adding all areas would give the total area or surface area.

- Discuss how using just letters rather than whole words as labels can be simpler.
- Have students come up with a formula for surface area using only letters:

SA = a + a + b + b + c + c

(f=area of floor and ceiling; b=area of front and back wall; c=area of side walls)

- Plug in actual area for each face underneath formula. Give students *The Great Cover Up* to record data with you.
- Repeat this process of creating nets together using different rectangular prisms.

2. Next, pass out individual *9x11 Double sided X/Y-Axis Dry Erase Mats* and dry erase markers so students can draw more nets with you. Continue to use worksheet to determine and record the surface areas.
3. Now using graph paper and the cubes.
   - Have students work in small groups to reconstruct the original four 12-cube prisms and have them draw nets for each one.
   - Fill out *The Great Cover Up* for each prism.
   - Save their nets for the next activity.

Cut it Out!

1. Use chart made in Invitation to Learn and referring to the first prism listed, have all students find that net and cut it out.
2. Fill out #1 on *Cut it Out* together.
3. Students will repeat the process for the remaining three prisms.
4. Review their predictions of the connection between surface area and volume.
5. In their math journals, have students write what they learned about surface area and volume. Have them analyze the pattern they learned from their *Cut it Out*. Have the students explain how to use a formula to find volume, and how they would find the surface area of a prism.

Materials

- ❑ Graph paper
- ❑ Cubes
- ❑ Scissors
- ❑ Tape
- ❑ *Cut it Out*

Assessment Suggestions

- Informal assessment includes class discussion, math journals and observation of group/partner work.

- *The Great Cover Up*
- *Cut it Out*
- *Birthday Boxes*

Curriculum Extensions/Adaptations/ Integration

- Prism Race
- Challenge students to find out how many different rectangular prisms they can make with a volume of 36 cubes. Encourage them to think of ways to make sure they have made all possible prisms.
- Challenge students to make as many prisms as possible with the same surface area. Have them record the volumes of their solids and note any patterns.
- Art: Draw three-dimensional rectangular prisms on isometric dot paper.
- Design a net that can be folded into a rectangular prism that can hold 24 Multilink cubes.

Family Connections

- Have students find at least five rectangular prisms from household items. Have them measure the length, height, and width of each item. Record the measurements in their math journals, and then find the volume and surface area of each of the items.

Additional Resources

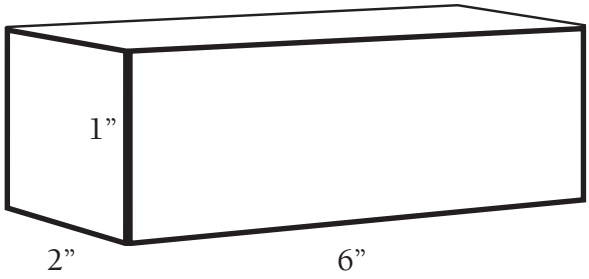
Books

Math Dictionary-The Easy, Simple, Fun Guide to Help Math Phobics Become Math Lovers, by Eula Ewing Monroe; ISBN 978-1-59078-413-6

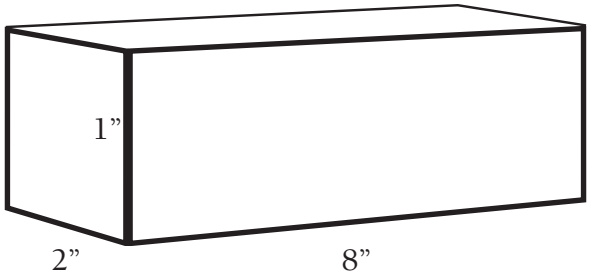
Birthday Boxes

During Jeff's birthday party, all of the tags on his presents were lost. Use the clues to figure out who gave each present. You may use graph paper to draw nets. Show all work.

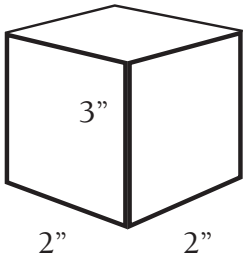
Box A



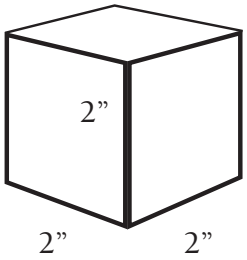
Box B



Box C



Box D



Clues

- The box with Lyla's present has the smallest surface area and the smallest volume.
- Sarah's and Ben's presents have the same volume.
- Ben's present is in the box that has a surface area of 160 square inches.
- The box with Skylar's present has the greatest surface area.

1. Box A was given by _____

2. Box B was given by _____

3. Box C was given by _____

4. Box D was given by _____

Name _____ Date _____

Cut It Out

Dimensions of prism: _____*(l)* x _____*(w)* x _____*(h)*

Volume Net (attach below)

Surface Area

Dimensions of prism: _____ *(l)* x _____ *(w)* x _____ *(h)*

Volume Net (attach below)

Surface Area

Name _____ Date _____

The Great Cover Up

Formula for Area =

Formula for Surface Area =

	length	width	height	Area
floor				
ceiling				
front wall				
back wall				
side wall				
side wall				
Surface Area:				

	length	width	height	Area
floor				
ceiling				
front wall				
back wall				
side wall				
side wall				
Surface Area:				

	length	width	height	Area
floor				
ceiling				
front wall				
back wall				
side wall				
side wall				
Surface Area:				

	length	width	height	Area
floor				
ceiling				
front wall				
back wall				
side wall				
side wall				
Surface Area:				

	length	width	height	Area
floor				
ceiling				
front wall				
back wall				
side wall				
side wall				
Surface Area:				

	length	width	height	Area
floor				
ceiling				
front wall				
back wall				
side wall				
side wall				
Surface Area:				

	length	width	height	Area
floor				
ceiling				
front wall				
back wall				
side wall				
side wall				
Surface Area:				

Prism Race

1. Using cubes, secretly build a prism and record its length, width, height, and volume in the table.
2. At the same time, your and your partner show your prisms.
3. Calculate the volume of your partner's prism and write it down on a piece of paper. Put the slip of paper on the prism. The first player to put the correct volume down wins the round.
4. After each round, calculate the surface area for each of your prisms.
5. The player who wins the most rounds, wins the game.

Length x	Width x	Height =	Volume	Surface Area
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10.				

Math V-2

Activities

Probability

Probability

Standard V:

Students will construct, analyze, and construct reasonable conclusions from data and apply basic concepts of probability.

Objective 2:

Apply basic concepts of probability.

Intended Learning Outcomes:

3. Reason logically, using inductive and deductive strategies and justify conclusions.
4. Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.

Content Connections:

Math I-4; communicate parts of a whole
Math II-1; recongnize, anaylize and use patterns
Language Arts VIII; write to communicate

Math Standard V

Objective 2

Connections

Background Information

Students need to understand the correct terms and vocabulary while discussing probability. It is important for students to learn to distinguish between theoretical and experimental probability. Students can also be introduced to the mathematical formulas.

Theoretical probability:

- The numerical measure of the likelihood that an event will happen or the ratio of the number of ways the event can occur to the total number of possibilities.
- It is the fraction of times we expect an event to occur if the same experiment is repeated over and over.
- It is the represented by the fraction:

  $$\frac{\text{Number of ways the event can occur}}{\text{Total number of possible outcomes}}$$

- Theoretical probability does not change.

Example:

What is the probability of getting a number less than 3 when tossing an ordinary dice? There are six possible outcomes: 1, 2, 3, 4, 5, or 6; all of which are equally likely to occur. Two of these, 1 and 2, are less than 3; so the theoretical probability of getting a number less than 3 is: 2/6 = 1/3.

Experimental probability:

- The numerical measure of what actually happens in an experiment.
- It is the fraction of times an event actually occurs when the same experiment is repeated over and over.
- It is represented by the fraction: $\frac{\text{Number of actual outcomes}}{\text{Total number of possible outcomes}}$
- The experimental probability may vary from the theoretical probability, but the more times the experiment is repeated, the closer the experimental probability approaches the theoretical probability.

Research Basis

Rivero, V. (2006) let technology be your guide. ***American school board journal,*** November, p52-53.

The author gives seven tips for integrating tools of technology to help bolster students' knowledge in math and science education in the classrooms and schools.

Blessman, J., Myszczak, B. (2001). Mathematics vocabulary and its effect on student comprehension. ***ERIC Source*** (ED455112). Retrieved January 12, 2007, from http://www.eric.ed.gov.

In this action research project, interventions were used for improving fifth grade students' comprehension of mathematical vocabulary. The following were used: math journals, student-created math dictionaries, children's literature to introduce and reinforce mathematical concepts, graphic organizers, visual aids, and written explanations of open-ended word problems. These interventions resulted in an increase in comprehension and use of mathematical vocabulary in math performance and in communication of mathematical ideas.

Invitation to Learn

Provide four slips of paper for each student. Ask them to write their name on a paper each time they can answer "yes" to the following questions:

- Do you have black hair?
- Are you an only child in your family?
- Is your birthday in January or July?
- Is there the letter "w" found in your first, middle, or last name?

Materials

- ❑ Paper
- ❑ Container

Have students place the pieces of paper that have their names on them in a container.

Ask students to predict whether they think their name will be chosen. Draw one slip of paper out of the container. Compare students' predictions with the actual results. Tell students that today they will learn how to use mathematics to make better predictions.

Instructional Procedures

Flip the Coin Activity:

Conduct the following activity as a class:

- Trial 1: Hold up a coin and ask the students: "if I flip this coin one time, how many possible outcomes are there?" (2: heads or tails)
- Trial 2: Now ask: If I flip it ten times how many times would you predict that I would get heads? (1/2 of 10 or 5 times.) Pass one coin and paper to record to each student and direct them to flip the coin ten times and record the results. Ask: Did your *outcomes* match your prediction? Collect samplings from several students and record on board or overhead chart, pointing out that there was some variance.
- Trial 3: Have the students predict how many times they will get heads if they flip the coin 30 times. (1/2 of 30 or 15 times) Have them flip the coin 30 times and record the results. Ask: Did your outcomes match your prediction? Again collect samplings from several students and record on board or overhead chart.
- Trial 4: Have the students repeat the experiment, this time flipping the coin 100 times. Again take class samplings and record.

Class discussion:

- Through guided questioning, lead students to an understanding of the difference between what they predicted would occur, Theoretical Probability, and what actually occurred, Experimental Probability, then place words on the board.
- Also discuss the ways they used to record their results. Again through guided questioning, help students to determine the best ways to record results (e.g., tally marks, T-charts, boxes, or columns).

Materials

- ❑ Coins
- ❑ Paper
- ❑ *Theoretical Probability* word card
- ❑ *Experimental Probability* word card

- Be sure to use vocabulary such as: *event, likely, unlikely, possible, impossible, outcomes, theoretical probability, and experimental probability* during discussion.
- Introduce the Theoretical Probability formula:

  $$\frac{\text{Number of ways the event can occur}}{\text{Total number of possible outcomes}}$$

  and the Experimental Probability formula:

  $$\frac{\text{Number of actual outcomes}}{\text{Total number of possible outcomes}}$$
- Lead the discussion to an understanding of the idea that the experimental probability may vary from the theoretical probability, but the more times the experiment is repeated, the closer the experimental probability approaches the theoretical probability. Use the classes total results to illustrate this concept.

Materials

- ❑ *Bag of Colors*
- ❑ Tiles
  - 4 red
  - 3 yellow
  - 2 blue
  - 1 green

Bag of Colors

1. Put students into cooperative learning groups with no more than four to a group
2. Give each group a bag of tiles and a recording sheet for each student
3. With the class, go through the three steps in writing a Theoretical Probability, found at the top of the *Bag of Colors* Recording Sheet.
   - Step 1: Count the number of red tiles.
   - Step 2: Count all of the tiles.
   - Step 3: Write a fraction-Theoretical Probability.
   - Allow each group time to write the Theoretical Probability fraction for the remaining three colors. Do a quick check to make sure they are correct.
4. On their own, each group will fill out the chart and conduct their experiment.

Probability Posters

A collection of various manipulatives to be used in conducting experiments with probability such as:

1. Put students into cooperative learning groups with no more than four to a group.
2. Each group will be given a different set of manipulatives to conduct probability experiments.

3. Each group will then prepare a short visual presentation of their experiment.
4. Give them time to conduct experiments, record results, and prepare presentation.

Materials

- ❑ Colored chips
- ❑ Dice
- ❑ Colored marbles
- ❑ Colored centimeter cubes
- ❑ Small circles or tiles (numbered 1-20)
- ❑ Spinners
- ❑ *Probability Recording Sheet*
- ❑ Poster board / chart paper

Assessment Suggestions

- Completion of *Bag of Colors* and *Probability Recording Sheet*.
- Group presentation of poster or chart displaying results and findings using probability journal.
- Write a journal entry about what they have learned about theoretical and experimental probability.

Curriculum Extensions/Adaptations/ Integration

- Put manipulatives from Probability Station in a center and have students continue conducting probability experiments with materials that they didn't use before. Use *Probability Recording Sheet*.
- In a center, provide manipulatives for students to create new probability experiments and share with the class.

Family Connections

- Share experiments with family.
- Look for ways at home where probability can be used.

Additional Resources

Books

Probability, by Sarah Jane Brian; ISBN 0590373676

Web sites

http://www.brainpop.com/

http://www.rainforestmaths.com/

http://argyll.epsb.ca/jreed/math7/strand4/4201.htm (Theoretical/Experimental Probability Web lesson with activities)

http://www.amathsdictionaryforkids.com

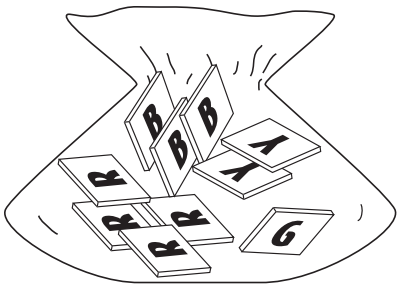
http://www.shodor.org/interactivate/lessons/PlayWithProbability/ a site where students can learn how to calculate both theoretical and experimental probability by rotating through a series of work stations.

Word Cards

Theoretical Probability	**Experimental Probability**

Bag of Colors Recording Sheet

R=red
B=blue
Y=yellow
G=green



Step 1: Count the number of red tiles.
Step 2: Count all the tiles.
Step 3: Write a fraction.

$$\frac{\text{Number of red tiles}}{\text{Total number of tiles}} = \_\_\_\_\_$$

The Theoretical probability of picking a red tile is $\frac{4}{10}$

Use the following steps to complete the table:

Step 1: Decide how many times you are going to conduct the experiment.
Step 2: Write down the number and color of each tile.
Step 3: Write the Theoretical Probability fraction for each color.
Step 4: Pull out one tile at a time, tally the result; remember to replace the tile in the bag.
Step 5: After conducting the experiment, write the fractions for the actual outcomes.

Step 1: Total number of events:_____	Theoretical Probability	Experimental Probability	
Step 2: Objects	Step 3: Write the Fraction	Step 4: Tallies	Step 5: Write the Fraction

How did the Theoretical Probability compare to the Experimental Probability?

_____

_____

THINK ABOUT IT: What could you do to bring the Experimental Probability results closer to the Theoretical Probability results?

_____

_____

Probability Recording Sheet

Remember the following steps to conduct your own experiment as you complete the table:

Step 1: Decide how many times you are going to conduct the experiment.
Step 2: Write down the number and color of each tile.
Step 3: Write the Theoretical Probability fraction for each color.
Step 4: Pull out one tile at a time, tally the result; remember to replace the tile in the bag.
Step 5: After conducting the experiment, write the fractions for the actual outcomes.

Step 1: Total number of events:_____	Theoretical Probability	Experimental Probability	
Step 2: Objects	Step 3: Write the Fraction	Step 4: Tallies	Step 5: Write the Fraction

How did the Theoretical Probability compare to the Experimental Probability?

_____

_____

THINK ABOUT IT: What could you do to bring the Experimental Probability results closer to the Theoretical Probability results?

_____

_____

Probability

Standard V:

Students will construct, analyze, and construct reasonable conclusions from data and apply basic concepts of probability.

Objective 2:

Apply basic concepts of probability.

Intended Learning Outcomes:

2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.
5. Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.

Content Connections:

Math I-4; communicate parts of a whole.
Math II-1; recognize, analyze, and use patterns.

Math Standard V

Objective 2

Connections

Background Information

It is important for students to make connections between probability and real life applications. Through the use of games, the students can learn to make better predictions, increasing their probability to win. The terms: likely, unlikely, certain, possible, and impossible become more meaningful because of the variance of outcomes. The students need to have a prior understanding of the terms theoretical and experimental probability. Probability is not just a guess; it is based on mathematical formulas.

Research Basis

Moch, P.L (2001). Manipulatives Work!. *The Educational Forum*. 66, 81-87.

Under current reforms of mathematical curriculum much debate has arisen over the issues of depth being more important than breadth; and the pressures of how inclusion affects standardized testing results. The author discusses the use of manipulatives and how it strengthens current mathematical curriculum and reinforces the notion that worksheets should not be the beginning or the end of any student's mathematical experience. She notes, however, that is important that teachers gain proficiency in appropriate usage to gain maximum benefits.

Koirala, H.P. (2002). Facilitating Student Learning Through Math Journals. *ERIC Source* (ED476099). Retrieved January 12, 2007, from http://www.eric.ed.gov.

The author, in analyzing over 1800 math journal entries over a period of five years, has determined that journal writing has a great potential in aiding student mathematical learning. Math journals

help students demonstrate their mathematical thinking processes and understanding, and aid teachers in better understanding students' grasp of the concepts.

Invitation to Learn

Tell student that you are going to play a short game of "Even I Win, Odd You Lose." When they argue that that isn't fair then concede that you made a mistake and rename the game "Even I Win, Odd You Win." Then tell them that they're going to use the four basic operations on a number and if the final answer is "even", you win; if it is "odd," they win.

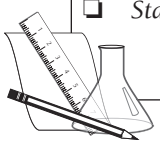
1. Have students pick a number
2. Add *3 to it *(*this number can be even or odd)*
3. Multiply the sum by *6 *(*must always be an even number)*
4. Subtract *6 from the product *(*must always be the same number that is used in step 3)*
5. Divide by *2 *(*don't change this number)*
6. Ask "What is the quotient?"
7. Ask "Even or Odd?" State: "I win!!!" *(*You will always win. If any student comes up with an odd number, challenge their answer by walking through each step and checking their answers. You can change the numbers, but note the above teacher notes.)*

After several rounds, or when the students begin to realize that they will never win, begin discussion of why odd numbers will never win.

As a final note, you might point out that you really didn't make a mistake with the original title of the game, "Even I Win, Odd You Lose."

Materials

- ❑ *Escape From the Core*
- ❑ Markers
- ❑ Dice
- ❑ Direction card
- ❑ *Zip, Zap, Zonk, You Win*
- ❑ Pencils
- ❑ Scratch paper
- ❑ *Ya' Don't Know Beans*
- ❑ Beans
- ❑ *Do You Feel Lucky?*
- ❑ Bags to hold counters
- ❑ 20 counters or tiles
- ❑ *Math Journal*
- ❑ *Station Rules*



Instructional Procedures

"Playing Around With Probability" Stations

These games are designed for two to three players each. Have enough materials at each station for at least three to four groups depending on the size of your class. Groups may rotate through each station independently or as directed by teacher. Remind the students to respond to the questions in their math journal at the end of each activity before advancing to the next station.

Station 1: "Escape from the Core"

1. All players place their markers in the center box.
2. The goal of the game is to be the first player to move a marker off the board.
3. On each turn, a player determines which way they want to rotate the *Direction Card*. This and the number rolled on the die will determine the direction moved for that turn.
4. Roll the die and move in the direction indicated by the number on the *Direction Card*. Each player only moves one square (regardless of the number on the die).
5. Continue playing until the first player moves off the board.
6. The winning player scores one point for each victory.
7. Play until time is over.
8. Respond in *Math Journal*

Station 2: "Zip, Zap, Zonk, You Win!"

1. Each player gets a game board.
2. First player, The Zap Master, thinks of a 2-digit number, writes it on scratch paper, and keeps it hidden from other player.
3. The second player, The Guesser, writes their guess on line #1 of the game board.
4. The Zap Master puts an x in the correct clue column on line #1.
   Clue guidelines:
   - Zip = The Guesser has nothing correct
   - Zap = The Guesser has correct digit(s), but in the wrong position
   - Zonk = The Guesser has one correct digit in the correct position
   - You Win = The Guesser has guessed the correct number
5. The Guesser continues guessing numbers until the correct number is guessed.
6. Players then switch roles and continue playing.
7. Respond in *Math Journal*

Station 3: "Ya' Don't Know Beans"

1. This game is for two players. *(*Teacher note: After first round, can be played with up to five players.)*

2. Each group will get a game board, "*Ya' Don't Know Beans,*" 21 beans, pair of dice, and a pencil.
3. Each player rolls one die to determine who goes first high number writes first.
4. Players take turns writing their initials in one box on the bottom until an *equal* number of boxes have been filled in. Depending on number of players, there will be empty boxes.
5. Players take turns rolling the dice, adding the sums, and placing a bean in that column until all 21 beans have been placed.
6. Each player then totals the number of beans placed in each of the columns with their initials. Player with the most beans wins.
7. Respond in *Math Journal*.

Station 4: "Do You Feel Lucky?"

1. Each group gets a game board and a bag with 20 counters (10 of each color). They determine which color moves two spaces and which moves three spaces by writing in the blanks.
2. They then decide which player will go first.
3. Before drawing out a counter, the player first predicts which color they will draw.

   Movement guidelines:

   - An incorrect guess……...move one space.
   - A correct guess (_Players write in which color_)…….move two spaces.
   - A correct guess (_Players write in which color_)…….move three spaces
4. Do not return counters to the bag after drawing.
5. First player to reach the center wins!
6. Respond in *Math Journal*.

Math Journal Response Questions

Station 1: "Escape from the Core"

- Before beginning the game, predict how many turns it will take to win the game. _____. How close was your prediction? _____ ___
- How did knowing about probability affect your decisions on rotating the direction card?
- How often did your experimental probability match the theoretical probability?

- How could you change the rules to increase your probability of winning?

Station 2*:* "Zip, Zap, Zonk, You Win!"

- Before beginning the game as The Guesser, predict how many turns it will take to guess the number? _____ How close was your prediction? _____
- Before beginning the game as Zap Master, predict how many turns it will take your partner to guess the number? _____ How close was your prediction? _____
- As The Guesser, what strategies did you use to select your numbers?
- As the Zap Master, what strategies did you use to pick your number?
- How does knowing about probability increase your chances of winning?
- How could you change the rules to make it more challenging?

Station 3: "Ya' Don't Know Beans"

- What strategies did you use in choosing which boxes to initial?
- After you and your partner have placed your initials in the boxes, predict how many beans you will win._____ How close was your prediction? _____
- On the next round, did you change your strategies in choosing where to place your initials? Why or why not?
- How does knowing about probability increase your chances of winning?

Station 4: "Do You Feel Lucky?"

- What strategies did you use in guessing which color you would draw?
- Did the different values of the colors influence your guess? Why or why not?
- How does knowing about probability increase your chances of winning?
- How would the game change if you returned the tiles after each draw?

Assessment Suggestions

- *Math Journal*

Curriculum Extensions/Adaptations/ Integration

- Using the activity *"Dice Sums"* from the 2003 Elementary CORE Academy in connection with Station 3: Ya' Don't Know Beans, may help students better apply probability in their selections.
- These activities can be put in a math center and allow students to continue to playing these games.
- Have students investigate how changing the rules to any of these game changes the probability of winning.

Family Connections

- Play a game that uses probability with the family.
- Allow students to take home the station games to play with their families.
- The book, *Family Math*, has a variety of probability games that could be used as a grade-level or school wide Family Math Night.

Additional Resources

Books

Family Math, by Ruth Cossey, Jean Kerr Stenmark, and Virginia Thompson; ISBN 0912511060

Probability, by Sarah Jane Brian; ISBN 0590373676

CDs

Mrs. Glosser's The Math Goodies CD contains over 275 pages of self-paced instruction to make math fun! This can be purchased online at the following website: http://www.mathgoodies.com/cd/

Web sites

http://www.shodor.org/interactivate/activities/CrazyChoicesGame/ - *Crazy Choices Game*-an interactive site where students can use different random number-generating devices

http://www.shodor.org/interactivate/lessons/Probability/ - *Race Cars* - an interactive site where students learn about probability by predicting the outcome of planned experiments and playing racing games.

http://www.shodor.org/interactivate/lessons/PlayingWithFire/ - *Playing with Fire*-Students use probability to set a simulated forest on fire.

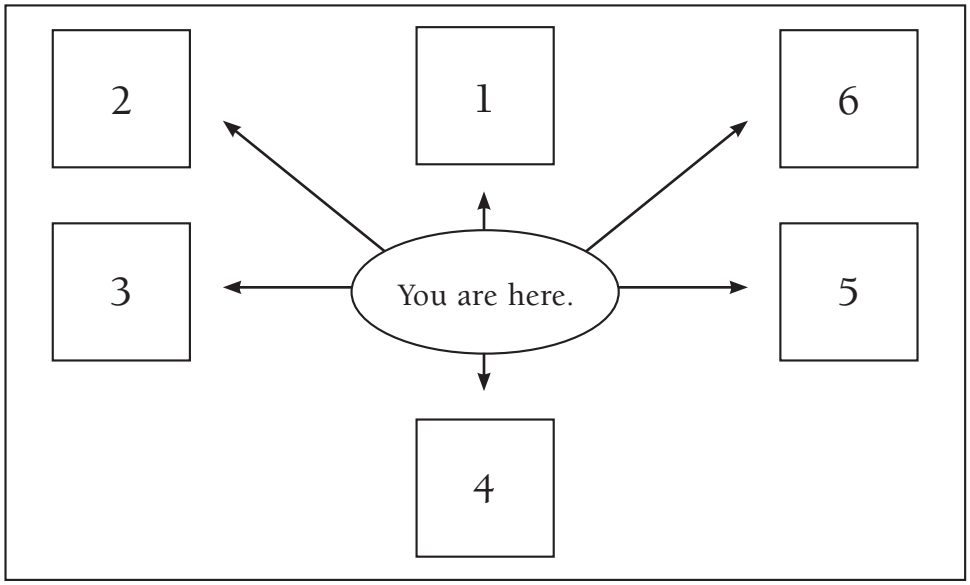
http://illuminations.nctm.org/LessonDetail.aspx?id=L585 - *Sticks and Stones*-Students can investigate probability with a Native American game

http://www.mathgoodies.com/lessons/vol6/complement.html - *Complement of an Event*-a series of five probability experiments using different manipulatives.

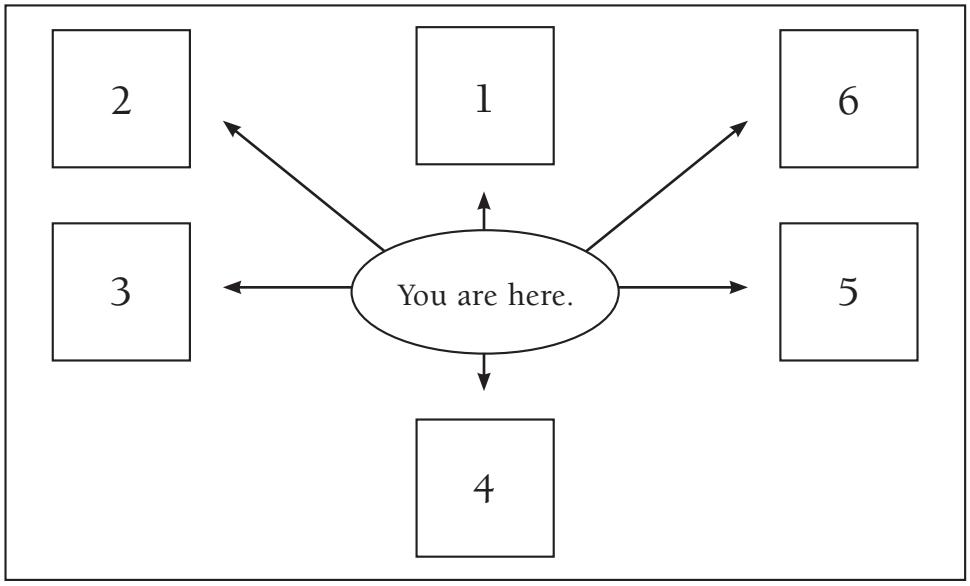
Escape from the Core

				Core (start)				

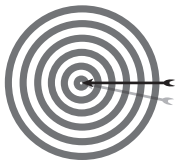
Direction Card



Direction Card



Zip, Zap, Zonk, You Win



	Guess	Zip Nothing correct	Zap Correct digit, Wrong position		Zonk Correct digit, Correct position	You Win!
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						

Ya' Don't Know Beans

Initials or markers for each player	Sum of dice	2	3	4	5	6	7	8	9	10	11	12

Do You Feel Lucky?

An incorrect guess....move 1 space
A correct guess (_____)....move 2 spaces
A correct guess (_____)....move 3 spaces

	Start Player 1					Sorry, lose a turn			
				Take 1 more turn			Go back 1 space		
Go ahead 1 space									
								Go back 1 space	
Go back 1 space									
									Go ahead 1 space
	Go back 1 space			Take 1 more turn					
Sorry, lose a turn					**Start** Player 2				

Do You Feel Lucky?

Finish

Math Probability Journal

Name: _____

"Do You Feel Lucky?"

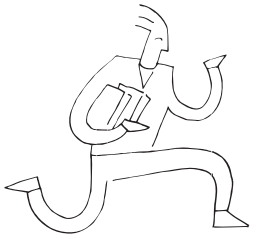
1. What strategies did you use in guessing which color you would draw?

2. Did the different values of the colors influence your guess? Why or why not?

3. How does knowing about probability increase your chances of winning?

4. How would the game change if you returned the tiles after each draw?

Escape from the Core



1. Before beginning the game, predict how many turns it will take to win the game _____.

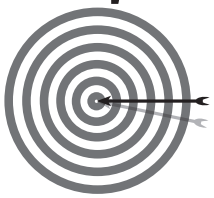
2. How close was your prediction? _____

3. How did knowing about probability affect your decisions on rotating the direction card?

4. How often did your experimental probability match the theoretical probability?

5. How could you change the rules to increase your probability of winning?

Zip, Zap, Zonk, You Win!



1. Before beginning the game as "The Guesser," predict how many turns it will take to guess the number? _____ How close was your prediction? _____

2. Before beginning the game as "Zap Master," predict how many turns it will take your partner to guess the number? _____ How close was your prediction? _____

3. As "The Guesser" what strategies did you use to select your numbers?

_____

_____

4. As the "Zap Master" what strategies did you use to pick your number?

_____

_____

5. How does knowing about probability increase your chances of winning?

_____

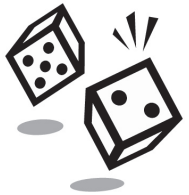
_____

6. How could you change the rules to make it more challenging?

_____

_____

Ya' Don't Know Beans



1. What strategies did you use in choosing which boxes to initial?

_____

_____

2. After you and your partner have placed your initials in the boxes, predict how many beans you will win._____ How close was your prediction? _____

3. On the next round, did you change your strategies in choosing where to place your initials? _____ Why or why not?

_____

_____

4. How does knowing about probability increase your chances of winning?

_____

_____

Station Rules

Station 1 Rules: "Escape from the Core"

1. All players place their markers in the center box.
2. The goal of the game is to be the first player to move their marker off the board.
3. On each turn, a player determines which way they want to rotate the Direction Card. This and the number rolled on the die will determine the direction moved for that turn.
4. Roll the die, and move in the direction indicated by the number on the Direction Card. Each player only moves 1 square (regardless of the number on the die).
5. Continue playing until the first player moves off the board.
6. The winning player scores 1 point for each victory.
7. Play again if there is time.
8. Respond in Math Journal.

Station 2 Rules: "Zip, Zap, Zonk, You Win!"

1. Each player gets a game board.
2. First player, "The Zap Master," thinks of a 2-digit number, writes it on scratch paper, and keeps it hidden from the other player.
3. The second player, "The Guesser", writes their guess on line # 1 of the game board.
4. "The Zap Master" puts an x in the correct clue column on line # 1. Clue guidelines:
   - Zip = "The Guesser" has nothing correct.
   - Zap = "The Guesser" has correct digit(s), but in the wrong position.
   - Zonk = "The Guesser" has one correct digit in the correct position.
   - You Win = "The Guesser" has guessed the correct number
5. "The Guesser" continues guessing numbers until the correct number is guessed.
6. Players then switch roles and continue playing.
7. Respond in Math Journal.

Station 3 Rules: "Ya' Don't Know Beans"

1. This game is for 2 players
2. Get a game board, "Ya' Don't Know Beans", 21 beans, pair of dice, and a pencil.
3. Each player rolls one die to determine who goes first (High number writes first)
4. Players take turns writing their initials in one box on the bottom until an <u>equal</u> number of boxes have been filled in.
5. Take turns rolling the dice, adding the sums, and placing a bean in that column until all 21 beans have been placed.
6. Each player then totals the number of beans placed in each of the columns with their initials. Player with the most beans wins.
7. Respond in Math Journal.

Station 4 Rules: "Do You Feel Lucky?"

1. Get a game board and a bag with 20 counters (10 of each color). Determine which color moves 2 spaces and which moves 3 spaces by writing in the blanks.
2. Decide which player will go first
3. Before drawing out a counter, predict which color you will draw. Movement guidelines:
   - An incorrect guess……..move one space
   - A correct guess (_color_)…….move two spaces
   - A correct guess (_color_)…….move three spaces
4. <u>Do not</u> return counters to the bag after drawing
5. First player to reach the center wins!
6. Respond in Math Journal.

Appendix

Name _____ Date _____

Playing with Remainders

Listen to the play *"Round-up!"* and then answer the following four questions:

1. 23 people went to the family reunion. If 5 people could fit in each car, how many cars were needed to take people to the *No-Remainder Ranch*?

2. At the *No-Remainder Ranch* 4 people were assigned to each cabin. How many cabins were needed for the 23 people?

3. While they were at the ranch, the family members went on a river-rafting trip. If 6 people could fit on each raft, how many rafts were needed for the 23 people?

4. Why did Tyler and Nikki have to round up their remainders each time in this story?

Listen to the play *"You Just Drop It!"* and then answer the following four questions:

1. How many bunches of 7 flowers could Shailee and Marisol make with 37 flowers?

2. How many plates of 6 brownies each could be made with 34 brownies?

3. How many plates of 8 cookies each could be made with 50 cookies?

4. Why did Shailee and Marisol have to drop their remainders each time in this story?

Listen to the play "*Sharing is Very Important?* and then answer the following three questions:

1. Scott and Travis cut an 11-foot rope into two equal pieces. How long was each piece?

2. Four boys are sharing 17 Airheads equally. How many Airheads does each boy get to eat?

3. Why didn't the remainders in this story need to be rounded up or dropped?

Write a good rule for what to do with remainders.

when you need to round up the remainder	when you need to drop the remainder	when you need to share the remainder equally

Name _____ Date _____

Remainder Stories

Answer each question with a complete sentence. Then tell how you used the remainder (whether you rounded up, dropped, or shared the remainder equally). Last, tell why you used the remainder the way you did. Each problem is worth four points (1 point = correct answer; 1 point = complete sentence; 1 point = correct use of remainder; 1 point = explanation for use of remainder).

1. Skyler is helping his mother plan a wedding breakfast for his older sister, Jessica. They are expecting 63 family members to attend, and they are using round tables that seat 8 guests each. How many tables will be needed to seat 63 people?

2. Skyler and his sister, Rylie, are preparing flower bouquets as centerpieces for each table at the wedding breakfast. They hope to have enough to decorate the table that is displaying the wedding cake as well. They have 67 carnations and wish to put 6 carnations in each bouquet. How many bouquets can they make with 67 carnations?

3. Rylie is going to the zoo for her 12th birthday party, and she is taking 9 friends. The zoo has a new baby giraffe, and groups of 3 children are allowed at a time in a special viewing room to see the giraffe and his mother. How many tours will it take for Rylie and her 8 friends to see the giraffe?

4. At Rylie's 12th birthday party, she wants to give each of her friends a jar with a variety of candy from the candy store. She has 5 friends coming, and she has 113 individually wrapped pieces of candy. If she gives each person the same number of pieces of candy, how many pieces will each friend receive in her candy jar?

5. While Rylie is celebrating with her friends, Skyler's mom gave him $10.00 to share equally with his three best friends so they could buy candy too. How much money do Skyler and his friends each get to spend?

6. Jessica is making curtains for her new apartment. She has 15 yards of material to make 2 sets of curtains. How much fabric can she use for each set?

7. Jessica's mother is serving punch at the wedding reception. She has a punch bowl that holds 106 ounces of punch. How many 8-ounce servings can be poured from the punch bowl when it is full?

Name _____ Date _____

Divisibility Test

1. Write the 24 numbers you created in the first column.
2. Decide if your numbers are divisible by 2, 3, 5, 6, 9, or 10. Write yes or no in the correct colums.

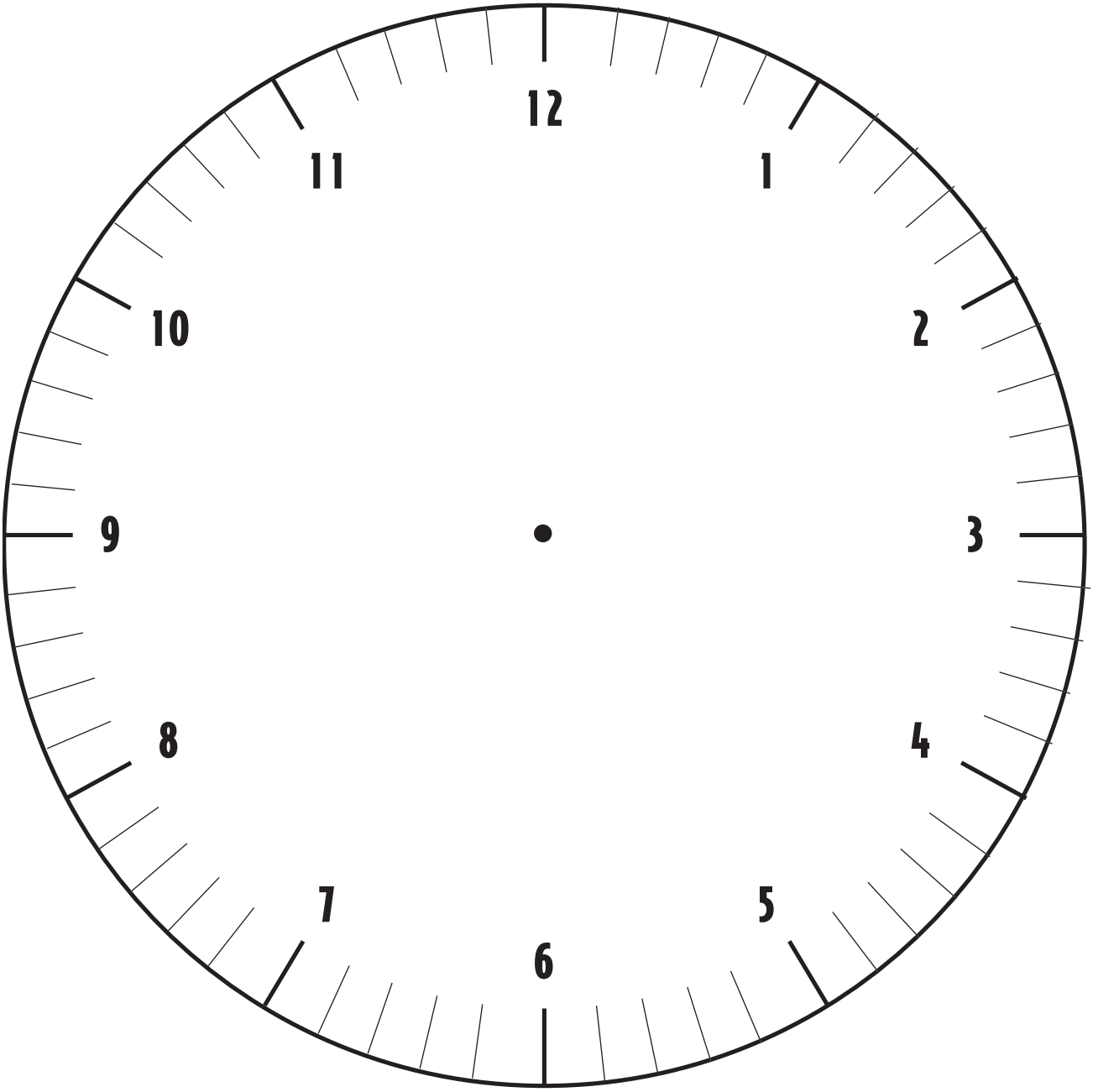
Number	2	3	5	6	9	10
1.						
2.						
3.						
4.						
5.						
6.						
7.						
8.						
9.						
10.						
11.						
12.						
13.						
14.						
15.						
16.						
17.						
18.						
19.						
20.						
21.						
22.						
23.						
24.						

Name _____ Date _____

Divisibility Rules

Number Divisible By	My Hypothesis	The Actual Rule
2		
3		
5		
6		
9		
10		

Clock



Cake

Can You Make?

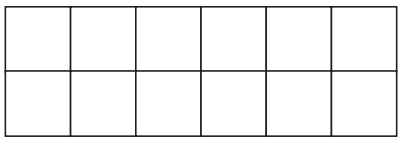
Can You Make?	with wholes	with halves	with thirds	with fourths	with fifths	with sixths
1						
1/2						
1/3						
1/4						
1/5						
1/6						
1/7						
1/8						
1/9						
1/10						
1/11						
1/12						

Can You Make?

Can You Make?	with sevenths	with eighths	with ninths	with tenths	with elevenths	with twelfths
1						
1/2						
1/3						
1/4						
1/5						
1/6						
1/7						
1/8						
1/9						
1/10						
1/11						
1/12						

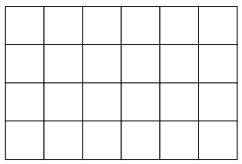
Share Equally

Share each of the items below as directed. Be prepared to explain how you did the sharing.

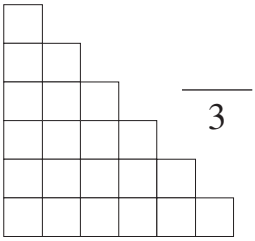


3

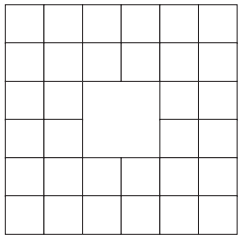
4



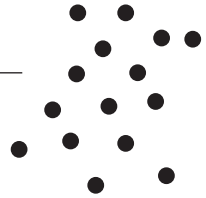
12



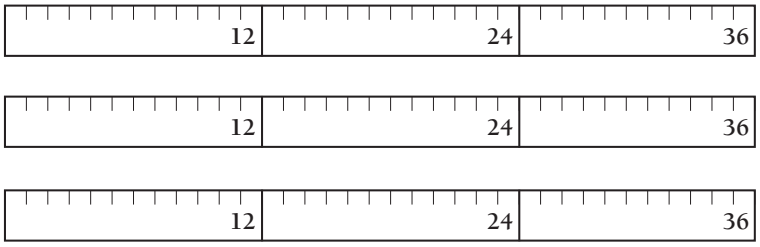
3



3



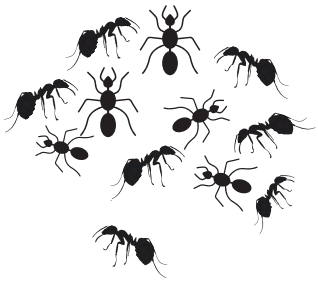
5



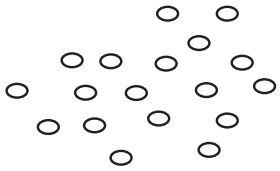
9

8

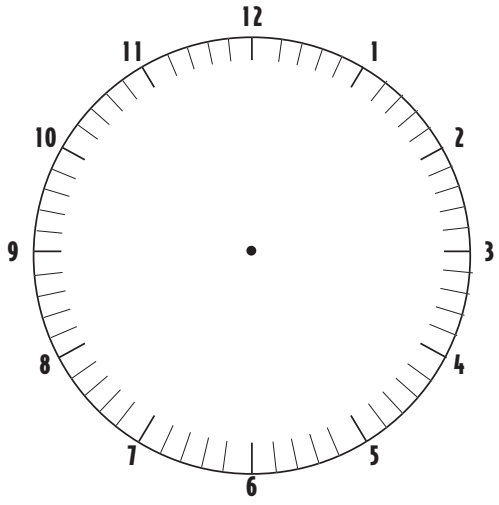
4



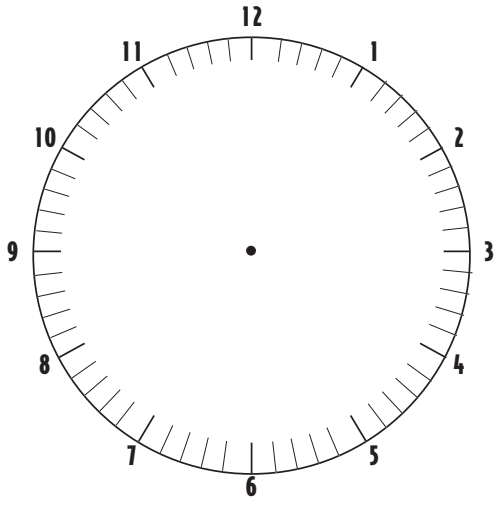
4



6



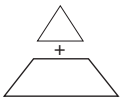
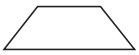
8

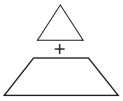


5

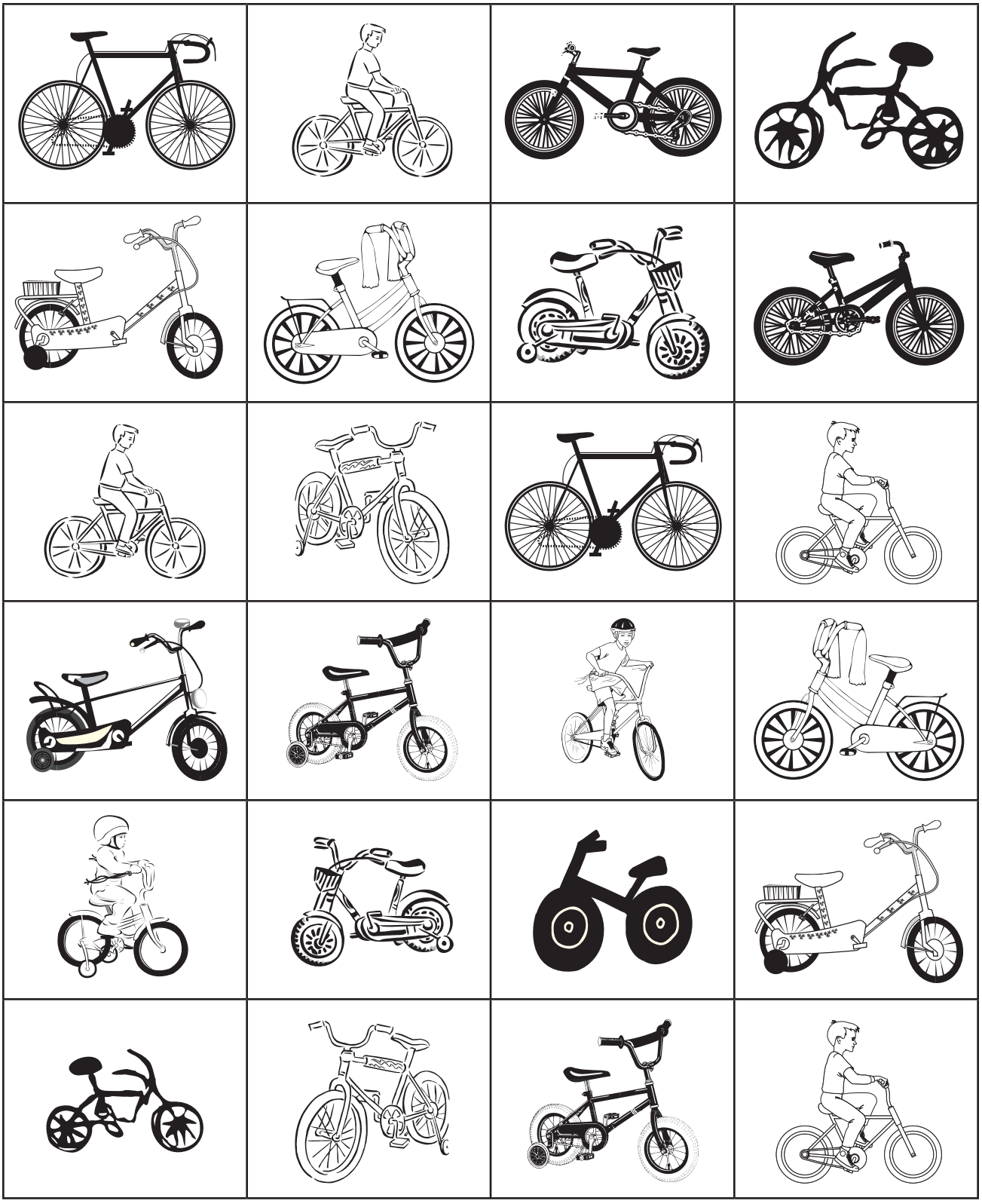
"If This Is...?

Challenge yourself to find the answers without using pattern blocks. Then if you need check with pattern blocks. Happy thinking!

If  is					
1/2					
1/4					
1/8					

If  is					
1					
2					
1/2					

BIKE Picture Cards



Name _____ Date _____

Milky Way Fraction Hunt

Write the appropriate parts of the words on the line to form a new word.

1. The first half of food + the last quarter of door.

_____

2. The last third of hat + the first 2/5 of heavy.

_____

3. The second 1/3 of office + the last 1/4 of door + the first 1/3 of street.

_____

4. The last half of go + the last 1/2 of done.

_____

5. The last 1/8 of elephant + the first 1/5 of order.

_____

6. The first 3/4 of fine + the last 3/4 of dish.

_____

7. The last 1/6 of cement + the first of 3/7 of history.

_____

8. The last half of bath + the finest 1/3 of end + the last 2/7 of require.

_____

9. The first 2/5 of water + the last 3/4 of fits.

_____

10. The last 1/6 of Glenda.

_____

11. The first 1/3 of principal + the first half of zero.

_____

12. The first 1/7 of instant + the first third of fat.

_____

13. The first 2/5 of young + the first 1/10 of understand.

_____

14. The first 1/4 of ugly + the first 1/5 of settlement.

_____

15. The first 1/4 of youthful + the last half of pour.

_____

16. The first 1/4 of hesitate + the last 2/3 of sad.

_____

17. The last 1/3 of rat + the first 2/5 of heart.

_____

18. The first 3/7 of mileage + the last 2/3 of sky.

_____

19. The first 1/5 of white + the last 1/3 of Friday.

_____

20. The last 1/4 of Meri + the first 1/5 of Susan.

_____

21. The first 3/5 of dirty + the last 3/7 of perfect + the first 2/5 of Lynda.

_____

22. The first 3/4 of bent + the last 2/3 of breath.

_____

23. The first 1/3 of Thomas + the first 1/8 of Endicott.

_____

24. The first 3/5 of sound + the last 2/9 of Aylsworth.

_____

25. The first quarter of positive + the first two thirds of Lee.

_____

26. The first 3/5 of quick + the second 1/4 of meat + the last 1/3 of patiently.

_____

27. The first third of get + the second fourth of Jody.

_____

28. The first half of loud + the last half of book.

_____

Write the clues in numerical order:

_____

_____

_____

The ANSWER KEY for the activity is: FOR THE FIRST ONE TO FINISH THIS THERE WAITS A PRIZE IF YOU USE YOUR HEAD THE MILKY WAY IS DIRECTLY BENEATH THE SOUTH POLE QUIETLY GO LOOK

BIKES

What is the whole unit that you are considering? _____

What fraction of the bikes are mountain bikes? _____

What fraction of the bikes have gears? _____

What fraction of the bikes have training wheels? _____

The boy style frames on the bikes represent _____ of the bikes.

How many of the handle bars have streamers on them. _____

How many of the bikes have fenders? _____

Make **two** true statements about the bike colors using the terms: **half, third, fourth, fifth, sixth, eighth, and/or tenth.**

1. _____

_____

_____

2. _____

_____

_____

BIKES

What is the whole unit that you are considering? _____

What fraction of the bikes are mountain bikes? _____

What fraction of the bikes have gears? _____

What fraction of the bikes have training wheels? _____

The boy style frames on the bikes represent _____ of the bikes.

How many of the handle bars have streamers on them. _____

How many of the bikes have fenders? _____

Make **two** true statements about the bike colors using the terms: **half, third, fourth, fifth, sixth, eighth, and/or tenth.**

1. _____

_____

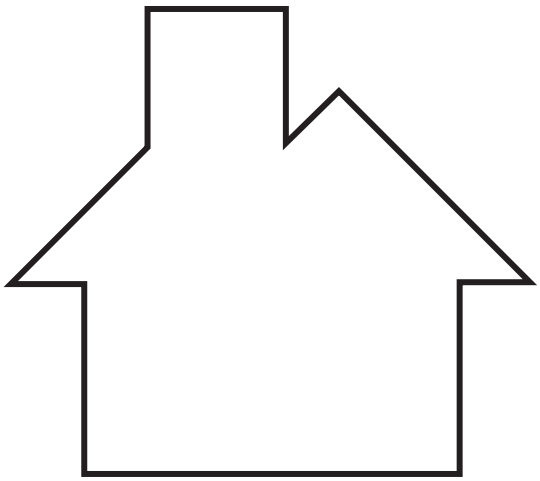
_____

2. _____

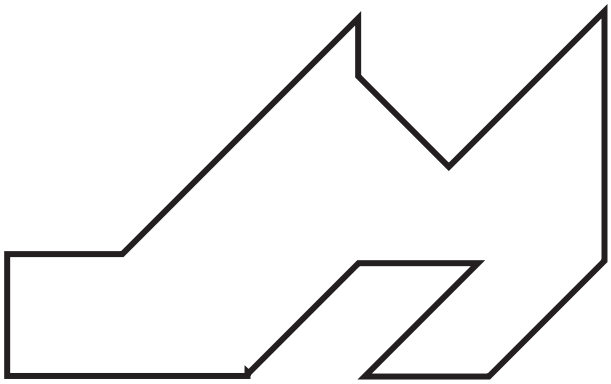
_____

_____

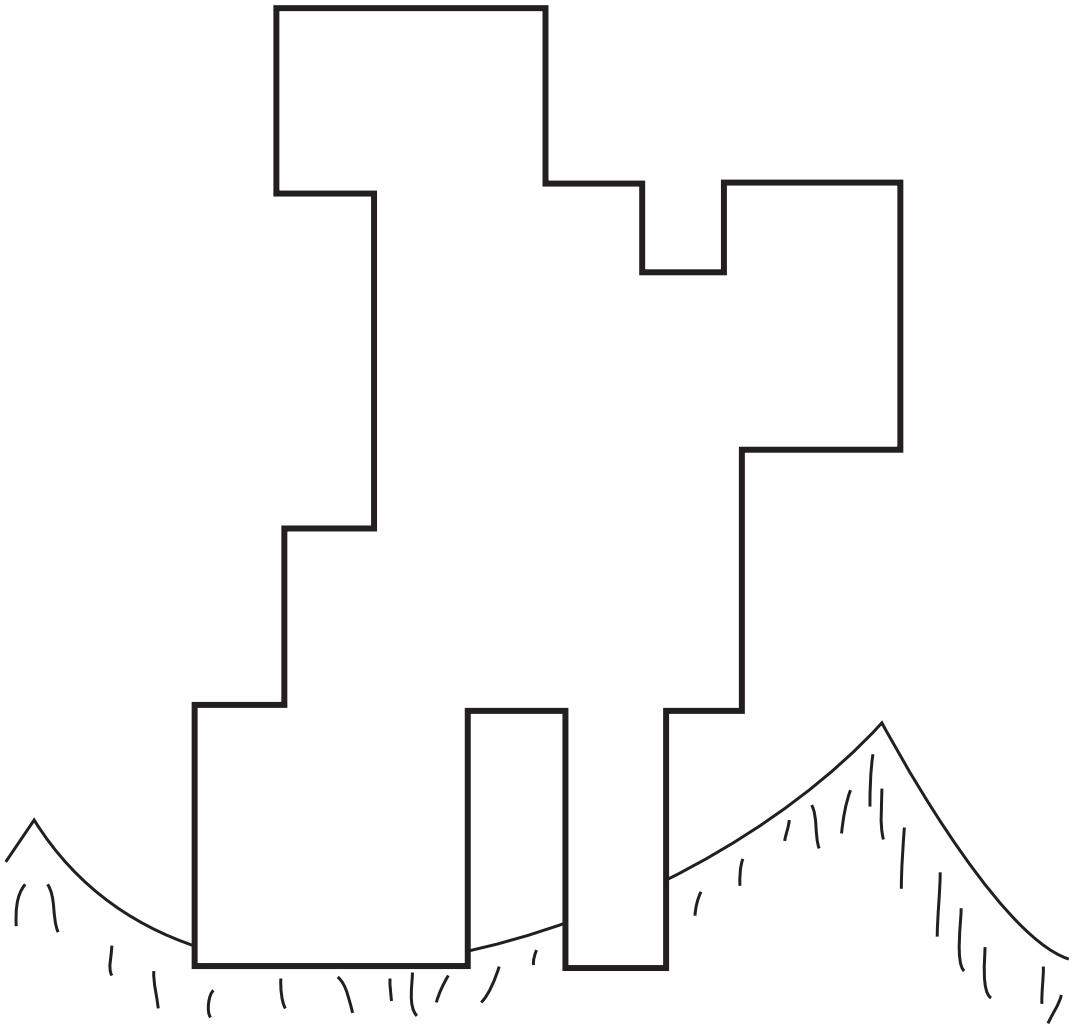
Tangram Puzzle



House



Shoe



One-on-the- Mountain

How Much Will It Hold??

List least to greatest, which solid has the largest volume.

Estimation	Actual after Measurement
1. _____	1. _____
2. _____	2. _____
3. _____	3. _____
4. _____	4. _____
5. _____	5. _____
6. _____	6. _____
7. _____	7. _____
8. _____	8. _____

SHAPE	Volume Measurement 1 # ml	Volume Measurement 2 # ml
Large Square prism		
Small Square prism		
Small rectangular prism		
Square pyramid		
Large Rectangular prism		
Cone		
Large cylinder		
Sphere		

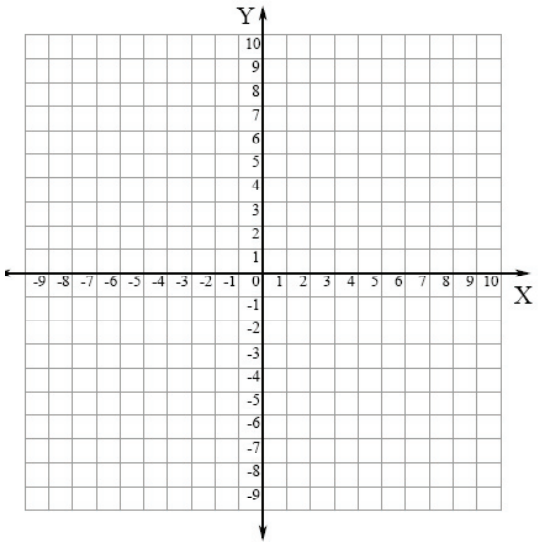
Type of material used for measurement:_____

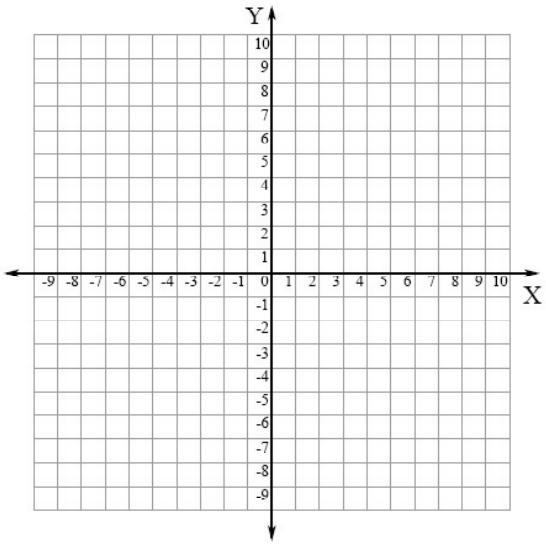
Look at results for possible relationships between different solids. Write and describe your findings below-

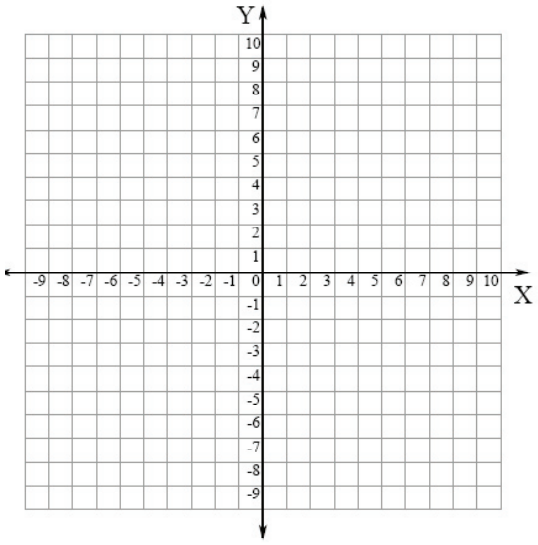
Tic Tac Toe

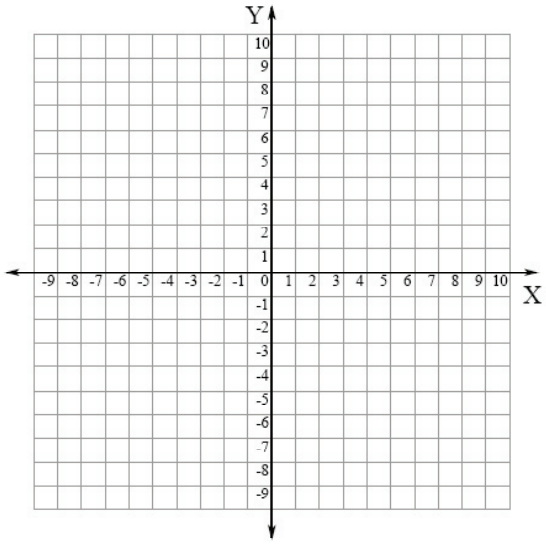
The object of the game is to get four X's or four O's in a row vertically, horizontally, or diagonally.

1. Play rock, paper, and scissors to decide who starts.
2. The winner begins the game, while the other picks X or O.
3. To start, write down the ordered pairs on paper. Point to that location. Other player checks to see if it is correct.
4. If you are right, place your mark. If you are wrong, you lose your turn.
5. Take turns until one player has four in a row.
6. Play four games, one in each quadrant.

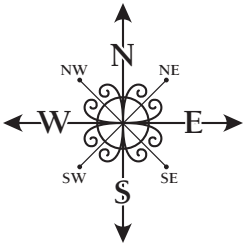


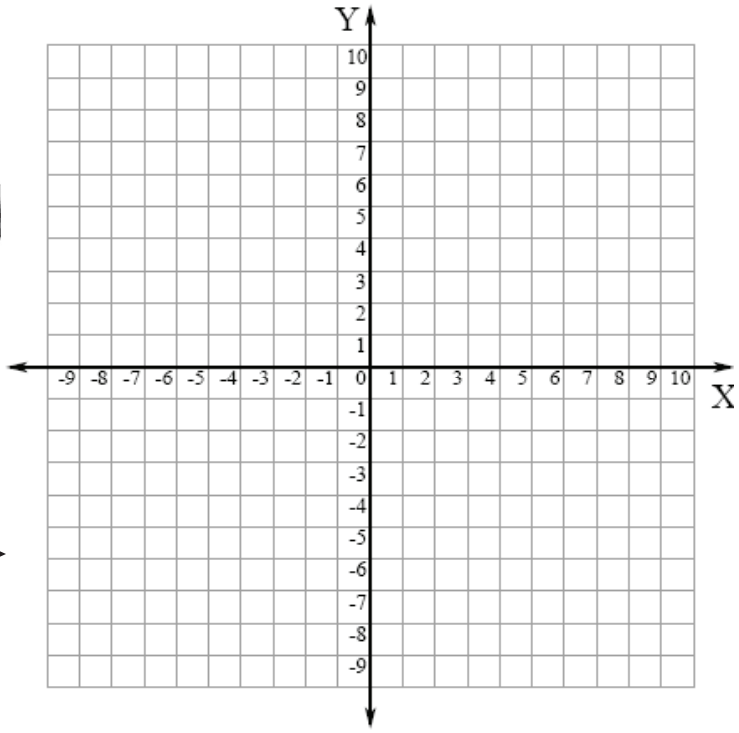






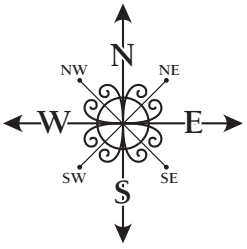
In Search of Buried Treasure

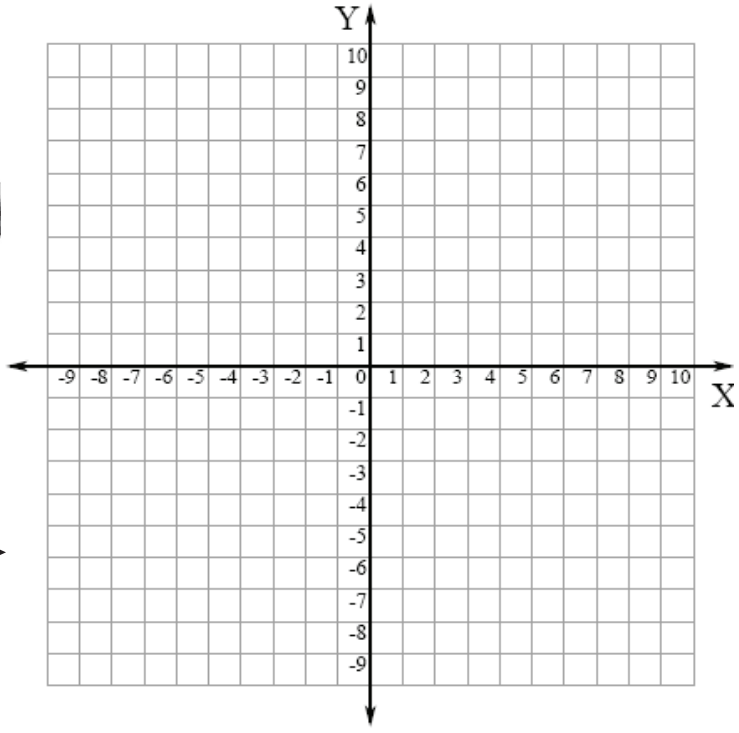




Guess	Clue

In Search of Buried Treasure





Guess	Clue

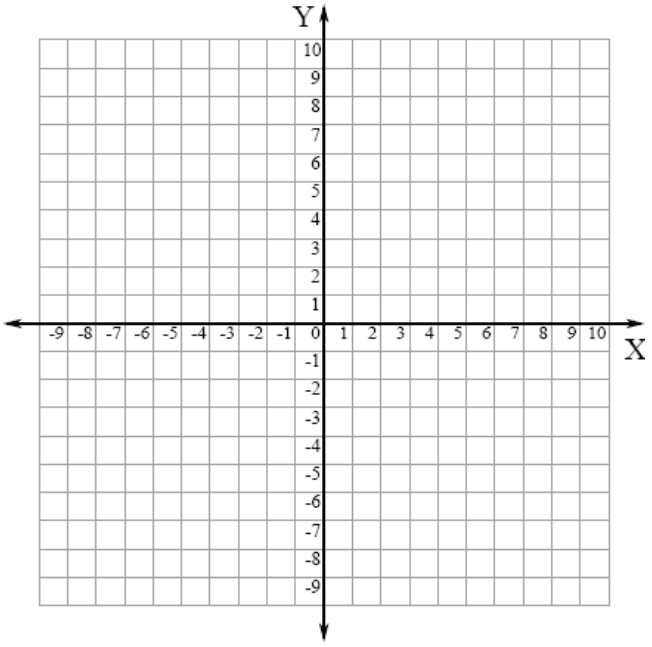
Space Wars

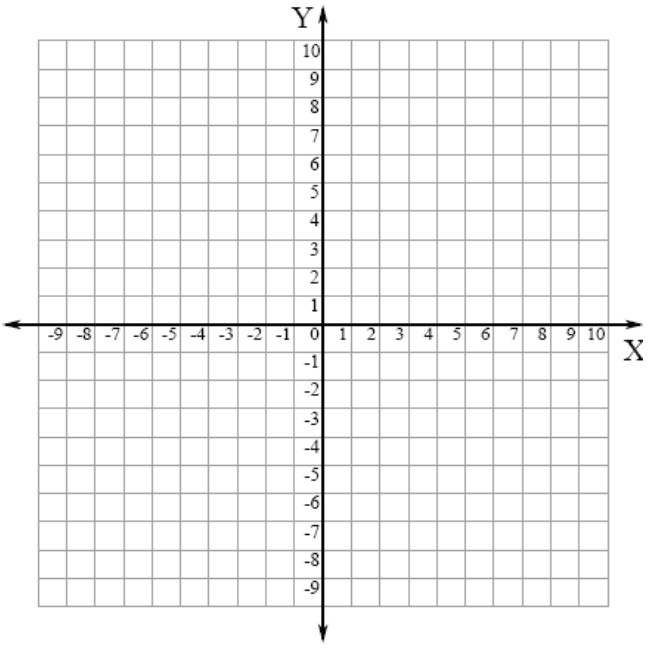
Object: Find and destroy each others' hidden spaceships.
Rules:

1. Players each mark (vertically or horizontally only) their "Fleet" of five ships on their "Air Space" on the coordinate plane. There must be a least one ship in each quadrant.
2. The ships should remain hidden from the opponent's view. A book works well.
3. Taking turns, players call out their "shots" attempting to get "hits" on the opponent's spaceships and destroy them.
4. "Hits" or "misses" should be marked on the other coordinate plane.
5. Use an X for a hit and an O for a miss.
6. A spaceship is destroyed when all points on the craft are hit.
7. A player wins when all 5 opponent's ships are destroyed.

Fleet

Length	Name
5 points	Death Star
4 points	Warbirds
3 points	Starship
3 poin11:37:56 AM	Fighters
2 points	Starbase





"Do You Feel Lucky?"

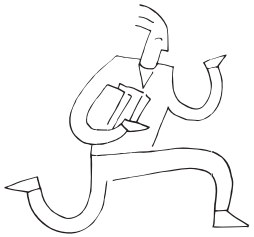
1. What strategies did you use in guessing which color you would draw?

2. Did the different values of the colors influence your guess? Why or why not?

3. How does knowing about probability increase your chances of winning?

4. How would the game change if you returned the tiles after each draw?

Escape from the Core



1. Before beginning the game, predict how many turns it will take to win the game _____.

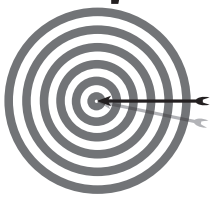
2. How close was your prediction? _____

3. How did knowing about probability affect your decisions on rotating the direction card?

4. How often did your experimental probability match the theoretical probability?

5. How could you change the rules to increase your probability of winning?

Zip, Zap, Zonk, You Win!



1. Before beginning the game as "The Guesser," predict how many turns it will take to guess the number? _____ How close was your prediction? _____

2. Before beginning the game as "Zap Master," predict how many turns it will take your partner to guess the number? _____ How close was your prediction? _____

3. As "The Guesser" what strategies did you use to select your numbers?

_____

_____

4. As the "Zap Master" what strategies did you use to pick your number?

_____

_____

5. How does knowing about probability increase your chances of winning?

_____

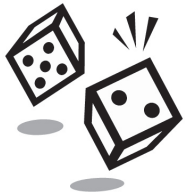
_____

6. How could you change the rules to make it more challenging?

_____

_____

Ya' Don't Know Beans



1. What strategies did you use in choosing which boxes to initial?

_____

_____

2. After you and your partner have placed your initials in the boxes, predict how many beans you will win._____ How close was your prediction? _____

3. On the next round, did you change your strategies in choosing where to place your initials? _____ Why or why not?

_____

_____

4. How does knowing about probability increase your chances of winning?

_____

_____

Station Rules

Station 1 Rules: "Escape from the Core"

1. All players place their markers in the center box.
2. The goal of the game is to be the first player to move their marker off the board.
3. On each turn, a player determines which way they want to rotate the Direction Card. This and the number rolled on the die will determine the direction moved for that turn.
4. Roll the die, and move in the direction indicated by the number on the Direction Card. Each player only moves 1 square (regardless of the number on the die).
5. Continue playing until the first player moves off the board.
6. The winning player scores 1 point for each victory.
7. Play again if there is time.
8. Respond in Math Journal.

Station 2 Rules: "Zip, Zap, Zonk, You Win!"

1. Each player gets a game board.
2. First player, "The Zap Master," thinks of a 2-digit number, writes it on scratch paper, and keeps it hidden from the other player.
3. The second player, "The Guesser", writes their guess on line # 1 of the game board.
4. "The Zap Master" puts an x in the correct clue column on line # 1. Clue guidelines:
   - Zip = "The Guesser" has nothing correct.
   - Zap = "The Guesser" has correct digit(s), but in the wrong position.
   - Zonk = "The Guesser" has one correct digit in the correct position.
   - You Win = "The Guesser" has guessed the correct number
5. "The Guesser" continues guessing numbers until the correct number is guessed.
6. Players then switch roles and continue playing.
7. Respond in Math Journal.

Station 3 Rules: "Ya' Don't Know Beans"

1. This game is for 2 players
2. Get a game board, "Ya' Don't Know Beans", 21 beans, pair of dice, and a pencil.
3. Each player rolls one die to determine who goes first (High number writes first)
4. Players take turns writing their initials in one box on the bottom until an <u>equal</u> number of boxes have been filled in.
5. Take turns rolling the dice, adding the sums, and placing a bean in that column until all 21 beans have been placed.
6. Each player then totals the number of beans placed in each of the columns with their initials. Player with the most beans wins.
7. Respond in Math Journal.

Station 4 Rules: "Do You Feel Lucky?"

1. Get a game board and a bag with 20 counters (10 of each color). Determine which color moves 2 spaces and which moves 3 spaces by writing in the blanks.
2. Decide which player will go first
3. Before drawing out a counter, predict which color you will draw. Movement guidelines:
   - An incorrect guess……..move one space
   - A correct guess (_color_)…….move two spaces
   - A correct guess (_color_)…….move three spaces
4. <u>Do not</u> return counters to the bag after drawing
5. First player to reach the center wins!
6. Respond in Math Journal.