ELEMENTARY
CORE Academy
UTAH STATE OFFICE OF EDUCATION & UTAH STATE UNIVERSITY

ELEMENTARY CORE ACADEMY • ELEMENTARY CORE ACADEMY •

6TH GRADE

2005

PARTICIPANT HANDBOOK

UTAH STATE OFFICE OF EDUCATION

UtahState UNIVERSITY

ELEMENTARY CORE ACADEMY

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Organizations:

Utah State Office of Education (USOE)
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Dear CORE Academy Teachers:

Thank you for your investment in children and in building your own expertise as you participate in the CORE Academy. I hope your involvement helps you to sustain a laser-like focus on student achievement.

Teachers in Utah are superb. By participating in the Academy, you join a host of teachers throughout the state who understand that teaching targeted on the core curricula, across a spectrum of subjects, will produce results of excellence. The research is quite clear: the closer the match of explicit instruction to core standards, the better the outcome on core assessments.

I personally appreciate your excellence and your desire to create wonderful classrooms of learning for students. Thank you for your dedication. I feel honored to associate with you and pledge my support to lead education in ways that benefit all of our children.

Sincerely,

Patti Harrington, Ed.D.
State Superintendent
of Public Instruction

Funding Sources

Appreciation is expressed for the tremendous educational input and monetary commitment of several organizations for the successful delivery of the Elementary CORE Academy. This year's Elementary CORE Academy was developed and funded through a variety of sources. The Utah State Office of Education (USOE), in collaboration with Utah State University (USU) and local school districts of Utah, have supported kindergarten through sixth grade teachers with professional development experiences that will enhance the educational experience for Utah children.

Major funding for the Academy comes from the following sources:

Federal/State Funds:

Utah State Office of Education
Staff Development Funds
Special Education Services Unit

ESEA Title II

Utah Math Science Partnership

WestED Eisenhower Regional Consortium

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Various sources including Quality Teacher Block, Federal ESEA Title II, and District Professional Development Funds

School Funds:

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Utah State Office of Education Special Education Services

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Additionally, numerous school districts, individual schools, and principals in Utah have sponsored teachers to attend the Academy. Other educational groups such as the Utah Division of Water Resources, National Energy Foundation, Utah Energy Office, and the Utah Mining Association have assisted in the development and delivery of resources in the Academy.

Most important is the thousands of teachers who take time from their summer to attend these professional development workshops. It is these teachers who make this program possible.

Goals of the Elementary CORE Academy

Overall

The purpose of the Elementary CORE Academy is to create high quality teacher instruction and improve student achievement through the delivery of professional development opportunities and experiences for teachers across Utah.

The Academy will provide elementary teachers in Utah with:

1. Models of exemplary and innovative instructional strategies, tools, and resources to meet newly adopted Core Curriculum standards, objectives, and indicators.
2. Practical models and diverse methods of meeting the learning needs of all children, with instruction implementation aligned to the Core Curriculum.
3. Meaningful opportunities for collaboration, self-reflection, and peer discussion specific to innovative and effective instructional techniques, materials, teaching strategies, and professional practices in order to improve classroom instruction.

Learning a limited set of facts will no longer prepare a student for real experiences encountered in today's world. It is imperative that educators have continued opportunities to obtain instructional skills and strategies that provide methods of meeting the needs of all students. Participants of the Academy experience will be better equipped to meet the challenges faced in today's classrooms.

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Sixth Grade Mathematics Core Curriculum

Utah Elementary Mathematics Core Curriculum

Introduction

Most students enter school confident in their own abilities; they are curious and eager to learn more. They make sense of the world by reasoning and problem solving. Young students are active, resourceful individuals who construct, modify, and integrate ideas by interacting with the physical world as well as with peers and adults. They learn by doing, collaborating, and sharing their ideas. Students' abilities to communicate through language, pictures, sound, movement, and other symbolic means develop rapidly during these years.

Young students are building beliefs about what mathematics is, about what it means to know and do mathematics, and about themselves as mathematical learners. Mathematics instruction needs to include more than short-term learning of rote procedures. Students must use technology and other mathematical tools, such as manipulative materials, to develop conceptual understanding and solve problems as they do mathematics. Students, as mathematicians, learn best with hands-on, active experiences throughout the instruction of the mathematics curriculum.

Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, the term mathematical proficiency has been chosen to capture what it means to learn mathematics successfully. Mathematical proficiency has five strands: computing (carrying out mathematical procedures flexibly, accurately, efficiently, and appropriately), understanding (comprehending mathematical concepts, operations, and relations), applying (ability to formulate, represent, and solve mathematical problems), reasoning (using logic to explain and justify a solution to a problem), and engaging (seeing mathematics as sensible, useful, and doable, and being able to do the work).

The most important observation about the five strands of mathematical proficiency is that they are interwoven and interdependent. This observation has implications for how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve that goal. At any given moment during a mathematics lesson or unit, one or two strands might be emphasized. But all the strands must eventually be addressed so that the links among them are strengthened. The integrated and balanced development of all five strands of mathematical proficiency should guide

the teaching and learning of school mathematics. Instruction should not be based on extreme positions that students learn solely by internalizing what a teacher or book says or solely by inventing mathematics on their own.

The Elementary Mathematics Core describes what students should know and be able to do at the end of each of the K-6 grade levels. It was developed, critiqued, and revised by a community of Utah mathematics teachers, university mathematics educators, State Office of Education specialists, mathematicians, and an advisory committee representing a wide variety of people from the community. The Core reflects the current philosophy of mathematics education that is expressed in national documents developed by the National Council of the Teachers of Mathematics, the American Association for the Advancement of Science, and the National Research Council. This Mathematics Core has the endorsement of the Utah Council of Teachers of Mathematics Association. The Core reflects high standards of achievement in mathematics for all students.

Organization of the Elementary Mathematics Core

The Core is designed to help teachers organize and deliver instruction.

- The INTENDED LEARNING OUTCOMES (ILOs) describe the goals for mathematical skills and attitudes. They are found at the beginning of each grade level, are an integral part of the Core, and should be included as part of instruction.
- A STANDARD is a broad statement of what students are expected to understand. Several Objectives are listed under each Standard.
- An OBJECTIVE is a more focused description of what students need to know and be able to do at the completion of instruction. If students have mastered the Objectives associated with a given Standard, they have mastered that Standard at that grade level. Several Indicators are described for each Objective.
- An INDICATOR is a measurable or observable student action that enables one to assess whether a student has mastered a particular Objective. Indicators are not meant to be classroom activities, but they can help guide classroom instruction.

Guidelines Used in Developing the Elementary Mathematics Core

The Core is:

Consistent With the Nature of Learning

The main intent of mathematics instruction is for students to value and use mathematics as a process to understand the world. The Core is designed to produce an integrated set of Intended Learning Outcomes for students.

Coherent

The Core has been designed so that, wherever possible, the ideas taught within a particular grade level have a logical and natural connection with each other and with those of earlier grades. Efforts have also been made to select topics and skills that integrate well with one another and with other subject areas appropriate to grade level. In addition, there is an upward articulation of mathematical concepts, skills, and content. This spiraling is intended to prepare students to understand and use more complex mathematical concepts and skills as they advance through the learning process.

Developmentally Appropriate

The Core takes into account the psychological and social readiness of students. It builds from concrete experiences to more abstract understandings. The Core focuses on providing experiences with concepts that students can explore and understand in depth to build the foundation for future mathematical learning experiences.

Reflective of Successful Teaching Practices

Learning through play, movement, and adventure is critical to the early development of the mind and body. The Core emphasizes student exploration. The Intended Learning Outcomes are central in each standard. The Core is designed to encourage instruction with students working in cooperative groups. Instruction should include recognition of the role of mathematics in the classroom, school, and community.

Comprehensive

The Elementary Mathematics Core does not cover all topics that have traditionally been in the elementary mathematics curriculum; however, it provides a comprehensive background in mathematics. By emphasizing depth rather than breadth, the Core seeks to empower students rather than intimidate them with a

The Core is:

- **Consistent**
- **Coherent**
- **Developmentally Appropriate**
- **Reflective of Successful Teaching Practices**
- **Comprehensive**
- **Feasible**
- **Useful and Relevant**
- **Reliant Upon Effective Assessment Practices**
- **Engaging**

collection of isolated and eminently forgettable facts. Teachers are free to add related concepts and skills, but they are expected to teach all the standards and objectives specified in the Core for their grade level.

Feasible

Teachers and others who are familiar with Utah students, classrooms, teachers, and schools have designed the Core. It can be taught with easily obtained resources and materials. A Teacher Handbook is also available for teachers and has sample lessons on each topic for each grade level. The Teacher Handbook is a document that will grow as teachers add exemplary lessons aligned with the new Core.

Useful and Relevant

This curriculum relates directly to student needs and interests. Relevance of mathematics to other endeavors enables students to transfer skills gained from mathematics instruction into their other school subjects and into their lives outside the classroom.

Reliant Upon Effective Assessment Practices

Student achievement of the standards and objectives in this Core is best assessed using a variety of assessment instruments. Performance tests are particularly appropriate to evaluate student mastery of mathematical processes and problem-solving skills. Teachers should use a variety of classroom assessment approaches in conjunction with standard assessment instruments to inform instruction. Sample test items, keyed to each Core Standard, may be located on the "Utah Mathematics Home Page" at http://www.usoe.k12.ut.us/curr/math. Observation of students engaged in instructional activities is highly recommended as a way to assess students' skills as well as attitudes toward learning. The nature of the questions posed by students provides important evidence of their understanding of mathematics.

Engaging

In the early grades, children are forming attitudes and habits for learning. It is important that instruction maximizes students' potential and gives them understanding of the intertwined nature of learning. Effective elementary mathematics instruction engages students actively in enjoyable learning experiences. Instruction should be as thrilling an experience for a child as seeing a rainbow, growing a flower, or describing a toad. In a world of rapidly expanding knowledge and technology, all students must gain the skills they will need to understand and function responsibly and successfully in the world. The Core provides skills in a context that enables students to experience the joy of learning.

Intended Learning Outcomes for Sixth Grade Mathematics

The main intent of mathematics instruction is for students to value and use mathematics and reasoning skills to investigate and understand the world.

The Intended Learning Outcomes (ILOs) describe the skills and attitudes students should learn as a result of mathematics instruction. They are an essential part of the Mathematics Core Curriculum and provide teachers with a standard for evaluation of student learning in mathematics. Significant mathematics understanding occurs when teachers incorporate ILOs in planning mathematics instruction.

By the end of sixth grade students will be able to:

1. **Demonstrate a positive learning attitude toward mathematics.**
   a. Display a sense of curiosity about numbers and patterns.
   b. Pose mathematical questions about objects, events, and processes.
   c. Demonstrate persistence in completing tasks.
   d. Apply prior knowledge and processes to construct new knowledge.
   e. Maintain an open and questioning mind toward new ideas and alternative points of view.
2. **Become mathematical problem solvers.**
   a. Determine the approach, materials, and strategies to be used in setting up a problem.
   b. Model problem situations in a variety of ways.
   c. Develop understanding of new mathematical concepts and vocabulary by answering questions such as: What made you think that? Did anyone think of this in a different way? Where have we seen a problem like this before?
   d. Construct and use concrete, pictorial, symbolic, and graphical models to represent problem situations.
   e. Know when to select and how to use grade-appropriate mathematical tools and methods as a natural and routine part of the problem-solving process.
   f. Build new mathematical knowledge through problem solving.
   g. Solve problems in both mathematical and everyday contexts.

- **Demonstrate a positive learning attitude toward mathematics**
- **Become mathematical problem solvers**
- **Reason mathematically**
- **Communicate mathematically**
- **Make mathematical connections**
- **Represent mathematical situations**

h. Recognize that there may be multiple ways to solve a problem.

i. Persevere in developing alternative problem-solving strategies if initially selected approaches do not work.

3. Reason mathematically.

a. Draw logical conclusions and make generalizations.

b. Determine the approach, materials, and strategies to be used in solving problems.

c. Use models, known facts, and relationships to explain reasoning.

d. Make precise calculations and check the validity of the results in the context of the problem.

e. Make conjectures based on observation and information and test mathematical conjectures and arguments.

f. Follow and construct logical arguments and judge their validity.

g. Analyze mathematical situations by recognizing and using patterns and relationships.

h. Justify answers and solution processes.

4. Communicate mathematically.

a. Represent mathematical ideas with objects, pictures, and symbols.

b. Express mathematical ideas to peers, teachers, and others through oral and written language.

c. Engage in mathematical discussions through brainstorming, asking questions, and sharing strategies for solving problems.

d. Explain mathematical work and justify reasoning and conclusions.

e. Analyze, evaluate, and explain mathematical arguments and conclusions presented by others.

5. Make mathematical connections.

a. Use one mathematical idea to extend understanding of another.

b. Recognize the role of mathematics in the classroom, school, and community.

c. Explore problems and describe and confirm results using various representations.

d. Recognize the connections between mathematics and other content areas and apply mathematical thinking and problem solving in those areas.

6. Represent mathematical situations.

a. Create and use representations to organize and communicate mathematical ideas.

b. Represent mathematical concepts using concrete, pictorial, and symbolic models.

Sixth Grade Math Standards

Standard I:
Students will acquire number sense and perform operations with rational numbers.

Standard I: Students will acquire number sense and perform operations with rational numbers.

Objective 1: Represent whole numbers and decimals in a variety of ways.

a. Change *whole numbers* with *exponents* to *standard form* (e.g., $2^4 = 16$) and recognize that $10^0 = 1$.

b. Read and write *numerals* from thousandths to one billion.

c. Write a whole number to 999,999 in *expanded form* using *exponents* (e.g., $876{,}539 = 8 \times 10^5 + 7 \times 10^4 + 6 \times 10^3 + 5 \times 10^2 + 3 \times 10^1 + 9 \times 10^0$).

d. Express numbers in *scientific notation* using positive powers of ten.

e. Classify whole numbers to 100 as *prime, composite*, or neither.

f. Determine the *prime factorization* for a whole number up to 50.

Objective 2: Identify relationships among whole numbers, fractions (rational numbers), decimals, and percents.

a. Find the *greatest common factor* and *least common multiple* for two numbers using a variety of methods (e.g., list of multiples, prime factorization).

b. Compare and order *rational numbers*, including mixed fractions, using a variety of methods and symbols.

c. Locate positive rational numbers on a number line.

d. Convert common fractions, decimals, and percents from one form to another (e.g., 3/4= 0.75 = 75%).

Objective 3: Model and illustrate meanings of operations and describe how they relate.

a. Represent division of a multi-digit *dividend* by two-digit *divisors*, including decimals, using models, pictures, and symbols.

b. Model addition, subtraction, multiplication, and division of fractions and decimals in a variety of ways (e.g., objects, a number line).

c. Apply *rules of divisibility*.

d. Select or write a number sentence that can be used to solve a multi-step problem and write a word problem when given a two-step expression or equation.

Objective 4: Use fractions and percents to communicate parts of the whole.

a. Divide regions, sets of objects, and *line segments* into equal parts using a variety of models and illustrations.

b. Name and write a fraction to represent a portion of a unit whole for halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and sixteenths.

c. Write a fraction or ratio in simplest form.

d. Name equivalent forms for fractions (halves, thirds, fourths, fifths, tenths), ratios, percents, and decimals, including repeating or terminating decimals.

e. Relate percents less than 1% or greater than 100% to equivalent fractions, decimals, *whole numbers*, and mixed numbers.

Objective 5: Solve problems using the four operations with whole numbers, decimals, and fractions.

a. Determine when it is appropriate to use estimation, mental math strategies, paper and pencil, or a calculator.

b. Use estimation strategies to determine whether results obtained using a calculator are reasonable.

c. Multiply up to a three-digit *factor* by a one- or two-digit factor including decimals.

d. Divide up to a four-digit *dividend* by a one- or two-digit *divisor* including decimals.

e. Add and subtract decimals to the thousandths place (e.g., 34.567+3.45; 65.3-5.987).

f. Add, subtract, multiply, and divide fractions and mixed numbers.

g. Solve problems using ratios and proportions.

h. Simplify *expressions* with *exponents*, using the *order of operations*.

Objective 6: Model, illustrate, and perform the operations of addition and subtraction of integers.

a. Recognize that the sum of an *integer* and its opposite is zero.

b. Model addition and subtraction of integers using manipulatives and a number line.

c. Add and subtract integers.

Standard II: Students will use patterns, relations, and functions to represent and analyze mathematical situations using algebraic symbols.

Objective 1: Recognize, analyze, and use multiple representations of patterns and functions and describe their attributes.

a. Analyze patterns on graphs and tables and write a generalization to predict how the patterns will continue.

b. Create tables and graphs to represent given patterns and algebraic *expressions*.

c. Write an algebraic expression from a graph or a table of values.

d. Draw a graph from a table of values or to represent an equation.

Objective 2: Represent, solve, and analyze mathematical situations using algebraic symbols.

a. Recognize that a number in front of a variable indicates multiplication (e.g., 3y means 3 times the quantity y).

b. Solve two-step equations involving *whole numbers* and a single variable (e.g., 3x+4=19).

c. Recognize that "≈" indicates a relationship in which the quantities on each side are approximately of equal value (e.g., $\Pi \approx 3.14$).

d. Recognize that an *exponent* can be represented in the following ways: 4^3 or 4^3.

e. Evaluate *expressions* and formulas, substituting given values for the variables (e.g., 2x+4; x=2; therefore, 2(2)+4=8).

f. Recognize that if the *product* is zero, then one or more factors equal zero (i.e., if a*b=0 then either a=0 or b=0 or a and b=0).

Standard II: Students will use patterns, relations, and functions to represent and analyze mathematical situations using algebraic symbols.

Standard III:
Students will use spatial and logical reasoning to recognize, describe, and identify geometric shapes and principles.

Standard III: Students will use spatial and logical reasoning to recognize, describe, and identify geometric shapes and principles.

Objective 1: Identify and analyze characteristics and properties of geometric shapes.

a. Identify the *midpoint* of a line *segment*.

b. Identify *concave* and *convex polygons*.

c. Identify the center, *radius*, *diameter*, and *circumference* of a circle.

d. Identify the number of *faces*, *edges*, and *vertices* of *prisms* and *pyramids*.

Objective 2: Specify locations and describe spatial relationships using coordinate geometry.

a. Graph points defined by ordered pairs in all four *quadrants*.

b. Write the ordered pair for a point in any quadrant.

Objective 3: Visualize and identify geometric shapes after applying transformations.

a. *Turn (rotate)* a shape around a point and identify the location of the new vertices.

b. *Slide (translate)* a polygon either horizontally or vertically on a coordinate grid and identify the location of the new vertices.

c. *Flip (reflect)* a shape across either the x- or y-axis and identify the location of the new vertices.

Standard IV: Students will understand and apply measurement tools and techniques.

Objective 1: Identify and describe measurable attributes of objects and units of measurement.

a. Compare a meter to a yard, a liter to a quart, and a kilometer to a mile.

b. Identify *pi* as the ratio of the *circumference* to *diameter* of a circle.

c. Explain how the size of the unit used in measuring affects the precision.

d. Estimate length, volume, weight, and area using *metric* and *customary* units.

Objective 2: Determine measurements using appropriate tools and formulas.

a. Measure length to the nearest one-sixteenth of an inch and to the nearest millimeter.

b. Estimate and measure an angle to the nearest degree.

c. Calculate the *circumference* of a circle using a given formula.

d. Calculate *elapsed time* across a.m. and p.m. time periods.

e. Calculate the *areas* of triangles, rectangles, and *parallelograms* using given formulas.

f. Calculate the *surface area* and *volume* of right, rectangular prisms using given formulas.

Standard IV: Students will understand and apply measurement tools and techniques.

Standard V:
Students will collect, analyze, and draw conclusions from data and apply basic concepts of probability.

Standard V: Students will collect, analyze, and draw conclusions from data and apply basic concepts of probability.

Objective 1: Design investigations to reach conclusions using statistical methods to make inferences based on data.

a. Design investigations to answer questions by collecting and organizing data in a variety of ways (e.g., bar graphs, line graphs, frequency tables, stem and leaf plots).

b. Collect, compare, and display data using an appropriate format (i.e., bar graphs, line graphs, *line plots*, circle graphs, scatter plots).

c. Compare two similar sets of data on the same graph and compare two graphs representing the same set of data.

d. Recognize that changing the scale influences the appearance of a display of data.

e. Develop and evaluate inferences and predictions based on data.

Objective 2: Apply basic concepts of probability.

a. Write the results of a probability experiment as a fraction, ratio, or percent between zero and one.

b. Compare experimental results with anticipated results (e.g., experimental: 7 out of 10 tails; whereas, anticipated 5 out of 10 tails).

c. Compare individual, small group, and large group results for a probability experiment.

K-6 Elementary Mathematics Core Curriculum in Table Format

Kindergarten	1st Grade	2nd Grade	3rd Grade	4th Grade	5th Grade	6th Grade
Standard I: **Students will understand simple number concepts and relationships.**	**Standard I:** **Students will acquire number sense and perform simple operations with whole numbers.**	**Standard I:** **Students will acquire number sense and perform operations with whole numbers.**	**Standard I:** **Students will acquire number sense and perform operations with whole numbers and simple fractions.**	**Standard I:** **Students will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.**	**Standard I:** **Students will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.**	**Standard I:** **Students will acquire number sense and perform operations with rational numbers.**
Objective 1: **Identify and use whole numbers.** a. Relate a *numeral* to the number of objects in a set (e.g., ☐ ☐ ☐ = 3). b. Construct models of numbers to 10 with physical objects or manipulatives. c. Make pictorial representations of numbers to 10 (e.g., draw four circles, draw six squares). d. Recognize and write numerals from 0 to 10. e. Manipulate objects to demonstrate and describe multiple ways of representing a number (e.g., 5 can be 3 and 2 more, 5 can also be 2 and 2 and 1).	**Objective 1:** **Represent whole numbers in a variety of ways.** a. Relate number words to the *numerals* that represent the quantities 0 to 10. b. Sort objects into groups of tens and ones and write the numeral representing the set. c. Represent *whole numbers* up to 100 in groups of tens and ones using objects. d. Write a numeral when given the number of tens and ones. e. Write a numeral to 99 in *expanded form* (e.g., 39 is 3 tens and 9 ones or 30+9). f. Use zero to represent the number of elements in the empty set or as a placeholder in a two-digit numeral.	**Objective 1:** **Represent whole numbers in a variety of ways.** a. Relate number words to the *numerals* that represent the quantities 0-100. b. Represent *whole numbers* up to 1,000 in groups of hundreds, tens, and ones using base ten models, and write the numeral representing the set. c. Read and write a three-digit numeral, relating it to a set of objects and a pictorial representation. d. Write a numeral to 999 in *expanded form* (e.g., 539 is 5 hundreds, 3 tens, 9 ones or 500+30+9). e. Identify the place and the value of a given digit in a three-digit numeral (e.g., the two in 281 means 2 hundreds or 200). f. Demonstrate multiple ways to represent numbers using symbolic representations (e.g., thirty is the same as two groups of 15, the number of pennies in three dimes, or 58-28).	**Objective 1:** **Represent whole numbers in a variety of ways.** a. Model, read, and write *whole numbers* up to 10,000 using base ten models, pictures, and symbols. b. Write a *numeral* when given the number of thousands, hundreds, tens, and ones. c. Write a number up to 9,999 in expanded form (e.g., 6,539 is 6 thousands, 5 hundreds, 3 tens, 9 ones or 6000+500+30+9). d. Identify the place and the value of a given digit in a four-digit numeral. e. Demonstrate multiple ways to represent numbers using models and symbolic representations (e.g., fifty is the same as two groups of 25, the number of pennies in five dimes, or 75-25).	**Objective 1:** **Represent whole numbers and decimals in a variety of ways.** a. Model, read, and write numerals from tenths to 100,000. b. Write a *whole number* up to 99,999 in *expanded form* (e.g., 76,539 is 7 ten-thousands, 6 one-thousands, 5 hundreds, 3 tens, 9 ones or 70,000+6,000+500+30+9). c. Identify the place and the value of a given digit in a five-digit numeral, including decimals to tenths. d. Demonstrate multiple ways to represent numbers by using models and symbolic representations (e.g., 36 is the same as the square of six, three dozen, or 9x4). e. Identify *square numbers* using models.	**Objective 1:** **Represent whole numbers and decimals in a variety of ways.** a. Model, read, and write numerals from hundredths to one millions. b. Write a *whole number* up to 999,999 in *expanded form* (e.g., 876,539 = 8 hundred-thousands, 7 ten-thousands, 6 thousands, 5 hundreds, 3 tens, 9 ones or 8x100,000 + 7x10,000 + 6x1,000 + 5x100 + 3x10 + 9). c. Demonstrate multiple ways to represent whole numbers by using models and symbolic representations (e.g., 108=2x50+8; 108=10^2 + 8). d. Classify whole numbers from 2 to 20 as *prime* or *composite* and 0 and 1 as neither prime nor composite, using models. e. Represent repeated factors using *exponents* up to three (e.g., 8=2x2x2=2^3).	**Objective 1:** **Represent whole numbers and decimals in a variety of ways.** a. Change *whole numbers* with *exponents* to *standard form* (e.g., 2^4 = 2^4=16) and recognize that 10^0 = 1. b. Read and write *numerals* from thousandths to one billion. c. Write a whole number to 999,999 in *expanded form* using *exponents* (e.g., 876,539 = 8 x 10^5 + 7 x 10^4 + 6 x 10^3 + 5 x 10^2 + 3 x 10^1 + 9 x 10^0). d. Express numbers in *scientific notation* using positive powers of ten. e. Classify whole numbers to 100 as *prime*, *composite*, or neither. f. Determine the *prime factorization* for a whole number up to 50.

Kindergarten	1st Grade	2nd Grade	3rd Grade	4th Grade	5th Grade	6th Grade
Objective 2: Identify simple relationships among whole numbers. a. Develop strategies for *one-to-one* correspondence and keeping track of quantities. b. Compare two sets of objects to determine whether they have the same, fewer, or more elements. c. Order sets of objects from 1 to 9. d. Estimate quantities less than 10.	**Objective 2: Identify simple relationships among whole numbers.** a. Identify the number that is one more or one less than any *whole number* from 1 to 99. b. Use the vocabulary "greater than," "less than," and "equal to" when comparing sets of objects or numbers. c. Order sets of objects and numbers from 0 to 20. d. Use ordinal numbers 1st through 5th (i.e., 1st, 2nd, 3rd, 4th, 5th).	**Objective 2: Identify simple relationships among whole numbers.** a. Identify the number that is one more, one less, ten more, or ten less than any *whole number* up to 100. b. Write number sentences using the terms "greater than," "less than," or "equal to," to compare numbers. c. Order four whole numbers less than 100 from least to greatest and from greatest to least. d. Use *ordinal numbers* 1st through 10th.	**Objective 2: Identify relationships among whole numbers.** a. Use a variety of strategies to determine whether a number is even or odd. b. Identify the number that is ten more, ten less, 100 more, or 100 less than any *whole number* up to 1,000. c. Compare the relative size of numbers (e.g., 31 is large compared to 4, about half as big as 60, close to 27). d. Compare whole numbers up to four digits using the symbols <, >, and =. e. Order and compare whole numbers on a number line.	**Objective 2: Identify relationships among whole numbers and decimals.** a. Identify the number that is 100 more, 100 less, 1,000 more, or 1,000 less than any *whole number* up to 10,000. b. Compare the relative size of numbers (e.g., 100 is small compared to a million, but large compared to 5). c. Compare whole numbers up to five digits using the symbols <, >, and =. d. Identify a whole number that is between two given whole numbers. e. Order and compare whole numbers and decimals to tenths on a number line.	**Objective 2: Identify relationships among whole numbers, fractions, decimals, and percents.** a. Order and compare *whole numbers*, fractions (including mixed numbers), and decimals using a variety of methods and symbols. b. Rewrite mixed numbers and improper fractions from one form to the other. c. Find the least common denominator for two fractions. d. Represent commonly used fractions as decimals and percents in various ways (e.g., objects, pictures, calculators).	**Objective 2: Identify relationships among whole numbers, fractions (rational numbers), decimals, and percents.** a. Find the *greatest common factor* and *least common multiple* for two numbers using a variety of methods (e.g., list of multiples, prime factorization). b. Order and compare *rational numbers*, including mixed numbers, using a variety of methods and symbols. c. Locate positive rational numbers on a number line. d. Convert common fractions, decimals, and percents from one form to another (e.g., 3/4= 0.75 = 75%).

Kindergarten	1st Grade	2nd Grade	3rd Grade	4th Grade	5th Grade	6th Grade
Objective 3: **Model and illustrate meanings of the operations of addition and subtraction and describe how they relate.** a. Demonstrate the joining and separating of sets of objects to solve problems. b. Describe the joining or separating of sets with informal language when using models. c. Record pictorially the results from the joining or separating of sets.	**Objective 3:** **Model and illustrate meanings of the operations of addition and subtraction and describe how they relate.** a. Demonstrate the joining and separating of sets with twelve or fewer objects and record the results with pictures or symbols. b. Model two meanings of subtraction: separating of sets ("take away") and comparison of sets ("how many more/fewer") using objects, pictorial representations, and symbols. c. Use correct vocabulary and symbols to describe addition (i.e., add, "and," plus, +, sum), subtraction (i.e., subtract, minus, -, take away, how many more/fewer), and equals (i.e., =, same as). d. Use zero in addition and subtraction sentences.	**Objective 3:** **Model and illustrate meanings of the operations of addition and subtraction and describe how they relate.** a. Demonstrate the joining and separating of sets with eighteen or fewer objects and record the results with pictures or symbols. b. Model three meanings of subtraction: separating of sets ("take away"), comparison of sets ("how many more/fewer"), and missing addends using objects, pictorial representations, and symbols. c. Separate a given set of objects into two, three, five, or ten groups of equal size. d. Model addition and subtraction of two-digit whole numbers in a variety of ways. e. Select an addition or subtraction sentence to solve a problem involving joining or separating of sets with eighteen or fewer objects. f. Recognize that addition number sentences have related subtraction sentences (e.g., 8-5=3, 3+5=8).	**Objective 3:** **Model and illustrate meanings of the operations of addition, subtraction, multiplication, and division and describe how they relate.** a. Model addition and subtraction of two- and three-digit *whole numbers* in a variety of ways. b. Model multiplication of a one-digit *factor* by a one-digit factor using various methods (e.g., repeated addition, rectangular *arrays*, manipulatives, pictures) and connect the representation to an *algorithm*. c. Model division as sharing equally and as repeated subtraction using various methods (e.g., rectangular arrays, manipulatives, number lines, pictorial representations). d. Demonstrate, using objects, that multiplication and division are inverse operations (e.g., 3x4=12; thus, 12÷4=3 and 12÷3=4). e. Select and write an addition, subtraction, or multiplication sentence to solve a problem related to the students' environment, and write a story problem that relates to a given equation. f. Demonstrate the effects of place value when multiplying whole numbers by 10.	**Objective 3:** **Model and illustrate meanings of the four operations and describe how they relate.** a. Use models to represent multiplication of a one- or two-digit *factor* by a two-digit factor (up to 30) using a variety of methods (e.g., rectangular *arrays*, manipulatives, pictures) and connect the representation to an *algorithm*. b. Recognize that division by zero is not possible (e.g., 6÷0 is undefined). c. Select and write a multiplication or division sentence to solve a problem related to the students' environment and write a story problem that relates to a given equation. d. Represent division of a two-digit *dividend* by a one-digit *divisor*, including whole number remainders, using various methods (e.g., rectangular arrays, manipulatives, pictures) and connect the representation to an algorithm. e. Demonstrate that multiplication and division are inverse operations (e.g., 3x4=12; thus, 12÷4=3 and 12÷3=4). f. Describe the effect of place value when multiplying whole numbers by 10 and 100.	**Objective 3:** **Model and illustrate meanings of operations and describe how they relate.** a. Identify the *dividend*, *divisor*, and *quotient* regardless of the division symbol used. b. Determine whether a whole number is divisible by 2, 3, 5, 9, and/or 10, using the *rules of divisibility*. c. Represent remainders as *whole numbers*, decimals, or fractions and describe the meaning of remainders as they apply to problems from the students' environment (e.g., If there are 53 people, how many vans are needed if each van holds 8 people?). d. Model addition, subtraction, and multiplication of fractions and decimals in a variety of ways (e.g., using objects and a number line). e Select or write the number sentences that can be used to solve a two-step problem. f. Model different strategies for whole number multiplication (e.g., partial product, lattice) and division (e.g., partial quotient). g. Describe the effect on place value when multiplying and dividing whole numbers and decimals by 10, 100, and 1,000.	**Objective 3:** **Model and illustrate meanings of operations and describe how they relate.** a. Represent division of a multi-digit *dividend* by two-digit *divisors*, including decimals, using models, pictures, and symbols. b. Model addition, subtraction, multiplication, and division of fractions and decimals in a variety of ways (e.g., objects, a number line). c. Apply *rules of divisibility*. d. Select or write a number sentence that can be used to solve a multi-step problem and write a word problem when given a two-step expression or equation.

Kindergarten	1st Grade	2nd Grade	3rd Grade	4th Grade	5th Grade	6th Grade
	Objective 4: **Use fractions to identify parts of the whole.** a. Share sets of up to ten objects between two students and identify each part as half. b. Divide geometric shapes into equal parts, identifying halves and fourths.	**Objective 4:** **Use fractions to identify parts of the whole.** a. Separate geometric shapes and sets of objects into halves, thirds, and fourths using a variety of models and illustrations. b. Specify a region of a geometric shape (e.g., as "___ out of ___ equal parts" when given four or fewer equal parts. c. Represent the unit fractions 1/2, 1/3, and 1/4 with objects, pictures, and symbols.	**Objective 4:** **Use fractions to communicate parts of the whole.** a. Identify the denominator of a fraction as the number of equal parts in the whole region or set. b. Identify the numerator of a fraction as the number of equal parts being considered. c. Divide *regions* and sets of objects into equal parts using a variety of models and illustrations. d. Name and write a fraction to represent a portion of a unit whole for halves, thirds, fourths, sixths, and eighths. e. Determine which of two fractions is greater using models or illustrations.	**Objective 4:** **Use fractions to communicate parts of the whole.** a. Divide regions and sets of objects into equal parts using a variety of models and illustrations. b. Name and write a fraction to represent a portion of a unit whole for halves, thirds, fourths, fifths, sixths, eighths, and tenths. c. Relate fractions to decimals that represent tenths. d. Determine which of two fractions is greater using models or illustrations. e. Find equivalent fractions for one-half, one-third, and one-fourth using manipulatives and pictorial representations.	**Objective 4:** **Use fractions to communicate parts of the whole.** a. Divide regions, sets of objects, and line segments into equal parts using a variety of models and illustrations. b. Name and write a fraction to represent a portion of a unit whole for halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths. c. Represent the simplest form of a fraction in various ways (e.g., objects, pictorial representations, symbols). d. Represent mixed numbers and improper fractions in various ways (e.g., rulers, objects, number lines, symbols). e. Rename *whole numbers* as fractions with different denominators (e.g., 5=5/1, 3=6/2, 1=7/7). f. Model and calculate equivalent forms of a fraction and describe the process used.	**Objective 4:** **Use fractions and percents to communicate parts of the whole.** a. Divide regions, sets of objects, and *line segments* into equal parts using a variety of models and illustrations. b. Name and write a fraction to represent a portion of a unit whole for halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and sixteenths. c. Write a fraction or ratio in simplest form. d. Name equivalent forms for fractions (halves, thirds, fourths, fifths, tenths), ratios, percents, and decimals, including repeating or terminating decimals. e. Relate percents less than 1% or greater than 100% to equivalent fractions, decimals, *whole numbers*, and mixed numbers.

Kindergarten	1st Grade	2nd Grade	3rd Grade	4th Grade	5th Grade	6th Grade
	Objective 5: **Solve whole number problems using addition and subtraction in horizontal and vertical notation.** a. Compute addition and subtraction facts to twelve. b. Add three whole numbers with sums to twelve.	**Objective 5:** **Solve whole number problems using addition and subtraction in vertical and horizontal notation.** a. Use a variety of methods and tools to facilitate computation (e.g., estimation, mental math strategies, paper and pencil, calculator). b. Compute accurately with basic number combinations for addition and subtraction facts to eighteen. c. Add three *whole numbers* with *sums* to eighteen. d. Find the sum of two-digit whole numbers and describe the process used.	**Objective 5:** **Solve whole number problems using addition, subtraction, multiplication, and division in vertical and horizontal notation.** a. Use a variety of methods and tools to facilitate computation (e.g., estimation, mental math strategies, paper and pencil, calculator). b. Find the sum of any two *addends* with three or fewer digits, including monetary amounts, and describe the process used. c. Find the *difference* of two-digit *whole numbers* and describe the process used. d. Find the *product* for multiplication facts through ten times ten and describe the process used.	**Objective 5:** **Solve whole number problems using addition, subtraction, multiplication, and division in vertical and horizontal notation.** a. Determine when it is appropriate to use estimation, mental math strategies, paper and pencil, or a calculator. b. Find the sum and difference of four-digit numbers, including monetary amounts, and describe the process used. c. Multiply two- and three-digit *factors* by a one-digit factor and describe the process used. d. Divide a two-digit *whole number dividend* by a one-digit *divisor*, with a *remainder* of zero and describe the process used.	**Objective 5:** **Solve problems using the four operations with whole numbers, decimals, and fractions.** a. Determine when it is appropriate to use estimation, mental math strategies, paper and pencil, or a calculator. b. Use estimation strategies to determine whether results obtained using a calculator are reasonable. c. Multiply up to a three-digit *whole number* by a one- or two-digit whole number. d. Divide up to a three-digit whole number *dividend* by a one-digit *divisor*. e. Add and subtract decimals with digits to the hundredths place (e.g., 35.42+7.2; 75.2–13.45). f. Add, subtract, and multiply fractions. g. Simplify *expressions*, without *exponents*, using the *order of operations*.	**Objective 5:** **Solve problems using the four operations with whole numbers, decimals, and fractions.** a. Determine when it is appropriate to use estimation, mental math strategies, paper and pencil, or a calculator. b. Use estimation strategies to determine whether results obtained using a calculator are reasonable. c. Multiply up to a three-digit *factor* by a one- or two-digit factor including decimals. d. Divide up to a three-digit *dividend* by a one- or two-digit *divisor* including decimals. e. Add and subtract decimals to the thousandths place (e.g., 34.567+3.45; 65.3-5.987). f. Add, subtract, multiply, and divide fractions and mixed numbers. g. Solve problems using ratios and proportions. h. Simplify *expressions*, with *exponents*, using the *order of operations*.

Kindergarten	1st Grade	2nd Grade	3rd Grade	4th Grade	5th Grade	6th Grade
					Objective 6: **Model and illustrate integers.** a. Identify, read, and locate *integers* on a number line. b. Describe situations where integers are used in the students' environment.	**Objective 6:** **Model, illustrate, and perform the operations of addition and subtraction of integers.** a. Recognize that the sum of an *integer* and its opposite is zero. b. Model addition and subtraction of integers using manipulatives and a number line. c. Add and subtract integers.
Standard II: **Students will identify and use patterns to represent mathematical situations.**	**Standard II:** **Students will identify and use patterns and relations to represent mathematical situations.**	**Standard II:** **Students will identify and use patterns and relations to represent mathematical situations.**	**Standard II:** **Students will use patterns and relations to represent mathematical situations.**	**Standard II:** **Students will use patterns and relations to represent mathematical situations.**	**Standard II:** **Students will use patterns and relations to represent and analyze mathematical situations using algebraic symbols.**	**Standard II:** **Students will use patterns, relations, and functions to represent and analyze mathematical situations using algebraic symbols.**
Objective 1: **Identify and sort objects according to common attributes.** a. Sort objects into groups by color, shape, size, number, or other *attributes*. b. Identify which attribute was used to sort objects into a group. c. Find multiple ways to sort and classify a group of objects.	**Objective 1:** **Recognize and represent patterns with one or two attributes.** a. Sort and classify objects by one or two *attributes*. b. Identify, create, and label simple patterns using manipulatives, pictures, and symbolic notation (e.g., ABAB. . ., □ ○ △ □ ○ △ . . .). c. Identify patterns in the environment. d. Identify horizontal and vertical patterns on hundreds charts. e. Use patterns to establish skip counting by twos to 20 and by fives and tens to 100. f. Count backward from 10 to 0 and identify the pattern.	**Objective 1:** **Recognize and represent patterns having multiple attributes.** a. Sort, classify, and label objects by three or more *attributes*. b. Identify and label repeating and *growing patterns* using objects, pictures, and symbolic notation (e.g., ABAABBAAABBB...). c. Identify repeating and growing patterns in the environment. d. Construct models and skip count by twos, threes, fives, and tens and relate to repeated addition.	**Objective 1:** **Recognize and create patterns with given attributes.** a. Create and extend *repeating* and *growing* patterns using objects, numbers, and tables. b. Record results of patterns created using manipulatives, pictures, and numeric representations and describe how they are extended.	**Objective 1:** **Recognize, describe, and use patterns and identify the attributes.** a. Represent and analyze repeating and growing patterns using objects, pictures, numbers, and tables. b. Recognize and extend multiples and other number patterns using a variety of methods.	**Objective 1:** **Recognize, analyze, and use patterns and describe their attributes.** a. Analyze and make predictions about patterns involving *whole numbers*, decimals, and fractions using a variety of tools including organized lists, tables, objects, and variables. b. Extend patterns and describe a rule for predicting the next element.	**Objective 1:** **Recognize, analyze, and use multiple representations of patterns and functions and describe their attributes.** a. Analyze patterns on graphs and tables and write a generalization to predict how the patterns will continue. b. Create tables and graphs to represent given patterns and algebraic *expressions*. c. Draw a graph from a table of values or to represent an equation. d. Write an algebraic expression from a graph or a table of values.

Kindergarten	1st Grade	2nd Grade	3rd Grade	4th Grade	5th Grade	6th Grade
Objective 2: **Identify and use patterns to describe numbers or objects.** a. Use patterns to count orally from 1 to 20 and backward from 10 to 0. b. Identify simple patterns in the environment. c. Predict what comes next in an established pattern and justify thinking. d. Duplicate, extend, and create simple patterns using objects and pictorial representations.	**Objective 2:** **Recognize and represent relations using mathematical symbols.** a. Recognize that "=" indicates a relationship in which the quantities on each side of an equation are equal. b. Recognize that symbols such as □, △, or ◇ in an addition or subtraction equation represent a missing value that will make the statement true (e.g., □ + 3 = 6, 5 +7 = △, 4 = 5 – ◇). c. Demonstrate that changing the order of *addends* does not change the *sum* (e.g., 3+2=5 and 2+3=5).	**Objective 2:** **Recognize and represent relations using mathematical symbols.** a. Recognize that "≠" indicates a relationship in which the quantities on each side are not of equal value. b. Recognize that symbols such as □, △, or ◇ in an addition or subtraction equation represent a value that will make the statement true (e.g., □ +3=6, 5+7=△, 7=9-◇). c. Demonstrate that changing the order of *addends* does not change the *sum* (e.g., 3+2+7=12, 7+3+2=12) and that changing the grouping of three or more addends does not change the sum (e.g., (2+3)+7=12, 2+(3+7)=12).	**Objective 2:** **Recognize and represent mathematical situations using patterns and symbols.** a. Recognize that symbols such as □, △, or ◇ in an addition, subtraction, or multiplication equation, represent a value that will make the statement true (e.g., 5+7=△, □-3=6, ◇=2x4). b. Solve equations involving equivalent expressions (e.g., 6+4 = □+7). c. Use the >, <, and = symbols to compare two *expressions* involving addition and subtraction (e.g., 4+6 □ 3+2; 3+5 ◇ 16-9). d. Demonstrate that grouping three or more *addends* does not change the *sum* (e.g., 3+(2+7)=12, (7+3)+2=12) and changing the order of *factors* does not change the *product* (e.g., 3x7=21, 7x3=21). e. Use a variety of manipulatives to model the *identity property of addition* (e.g., 3+0=3), the *identity property of multiplication* (e.g., 7x1=7), and the *zero property of multiplication* (e.g., 6x0=0).	**Objective 2:** **Recognize, represent, and solve mathematical situations using patterns and symbols.** a. Solve equations involving equivalent expressions (e.g., 6x2= □x3 or 6x□=9+9). b. Use the <, >, = symbols to compare two expressions involving addition, subtraction, multiplication, and division (e.g., 5x4◇9÷3). c. Recognize that a given variable maintains the same value throughout an equation or expression (e.g., □+□=8; □=4). d. Demonstrate that changing the order of *factors* does not change the *product* (e.g., 2x3=6, 3x2=6) and that the grouping of three or more factors does not change the product (e.g., (2x3)x1=6; 2x(3x1)=6). e. Demonstrate the distribution of multiplication over addition using a rectangular *array* (e.g., 8x14=8 rows of 10 plus 8 rows of 4).	**Objective 2:** **Represent, solve, and analyze mathematical situations using algebraic symbols.** a. Recognize a variety of symbols for multiplication and division including x, •, and * as symbols for multiplication and ÷, ⌐, and a fraction bar (/ or -) as division symbols. b. Recognize that a variable (◇, n, x) represents an unknown quantity. c. Solve one-step equations involving *whole numbers* and a single variable (e.g., n÷7=3). d. Recognize that the answer to a multiplication problem involving a factor of zero is equal to zero (e.g., 0x45=0). e. Use expressions or one-step equations to represent real-world situations. f. Use the *associative*, *commutative*, and *distributive properties* to compute with whole numbers.	**Objective 2:** **Represent, solve, and analyze mathematical situations using algebraic symbols.** a. Recognize that a number in front of a variable indicates multiplication (e.g., 3y means 3 times the quantity y). b. Solve two-step equations involving *whole numbers* and a single variable (e.g., 3x+4=19). c. Recognize that "≈" indicates a relationship in which the quantities on each side are approximately of equal value (e.g., π ≈3.14). d. Recognize that an *exponent* can be represented in the following ways: 4^3 or 4^3. e. Evaluate *expressions* and formulas, substituting given values for the variables (e.g., 2x+4; x=2; therefore, 2(2)+4=8). f. Recognize that if the *product* is zero, then one or more *factors* equal zero (i.e., if ab=0 then either a=0 or b=0 or a and b=0).

Kindergarten	1st Grade	2nd Grade	3rd Grade	4th Grade	5th Grade	6th Grade
Standard III: **Students will identify and create simple geometric shapes and describe spatial relationships.**	**Standard III:** **Students will describe, identify, and create and simple geometric shapes and describe spatial relationships.**	**Standard III:** **Students will describe, identify, and create geometric shapes and describe spatial relationships.**	**Standard III:** **Students will use spatial reasoning to describe, identify, and create geometric shapes.**	**Standard III:** **Students will use spatial reasoning to recognize, describe, and identify geometric shapes.**	**Standard III:** **Students will use spatial reasoning to recognize, describe, and identify geometric shapes and principles.**	**Standard III:** **Students will use spatial and logical reasoning to recognize, describe, and identify geometric shapes and principles.**
Objective 1: **Identify and create simple geometric shapes.** a. Identify circles, triangles, rectangles, and squares. b. Combine shapes to create *two-dimensional* objects. c. Draw circles, triangles, rectangles, and squares. d. Recognize circles, triangles, rectangles, and squares in the students' environment.	**Objective 1:** **Describe, identify, and create simple geometric shapes.** a. Identify, name, draw, create, and sort circles, triangles, rectangles, and squares. b. Identify circles, triangles, rectangles, and squares in the students' environment. c. Recognize that combining simple geometric shapes can create more complex geometric shapes.	**Objective 1:** **Describe, identify, and create geometric shapes.** a. Identify, name, draw, sort, and compare circles, triangles, and *parallelograms*. b. Identify and name spheres, cones, and cylinders. c. Find and identify familiar geometric shapes in the students' environment. d. Determine whether a circle, triangle, square, or rectangle has a *line of symmetry*.	**Objective 1:** **Describe, identify, and create geometric shapes.** a. Identify and draw *points*, *lines*, *line segments*, and *endpoints*. b. Identify and draw *lines of symmetry* on triangles, squares, circles, and rectangles. c. Determine whether an angle is *right*, *obtuse*, or *acute* by comparing the angle to the corner of a rectangle. d. Classify polygons (e.g., *quadrilaterals*, pentagons, hexagons, octagons) by the number of sides and corners. e. Identify, make, and describe cubes (e.g., a cube has 6 square *faces*, 8 *vertices*, and 12 *edges*).	**Objective 1:** **Describe, identify, and analyze characteristics and properties of geometric shapes.** a. Identify and draw *parallel lines* and *intersecting lines*. b. Identify and draw lines of symmetry on a variety of *polygons*. c. Identify and describe *quadrilaterals* (i.e., rectangles, squares, *rhombuses*, *trapezoids*, kites). d. Identify *right*, *obtuse*, and *acute* angles. e. Compare two polygons to determine whether they are *congruent* or *similar*. f. Identify and describe *cylinders* and *rectangular prisms*.	**Objective 1:** **Describe, identify, and analyze characteristics and properties of geometric shapes.** a. Identify and draw *perpendicular* lines. b. Draw, label, and describe rays and describe an angle as two rays sharing a common endpoint. c. Label an angle as acute, *obtuse*, *right*, or *straight*. d. Identify and describe *equilateral*, *isosceles*, *scalene*, *right*, *acute*, and *obtuse* triangles. e. Identify the *vertex* of an angle or the *vertices* of a polygon. f. Compare *corresponding angles* of two triangles and determine whether the triangles are *similar*. g. Identify and describe *pyramids* and *prisms*.	**Objective 1:** **Identify and analyze characteristics and properties of geometric shapes.** a. Identify the *midpoint* of a *line segment*. b. Identify concave and *convex polygons*. c. Identify the center, *radius*, *diameter*, and *circumference* of a circle. d. Identify the number of *faces*, *edges*, and *vertices* of *pyramids* and *prisms*.

Kindergarten	1st Grade	2nd Grade	3rd Grade	4th Grade	5th Grade	6th Grade
Objective 2: Describe simple spatial relationships. a. Visualize how to fit a shape into a design. b. Use and demonstrate words to describe position with objects (i.e., on, over, under, above, below, top, bottom, up, down, in front of, behind, next to, beside). c. Use and demonstrate words to describe distance with objects (i.e., far, near).	**Objective 2: Describe simple spatial relationships.** a. Use and demonstrate words to describe position (i.e., between, before, after, middle, left, right). b. Use and demonstrate words to describe distance (i.e., closer, farther).	**Objective 2: Describe spatial relationships.** a. Create and use verbal or written instructions to move within the environment. b. Find and name locations using coordinates (A, 1). c. Identify shapes in various orientations (e.g., △ and ▽).	**Objective 2: Describe spatial relationships.** a. Give directions to reach a location. b. Use coordinates (A, 1) or regions to locate positions on a map. c. Demonstrate and use horizontal and vertical lines.	**Objective 2: Specify locations and describe spatial relationships using grids and maps.** a. Locate positions on a map of Utah using coordinates or regions. b. Give the *coordinates* or *regions* of a position on a map of Utah.	**Objective 2: Specify locations and describe spatial relationships using coordinate geometry.** a. Locate points defined by ordered pairs in the first *quadrant*. b. Write an ordered pair for a point in the first quadrant. c. Specify possible paths between locations on a *coordinate grid* and compare distances of the various paths.	**Objective 2: Specify locations and describe spatial relationships using coordinate geometry.** a. Graph points defined by ordered pairs in all four quadrants. b. Write the ordered pair for a point in any quadrant.
			Objective 3: Visualize and identify geometric shapes after applying transformations. a. Demonstrate the effect of a slide (translation) or flip (reflection) on a figure, using manipulatives. b. Determine whether two polygons are *congruent* by sliding, flipping, or turning to physically fit one object on top of the other. c. Identify *two-dimensional* shapes (*nets*) that will fold to make a cube. d. Create a *polygon* that results from combining other polygons.	**Objective 3: Visualize and identify geometric shapes after applying transformations.** a. Identify a *slide* (*translation*) or *flip* (*reflection*) on a figure using manipulatives. b. Relate *cubes*, *cylinders*, *cones*, and *rectangular prisms* to the *two-dimensional* shapes (*nets*) from which they were created.	**Objective 3: Visualize and identify geometric shapes after applying transformations.** a. Identify a *slide* (*translation*) or *flip* (*reflection*) on a figure across a line. b. Demonstrate the effect of a *turn* (*rotation*) on a figure using manipulatives. c. Relate *pyramids* and *prisms* to the *two-dimensional* shapes (*nets*) from which they were created.	**Objective 3: Visualize and identify geometric shapes after applying transformations.** a. *Turn* (*rotate*) a shape around a point and identify the location of the new vertices. b. *Slide* (*translate*) a polygon either horizontally or vertically on a coordinate grid and identify the location of the new vertices. c. *Flip* (*reflect*) a shape across either the x- or y-axis and identify the location of the new vertices.

Kindergarten	1st Grade	2nd Grade	3rd Grade	4th Grade	5th Grade	6th Grade
Standard IV: **Students will understand and use simple measurement tools and techniques.**	**Standard IV:** **Students will understand and use simple measurement tools and techniques.**	**Standard IV:** **Students will understand and use measurement tools and techniques.**	**Standard IV:** **Students will understand and use measurement tools and techniques.**	**Standard IV:** **Students will understand and use measurement tools and techniques.**	**Standard IV:** **Students will understand and apply measurement tools and techniques.**	**Standard IV:** **Students will understand and apply measurement tools and techniques.**
Objective 1: **Identify measurable attributes of objects and units of measurement.** a. Identify clocks and calendars as tools that measure time. b. Identify a day, week, and month on a calendar. c. Identify pennies, nickels, dimes, and quarters as units of money.	**Objective 1:** **Identify measurable attributes of objects and units of measurement.** a. Identify the appropriate tools for measuring length, weight, capacity, temperature, and time. b. Identify the values of a penny, nickel, dime, and quarter. c. Estimate the length of an object by comparing to a nonstandard unit (e.g., How many new pencils wide is your desk?).	**Objective 1:** **Identify measurable attributes of objects and units of measurement.** a. Sequence a series of events of a day in order by time (e.g., breakfast at 7:00, school begins at 9:00). b. Identify the name and value of a penny, nickel, dime, quarter, and dollar. c. Estimate length, *capacity*, and weight using customary units.	**Objective 1:** **Identify and describe measurable attributes of objects and units of measurement.** a. Recognize the two systems of measurement: *metric* and *customary*. b. Describe the relationship between metric units of length (i.e., centimeter, meter). c. Describe the relationship among customary units of length (i.e., inch, foot, yard) and the relationship between customary units of *capacity* (i.e., cup, quart). d. Estimate length, capacity, and weight using metric and customary units.	**Objective 1:** **Identify and describe measurable attributes of objects and units of measurement.** a. Describe the relationship among *metric* units of length (i.e., millimeter, centimeter, meter), between metric units of capacity (i.e., milliliter, liter), and between metric units of weight (i.e., gram, kilogram). b. Identify a mile as a measure of distance and its relationship to other *customary* units of length. c. Describe the relationship among customary units of *capacity* (i.e., cup, pint, quart, gallon). d. Estimate length, capacity, and weight using metric and customary units.	**Objective 1:** **Identify and describe measurable attributes of objects and units of measurement.** a. Describe the relationship among *metric* units of length (i.e., millimeter, centimeter, meter, kilometer). b. Describe the relationship among *customary* units of weight (i.e., ounce, pound). c. Identify the correct units of measurement for *volume*, *area*, and *perimeter* in both metric and customary systems. d. Estimate length, volume, weight, and area using metric and customary units. e. Convert units of measurement within the metric system and convert units of measurement within the customary system.	**Objective 1:** **Identify and describe measurable attributes of objects and units of measurement.** a. Compare a meter to a yard, a liter to a quart, and a kilometer to a mile. b. Identify *pi* as the ratio of the *circumference* to *diameter* of a circle. c. Explain how the size of the unit used in measuring affects the precision. d. Estimate length, volume, weight, and area using *metric* and customary units.

Kindergarten	1st Grade	2nd Grade	3rd Grade	4th Grade	5th Grade	6th Grade
Objective 2: **Use appropriate techniques and tools to determine measurements.** a. Compare two objects (e.g., shorter/longer, heavier/lighter, larger/smaller, more/less). b. Find the length of an object using nonstandard units (e.g., pencils, paper clips). c. Name the days of the week in order. d. Sort pennies, nickels, dimes, and quarters.	**Objective 2:** **Use appropriate techniques and tools to determine measurements.** a. Compare objects, using nonstandard units, according to their length, weight, or volume (e.g., pencils/length, books/weight, boxes/volume). b. Read and tell time to the nearest hour. c. Name the days of the week, months of the year, and seasons in order. d. Determine the value of a set of the same coins that total 25¢ or less (e.g., a set of 14 pennies equals 14¢, a set of 5 nickels equals 25¢, a set of 2 dimes equals 20¢).	**Objective 2:** **Use appropriate techniques and tools to determine measurements.** a. Compare and order objects, using nonstandard units, according to their length, weight, or capacity. b. Measure length using inches and feet, weight using pounds, and *capacity* using cups. c. Determine the value of a set of up to five coins that total $1.00 or less (e.g., two quarters and one dime equals 60¢; three dimes, one nickel, and one penny equals 36¢). d. Read, tell, and write time to the hour and half-hour. e. Use a calendar to determine the day of the week and date. f. Determine the *perimeter* of a square, triangle, and rectangle by measuring with nonstandard units.	**Objective 2:** **Use appropriate techniques and tools to determine measurements.** a. Measure the length of objects to the nearest centimeter, meter, half-inch, foot, and yard. b. Measure *capacity* using cups and quarts, and measure weight using pounds. c. Determine the value of a combination of coins and bills that total $5.00 or less and write the monetary amounts using the dollar sign and decimal notation. d. Identify the number of hours in a day, the number of days in a year, and the number of weeks in a year. e. Read, tell, and write time to the quarter-hour. f. Identify any given day of the month (e.g., the third Wednesday of the month is the 18th). g. Read and record the temperature to the nearest ten degrees using a Fahrenheit thermometer. h. Estimate and measure the *perimeter* and area of rectangles by measuring with nonstandard units.	**Objective 2:** **Determine measurements using appropriate tools and formulas.** a. Measure the length of objects to the nearest centimeter, meter, quarter-inch, foot, and yard. b. Measure *capacity* using milliliters, liters, cups, pints, quarts, and gallons and measure weight using grams, kilograms, and pounds. c. Read, tell, and write time to the nearest minute, identifying a.m. and p.m. d. Read and record the temperature to the nearest degree, in Fahrenheit, using a thermometer. e. Determine the value of a combination of coins and bills that total $20.00 or less. f. Count back change for a single-item purchase and determine the amount of change to be received from a multiple-item purchase. g. Determine possible *perimeters*, in whole units, for a rectangle with a fixed *area* and determine possible areas when given a rectangle with a fixed perimeter.	**Objective 2:** **Determine measurements using appropriate tools and formulas.** a. Measure length to the nearest 1/8 of an inch and to the nearest centimeter. b. Measure *volume* and weight using *metric* and *customary* units. c. Measure angles using a protractor. d. Calculate *elapsed time* within a.m. or p.m. time periods. e. Read and record the temperature to the nearest degree (above and below zero) when using a thermometer with a Celsius or Fahrenheit scale. f. Calculate the *perimeter* of rectangles and triangles. g. Calculate the *area* of squares and rectangles using a formula.	**Objective 2:** **Determine measurements using appropriate tools and formulas.** a. Measure length to the nearest one-sixteenth of an inch and to the nearest millimeter. b. Estimate and measure an angle to the nearest degree. c. Calculate the *circumference* of a circle using a given formula. d. Calculate *elapsed time* across a.m. and p.m. time periods. e. Calculate the *areas* of triangles, rectangles, and *parallelograms* using given formulas. f. Calculate the *surface area* and *volume* of right, rectangular prisms using given formulas.

Kindergarten	1st Grade	2nd Grade	3rd Grade	4th Grade	5th Grade	6th Grade
Standard V: **Students will collect and draw conclusions from data and understand basic concepts of probability.**	**Standard V:** **Students will collect and draw conclusions from data and understand basic concepts of probability.**	**Standard V:** **Students will collect and draw conclusions from data and understand basic concepts of probability.**	**Standard V:** **Students will collect and organize data to make predictions and identify basic concepts of probability.**	**Standard V:** **Students will collect and organize data to make predictions and use basic concepts of probability.**	**Standard V:** **Students will collect, analyze, and draw conclusions from data and apply basic concepts of probability.**	**Standard V:** **Students will collect, analyze, and draw conclusions from data and apply basic concepts of probability.**
Objective 1: **Collect, organize, and display simple data.** a. Collect, organize, and record data using objects and pictures. b. Represent data in a variety of ways (e.g., graphs made from people, *pictographs*, bar graphs) and interpret the data (e.g., more people like red than blue).	**Objective 1:** **Collect, organize, and display simple data.** a. Collect physical objects to use as data. b. Collect, represent, and interpret data using tables, tally marks, *pictographs*, and bar graphs.	**Objective 1:** **Collect, organize, and display simple data.** a. Gather data by vote or survey. b. Sort, classify, and organize data in a variety of ways. c. Use a variety of methods to organize, display, and label information, including keys, using *pictographs*, tallies, bar graphs, and organized tables. d. Report information from a data display.	**Objective 1:** **Collect, organize, and display data to make predictions.** a. Collect, read, represent, and interpret data using tables, graphs, and charts, including keys (e.g., pictographs, bar graphs). b. Make predictions based on a data display.	**Objective 1:** **Collect, organize, and display data to make predictions and answer questions.** a. Identify a question that can be answered by collecting data. b. Collect, read, and interpret data from tables, graphs, charts, surveys, and observations. c. Represent data using tables, line plots, line graphs, and bar graphs. d. Identify and distinguish between *clusters* and *outliers* of a data set.	**Objective 1:** **Formulate and answer questions using statistical methods to compare data.** a. Formulate a question that can be answered by collecting data. b. Collect, compare, and display data using an appropriate format (i.e., *line plots*, bar graphs, pictographs, circle graphs, line graphs). c. Identify minimum and *maximum* values for a set of data. d. Identify or calculate the *mean*, *mode*, and *range*. e. Propose and justify inferences based on data.	**Objective 1:** **Design investigations to reach conclusions using statistical methods to make inferences based on data.** a. Design investigations to answer questions by collecting and organizing data in a variety of ways (e.g., bar graphs, line graphs, frequency tables, stem and leaf plots). b. Collect, compare, and display data using an appropriate format (i.e., *line plots*, circle graphs, scatter plots). c. Compare two similar sets of data on the same graph and compare two graphs representing the same set of data. d. Recognize that changing the scale influences the appearance of a display of data. e. Develop and evaluate inferences and predictions based on data.

Kindergarten	1st Grade	2nd Grade	3rd Grade	4th Grade	5th Grade	6th Grade
Objective 2: **Determine the likelihood of events.** a. Describe events encountered in books read as possible or not possible. b. Describe events as likely or unlikely (e.g., It is likely to snow today. It is unlikely an elephant will be in school).	**Objective 2:** **Determine the likelihood of an event.** a. Compare events to decide which are more likely, less likely, and equally likely. b. Relate past events to future events (e.g., The sun set about 6:00 last night, so it will set about the same time tonight).	**Objective 2:** **Determine the likelihood of an event.** a. Predict events that will be the same in one day or one week. b. Predict the outcome when there are only two possible outcomes (e.g., tossing a coin).	**Objective 2:** **Identify basic concepts of probability.** a. Describe the results of events using the terms "certain," "equally likely," and "impossible." b. Predict outcomes of simple activities (e.g., a bag contains three red marbles and five blue marbles. If one marble is selected, is it more likely to be red or blue?).	**Objective 2:** **Use basic concepts of probability.** a. Describe the results of investigations involving random outcomes as simple ratios (e.g., 4 out of 9, 4/9). b. Predict outcomes of simple experiments, including with and without replacement, and test the predictions.	**Objective 2:** **Apply basic concepts of probability.** a. Describe the results of investigations involving random outcomes using a variety of notations (e.g., 4 out of 9, 4/9, 4:9). b. Recognize that outcomes of experiments and samples are fractions between 0 and 1. c. Predict the probability of an outcome in a simple experiment.	**Objective 2:** **Apply basic concepts of probability.** a. Write the results of a probability experiment as a fraction, ratio, or percent between zero and one. b. Compare experimental results with anticipated results (e.g., experimental: 7 out of 10 tails; whereas, anticipated 5 out of 10 tails). c. Compare individual, small group, and large group results for a probability experiment.

Mathematics Glossary

acute angle	An angle with a measure less than 90º.
addend	Any number being added. In 32+4=36, 32 and 4 are *addends*.
algorithm	A step-by-step method for computing.
area	The measure, in square units, of the inside of a plane figure.
array	An arrangement of objects in equal rows.
Associative Property	Changing the grouping of three or more *addends* does not change the *sum*. Changing the grouping of three or more *factors* does not change the *product*.
attribute	A characteristic of an object, such as color, shape, size, etc.
capacity	The maximum amount that can be contained by an object. Often refers to measurement of a liquid.
chord	Any *line segment* that joins two *points* on a circle.
circumference	The *perimeter* of a circle.
cluster	Data that are grouped together.
Commutative Property	Changing the order of the *addends* does not change the *sum*. Changing the order of the *factors* does not change the *product*.
composite number	A number greater than 0 that has more than two different factors. The number 9 is a *composite number* because it has three *factors*: 1, 3, and 9.
concave polygon	A *polygon* with one or more diagonals that have points outside the polygon.
cone	A solid bounded by a circular base and a curved surface with one *vertex*.
congruent	Having exactly the same size and shape.
convex polygon	A *polygon* with all interior angles measuring less than 180º. All diagonals of a *convex polygon* are inside the figure.

coordinate grid	A *two-dimensional* system in which the *coordinates* of a point are its distances from two intersecting, usually *perpendicular*, straight lines called axes.
coordinates	An ordered pair of numbers that identify a point on a coordinate plane or grid.
corresponding angles	Angles in the same position from one line to another.
cube (solid figure)	A regular solid with six congruent square faces.
customary system	A system of measurement used in the U.S. The system includes units for measuring length, capacity, and weight.
cylinder	A three-dimensional figure with two circular bases that are *parallel* and *congruent*.
diameter	A *chord* that goes through the center of a circle.
difference	The amount that remains after one quantity is subtracted from another.
Distributive Property	When one of the *factors* of a *product* is a *sum*, multiplying each *addend* before adding does not change the *product*. For example: 6x(2+3)=(6x2)+(6x3)
dividend	A number that is divided by another number.
divisor	The number by which another number is divided.
e.g.	This abbreviation means "for example." When used in the Core, *e.g.* is not limited to the examples given.
edge	The *line segment* where two *faces* of a solid figure meet.
elapsed time	The amount of time that passes between two times.
endpoint	A point at either end of a *line segment*, arc, or a point at one end of a *ray*.
equilateral triangle	A triangle with all sides the same length.
expanded form	A way to write numbers that shows the place value of each digit. 263 = 200 + 60 + 3 or 263 is 2 hundreds, 60 tens, and 3 ones.
exponent	The number that tells how many equal *factors* there are.

expression	A variable or combination of variables, numbers, and operation symbols that represents a mathematical relationship. 6, 2 + 3, x, x + 4, and x + 2y are all *expressions*.
face	A plane figure that serves as one side of a solid figure. The *faces* of a *cube* are squares.
factors	The *whole numbers* that are multiplied to get a *product*. In 6x3=18, 6 and 3 are factors of 18.
flip	A transformation creating a mirror image of a figure on the opposite side of a line. A *flip* is also called a *reflection*.
greatest common factor	The greatest number that is a *factor* of every number in a set of numbers. 3 is the *greatest common factor* of 9 and 15.
growing pattern	A pattern that grows or increases.
horizontal line	A line that is *parallel* to the horizon. A *horizontal line* is straight across.
i.e.	This abbreviation means "that is to say." When used in the Core, *i.e.* is limited to the specific examples given.
Identity Property of Addition	If you add zero to a number, the *sum* is the same as that number. For example, 8+0=8.
Identity Property of Multiplication	If you multiply a number by one, the *product* is the same as that number. For example, 18x1=18.
integers	*Whole numbers* and their opposites.
intersect	To meet or cross.
isosceles triangle	A triangle that has exactly two *congruent* sides.
least common multiple	The least common multiple of a set of two or more numbers. For example, the *least common multiple* of 3 and 5 is 15.
line	A set of connected points continuing without end in both directions.
line of symmetry	A line that divides a figure into two *congruent* halves that are mirror images of each other.
line plot	A graph showing frequency of data on a number line.

line segment	A part of a line with two *endpoints*.
mean	A number found by dividing the sum of two or more numbers by the number of *addends*. The *mean* is often referred to as the average.
metric system	A system of measurement based on tens. The basic unit of length is the meter. The basic unit of mass is the gram. The basic unit of *capacity* is the liter.
midpoint	The point on a *line segment* that divides it into two *congruent* segments.
mode	The number that appears most frequently in a set of numbers. There may be one, more than one, or no mode.
net	A *two-dimensional* shape that can be folded into a three-dimensional figure is a *net* of that figure.
numeral	A symbol used to represent a number.
obtuse angle	An angle with a measure greater than 90^{o} and less than 180^{o}.
obtuse triangle	A triangle with one *obtuse angle*.
one-to-one correspondence	The relationship between the spoken word and the written symbol.
Order of Operations	A set of rules that tells the order in which to compute.
ordinal number	A *whole number* that names the position of an object in sequence. First, second, and third are *ordinal numbers*.
outlier	A number in a set of data that is much larger or smaller than most of the other numbers in the set.
parallel lines	Lines in the same plane that are always the same distance apart.
parallelogram	A *quadrilateral* with two pairs of *parallel* and *congruent* sides.
perimeter	The distance around a figure.
perpendicular	Forming *right angles*.

pi	The ratio of the *circumference* of any circle to its *diameter*, approximately equal to 3.14.
pictograph	A graph that uses pictures to show data.
plane	A flat surface that extends infinitely in all directions.
point	An exact location in space represented by a dot.
polygon	A closed plane figure made by *line segments*.
prime factorization	A way to show a number as the *product* of *prime factors*. The *prime factorization* of 12 is 2x2x3.
prime number	A *whole number* greater than 0 that has exactly two different *factors*, 1 and itself. 5 is a *prime number* because its only *factors* are 1 and 5.
prism	A three-dimensional figure that has two *congruent* and *parallel* faces that are *polygons*. The rest of the faces are *parallelograms*.
product	The answer to a multiplication problem. For example, 6x3=18, 18 is the *product* of 6x3.
pyramid	A polyhedron whose base is a *polygon* and whose other *faces* are triangles that share a common *vertex*.
quadrants	The four sections of a *coordinate grid* that are separated by the axes.
Quadrilateral	A four-sided *polygon*.
quotient	The answer to a division problem.
radius	The segment, or the length of the segment, from the center of a circle to any point on the circle.
Range	The difference between the greatest number and the least number in a set of numbers.
rational number	A number that can be expressed as a ratio of two non-zero *integers*.
ray	A part of a line that has one *endpoint* and goes on forever in one direction.
rectangular prism	A *prism* with six rectangular faces.
reflection	A transformation creating a mirror image of a figure on the opposite side of a line. A *reflection* is also called a *flip*.

region	A part of a plane.
remainder	In *whole number* division, when you have divided as far as you can without using decimals, what has not been divided yet is the remainder.
repeating pattern	A pattern of a group of items that repeats over and over.
rhombus	A *parallelogram* with all four sides equal in length.
right angle	An angle that measures exactly 90º.
right triangle	A triangle that has one 90º angle.
rotation	The transformation that occurs when a figure is turned a certain angle and direction around a point. A rotation is also called a turn.
Rules of Divisibility	Patterns that make it easier to tell whether one number is *divisible* by another.
scalene triangle	A triangle that has no *congruent* sides.
scientific notation	A form of writing numbers as the *product* of a power of 10 and a decimal number greater than or equal to 1 and less than 10.
similar figures	Figures that have the same shape, but not necessarily the same size.
slide	A transformation that slides a figure a given distance in a given direction. A *slide* is also called a *translation*.
square number	A number that is the result of multiplying an *integer* by itself. Any *square number* of dots can be arranged in a square array.
standard form	A number written with one digit for each place value. The *standard form* for the number three thousand three is 3,003.
straight angle	An angle with a measure of 180º.
sum	The answer to an addition problem. In 32+4=36, 36 is the *sum*.
surface area	The total *area* of the *faces* (including bases) and curved surfaces of a solid figure.
translation	A transformation that slides a figure a given distance in a given direction. A *translation* is also called a *slide*.

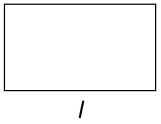
trapezoid	A *quadrilateral* with one pair of *parallel* sides and one pair of sides that are not parallel.
turn	The transformation that occurs when a figure is turned a certain angle and direction around a point. A *turn* is also called a *rotation*.
two-dimensional	A figure that has length and width, but not height. Having *area*, but not *volume*. The image on a movie screen is two-dimensional.
vertex	The point at which two *line segments*, *lines*, or *rays* meet to form an angle.
vertical line	A line that has right angles to the horizon. A *vertical line* is straight up and down.
vertices	Plural of *vertex*.
volume	The number of cubic units it takes to fill a figure.
whole number	Any of the numbers 0, 1, 2, 3, 4, 5, and so on.
Zero Property of Multiplication	The *product* of any number and zero is zero. For example, 8x0=0.

Mathematics 6
Reference Sheet

Square

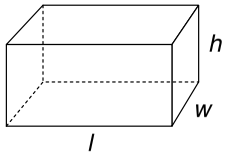
Area = s^2
Perimeter = 4s

Rectangle



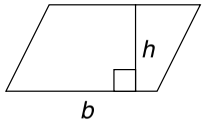
Area = lw
Perimeter = $2l + 2w$

Rectangular Prism



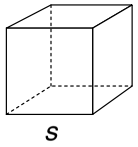
Volume = lwh
Surface Area = $2wl + 2lh + 2wh$

Parallelogram



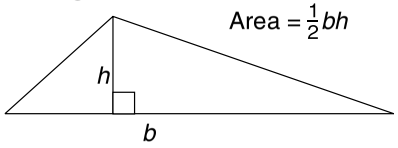
Area = bh

Cube



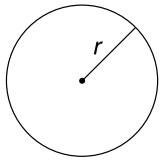
Volume = s^3
Surface Area = $6s^2$

Triangle



Area = $\frac{1}{2}bh$

Circle



Circumference = $2\pi r$

Metric Conversions						
kilo	hecto	deka	meter liter gram	deci	centi	milli
1000	100	10	1.0	0.1	0.01	0.001
K	H	D	M	D	C	M

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Customary Conversions

8 ounces = 1 cup
2 cups = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon
3 teaspoons = 1 tablespoon
16 tablespoons = 1 cup
16 ounces = 1 pint
8 pints = 1 gallon
16 ounces = 1 pound
5280 feet = 1 mile

Definitions

MEAN: In a collection of data, the sum of all the data divided by the number of data.

MEDIAN: The middle number or average of the two middle numbers in a collection of data when the data are arranged in order.

MODE: The number or numbers that occur most often in a collection of data.

RANGE: The difference between the greatest and the least numbers in a collection of data.

REFLECTION OR FLIP: A mirror image of a figure across its line of symmetry.

TRANSLATION OR SLIDE: A change in position of a figure. All points in the figure move in the same direction for the same distance.

ROTATION OR TURN: Turning a figure clockwise or counter-clockwise about a point.

Facilitated Activities

Resources for Facilitated Activities

Books

Burns, M. (2000). *About Teaching Mathematics*. Sausalito, CA: Math Solutions Publications; ISBN 0-941355-25-X

Van de Walle, J. A. (2004). *Elementary and Middle School Mathematics: Teaching Devlopmentally*. Pearson Education, Inc.; ISBN 0-205-38689-X

Hutchins, P. (1986). *The Doorbell Rang*, Mulberry Books, ISBN 0688092349.

Matthews, L. (1995). *Gator Pie*, Sundance Publishing, ISBN 0-7608-0005-7.

Tahan, M. (1993) *The Man who Counted: A Collection of Mathematical Adventures*. Norton & Co.; ISBN 0-393-30934-7

Web sites

Kids' Graphing Page: Create a Graph.
http://nces.ed.gov/nceskids/Graphing/

Online Manipulatives: National Library of Virtual Manipulatives (developed by USU).
http://matti.usu.edu/nlvm/nav/vlibrary.html

Professor Freedman's Math Helps
http://www.mathpower.com/index.htm

Blue Webn at SBC Knowledge network Explorer
(great resources for all disciplines)
http://www.kn.pacbell.com/

A good source of information about engaging students in mathematics is found in a booklet called *Helping Children Learn Mathematics*. The information is found in #5 "Engaging: Seeing mathematics as sensible, useful, and doable—if you work at it—and are willing to do the work." on pages 14 -16.
To access this book use the following URL.
http://www.nap.edu/books/0309084318/html/14.html

Notes

Notes

Notes

Math Standard I-1 & 2 Activities

Scientific Notation

Standard I:
Students will acquire number sense and perform operations with rational numbers.

Objective 1:
Represent whole numbers and decimals in a variety of ways.

Intended Learning Outcomes:
1. Demonstrate a positive learning attitude toward mathematics.
2. Become mathematical problem solvers.
3. Reason mathematically.

Content Connections:
Science IV-I

Math Standard I

Objective 1

Connections

Background Information

Students were introduced to the concept of exponents in fifth grade.

Scientific notation is writing a number as the product of a number (greater than or equal to 1 and less than 10) and a power of ten.

We use scientific notation because numbers can be hard to work with when they have so many zeros. Scientists use scientific notation as a simpler way to write these numbers.

Research Basis

Hatfield, M., Edwards, N., Bitter, G., & Morrow, J. (2000). *Mathematics Methods for Elementary and Middle School Teachers*. New York, NY. John Wiley & Sons Inc.

This resource includes the NCTM Principles and Standards for School Mathematics 2000, as well as the newest NAEP data and findings from the TIMSS. The book emphasizes considerations regarding cultural diversity and includes a CD-ROM with vignettes of real classroom situations to help the reader study teaching practices as they occur naturally in the classroom.

Assessment Suggestions

- Informal assessment includes observation of students as they complete the *Number Notation Table*. Class discussion and discovery is another form of assessment.
- Formal assessment includes completed *Number Notation Tables* with correct scientific notation and standard forms.

Invitation to Learn

Materials
- ❑ Stopwatch

Mercury is about 35 million miles from the sun.

Have students estimate how long they think it would take to write 35 million numerically (35,000,000).

Write down estimation.

Have each student (or one student) write out 35,000,000. Time to see how long it takes.

Instructional Procedures

Materials
- ❑ *Number Notation Table*
- ❑ *Planet Table*
- ❑ 3" x 5" cards

1. Give each student a *Number Notation Table*.
2. As a class (teacher directed), complete the first few lines together. Discuss what pattern can be seen.
3. Have class finish chart individually or in groups.

   Discuss what pattern can be seen.
4. Demonstrate how to change 35,000,000 (the distance Mercury is from the sun) into scientific notation (3.5×10^7).
5. Discuss how the number was changed and compare the pattern they discovered to the number. What is happening?

   Students will most likely assume that the exponent matches the number of zeroes. However, the exponent (power of ten) matches the number of places the decimal point moves (e.g., $1,000 = 1 \times 10^3$. The decimal moves three places to the left.).

   Change 3.5×10^7 back to standard form.
6. Compare 1×10^7 to 3.5×10^7

   Emphasize again that the exponent (power of ten) matches the number of places the decimal point moves.
7. Give more examples as needed.

   $2.87 \times 10^2 = 287$

   $3.982 \times 10^4 = 39,820$

   $5.843 \times 10^5 = 584,300$

   $1.457 \times 10^5 = 145,700$

   $5.47 \times 10^6 = 5,470,000$

   $38,700 = 3.87 \times 10^4$

   $16 \text{ billion} = 1.6 \times 10^{10}$

   $2,137,000 = 2.137 \times 10^6$

493,000,000,000 = 4.93×10^{11}

4,382,000,000,000 = 4.382×10^{12}

8. Have each student (or one student) write out 35,000,000, using scientific notation (3.5×10^{7}). Time to see how long it takes. (This shows that scientific notation is an efficient way to write large numbers.)
9. Give each student two 3" x 5" cards. Have each student write a self-selected (large) number in standard form on one card, and the equivalent number in scientific notation on the second card. Collect all the cards.
   - To play the game, use half of the pairs of cards to play one round of the game (since you now have twice as many cards as students).
   - Tape one card to each student's back, making sure that you use both the standard and scientific notation form cards of each number selected.

   Have students find the person with the equivalent number. They may attempt to identify the number by asking "yes" or "no" questions only. The round continues until all students have found their "partner." You may want to play a second round using the remaining cards.

Curriculum Extensions/Adaptations/Integration

- Just as a positive exponent designates how many spaces the decimal moves to the left, a negative exponent denotes how many spaces a decimal moves to the right. Although the concept of negative exponents is not in the sixth grade core curriculum, the study of microorganisms could be used to introduce the concept.
- Use *Number Cards* to create a human problem: Give a card to each student and have them stand to create a number (e.g., 2.87). Assign one student to be the decimal point. Have the "decimal point" move to the correct spot to form the number (2.87).

Materials

❑ *Number Cards*

Family Connections

- Students time a family member writing a number both in standard form and scientific notation and record the difference.
- Students search for a large number in a newspaper article and write the number in scientific notation.

Name _____

Number Notation Table

number	numeral	scientific notation
one		
ten		
hundred		
thousand		
ten thousand		
hundred thousand		
million		
ten million		
hundred million		
billion		
ten billion		
hundred billion		
trillion		

Describe any patterns you see in the table:

Number Notation Table–Key

number	numeral	scientific notation
one	1	1×10^0
ten	10	1×10^1
hundred	100	1×10^2
thousand	1,000	1×10^3
ten thousand	10,000	1×10^4
hundred thousand	100,000	1×10^5
million	1,000,000	1×10^6
ten million	10,000,000	1×10^7
hundred million	100,000,000	1×10^8
billion	1,000,000,000	1×10^9
ten billion	10,000,000,000	1×10^{10}
hundred billion	100,000,000,000	1×10^{11}
trillion	1,000,000,000,000	1×10^{12}

Describe any patterns you see in the table:

Name _____

Planet Table

Complete the following table:

Planet	Miles from the sun	Miles from the sun in scientific notation
Mercury	35,000,000	
Venus		6.5×10^7
Earth		9.3×10^7
Mars	137,000,000	
Jupiter	467,000,000	
Saturn	850,000,000	
Uranus		1.7×10^9
Neptune		2.7×10^9
Pluto	3,500,000,000	

Planet Table—Key

Complete the following table:

Planet	Miles from the sun	Miles from the sun in scientific notation
Mercury	35,000,000	3.5×10^7
Venus	65,000,000	6.5×10^7
Earth	93,000,000	9.3×10^7
Mars	137,000,000	1.37×10^8
Jupiter	467,000,000	4.67×10^8
Saturn	850,000,000	8.5×10^8
Uranus	1,700,000,000	1.7×10^9
Neptune	2,700,000,000	2.7×10^9
Pluto	3,500,000,000	3.5×10^9

Number Cards

0	1
2	3
4	5

6	7
8	9
.	,

Prime and Composite

Math Standard I

Objective 1

Connections

Standard I:
Students will acquire number sense and perform operations with rational numbers.

Objective 1:
Represent whole numbers and decimals in a variety of ways.

Intended Learning Outcomes:
1. Demonstrate a positive learning attitude toward mathematics.
2. Become mathematical problem solvers.
3. Reason mathematically.

Content Connections:
Social Studies IV-2

Background Information

Students learned to classify whole numbers from 2 to 20 as composite or prime, and 0 and 1 as neither prime nor composite, in fifth grade.

Every whole number greater than 1 is either a prime or composite number. A *prime* number has *exactly* two factors, 1 and itself. A *composite* number has more than two factors.

Commutative property is the fact that changing the order of addends or factors does not change the sum or product (e.g., 4 x 7 = 28 and 7 x 4 = 28).

Rules of divisibility

- A whole number is divisible by 2 if and only if the ones digit is even.
- A whole number is divisible by 3 if and only if the sum of its digits is divisible by 3.
- A whole number is divisible by 5 if and only if the ones digit is 5 or 0.
- A whole number is divisible by 6 if and only if it is divisible by 2 and 3.
- A whole number is divisible by 9 if and only if the sum of its digits are divisible by 9 (or 3).
- A whole number is divisible by 10 if and only if the ones digit is 0.

Research Basis

Hatfield, M., Edwards, N., Bitter, G. & Morrow, J. (2000). *Mathematics Methods for Elementary and Middle School Teachers*. New York, New York. John Wiley & Sons Inc.

"An activity-based approach to teaching with an emphasis on using manipulatives to build conceptual understanding! This valuable book combines practical teaching ideas, video examples, updated assessment techniques, and the NCTM Assessment Standards to give teachers all the background they need to introduce elementary and middle school students to the wonders of mathematics. Provides training and practicing teachers of kindergarten to the eight grade with ideas, techniques, and approaches to teaching mathematics such that their students will be prepared for later study."

Assessment Suggestions

- Observation of students, class discussion, and discovery.
- The *Prove or Disprove* worksheet may be used as an assessment tool.

Invitation to Learn

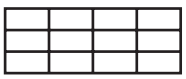
Pass out 12 centimeter cubes to each student. Have students make different shapes with the cubes. Share examples and point out examples and nonexamples of arrays. Write a definition of "array" as a class.

Instructional Procedures

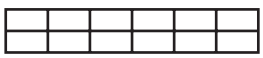
Day 1

1. Use the *Manipulative Master* to create an array for numbers from 3 to 20. For example, say to the class, "Using your manipulatives, show me what 4 looks like. How many ways can you make a complete rectangle using only four squares?"
2. Continue through all numbers, 3-20.
3. Transfer arrays onto graph paper and label each rectangle.

   Example:



3 x 4 = 12



2 x 6 = 12

Materials

For class:

- ❑ *We are Prime/ Composite Numbers* posters
- ❑ Sieve (or picture of)

For each student:

- ❑ Graph paper
- ❑ *Manipulative Master* or manipulatives
- ❑ *Sieve of Eratosthenes* worksheet
- ❑ *Prove or Disprove* worksheet
- ❑ Colored pencils

4. Circle each factor using one color. Put a circle around each product using a different color.

   Example:

   ③ x ④ = ⑫ ② x ⑥ = ⑫

5. Point out the commutative property—the fact that changing the order of addends or factors does not change the sum or product. *Example:*

   3 x 4 = 12 and 4 x 3 = 12

   factors 1, 2, 3, 4, 6, 12

6. Connect two factors using a curved line that resembles a rainbow. Draw a square around numbers that are the square root.

7. Numbers with exactly 2 factors are **prime** numbers.
8. Numbers with more than two factors are **composite** numbers.
9. The numbers 0 and 1 are neither prime nor composite. 1 is a unique counting number. 0 is not a counting number.

Day 2

1. Before class, place the *We Are Prime (Composite) Numbers* posters in different corners of the room.
2. Before class, prepare a sheet of graph paper for each student in your class by writing a different number at the top of each sheet. Begin with the number two and continue in order until you have enough for each student (e.g., If you have 36 students in your class, you would use the numbers 2-27 for this activity.).
3. Pass out a sheet to each student. Instruct students to complete their sheets as follows:
   - Draw all of the possible arrays for the number.
   - Label the length and width of each array.
   - Write multiplication sentences for each array and circle factors in red and products in blue.
   - Draw a factor rainbow for the number.
4. As the students finish, begin sending them to different corners of the room depending on whether they have a prime or composite number. Have the students take their completed papers with them.

5. Have the students in each corner compare their numbers with other members of their group and look for things that their numbers have in common. If the students are having a difficult time seeing similarities, prompt them to look at the number of factors and arrays. Discuss the findings as a class.
6. Have one member of each group hold up the *We Are Prime (Composite) Numbers* posters.
7. Have students return to their seats and write definitions of *prime* and *composite numbers* in math journals.
8. Gather papers from students and randomly pass them out again. After looking at their new number, have students move to the correct corner of the room again. Continue to do this until all students can correctly move it either the prime or composite corners of the room.
9. If you have been hanging the *Number Posters* up around your classroom, draw students' attention to them at this time. Look at the posters and discuss how the number *one* is different than the other numbers. Write a definition for *unique number* in math journals.

Day 3

1. Show a sieve. What is it? What is it used for? How does it work?
2. We can sort numbers just like a sieve separates and sorts material.
3. Using the *Sieve of Eratosthenes*, complete the hundreds chart as follows:
   a. As a class, cross out the 1 on the chart.
   b. Putting your finger on 2 count by twos and color the top left corner of each number square yellow.

      Teacher Note: As students begin coloring, they will most likely begin to see a pattern. Explain the rule of divisibility for 2—A whole number is divisible by 2 if the ones digit is even.

   c. Put your finger on 3. Counting by threes color in the top right corner of each number square orange.

      Teacher Note: As students begin coloring, they will most likely begin to see a pattern. Explain the rule of divisibility for 3—A whole number is divisible by 3 if the sum of its digits is divisible by 3.

   d. Place centimeter cubes on all multiples of 6. Discuss patterns and divisibility for 6.

e. Put your finger on 5. Count by fives and color the bottom left corner of each number square green.

Teacher Note: As students begin coloring, they will most likely begin to see a pattern. Explain the rules of divisibility for 5 and 10—A whole number is divisible by 5 if the ones digit is 5 or 0. A whole number is divisible by 10 if the ones digit is 0.

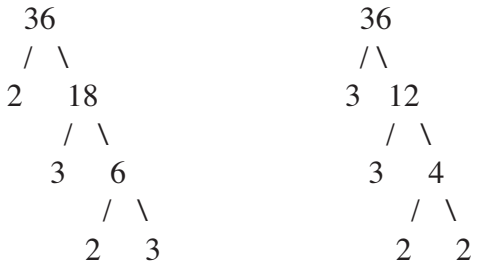
f. Put your finger on number 7. Count by sevens and color the bottom right corner of each number square blue.

g. The boxes with nothing colored are prime numbers. Color these boxes red.

Day 4

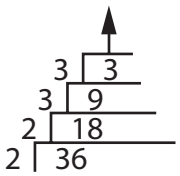
1. How can you tell people apart? How can you tell people apart when you cannot see them? (fingerprint)
2. Just like every person has a unique fingerprint, each number has a unique "factorprint." This factorprint is the number's prime factorization. It is the set of prime numbers whose product equals the number.

   Factor Trees



Birthday Cake

Start with lowest prime number (2).
If 2 will not work, move to 3, etc...



Curriculum Extensions/Adaptations/Integration

- Have students work in pairs or small groups to develop divisibility rules. Discuss as a class and complete rules as listed on p. 3-12.
- *Prove or Disprove* worksheet.
- Have students research Eratosthenes and tie to a Social Studies unit on Greece.

Resources

Book

Mathland, Journey Through Mathematics: Student Resource Book Grade 6, by Nancy Homan; ISBN 0762212411

Additional Resource

Shari Goodman, Granite School District.
shari.goodman@granite.k12.ut.us

Family Connections

- Students explain the S*ieve of Eratosthenes* to a family member.

We Are Prime/Composite Numbers Posters

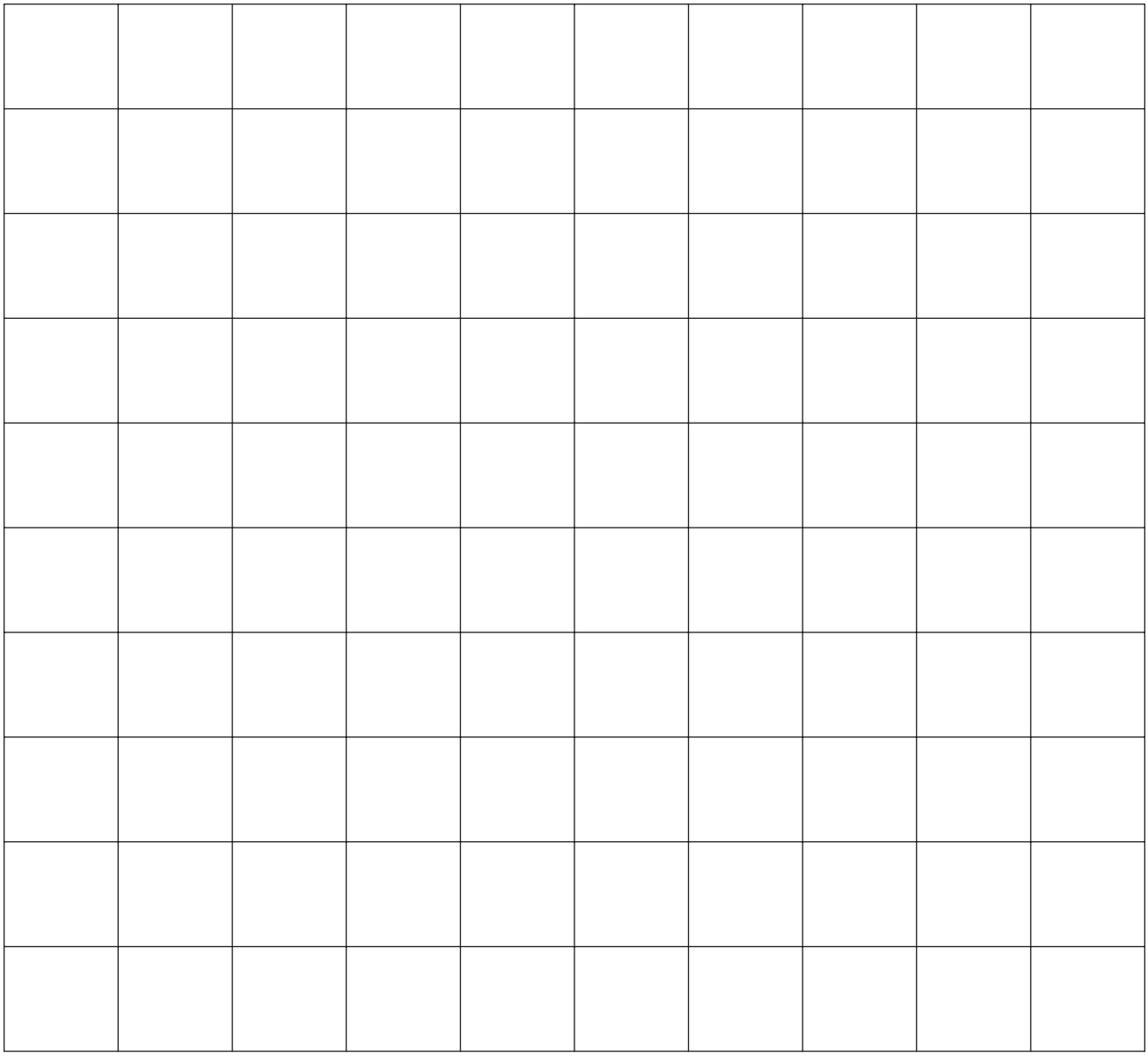
Name _____

Sieve of Eratosthenes

1. Put your finger on number 2. Count by twos and color the top left corner of each number square yellow.
2. Put your finger on number 3. Count by threes and color the top right corner of each number square orange.
3. Put your finger on number 5. Count by fives and color the bottom left corner of each number square green.
4. Put your finger on number 7. Count by sevens and color the bottom right corner of each number square blue.
5. Color the remaining boxes red.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Manipulative Master



Name _____

Prove or Disprove

Prove or disprove these statements:

1. Only three of the numbers below are prime numbers.

   123 635 587 247 198 233 267 307 7101

2. There are 25 prime numbers between 1 and 100 and 25 prime numbers between 100 and 200.

3. Each of the whole numbers between 1 and 50 can be written as a product of two or more prime numbers.

   Example: 20 = 2 x 2 x 5

4. Every number from 1 to 100 can be written as the sum of two prime numbers.

   Example: 18 = 7 + 11

Create three more prove or disprove statements of your own.

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

Smaller than One

Math Standard I

Objective 2

Connections

Standard I:
Students will acquire number sense and perform operations with rational numbers.

Objective 2:
Identify relationships among whole numbers, fractions (rational numbers), decimals, and percents.

Intended Learning Outcomes:
1. Demonstrate a positive learning attitude toward mathematics.
2. Become mathematical problem solvers.
3. Reason mathematically.

Content Connections:
Social Studies II-2, IV-2, V-2

Background Information

The fifth grade core curriculum has students order and compare *whole numbers*, fractions (including mixed numbers), and decimals using a variety of methods and symbols.

Percent is a ratio comparing a part to a whole using the number 100. The *percent* is the number of hundredths that part is equal to.

Research Basis

Farivar, S. & Webb, N.M. (1994). Helping and Getting Help–Essential Skills for Effective Group Problem Solving. *Arithmetic Teacher. 41*, 521-525.

Using examples drawn from a program of peer learning in middle school mathematics, this paper identifies a set of helping behaviors that best aid learning in collaborative small groups. The paper describes conditions that must be satisfied for helping behavior to be effective and the responsibilities of students seeking help, students giving help, and teachers to make helping productive for learning.

Hatfield, M., Edwards, N., Bitter, G., & Morrow, J. (2000). *Mathematics Methods for Elementary and Middle School Teachers*. New York, NY. John Wiley & Sons Inc.

This resource included the NCTM Principles and Standards for School Mathematics 2000, as well as the newest NAEP data and findings from the TIMSS. The book emphasizes considerations regarding cultural diversity and includes a CD-ROM with vignettes of real classroom situations to help the reader study teaching practices as they occur naturally in the classroom.

Assessment Suggestions

- Informal assessment includes observation of students, class discussion and discovery.
- Formal assessment includes completed diagram book with correct fractions and percents.

Invitation to Learn

1. Provide each student in the class with a card or paper with an assigned number (1-30, or the number of students in the class).
2. Have the student with 1 and the largest number stand on opposite sides of the room.
3. Instruct the rest of the class to place themselves in numerical order between the two numbers.
4. Repeat this process as needed by having the first two students hold 0 and 1. The other students should have numbers dealing with tenths, hundredths, or thousandths.
5. Repeat the game using fractions.

Instructional Procedures

1. Using the base ten blocks, show the 100 square. This equals 100/100 = 1/1 = 1.0 = 100%
2. Transfer information onto the *Base 10* worksheet by coloring all 100 squares.
3. Using base ten blocks, show 50 out of 100, 1/2 = 0.50 = 50 %.
4. Transfer information onto *Base Ten* worksheet by coloring 50 of the 100 squares.
5. Continue making diagrams using:

   10/100 = 1/10 = 0.1 = 10%
   20/100 = 1/5 = 0.2 = 20%
   25/100 = 1/4 = 0.25 = 25%
   30/100 = 3/10 = 0.3 = 30%
   40/100 = 4/10 = 0.4 = 40%
   60/100 = 3/5 = 0.6 = 60%
   70/100 = 7/10 = 0.7 = 70%
   75/100 = 3/4 = 0.75 = 75%
   80/100 = 4/5 = 0.8 = 80%
   90/100 = 9/10 = 0.9 = 90%

Materials

For each group:

- ❑ White paper for books
- ❑ Base ten blocks
- ❑ Copy of shapes for extension

For each student:

- ❑ Scissors
- ❑ Glue
- ❑ Colored pencils/crayons
- ❑ *Base 10* worksheet

6. Review how to convert a fraction into a decimal and then find the percent:
   - To convert a fraction to a decimal, divide the numerator by the denominator.
   - To convert a decimal to a fraction, write the place value of the decimal as the denominator. Write the digits of the decimal as the numerator.
   - To convert a percent into a fraction or a decimal, rewrite the percent as a fraction over 100.
   - To convert a fraction to a percent, use the following proportion:

     $$\frac{\text{part}}{\text{whole}} = \frac{\text{percent value}}{100}$$

7. Construct a book to keep diagrams.
   a. Fold a sheet of 8 1/2" x 11" paper in half, horizontally (paper A).
   b. Measure and mark one inch from each side along the fold.
   c. Cut one inch slits on either side of the mark. Unfold to full original size.
   d. Take a new sheet of paper (paper B).
   e. Fold in half, horizontally.
   f. Measure and mark one inch from each side along the fold.
   g. Cut **between** the inch marks to create a slit in the paper. Unfold to full size.
   h. Take paper A and roll vertically.
   i. Slide paper A through the slit in paper B.
   j. Unroll paper A carefully and match slits.
   k. Crease book again.
   l. Add as many pages (like paper A) as needed. To complete book for above conversions, use 1 paper A and 3 paper B (on last page write the steps to conversion).
8. Have students glue in their diagrams, in numerical order, and label each page.
9. Title the book.
10. Use book as a quick reference for the remainder of the unit on decimals, percents, and fractions.

11. As extra credit or an extension, students find:

    33 1/3 /100 = 1/3 = 0.3 =33 1/3%

    66 2/3/ 100 = 2/3 = 0.6 = 66 2/3 %

    15/100 = 3/20 = 0.15 = 15 %

    35/100 = 7/20 = 0.35 = 35%

    45/100 = 9/20 = 0.45 = 45%

    55/100 = 11/20 = 0.55 = 55%

    65/100 = 13/20 = 0.65 = 65%

    85/100 = 17/20 = 0.85 = 85%

    95/100 = 19/20 = 0.95 = 95%

Curriculum Extensions/Adaptations/Integration

- Using *Number Cards*, students could play War, Spoons, Memory, Gin, or Go Fish.

  (The cards equivalent to 1/3, 2/3, 1/8, 3/8, 5/8, and 7/8 are optional. 6/15 and 4/8 are also optional.)

Materials

❑ *Number Cards*

- Create "percentage pictures" from geometric shapes.
  1. Give each student two squares, two rectangles, two equilateral triangles and two circles.
  2. Ask students to "using your scissors cut 50% (25%, 75%, etc...) off your square (rectangle, triangle, circle)."
  3. When shapes have been cut use the shapes to create a picture.
  4. Have students create a worksheet to accompany their picture. For example, if there is a picture of a sun one question on the worksheet could be, "What percentage of the circle made the sun?"

Materials

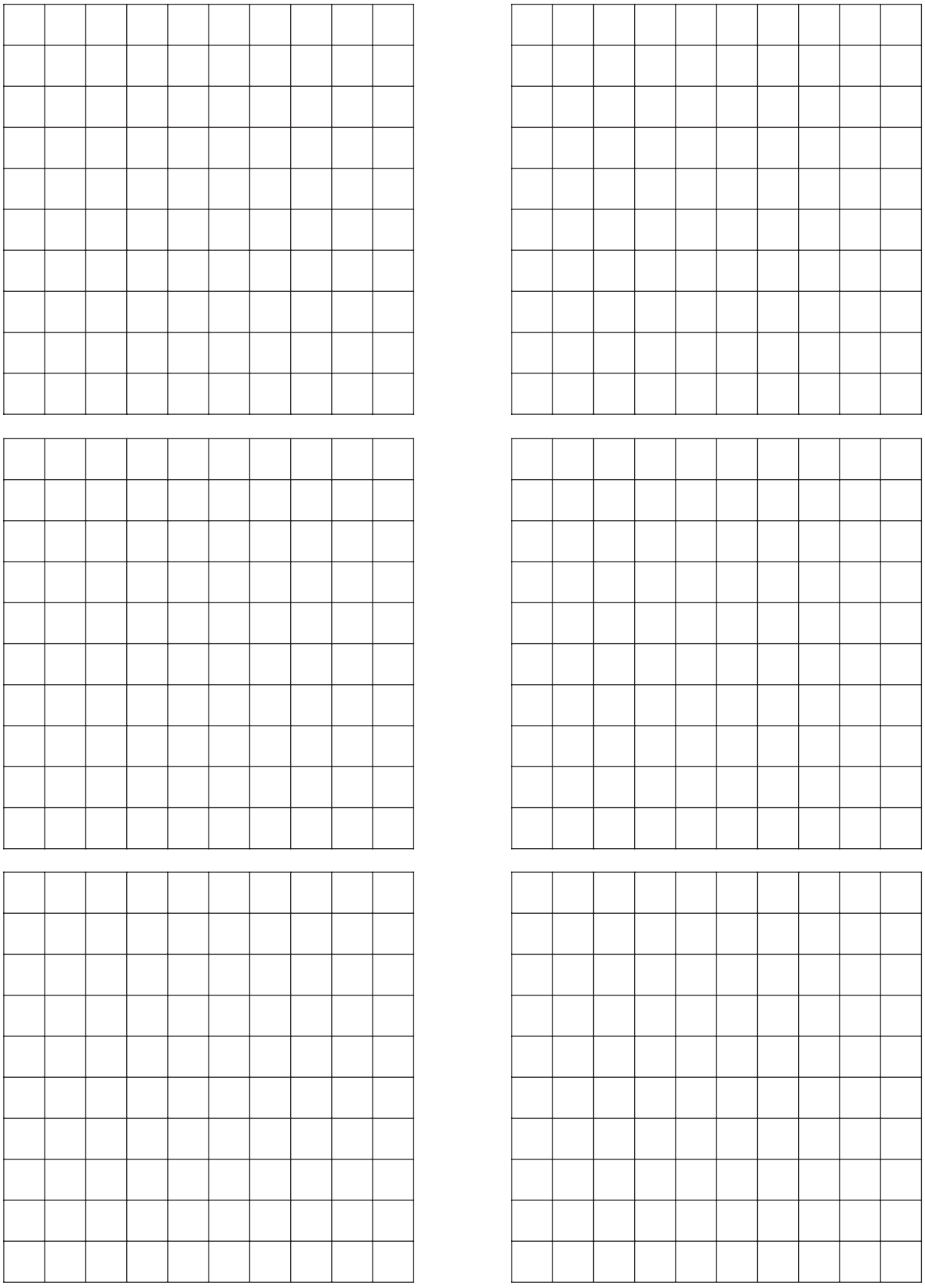
❑ *Geometric Shapes*

Family Connections

- Students take their diagram book home and explain it to a family member.
- Students make their own set of game cards and play one of the games with a family member.
- Students have a family member complete his/her "percentage picture."
- Students create their own answer key and test their family members.

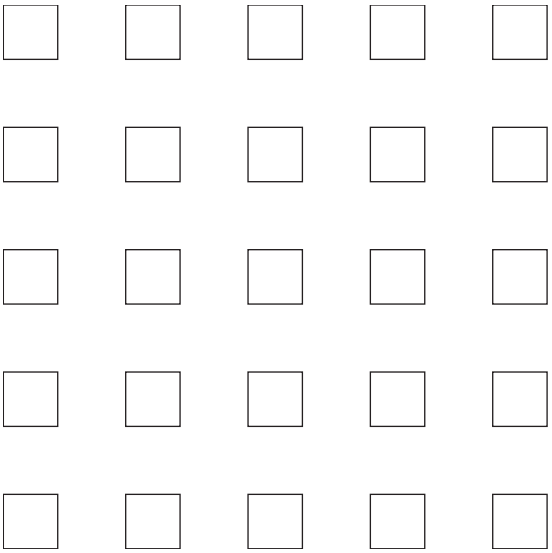
Name _____

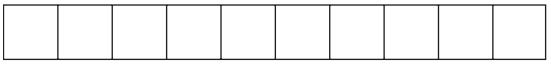
Base 10

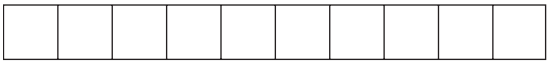


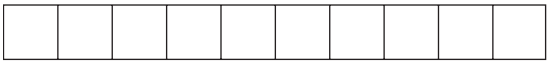
Name _____

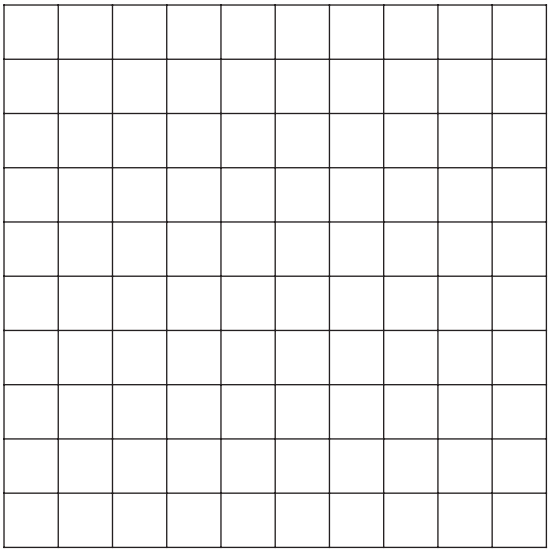
Base 10 Blocks

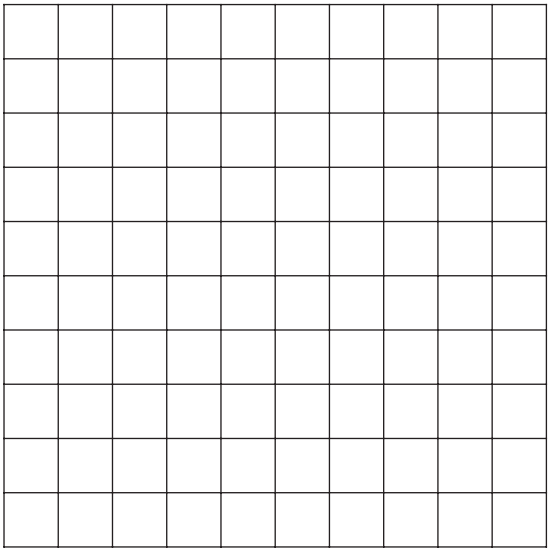


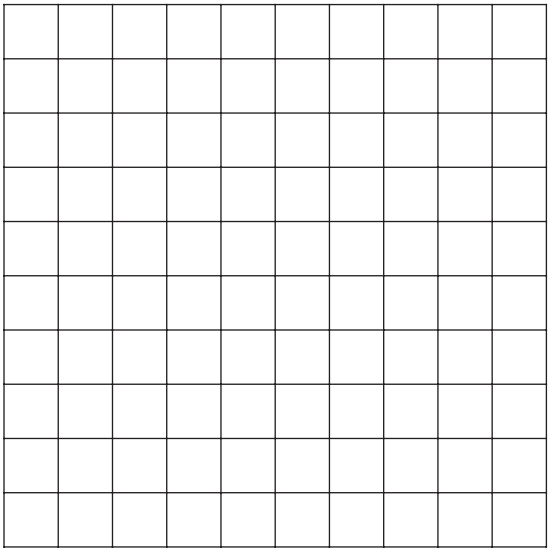




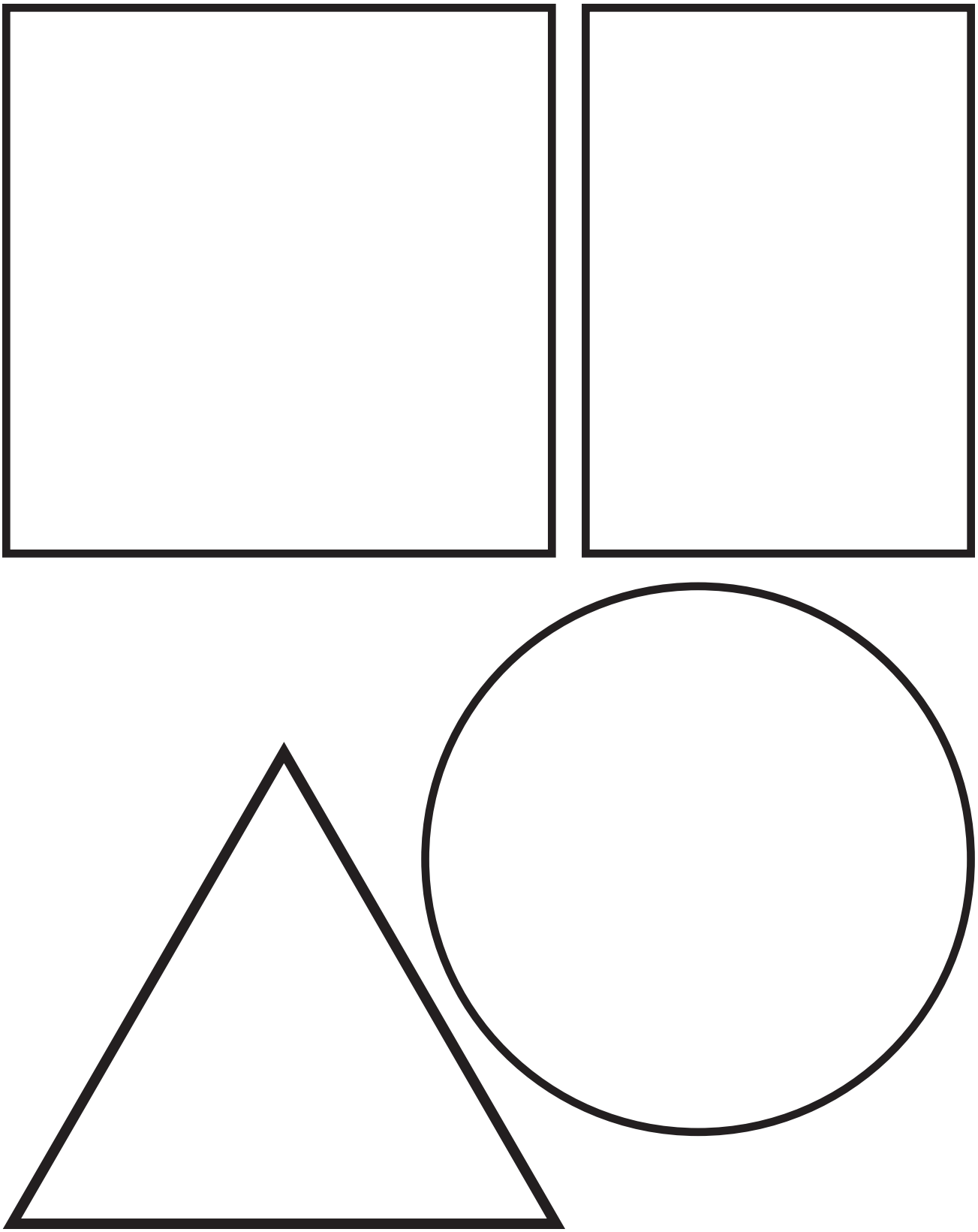








Geometric Shapes



Number Cards

1/10	0.1	10%
1/5	0.2	20%
1/4	0.25	25%

3/10	0.3	30%
2/5	0.4	40%
1/2	0.5	50%

3/5	0.6	60%
7/10	0.7	70%
3/4	0.75	75%

4/5	**0.8**	**80%**
9/10	**0.9**	**90%**
1	**1.0**	**100%**

3/20	**0.15**	**15%**
7/20	**.35**	**35%**
9/20	**.45**	**45%**

11/20	0.55	55%
13/20	0.65	65%
17/20	0.85	85%

19/20	0.95	95%
1/3	0.3	$33\frac{1}{3}\%$
2/3	0.6	$66\frac{2}{3}\%$

1/8	0.125	12.5%
3/8	0.375	37.5%
5/8	0.625	62.5%

7/8	0.875	87.5%
6/15	0.4	40%
4/8	0.5	50%

Math Standard II-1 & 2 Activities

Growing Patterns

Standard II:
Students will use patterns, relations, and functions to represent and analyze mathematical situations using algebraic symbols.

Objective 1:
Recognize, analyze, and use multiple representations of patterns and functions and describe their attributes.

Intended Learning Outcomes:
2. Become mathematical problem solvers.
3. Reason mathematically.
4. Communicate mathematically.

Content Connections:
Math III-2

Math Standard II

Objective 1

Connections

Background Information

Growing patterns can show increasing or decreasing sequences and can help students analyze mathematical change. In this activity, students graph growing patterns using ordered pairs on a coordinate grid. A *coordinate grid* is a two-dimensional system in which a location is described by its distances from two intersecting, perpendicular lines, called axes. *Coordinates* are ordered pairs of numbers that give the location of a point on a coordinate grid. Finally, an *ordered pair* is a pair of numbers that gives the coordinates of a point on a grid, the first number of the pair is the horizontal (x) coordinate, the second number is the vertical (y) coordinate.

Research Basis

Bryant, V.A. (1992) Improving Mathematics Achievement of At-Risk and Targeted Students in Grades 4-6 through the Use of Manipulatives. http://eric.ed.gov ERIC # ED355107

This document presents a practicum designed to improve mathematics achievement through the use of manipulatives. Results indicated an increase in test scores and letter grades.

Hinzman, K.P. (1997) Use of Manipulatives in Mathematics at the Middle School Level and Their Effects on Students' Grades and Attitudes. http://eric.ed.gov ERIC # ED411150

This paper reports on a study that examines mathematics when manipulatives are used in the classroom. Results indicate that student performance is enhanced by the use of manipulatives and students' attitudes toward mathematics was more positive than students who did not use manipulatives in the classroom.

Assessment Suggestions

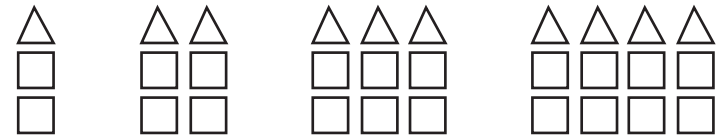
- Observation and class discussion of the students growing their patterns.
- Students' ability to predict what the growth pattern would look like and explain their reasoning.

Invitation to Learn

Materials

- ❑ Overhead pattern blocks—squares and triangles
- ❑ Pattern block triangles
- ❑ Square color tiles
- ❑ Math journal

1. On the overhead projector, show students the diagram of the first four houses.



2. How many pieces are needed to make each house? How many squares and triangles are needed for a given house?
3. Create a table (using x and y) in their math journals to show the information.
4. Ask students to use their squares and triangles to make house 5, record the information for house 5, and see if they notice a pattern.
5. Have students predict the total number of pieces they would need to build house 12, and explain their reasoning.
6. Write a rule (formula) that gives the total number of pieces needed to build any house in the sequence.

Instructional Procedures

Materials

- ❑ Square color tiles
- ❑ Centimeter graph paper

1. Students create a growing pattern using the initials in their name with square color tiles and centimeter graph paper.
2. On the overhead, give an example to the class using your own initial.
3. Students create their initial using as few squares as possible.
4. Copy that image down on the graph paper.
5. Students "grow" their initial to size 4, and copy the images down on the graph paper. Explain that they need to use more squares for each growth of their initial, but that it should still look like their original initial.

6. Create a table (using x and y) with the number of squares used for each size.
7. Look for a pattern.
8. Predict what the 10th size would look like. How many squares would be needed to build it? Write the prediction and reasoning in math journals.
9. Students create a formula for the *n*th size.
10. Have several students share their growth patterns with the class and discuss how they came up with their formulas.
11. Create a graph on a coordinate grid using the ordered pairs from the table.

Curriculum Extensions/Adaptations/Integration

- Leonardo Fibonacci, an Italian mathematician, who lived from about 1180 to about 1250, found this pattern.

  0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610 . . . .

  Each number in the series is the sum of the two numbers before it. Mathematicians are still finding things in nature that can be described with this sequence of numbers. Have students research Leonardo Fibonacci and Italy during the time he lived.
- Read *The Fly on the Ceiling, A Math Myth* and learn about Rene' Descartes, the founder of the coordinate grid.
- Add the words *ordered pair, coordinates,* and *coordinate grid* to your spelling and vocabulary units.

Resources

Books

The Fly on the Ceiling, A Math Myth, by Dr. Julie Glass; ISBN 0-679-88607-9

Navigating Through Algebra in Grades 6-8, by Susan Friel, Sid Rachlin, and Dot Doyle; ISBN 0-87353-501-4

Family Connections

- Students play Battleship with a family member. This gives them practice using ordered pairs and a coordinate grid.
- Students ask a family member to grow their initial to the sixth size on graph paper. Have the student find the pattern of growth and create a formula to find the *n*th size.

Best Salary

Standard II:
Students will use patterns, relations, and functions to represent and analyze mathematical situations using algebraic symbols.

Objective 1:
Recognize, analyze, and use multiple representations of patterns and functions and describe their attributes.

Intended Learning Outcomes:
2. Become mathematical problem solvers.
3. Reason mathematically.

Content Connections:
Math V-1

Math Standard II

Objective 1

Connections

Background Information

Finding patterns is the underlying theme of mathematics. Recognizing and using patterns is a valuable problem-solving tool. By exploring, discovering, and analyzing patterns, students can begin to make sense of things in mathematics. Searching for patterns begins with concrete activities, but moves to discovery, application, and a greater sense of understanding.

Research Basis

Brenner, M.E. (1995) The Role of Multiple Representaions in Learning Algebra. http://eric.ed.gov ERIC # ED391659

Prealgebra students learned about functions by representing problems in multiple formats. Students' learning that was anchored by a meaningful thematic context had gains in translating word problems into equations, tables, and graphs. The same results were found in lower-achieving students and language-minority students.

Capraro, R.M., Kulm, G. & Caprano, M.M. (2002) Investigating the Complexity of Middle Grade Students' Understanding of Mathematical Constructs: An example from Graphic Representation. http://eric.ed.gov ERIC # ED465799

This study examined four components of prior understanding required for graphic representation: coordinate relationships, graphs showing a variety of relationships, reading simple tables, and graphic displays.

Assessment Suggestions

- Observation of students creating their tables and graphs.
- Class discussion.
- *Which Salary is the Best?* worksheet and graph.

Invitation to Learn

Read *The King's Chessboard* to the point where the wise man makes his request and leaves the hall.

Ask the students how much rice they think the wise man will receive before the chessboard is full. Was it a good request for his reward? Why were the counselors and nobles laughing at the wise man's request?

The class will see if the wise man's request was truly wise after the activity.

Instructional Procedures

Materials

- ❑ *The King's Chessboard*
- ❑ *Which Salary is the Best?* worksheet
- ❑ Centimeter graph paper

1. Pass out copies of the *Which Salary is the Best?* worksheet and discuss the scenario:

   You want to buy a go-cart for $1,000 and need to find a job to raise the money. You found job openings to mow lawns with two different companies that have different pay scales. One company, Lawns Are Us, will double your salary each day. You will earn $1 the first day, $2 the second day, $4 the third day, $8 the fourth day, and so on. The second company, Smith Lawn Care will increase your salary by $4 each day. You will make $4 the first day, $8 the second day, $12 the third day, $16 the fourth day, and so on. Which company will help you reach the $1,000 needed to buy a go-cart the fastest?

2. After reading the scenario, ask students to predict which company would enable them to reach their $1,000 goal the fastest and explain their reasoning in their math journals.
3. Students create a table for each of the two companies and complete the tables until day 5. Which company pays the most at this point? Write a short paragraph in their journals about which company they would choose at day 5 and why.
4. Students complete the chart until $1,000 is made by both companies. Which company was the best choice? Why? Have students write a paragraph in their journals explaining what happened with the salaries.

5. As a class, write a function to find out which salary would pay more on the *n*th day.
6. Create a multiple line graph on the centimeter graph paper. The x-axis should be Total Earnings and the y-axis should be Number of Days. Have them graph the total earnings of each company in different colors.
7. Discuss the graph. For what days does Lawns Are Us yield better total earnings? For what days does Smith Lawn Care yield better total earnings?
8. Does the chart or graph illustrate the information more effectively? Why? Have students record their thoughts in their math journal.
9. Finish reading *The King's Chessboard* and discuss.

Curriculum Extensions/Adaptations/Integration

- Exponential growth is a number pattern that occurs in mitosis, or cell division. An *e-coli* cell is one of the fastest growing bacteria cells. It can reproduce itself in 15 minutes. Have students create a table showing mitosis of an *e-coli* cell in one hour. Have them find a pattern in the growth rate.
- Using Excel, have students create a double bar graph comparing the salary of the two different jobs.

Resources

Books

The King's Chessboard, by David Birch; ISBN 0-14-054880-7

One Grain of Rice: A Mathematical Folktale, by Hitz Demi; ISBN 059093998X

Navigating Through Algebra in Grades 3-5, by Gilbert J. Cuevas and Karol Yeatts; ISBN 0-87353-500-7

Family Connections

- Students poll their family members about which job they would choose. Would they rather get $1 the first day, $2 the second day, $4 the third day, $8 the fourth day, and so on? Or would they rather get $4 the first day, $8 the second day, $12 the third day, $16 the fourth day, and so on? After they get their families opinions, students explain which job is better and why.
- Students read *The King's Chessboard* or *One Grain of Rice* with their family.

Name _____

Which Salary is the Best?

You want to buy a go-cart for $1000 and need to find a job to raise the money. You found job openings to mow lawns with two different companies that have different pay scales. One company, Lawns Are Us, will double your salary each day. You will earn $1 the first day, $2 the second day, $4 the third day, $8 the fourth day, and so on. The second company, Smith Lawn Care will increase your salary by $4 each day. You will make $4 the first day, $8 the second day, $12 the third day, $16 the fourth day, and so on. Which company will help you reach the $1,000 needed to buy a go-cart the fastest?

Lawns Are Us			**Smith Lawn Care**		
Day	**Daily Salary (Dollars)**	**Total Earnings (Dollars)**	**Day**	**Daily Salary (Dollars)**	**Total Earnings (Dollars)**
1	1		1	4	
2	2		2	8	
3	4		3	12	
4	8		4	16	
5	16		5	20	
6			6		
7			7		
8			8		
9			9		
10			10		
11			11		
12			12		
13			13		
14			14		
15			15		
16			16		
17			17		
18			18		
19			19		
20			20		
21			21		
22			22		
23			23		
24			24		
25			25		
26			26		
*n*th			*n*th		

Patterns in Measurement

Math Standard II

Objective 1

Connections

Standard II:
Students will use patterns, relations, and functions to represent and analyze mathematical situations using algebraic symbols.

Objective 1:
Recognize, analyze, and use multiple representations of patterns and functions and describe their attributes.

Intended Learning Outcomes:
2. Become mathematical problem solvers.
3. Reason mathematically.
4. Communicate mathematically.

Content Connections:
Math IV-2; Visual Arts IV-1 & 2

Background Information

The ability to find and describe patterns found in numbers, charts, operations, geometric figures, and graphs is important for the development of a deep understanding of mathematics and algebra. This activity uses squares and cubes to find patterns in perimeter, area, and volume. Students then chart and graph the information found in the experiment.

Research Basis

Leinenbach, M., & Raymond, A.M. (1996) A Two-Year Collaborative Action Research Study on the Effects of a "Hands-On" Approach to Learning Algebra. http://eric.ed.gov ERIC # ED398081

This study was a two-year collaborative action research project that focused on the effects of the use of manipulatives in an algebra class. Findings indicates that students' confidence, interest, and ability in solving algebraic equations were very high when working with manipulatives.

Friel, S.N., & Bright, G.W. (1995) Graph Knowledge: Understanding How Students Interpret Data Using Graphs. http://eric.ed.gov ERIC # ED391661

This paper discusses a research study that focused on students' abilities to read and to move between graphical representations before and after instruction. Conclusions indicate that students need to talk more about graphs, and make predictions and inferences from graphs.

Assessment Suggestions

- Observation of the students as they are working on their charts and their graphs.
- *Patterns in Measurement* worksheet.

Invitation to Learn

1. Students use colored squares to make the following pattern: orange, orange, yellow, blue, orange, orange, yellow, blue. When there are 14 yellow squares, how many orange and blue squares will there be? How many squares will there be altogether?
2. Students record their work and explain how they calculated the number of orange and blue squares and the total number of squares in their math journals.

Instructional Procedures

1. Students use the plastic squares to create a 1 cm x 1 cm square. Calculate the perimeter and area of the square and record the results on the *Patterns in Measurement* worksheet.
2. Increase the length of the sides by 1 cm. Calculate the perimeter and area and record it on the worksheet.
3. Increase the square six more times (1 cm each time), recording the perimeter and area for each increase.
4. Demonstrate how to find volume with a cube.
5. Students calculate the volume for a 1 cm x 1 cm cube and record the information on the worksheet.
6. Increase each side of the cube by 1 cm. Calculate the volume and record it on the worksheet.
7. Increase the cube six more times, each time recording the volume on the worksheet.
8. Students complete the questions on the worksheet until the graphs section.
9. Have students graph the information on perimeter, area, and volume from the table.
10. Connect the dots on each graph and finish the questions on the worksheet.
11. Have students write about the patterns they found during this activity in their math journals.

Materials

- ❑ Square color tiles
- ❑ Interlocking cubes
- ❑ *Patterns in Measurement* worksheet
- ❑ Math journals

12. Discuss the patterns the students found during the activity. What were the differences in the patterns of the perimeter, area, and volume? How did the graphs differ from each other?

Curriculum Extensions/Adaptations/Integration

- One number pattern related to Ancient Greece is the Golden Ratio. Artists often use the Golden Ratio because it produces shapes that are pleasing to the eye. The Golden Ratio is a person's total height compared to waist height. The Golden Ratio is ≈ 1.618. One of the most famous buildings of Ancient Greece, the Parthenon, was designed using the Golden Ratio.
- Use Excel to create three graphs, one for perimeter, one for area, and one for volume, using information during the activity.

Resources

Book

Navigating Through Algebra in Grades 3-5, by Gilbert J. Cuevas and Karol Yeatts; ISBN 0-87353-500-7

Family Connections

- Students play the Input/Output game with a family member.
- Create a pattern using 10 colored squares. Have students ask a family member what color the 15th square will be. The 20th? The 25th?

Input/Output

n	?
2	8
3	12
8	32

What's the function?

(n x 4) or 4n

Name _____

Patterns in Measurement

Length of Side (cm)	Perimeter (cm)	Area (cm^2)	Volume (cm^3)
1			
2			
3			
4			
5			
6			
7			
8			

1. What pattern do you notice in the perimeter results? Write a rule for the pattern.

   _____

   _____

   _____

2. What pattern do you notice in the area results? Write a rule for the pattern.

   _____

   _____

   _____

3. What pattern do you notice in the volume results? Write a rule for the pattern.

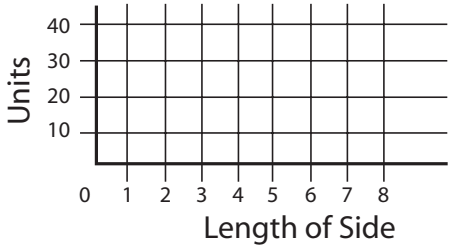
_____

_____

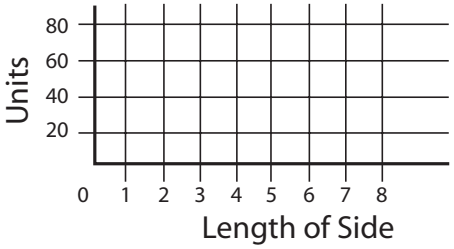
_____

Use the information from the table to draw the following graphs.

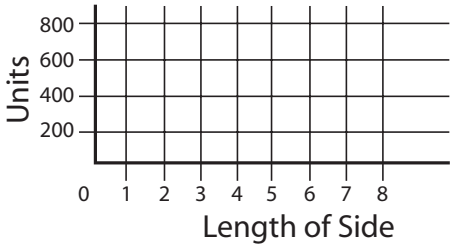
Perimeter



Area



Volume



4. Describe the shape of the perimeter graph.

_____

_____

_____

5. Describe the shape of the area graph.

_____

_____

_____

6. Describe the shape of the volume graph.

_____

_____

_____

Balance or Tilt?

Standard II:
Students will use patterns, relations, and functions to represent and analyze mathematical situations using algebraic symbols.

Objective 2:
Represent, solve, and analyze mathematical situations using algebraic symbols.

Intended Learning Outcomes:
2. Become mathematical problem solvers.
3. Reason mathematically.

Content Connections:
Math I-5

Math Standard II

Objective 2

Connections

Background Information

Variables, expressions, and *equations* are important concepts in the study of algebra. For this activity, students should know and use the correct terms. A *constant* is a quantity that stays the same. A *variable* is a quantity that can change. An *expression* is like a phrase, it has no equal sign. An *equation* is a mathematical statement that says two expressions are equal to each other.

Research Basis

Schifter, D. (1997) Developing operation sense as a foundation for algebra. http://eric.ed.gov ERIC # ED408152

This paper points out the importance of operation sense in the preparation for algebra. Implications of this work include the idea that diagrams, graphs, and tables are important for students to understand.

Swafford, J.O., & Langral, C.W. (2000). Grade 6 students' preinstructional use of equations to describe and represent problem situations. *Journal for Research in Mathematics Education, 31*(1), 89-112.

This study investigates sixth grade students' use of equations to describe and represent problem situations. The students showed a remarkable ability to generalize problem situations and to write equations using variables.

Assessment Suggestions

- Class discussion.
- *Balance or Tilt?* worksheet and the paragraph and definitions in their math journal.

Invitation to Learn

Materials
- ❑ Balance scale
- ❑ Small wooden blocks
- ❑ Math journals

Use a balance scale and small wooden blocks to demonstrate if the scale is balanced or if it is not. Model several different situations so the students know what happens to the side that weighs more, the side that weighs less, and when the amounts are equal. Have students write what happened in their math journals.

Instructional Procedures

Materials
- ❑ *Balance or Tilt?* worksheet
- ❑ Math journals

1. Explain to students that like the balance scale, equations need to be balanced, or equal.
2. Students write the definitions of *variable, constant, expression,* and *equation* in their journals.
3. Review with students how to solve two-step equations. Lesson plans for this are in the 2003 CORE Academy Handbook.
4. Draw a scale on the overhead using an expression on each side. Ask the students if the scale shows the correct balance. If not, how should we fix it? Should one side be lower than the other?
5. Give a few more examples to the class.
6. Students complete the *Balance or Tilt?* worksheet.
7. Discuss the worksheet as a class.
8. Students write a paragraph explaining the steps they use to solve equations in their math journals.

Curriculum Extensions/Adaptations/Integration

- Add the words constant, variable, expression and equation to your spelling and vocabulary units.
- Students brainstorm a list of variables and constants found in space:

  Variables: age of stars, distance between stars

  Constants: speed of light, distance light travels in one year, the size of each planet.
- Create word problems that represent given equations using variables. From given word problems, students write equations using variables.

Resources

Book

Navigating Through Algebra in Grades 3-5, by Gilbert J. Cuevas and Karol Yeatts; ISBN 0-87353-500-7

Family Connections

- Students make up three different equations to take home and have a family member solve. Explain to the family member how to solve the equation, if needed.
- Students make a list of five variables and five constants that they find in or around their home.

  Example: Number of people in family is a constant, number of hours they do chores each day is a variable.

Name _____

Balance or Tilt?

1. Find the values of the expressions on each side of the scale. Is the scale balanced or is it tilted? If the scale does not balance, write which side will tilt down. Explain your answer.

x = 5

_____

_____

s = 4

_____

_____

n = 4

s = 1

_____

_____

2. Find the variables that balance the scales.

s = _____

n = _____

f = _____

x = _____

Math Standard I-3 Activities

Domino Theory

Standard I:
Students will acquire number sense and perform operations with rational numbers.

Objective 3:
Model and illustrate meanings of operations and describe how they relate.

Intended Learning Outcomes:
1. Demonstrate a positive learning attitude toward mathematics.

Content Connections:
Math I-2, II-1

Math Standard I

Objective 3

Connections

Background Information

The use of dominos represents a graphical model for adding unlike fractions. A key concept for students to understand is the common denominator. A *common denominator* is a common multiple of two or more denominators. A *multiple* is the lowest number that both (or all) numbers (denominators) can "go into," or be divided into.

In the case of adding or subtracting, unlike fractions, the *Least Common Multiple* (LCM) needs to be found. The dominos in this activity illustrate the need for a common or matching set of dominos in order to add. That is where we need the common denominator.

As a teacher, you can even explain the use of the term common denominator in everyday speech. For example, the common denominator in the Civil War was hatred of each side for the other. That is what was shared in common—the common cause of the war.

Research Basis

Tankersley, K. (1993). Teaching Math Their Way. *Educational Leadership*, 50, 12-13.

This article follows the development of a methodology of using manipulatives for elementary math that fosters discovery and positive attitudes toward math. It also reflects an increase in testing results in math.

Rust, A. L. (1999). A study of the benefits of math manipulatives versus standard curriculum in the comprehension of mathematical concepts. Knoxville, TN, ERIC 436395.

This dissertation attempted to test first graders using both standard text-book based instruction and the use of manipulatives in math. Conclusions indicated little difference in tested results, though standard curriculum methods did show slightly higher results. However, the article points out that students' enjoyment of the different methods wasn't studied.

Assessment Suggestions

- An ideal assessment method for this activity combines a performance test with a traditional summative test. Students, given a set of problems, model and solve equations using the dominos in this activity. This assessment would indicate conceptual understanding of the process of addition and subtraction of fractions.

Invitation to Learn

Materials
- ❑ Set(s) of dominos

Kids enjoy dominos—whether playing a game or setting them up in a pattern only to tip them over. Students will use dominos to assist them in understanding addition of fractions during this activity.

Have pairs or small groups of students set up two unequal lines of dominos standing on their short end. They can use straight lines or curves—as long as the lines have an unequal amount with a difference of at least three.

When the lines are set up, have students tip them at the same time. What do they notice? Did the shorter line finish first? Of course. They are not even. One line has more than another. Explain that you will use the dominos to add unlike amounts of fractions together.

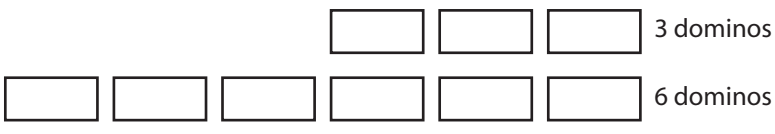
Instructional Procedures

Materials
- ❑ Set of dominos (ideally double twelves, but smaller sets will work)
- ❑ Set of coins or round plastic chips

1. This activity works best with unlike fractions with small denominators. How large the fractions are depends on how many dominos you have. Pass out sets of dominos and plastic chips or coins to pairs or small groups.
2. Explain that they will use the blank side of the dominos to add unlike fractions. Model the first problem with them.
   Example:

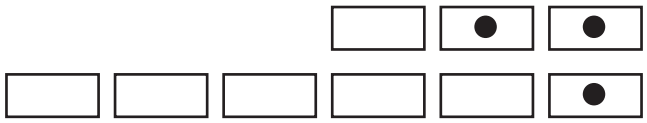
$$\frac{2}{3} + \frac{1}{6}$$

Remind students that the denominators represent the total number, in this case, of dominos. Have them arrange two lines parallel to each other that look like this, representing the denominators:

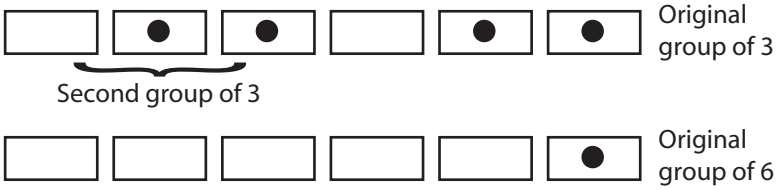


Another way to remember is to use the term "domino denominator."

3. Next, using the coins or plastic chips, place two chips on top of two dominos in the group of three, and one chip on top of one domino in the group of six, representing the numerators:



4. Ask the students how many dominos they would have to add to the group of three to equal the group of six. The answer is three. Remind them that if they add three more dominos, they also need to add a chip or coin to two of them, like this:



5. Now that both lines have the same total amount of dominos, six, ask the students to add up all of the dominos with chips or coins on the tops. The answer is five. So, five-sixths (5/6) of the dominos have coins or chips on top of them.
6. This process can be checked by adding the traditional way, as follows:

$$\begin{array}{r} \frac{2}{3} \quad \frac{4}{6} \\ +\ \frac{1}{6} \quad \frac{1}{6} \\ \hline \frac{5}{6} \end{array}$$

7. Again, this technique is more effective with "smaller fractions," as determined by the number of dominos and coins or chips.
8. This exercise may also be used to demonstrate simple subtraction of unlike fractions. The difference being you subtract the lesser amount of chips/coins from the greater amount.

Curriculum Extensions/Adaptations/Integration

- Another activity using the dominos is done with the numbered sides. With all dominos face down, have a student (in a pair or group) turn over two dominos. On each domino, the smaller number represents the numerator, and the larger number represents the denominator. Have students practice adding and subtracting these fractions on the dominos. Paper and pencil may be needed.

Resources

Web site

Online Domino Game
http://www.french.ucla.edu/faculty/gans/java/domino.htm

Family Connections

- Dominos is a popular game for many families. Many games involve grouping and patterns with a need for number sense and operations. Encourage students to play domino games with families. In the event a student has no access to dominos, at your discretion, they may check out your set.

Sticky Note Math

Standard I:
Students will acquire number sense and perform operations with rational numbers.

Objective 3:
Model and illustrate meanings of operations and describe how they relate.

Intended Learning Outcomes:
2. Become mathematical problem solvers.
4. Communicate mathematically.

Content Connections:
Math IV; Language Arts VII-3

Math Standard I

Objective 3

Connections

Background Information

This activity works best when students have a familiarity with division, even if only with whole numbers. Students should know such terms as *divisor*, the number by which another number is divided; *dividend*, a number that is divided by another number (divisor); and *quotient*, the answer to a division problem.

Having already learned operations with fractions, such as addition, subtraction, and multiplication, students should also understand the difference between the *numerator* (top number) and the *denominator* (bottom number). The line between the two means *divided by*. Therefore, the numerator represents the dividend, and the denominator represents the divisor, or the bottom number divides the top number.

Finally, a concept used in fractional division is *multiplicative inverse,* or the *reciprocal*. The product of a fraction and its multiplicative inverse equals one (e.g., 3/4 x 4/3 = 1).

Research Basis

Weisenberg, R.C. (1997). Appropriate technology for the classroom—using "Post-it® Notes" as an active learning tool. *Journal of College Science Teaching*. *26*(5), 339-44.

This article addresses the use of Post-it® notes as effective teaching tools. It lists activities using Post-it® notes, such as modeling, concept-mapping, and constructivist group activities.

Widmer, C. & Sheffield, L. (1998). Modeling mathematics concepts: using physical, calculator, and computer models to teach area and perimeter. *Learning and Leading with Technology*. *25*(5) 32-35.

This article examines ways middle school students use simple problems to gain a deeper understanding of mathematical concepts. It demonstrates the use of sample area and perimeter problems.

Assessment Suggestions

- Observation of students at they complete the Post-it® note activity. Do they understand how to divide and multiply fractions both mathematically and graphically?

Invitation to Learn

Give each pair of students a Post-it® note. Ask the class a series of questions, such as:

- What, if anything, do Post-it® notes have to do with math?
- How might a Post-it® note be used to demonstrate a mathematical idea?
- In what way(s) can Post-it® notes be used to represent operations of fractions?

Notice the questions get more specific.

List responses on the board. Explain and discuss how math is "all around us," even in the form of Post-it® notes.

Instructional Procedures

Materials

- ❑ 3" x 3" Post-it® notes
- ❑ 1 1/2" x 2" Post-it® notes
- ❑ Various sizes of rectangular-shaped paper
- ❑ Rulers and yardsticks
- ❑ Scissors

1. Show students a piece of 8 1/2" x 11" piece of paper. Ask, "How many 3" x 3" Post-it® notes would be needed to fit the width of the piece of paper?" Elicit responses from someone who has solved the problem. (2 5/6)
2. Model the process by taking the 3" x 3" Post-it® notes and carefully placing them along the 8 1/2" edge of the paper. Carefully cut the overlapping part of the Post-it® note. Measure the remaining part, which should be close to 5/6 of 3" or 2 1/2". Measure the cut segment, which should be 1/2".

   Note: To understand the concept of 5/6 of 3, ask the students to take the 1/2" amount they cut off and, using their ruler, determine how many 1/2" segments would be in 3". There should be 6. Since they have cut off one of those 6 segments, there are only 5 left, or 5/6 of the original 3". That means there are 2 full Post-it® notes and 5/6 of a third one that fit within the 8 1/2" side of the paper.

4. Perform the same procedure with the 11" side of the piece of paper by dividing 11 by 3. Then place the 3" x 3" Post-it® notes along the 11" edge, again trimming and measuring as above. Add the fractions together to get the total number of Post-it® notes to measure the length and width.

5. Do the same procedure with the 1 1/2" x 2" Post-it® notes for both dimensions. Make sure students select either the 1 1/2" side or the 2" side to measure with.
6. Explain the process mathematically, using traditional division, show how many 3" x 3" Post-it® notes would fit by dividing 8 1/2" by 3, like this: 8 1/2 ÷ 3, which is the same as

$$\frac{17}{2} \times \frac{1}{3} = \frac{17}{6} = 2\frac{5}{6}$$

7. After modeling the above, pass out two different sizes of rectangular paper to groups of students, depending on class size.
8. Have students determine mathematically how many of each kind of sticky notes it would take to cover both dimensions (length and width). After doing the math, students request the number of Post-it® notes they need and cover their paper. Cut the fractional part of the Post-it® by measuring it first.
9. Display the students' work and discuss strategies and steps for finding the answer.

Curriculum Extensions/Adaptations/Integration

- Students can use what they've learned about fractions, division, multiplication, and measurement to apply this process to a real world situation, such as hanging wallpaper. Give students the height and width measurements of a wall to determine how much wallpaper is needed. Using the concept of area, students determine how much wallpaper is needed to cover the wall.
- Have students write, using sequencing, the steps of the process for this activity. They can write the steps on sticky notes and place them on the board or a wall.
- Ensure that tactile and kinesthetic learners have the opportunity to place the Post-it® notes and cut the overlapping parts.

Resources

Web sites

eMINTS website covering a broad range of topics, activities, and online tools, including mathematics.
http://www.emints.org/ethemes/resources/index.shtml

SMILE Program Math Index
http://www.iit.edu/~smile/mathinde.html

Family Connections

- Encourage students to try this activity at home with family members, making sure they use the mathematical operations learned for division and multiplication of fractions.
- Instruct students to find three items at home, square or rectangular in nature, to measure with Post-it® notes. Share the results with the class.

A Newspaper?

Standard I:
Students will acquire number sense and perform operations with rational numbers.

Objective 3:
Model and illustrate meanings of operations and describe how they relate.

Intended Learning Outcomes:
5. Make mathematical connections.

Content Connections:
Language Arts VIII-6

Math Standard I

Objective 3

Connections

Background Information

When reading a newspaper, one might ask if the majority of the newspaper is really news, or advertising. Using newspapers, a teacher can involve students in a mathematical, as well as language arts, approach to learning.

Decide how best to divide your class and how many newspapers to use. You may find it best to use only one or two identical papers, dividing each paper up among groups. It is important to also assign a variety of pages per group so each group has an opportunity to work with different types of pages.

Students should understand the terms *numerator* and *denominator* and what each represents. They should also understand the process of converting fractions into decimals by taking the numerator and dividing it by the denominator. Remind students that the line in the fraction represents "divided by."

Research Basis

Rodine, J.A. (1986). Hook 'em using the newspaper as bait. Colorado Springs, CO. ERIC #ED296337.

This document shows how to use a newspaper to teach a variety of concepts, from reading and writing to mathematics.

Nelson, D.W. (1979). Extra! extra! newspaper math. *Instructor.* 88(9), 97-100.

This article illustrates ways in which newspapers can be used to teach critical mathematics and language arts concepts.

Assessment Suggestions

- Students write a reflective essay on the role of advertising, to persuade buyers, and how they respond to such advertising. They

may address, for instance, the role that advertising plays in their everyday lives—how exposed are they to it? Adapt this assessment to meet cross-curricular needs.

- Students submit their conversions of fractions to decimals to determine if they understand the operation.

Invitation to Learn

"How do newspapers make money?" Discuss several ideas, including subscriptions and advertising. How and why is advertising so important as a source of revenue? Draw parallels to television commercials, which occur frequently.

Instructional Procedures

Materials

- ❑ Newspapers
- ❑ Rulers
- ❑ Transparent centimeter grid paper
- ❑ Calculators (optional)
- ❑ Colored markers (for use in marking areas measured)

"How much of a newspaper is really news?"

1. Place students in groups of three to six. Each group is to define "news." Have a spokesperson from each group share their definitions so the class may reach a consensus.
2. Distribute a newspaper (or section) to each group.

   Note: Select a newspaper with a small number of pages (i.e., not the Sunday edition) so this activity is manageable. (You may use a selected number of pages for each group within one larger newspaper.)
3. Students should brainstorm, in groups, a list of the various types of content categories contained in the newspaper. A list might look like this:

News	Advertisements	Photographs	Entertainment
Sports	Weather	Empty Space	Classifieds

   Once again, have the class reach a consensus on the categories as a whole. It is possible that a class might determine only two categories: news (with attendant photos) and advertisements.
4. Using a transparent centimeter grid paper, students should determine how many square centimeter units it will take to cover an entire page. The class must decide on an approximation for the area of the page.
5. Each group will have a pre-assigned number of pages to work with. It is recommended that pages be scattered throughout the newspaper to give a variety to each group. Create jobs for each student in the group (e.g., recorder, measurer, spokesperson, etc.).

6. Reinforce that the total number of units per page represents the whole, or one whole page. Any areas, articles, etc. represent a part or fraction of the whole (less than one).
7. Students determine how the various articles should be categorized, then measure each article with the grid transparency, counting the squares. Express each area as a fraction first (number of centimeter squares of article over total number of centimeter squares for the whole page), then as a decimal (through division, denominator into the numerator). Decimals should be rounded to a specified place value. Discuss why the sum of areas for each page should add up to one.

   Sample of possible class data chart:

Page Number	Categories			
	News	Entertainment	Advertisements	Empty Space
1	.75	.00	.00	.25
2	.55	.00	.35	.10
3	.40	.00	.55	.05
4	.30	.00	.65	.05
5	.20	.00	.73	.07
6	.00	.00	.80	.20
7	.00	.50	.20	.30
8	.00	.50	.47	.03
Totals	**2.20**	**1.00**	**3.75**	**1.05**

8. Each group should calculate values for several categories. Remind students that if the articles in a category were reassembled, it would require one or more pages to account for all of them. For example, using the chart above, it would take 3.75 (or 3 3/4) pages to cover the advertisements.
9. Based on the newspaper used, the students should see that newspapers are predominantly advertisements. What else might the paper be called?

Curriculum Extensions/Adaptations/Integration

- If computers are available, this is a great activity to do in Excel, having students create their own spreadsheet for data.

Resources

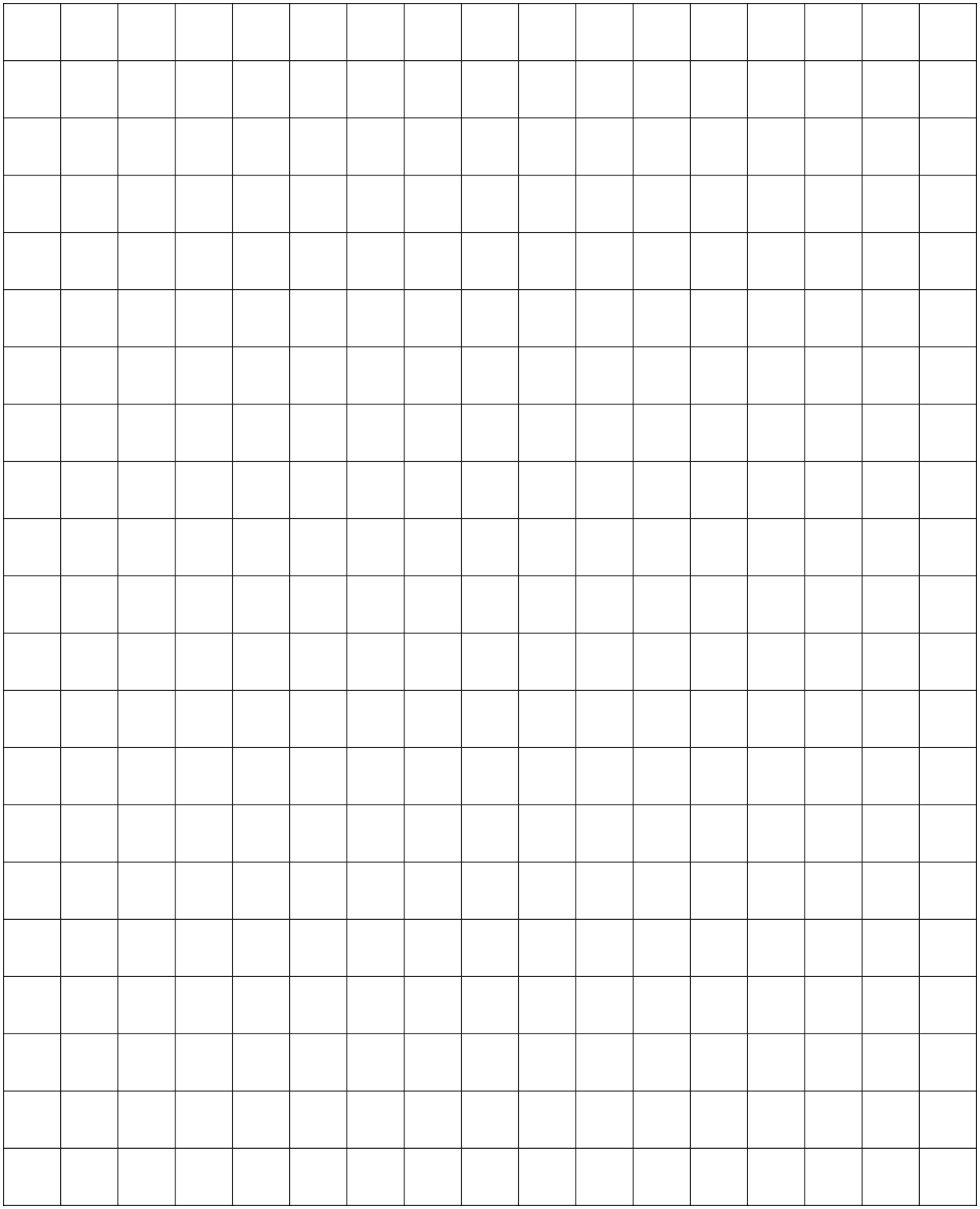
Web site

Another newspaper math activity
http://www.savannahnow.com/features/NIE/Reproducibles/math/m_3.htm

Family Connections

- Students can, while watching a favorite television program, time the length of commercials and express that as a fraction over the total time of the program. This can be done for two, say 30-minute (or one 60-minute) programs. Add up the fractions to indicate the total time commercials are aired in an hour of television viewing. Students report their findings to the class.

Centimeter Grid



Math Standard III-2 & 3 Activities

Coordinate Connections

Standard III:
Students will use spatial and logical reasoning to recognize, describe, and identify geometric shapes and principles.

Objective 2:
Specify locations and describe spatial relationships using coordinate geometry.

Intended Learning Outcomes:
4. Communicate mathematically.

Content Connections:
Art I-1; Language Arts VI-1; Social Studies VII-1

Math Standard III

Objective 2

Connections

Background Information

Analytic geometry, the branch of geometry that deals with lines, curves, and geometric figures plotted on a set of axes using coordinates, was first developed in the 17th century by the French mathematicians Pierre de Fermat and René Descartes.

You can use a *coordinate grid* to locate points on the plane. The x-axis and the y-axis are number lines. They intersect at right angles at their zero points, the *origin*. Any point can be located using an ordered pair. The first coordinate tells you how far to move on the x-axis from the origin. Coordinates of points to the right of the origin are positive numbers. Coordinates of points to the left are negative numbers. The second coordinate tells you how far to move on the y-axis from the origin. Coordinates of points up from the origin are positive numbers. Coordinates of points down from the origin have coordinates that are negative numbers.

Students should understand the following vocabulary for this activity:

number line—A line that shows numbers in order.

positive numbers—Numbers greater than zero.

negative numbers—Numbers less than zero.

coordinate grid—A set of lines used to locate points on a plane.

x-axis—The horizontal axis on a coordinate grid.

y-axis—The vertical axis on a coordinate grid.

origin—The point (0,0) where the x- and y-axes of a coordinate grid intersect.

quadrants—The four regions (labeled by Roman numerals) into which the two axes of a coordinate grid divide the plane (labeled in counter-clockwise order with quadrant I in upper right corner).

ordered pair—A pair of numbers used to locate a point on a coordinate grid. (The x-axis coordinate is always first because "x" comes before "y" alphabetically.)

coordinate—One of the numbers in an ordered pair.

x-coordinate—The first number in an ordered pair, locating a point on the x-axis of a coordinate grid.

y-coordinate—The second number in an ordered pair, locating a point on the y-axis of a coordinate grid.

Research Basis

Johnson, D. & Johnson R. (1975). *Learning together and Alone: Cooperation, Competition, and Individualization*. Englewood Cliffs: Prentice Hall.

In general, organizing students in cooperative learning groups has a powerful effect on learning regardless of whether groups compete with one another.

Kagan, S. (1992). *Cooperative Learning*. San Juan Capistrano, CA: Kagan Cooperative Learning.

Cooperative learning increases communication, trust, leadership, decision-making, and conflict resolution.

Assessment Suggestions

Evaluate students' understanding of the objective(s) using the following rubric:	
4 Full Accomplishment	Student accurately plots points and reads the coordinates of points on a coordinate grid.
3 Substantial Accomplishment	Student plots points and reads the coordinates of points on a coordinate plane, but not always accurately.
2 Partial Accomplishment	Student has difficulty plotting points and reading the coordinates of points on a coordinate plane.
1 Little Accomplishment	Student does not plot points or read coordinates of points on a coordinate grid accurately.

- Have students draw a four-quadrant grid with all quadrants labeled. Then plot two given points in each quadrant and label them with the correct ordered pairs. Example: A(3,5); B(3,-4); C(0, 4); D(-2,0); etc.

Invitation to Learn

Suppose you are having a birthday party and a friend you have invited has asked you for directions from the school to your house. You tell them it is five blocks away. Is this enough information for them to find your house?

Instructional Procedures

1. Discuss the importance of giving specific directions in real-life situations and have students give examples to illustrate.
2. Hold up a piece of graph paper and explain to students that you can also give specific directions to find an exact location on this piece of graph paper.
3. Distribute a *Coordinate Grid* worksheet to each student to use as the vocabulary is discussed.
4. Introduce or review the following vocabulary: number line, positive numbers, negative numbers, coordinate grid, x-axis, y-axis, origin, quadrants, ordered pair, coordinate, x-coordinate, y-coordinate.
5. Provide each student with 2 sheets of 1/2-inch graph paper and 1 sheet of lined paper.
6. On one piece of graph paper, instruct the students to draw and label a coordinate grid including the four quadrants and x- and y-axis lines.
7. Students draw a simple design that extends into all four quadrants of the coordinate grid.

   Note: The design should consist of a series of dots connected by straight lines; "dot-to-dot" style with no curved lines.
8. Distribute an *Ice Cream Sundae* worksheet to each student as an example.
9. On the blank side of another piece of graph paper, have students write directions for their design, consisting of sequential ordered pairs that, when graphed and connected with straight lines, will duplicate their design.

Materials

For each student:

- ❑ 2 sheets 1/2-inch graph paper
- ❑ 1 sheet 1/4" grarph paper
- ❑ 1 sheet lined paper
- ❑ Privacy folder (optional)
- ❑ *Coordinate Grid* worksheet
- ❑ *Ice Cream Sundae* worksheet
- ❑ Ruler
- ❑ Pencil

10. After students have completed writing the series of ordered pair directions for their design, have them bring both the designs and their directions to the teacher for evaluation.
11. Students then exchange their *directions only* with another classmate who has not seen their design. (You may want to use privacy folders so students cannot see each other's designs.)
12. On another piece of graph paper, instruct the students to draw and label a coordinate plane including the four quadrants and x- and y-axis lines.
13. Following their classmate's written directions, have them graph each ordered pair on the coordinate grid.
14. Sequentially connect the points with straight lines using a ruler.
15. Compare the completed design with the classmate who wrote the directions. If their designs are not congruent (same shape and size), have the two troubleshoot and problem solve whether the problem was in the directions, or in the way the student read and graphed the directions. Discuss the implications these scenarios could have in real-world situations if either the directions were incorrect or unclear, of if they were not followed properly (e.g, an engineer writing directions for a mechanic to build a machine the engineer designed, an architect drawing plans for a builder to follow, etc.
16. Students complete the *Ice Cream Sundae* picture on 1/4" graph paper as they have time.

Curriculum Extensions/Adaptations/Integration

- *Treasure Island* is a classic story of a search for pirate treasure. Stevenson based the story on a map drawn by his son, Lloyd. Read this story and create connections using coordinates and mapping activities.
- Make an entry in your daily math journal.
- A person who makes maps professionally is known as a cartographer. Invite students to research maps that were made at an earlier time and compare them to maps made today.
- Students find the location of latitude and longitude coordinates on a world map.
- Challenge advanced learners to draw and write directions for a more complicated design on 1/4" graph paper.

Resources

Book

Treasure Island, by Robert Louis Stevenson; ISBN 0-486-27559-0

Family Connections

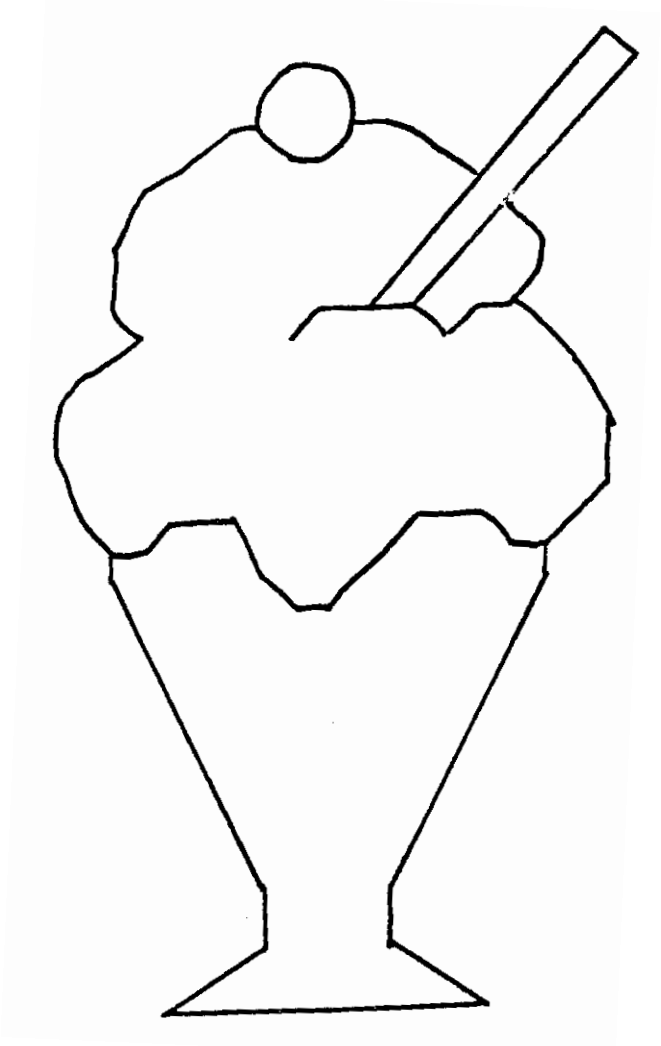
- Play Battleship as a family to reinforce the idea of coordinate grids and ordered pairs.
- Locate streets on a map of your city using coordinates.

Name _____

Coordinate Grid

Ice Cream Sundae

Dish	Ice Cream	Spoon & Finish Ice Cream	Cherry
(-7,-1)	(-1,6)	(1.5,7)	(-2,13)
(-7,-2)	(0, 7)	(9, 14.5)	(-2,14)
(-2,-12)	(3,7)	(10,14)	(-1,15)
(-2,-14)	(4,6)	(6,10)	(0,15)
(-5,-16)	(5,7)	(7,9)	(1, 14)
(5,-16)	(6,7)	(7,8)	(1, 13)
(2,-14)	(7,8)	(6,7)	
(2,-12)	(7,9)	(5,7)	
(7, -2)	(6,7)	(4,6)	
(7, -1)	(8,5)	(3,7)	
	(9,3)	(6,10)	
	(9,1)		
	(7,-1)		
	(6,-1)		
	(5,0)		
	(3,0)		
	(0,-3)		
	(-1,-3)		
	(-2,-2)		
	(-3,0)		
	(-5,0)		
	(-6,-1)		
	(-7,-1)		
	(-8,0)		
	(-9,2)		
	(-9,4)		
	(-8,5)		
	(-6,6)		
	(-7,7)		
	(-7,9)		
	(-6,11)		
	(-4,12)		
	(-3,13)		
	(-2,13)		
	(-1,12)		
	(0,12)		
	(1,13)		
	(2,13)		
	(3,12)		
	(5,11)		



Transforming Triangles

Math Standard III

Objective 3

Connections

Standard III:
Students will use spatial and logical reasoning to recognize, describe, and identify geometric shapes and principles.

Objective 3:
Visualize and identify geometric shapes after applying transformations.

Intended Learning Outcomes:
2. Become mathematical problem solvers.
6. Represent mathematical situations.

Content Connections:
Art I-1

Background Information

Students should understand the following vocabulary for this activity:

rotation (turn)—The image of a figure that has been "turned" as if on a wheel.

translation (slide)—The image of a figure that has been slid to a new position without flipping or turning.

reflection (flip)—The mirror image of a figure that has been "flipped" over a line.

transformation—The act of changing the form or appearance of an object.

clockwise—In the same direction as a clock's hands move.

counterclockwise—In the opposite direction as a clock's hands move.

In *Webster's Dictionary*, the word "transform" is defined as "to change or convert." Students should understand that rotations, translations, and reflections are all types of transformations.

Maurits Cornelis (M.C.) Escher (1898-1972) was an architect and graphic artist. He continually invented new visual constructions to challenge the conventional perception of spatial relationships. Escher's fascination with drawing figures that tile together perfectly led to numerous tessellation drawings that involve the repetition of one or more shapes that connect together in asymmetrical relationship.

Research Basis

Dickinson, D. (1996). *Learning Through Many Kinds of Intelligence*. Seattle: New Horizons for Learning.

It is important to encourage children to explore and exercise all of their intelligences. Bodily/Kinesthetic Intelligence involves physical coordination and dexterity, expressing oneself or learning through physical activities. Logical/Mathematical Intelligence involves number and computing skills, recognizing patterns and order, and the ability to solve different kinds of problems through classifying and sequencing activities, and solving various kinds of puzzles.

Marzano, R. J., Pickering, D. J., & Pollock, J. E. (2001). *Classroom Instruction that Works: Research-Based Strategies for Increasing Student Achievement*. Alexandria: Association for Supervision and Curriculum Development.

It has been shown that explicitly engaging students in the creation of nonlinguistic representations stimulates and increases activity in the brain. When students elaborate on knowledge, they not only understand it in greater depth, but they can recall it much more easily.

Wahl, M. (1999). *Math for Humans: Teaching Math Through 8 Intelligences*. Seattle: New Horizons for Learning.

With new pressures on teachers and students to meet higher standards, this book offers tools that make concepts concrete and understandable. It has many practical and creative methods that take into consideration different learning styles and kinds of intelligences. Wahl has developed strategies that all teachers can use to help their students become successful in math.

Assessment Suggestions

Evaluate students using the following rubric:	
4 Full Accomplishment	Student accurately graphs translations and reflections on the coordinate plane.
3 Substantial Accomplishment	Student graphs translations and reflections on the coordinate plane, but not always accurately.
2 Partial Accomplishment	Student has difficulty graphing translations and reflections on the coordinate plane.
1 Little Accomplishment	Student does not accurately graph translations and reflections on the coordinate plane.

- Have each student write a paragraph telling how graphing a translation is different from graphing a reflection.
- Have students draw and label a four quadrant grid. Then draw triangle ABC with A(-1,2); B(-3,4); C(-2,0). Translate it 3 units to the right and 1 unit down.
- Have students draw and label a four quadrant grid. Then draw triangle DEF with D(-3,2); E(-2,4); F(-1,1). Reflect it across the y-axis.

Invitation to Learn

(This activity requires adequate physical space.)

Have students stand where they can spread their arms out and not touch anyone. Give them the following directions:

1. Slide one step to the right.
2. Turn your body 1/2 turn to the left.
3. Slide three steps to the left.
4. Turn your body 1/2 turn to the right.
5. Lie on your back on the floor.
6. Flip over onto your stomach.
7. Stand up and turn completely around once.
8. Sit down in your seat.

Instructional Procedures

Materials

- ❑ *Transforming Triangles* worksheet
- ❑ *Mirror Drawing* worksheet
- ❑ Ruler
- ❑ Pencil

1. Explain to students that just as they were transforming their bodies by *sliding, turning,* and *flipping* them, they can also transform geometric shapes in math.
2. Introduce or review the following vocabulary: rotation, translation, reflection, transformation, clockwise, and counterclockwise (see background information).
3. Discuss that a reflection (flip) is accomplished by graphing the opposite of each coordinate (reflections across the x-axis change each y-coordinate into its opposite).
4. Distribute a *Transforming Triangles* worksheet to each student.
5. Instruct the class to graph and label a triangle with the following coordinate points in Grid 1: A(-5,4), B(-3,5), C(-2,1).

6. Also in Grid 1, have students create triangle ABC by translating (sliding) triangle ABC 7 units right and 6 units down. Label the points. Translation = A(2,-2), B(4,-1), C(5,-5).
7. Graph and label a triangle with the following coordinate points in Grid 2: D(1,1), E(4,5), F(5,3).
8. Reflect (flip) triangle DEF across the x-axis and label the points in Grid 2. Reflection = D(1,-1), E(4,-5), F(5,-3).
9. Graph and label a triangle with the following coordinate points in Grid 3: G(1,-4), H(4,-2), I(4,-5).
10. Reflect (flip) triangle GHI across the y-axis and label the points in Grid 3. Reflection = G(1,-4), H(4,-2), I(4,-5).
11. Pair students up and pass out a privacy folder to each pair.
12. Students draw and label triangle JKL in Quadrant IV of Grid 4 (they choose the coordinate points).
13. Have each student create triangle JKL by sliding triangle JKL into Quadrant II of Grid 4 (each students decides how many units to move triangle JKL up and left).
14. Each student takes turns giving his/her partner the coordinate points so s/he can duplicate his/her partner's triangles in Grid 5.
15. Partners compare their triangles. (Grid 4 of one partner should duplicate Grid 5 of the other partner.)
16. Students draw and label triangle MNO in Quadrant III of Grid 6 (they choose the coordinate points).
17. Each student reflects (flips) triangle MNO across the x-axis into Quadrant II.
18. His/her partner checks to see that s/he has reflected triangle MNO correctly.
19. Pass out a "Rotating Triangles" handout and brad to each student.
20. Cut the paper on the dotted line, then cut out the two triangles.
21. Push the brad through the black triangle, then the gray triangle.
22. Next, push the brad (with the triangles on it) into the center point of the square and secure.
23. Instruct the students to rotate (turn) the black triangle 1/4 turn (90 degrees). Continue to give instructions for the students to rotate the black triangle 1/2 turn, then 270 degrees, etc.

    Give directions in both degrees and fractions of a turn.

24. With a partner, have students take turns giving directions to their partner to rotate one of the triangles (have them give directions in both degrees and fractions of a turn).

Curriculum Extensions/Adaptations/Integration

- Students look for letters in the alphabet that look the same when reflected in a mirror. Try to find entire words that look the same, for example, "MOM" or "TOOT."
- The Kuba people of the Congo (Zaire) region of Africa use slides and flips when making patterns on cloth. Examine some samples of Kuba cloth.
- Tesselations (M.C. Escher)
- The famous Renaissance artist, Leonardo DaVinci, often wrote words in mirror-image in his journals so that others could not read them. Extend this concept into an art activity.

Resources

Book

Visions of Symmetry: Notebooks, Periodic Drawings, and Related Work of M.C. Escher, by Doris Schattschneider; ISBN 0-7167-2126-0

Video

The Fantastic World of M.C. Escher, directed by Michele Emmer; ASIN: 6303146767

Web site

http://www.worldofescher.com (this site sponsors a tessellation contest for kids)

Additional Media

M.C. Escher Sun and Moon Puzzle, by iproject (available from http://www.iproject.com/, P.O. Box 101, 3740 AC Baarn, Holland, Telephone: +31-35-5418041, Fax: +31-35-5411766); Item# ES-1020

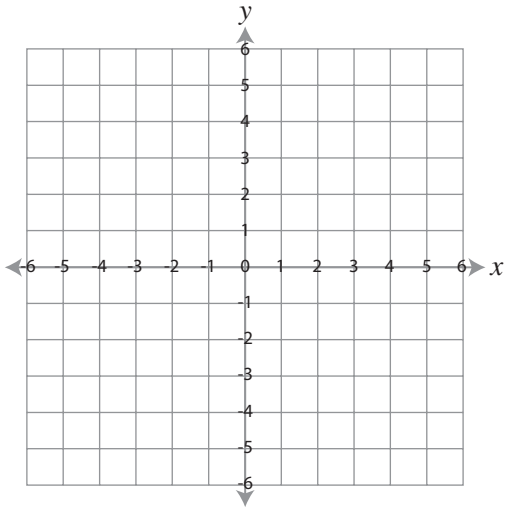
Family Connections

- Play with or discuss the popular "Transformer" toys. How do they change or transform?
- Using an everyday object from home (such as a spoon), practice sliding, turning, and flipping it on a flat surface.
- Play Simon Says and give instructions like "Simon says slide two steps forward," "Simon says turn around two and a half times," "Lie down on your back and flip over three times."

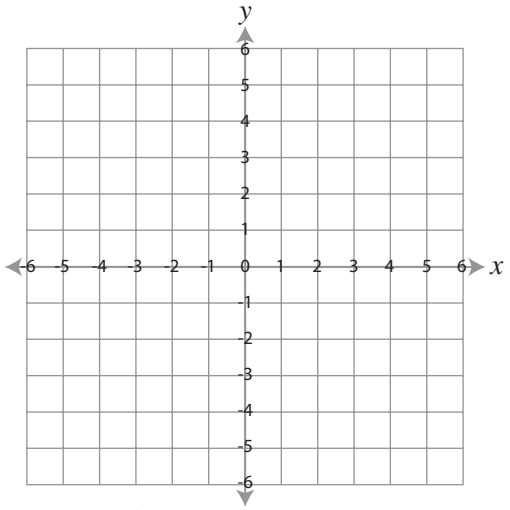
Name _____

Transforming Triangles

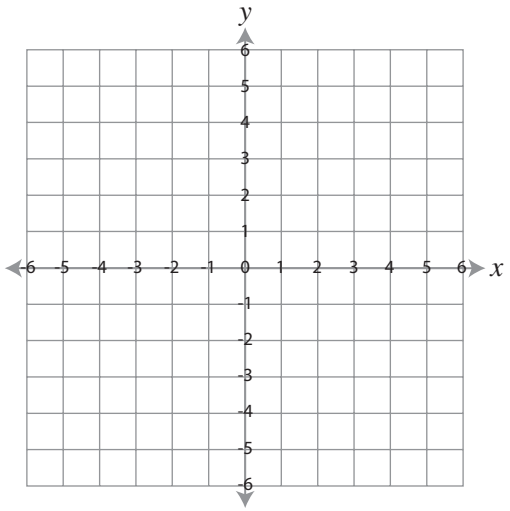
Grid 1—Translate (Slide)



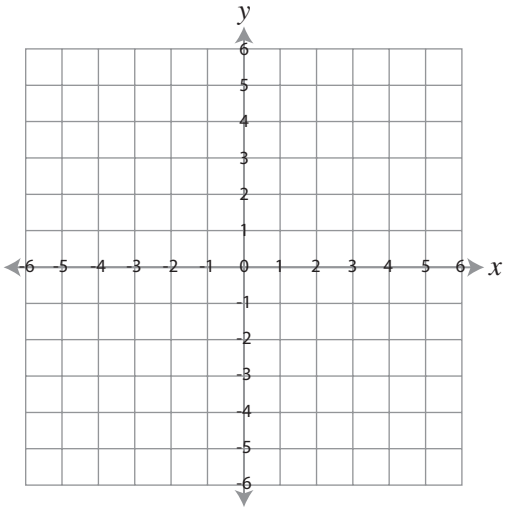
Grid 2—Reflect (Flip) across the x-axis



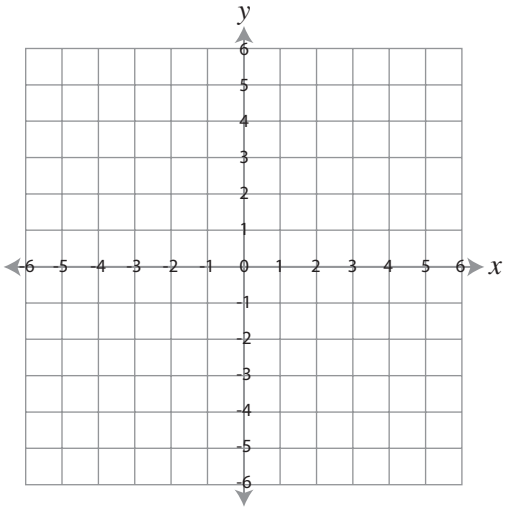
Grid 3—Reflect (FlipP across the y-axis



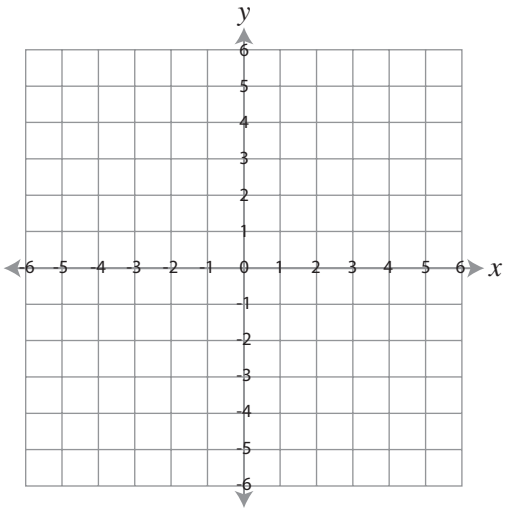
Grid 4—Translate (Slide)



Grid 5—Translate (Slide)

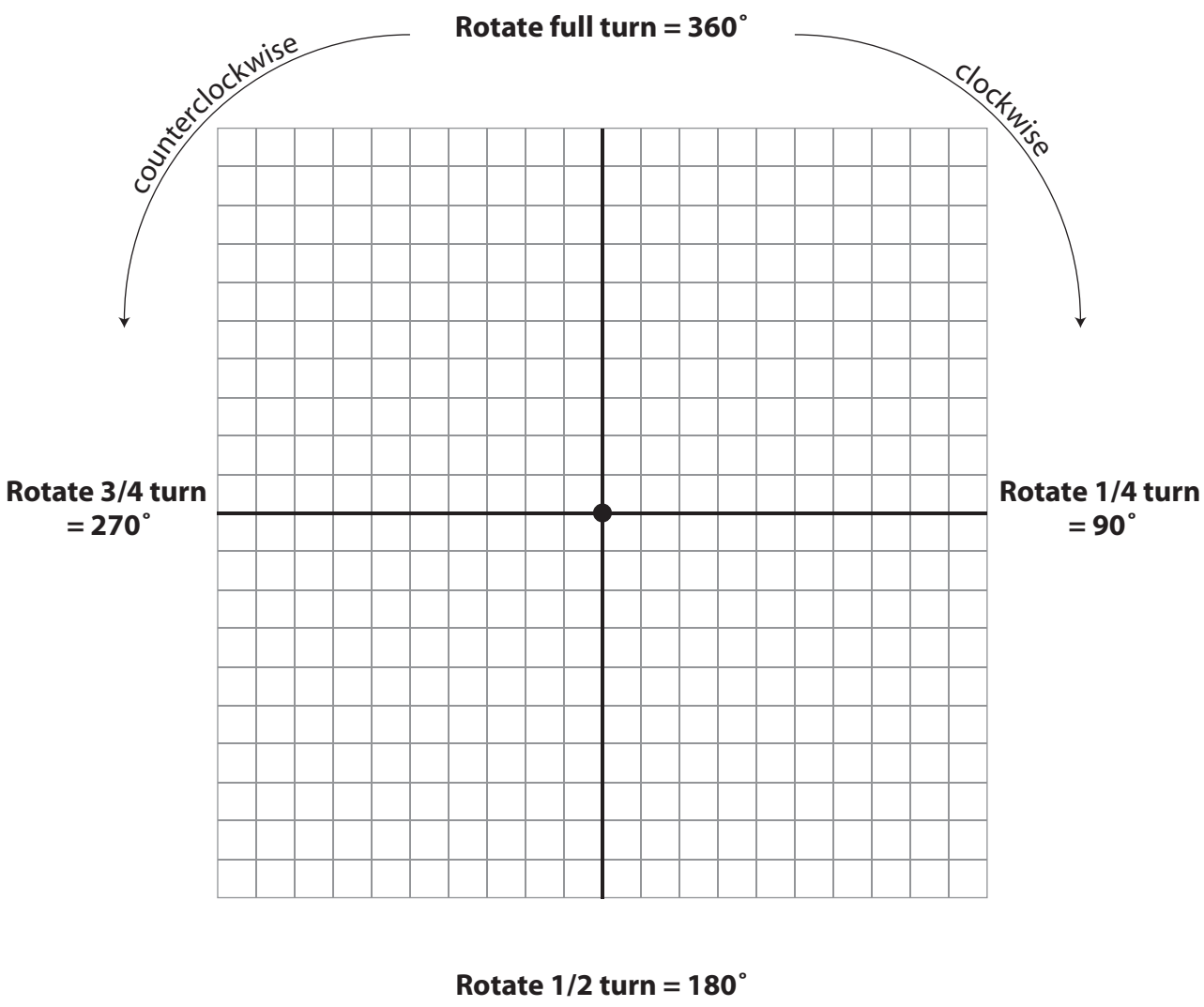


Grid 6—Reflect (Flip) across the x-axis

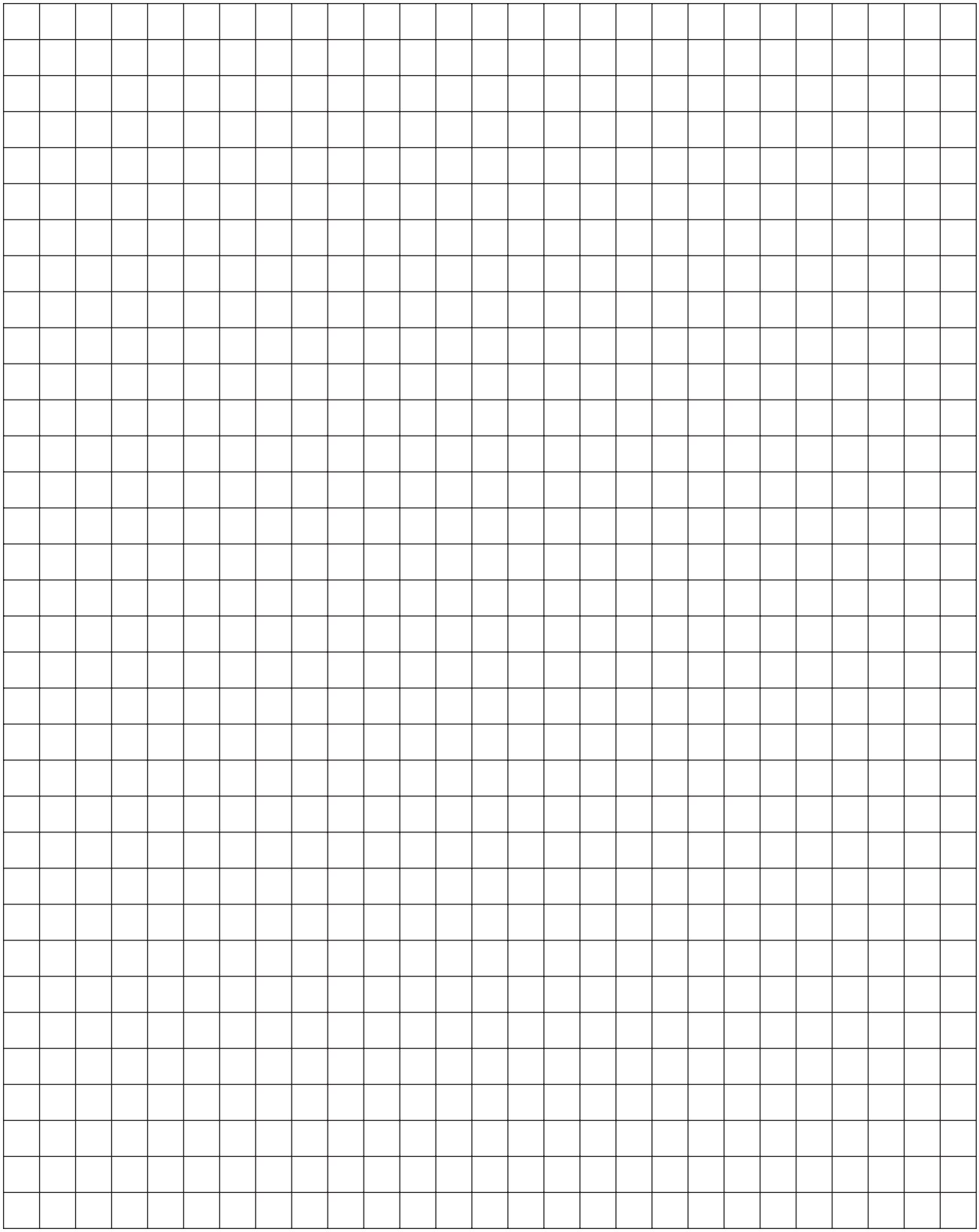


Name _____

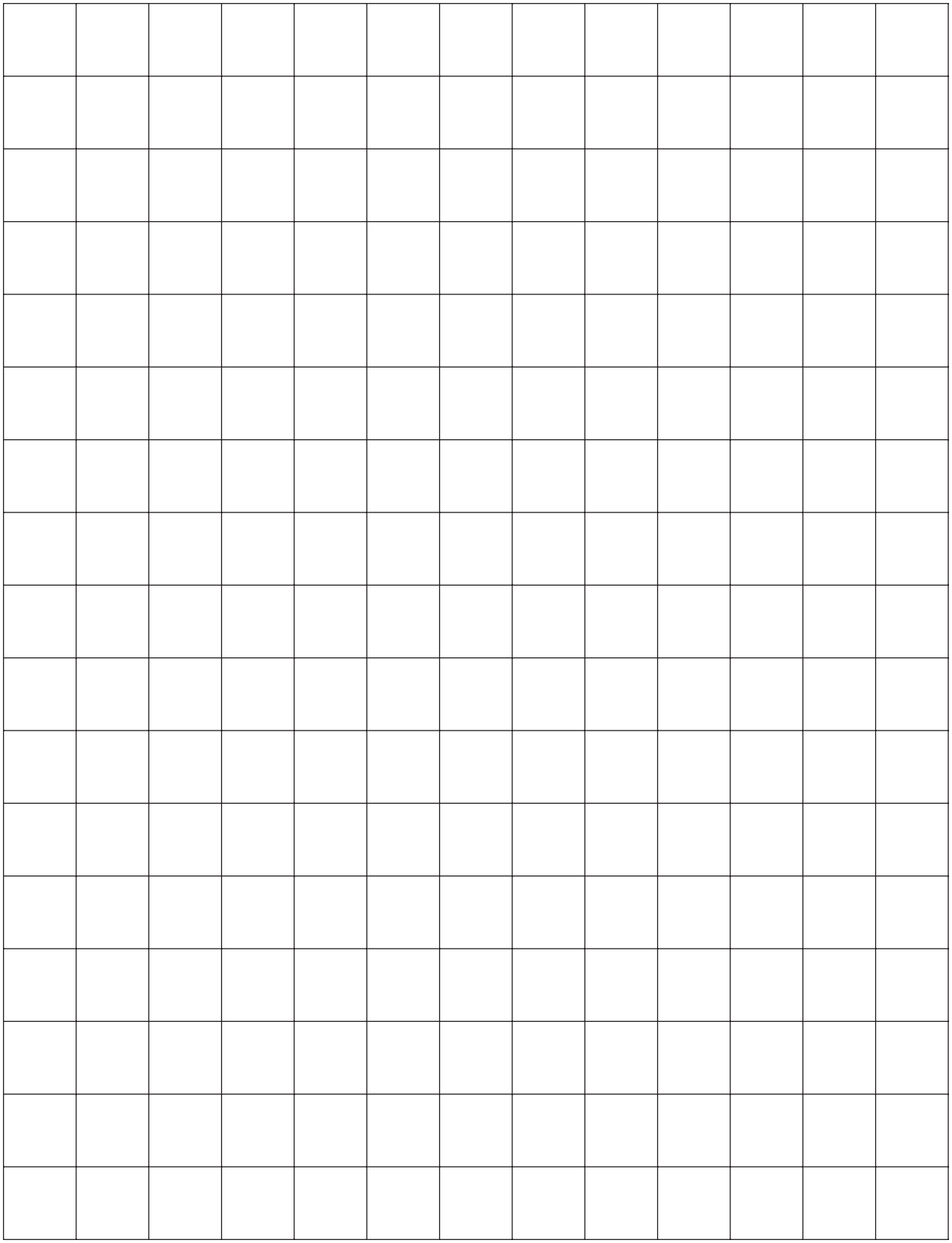
Rotating Triangles



1/4" Graph Paper



1/2" Graph Paper



Math Standard IV-2 Activities

Pennies for Pits

Standard IV:
Students will understand and apply measurement tools and techniques.

Objective 2:
Determine measurements using appropriate tools and formulas.

Intended Learning Outcomes:
2. Become mathematical problem solvers.
3. Reason mathematically.
4. Communicate mathematically.
6. Represent mathematical situations.

Content Connections:
Science IV-1, 2, & 3

Math Standard IV

Objective 2

Connections

Background Information

Teachers need to determine the level of each student's ability to determine percentages. This lesson is based on the assumption that the students have a working knowledge of mathematics involving percent.

The fruit experiment shows students that measurement is always an approximation. Weighing the edible portion and the nonedible portion separately may not provide the same weight measurement as when the fruit is weighed as a whole.

Research Basis

Thompson, P.W. & Lambdin, D. (1994). Research into practice: concrete materials and teaching for mathematical understanding. *Arithmetic Teacher. 41*(9), 556-558. ERIC EJ491834.

This article discusses the role of concrete materials in teaching for understanding. It includes research on the use of concrete materials, seeing mathematical ideas found in them, and identifying what teachers want students to understand.

Hartshorn, R. & Boren, S. (1990). Experiential Learning of Mathematics: Using Manipulatives. *ERIC Clearinghouse on Rural Education and Small Schools*. Charleston, WV. ERIC ED321967.

This article highlights much of the research on experiential learning in mathematics including the development and implementation of manipulatives in math instruction. It also addresses adherence to national math standards and directions for future research.

Assessment Suggestions

- Review completed *Let's Get Weighing* worksheets for correct computations and consistent measurements.
- Formal assessment should be done at end of measurement unit.

Invitation to Learn

Each student needs a piece of fresh fruit. You may want to have them bring their own fruit from home. Ask them to predict what percentage of their fruit is edible and what percent is not. Observe what fruit other students have and make a prediction about which fruit will have the highest and lowest percentages of edible portions. Which have similar percentages of nonedible and edible portions?

Instructional Procedures

Materials

- ❑ Balance scale
- ❑ Triple beam scale
- ❑ Digital scale
- ❑ Fruit
- ❑ *Let's Get Weighing* worksheet

1. Ask the students the following four true or false questions. They are to answer with a thumbs up (yes) or thumbs down (no). No sideways thumbs allowed.
   - Do bananas grow on trees? (no)
   - Are cucumbers fruit? (yes)
   - Is the pit of a peach good for you to eat? (no)
   - Are apples and roses in the same family? (yes)
2. Describe the parts of the fruit to the class. Students weigh the whole fruit using different types of scales, then record their findings. Weigh each of the fruits on the three scales and take an average of the weight. Record the weights on the *Let's Get to Weighing* worksheets. Cut the fruit, removing the portion that is not edible and weigh it against the edible part for comparison. Each portion is weighed three times and the three weights are averaged.
3. Have each group share their data and findings. Provide the first three columns of information to the rest of the class. The class can fillss in the fourth column using their own computations.

Curriculum Extensions/Adaptations/Integration

- Do this activity along with microorganisms in science. The non-edible portions of the fruits can be added to your decomposition chambers. Information on decomposition chambers is found in *Bottle Biology*.

Resources

Book

Bottle Biology: An Idea Book for Exploring the World Through Soda Bottles and Other Recyclable Materials, by Mrill Ingram; ISBN 084038601X

Family Connections

- Have students find four to six fruits or vegetables at home and estimate what portion of each is edible or nonedible. If possible, students weigh the whole item and its separate parts using a scale from home or checked out from the teacher. Look for other household items made up of parts and weigh or measure the various parts (e.g., The kitchen chair is 40" tall—of that height, 50% legs, 5% seat, and 45% back.).

Name _____

Let's Get Weighing

1. What type of fruit did you bring?_____
2. Estimate what percent of the fruit is edible._____
3. Estimate what percent of the fruit is nonedible._____
4. Do your two estimates equal 100? _____

   If they do not, adjust your estimates so they equal 100.

Scale Type	Try #1		Try #2		Try #2		Avg. Weight	
	Metric	Standard	Metric	Standard	Metric	Standard	Metric	Standard

What is the cost?

1. What was the cost of your fruit per pound?_____
2. How much did your piece of fruit cost?_____
3. How much did the edible portion cost? _____
4. How much did the nonedible portion cost? _____

Name _____

What Weighs More?

1. Which weighs more—
   A cup of uncooked rice or a cup of uncooked beans?

2. Which weighs more—
   A cup of liquid milk or a cup of powdered milk?

3. Which weighs more—
   A cup of sugar or a cup of flour?

4. Which weighs more—
   A cup of sand or a cup of powdered sugar?

5. Which weighs more—
   A cup of granulated sugar or a cup of brown sugar?

6. Which weighs more—
   A cup of salt or a cup of shredded newspaper?

Bonus Questions

1. Which weighs more—
   A cup of skim milk or a cup of whole milk?

2. Which weighs more—
   A cup of water or a cup of dehydrated water?

Name _____

Constructing a Scale

You can make a simple, sturdy, and useful scale from readily available materials. Here's how:

1. Students construct their own scales. Show them the completed scale and allow them to examine it, but do not provide specific instruction on how to build it. The students must construct the scale on their own.
2. Use a tall empty juice carton, coffee can, or milk carton for the base.
3. Clip two spring-type clothes pins to the top of your base.
4. You need a stiff piece of wire (a large paper clip works well) to suspend your balance.
5. If using a square base, put the wire through the spring on the clothespins. Make sure that each end of the wire is protected with a piece of pink eraser or clay.

How to make a balance scale:

1. Start with a wooden ruler or other similar object. Any length from 25 cm or up will suffice. The longer the rod, the more sensitive the balance.

   Rulers with predrilled holes are easiest, but if you do not have access to them, you can easily make your own.
2. Carefully mark and drill a hole in the center of the ruler, equidistant from both sides.
3. Check that your ruler balances well by placing a stiff piece of wire (such as a large opened paper clip) through the hole, suspending the ruler from it.

   If the balance is slightly off, stick a small piece of tape or pipe cleaner to the lighter of the two sides.
4. Drill holes on both ends of the ruler. Make sure they are equal distance from the center hole. These are the holes used to suspend the cups.
5. Check the balance of the scale with the pans in place. Adjust the balance as necessary.

Extension

- Have students measure small items from around the classroom or items from home (e.g., paperclip [large and small], eraser cap, dried beans, uncooked rice, powdered milk etc.). Provide the student with two sandwich bags (the fold over type work best) and about 20 centimeter cubes. A centimeter cube weighs 1gram. This becomes the control. Place the item being weighed into one baggie and then on the scale. Place the other baggie on the other side of the balance scale and slowly add centimeter cubes one at a time. Students need to track their data on a table that they create. Discuss outcome as a whole group.

Materials

Teacher provided:

- ❏ Several large paperclips
- ❏ Small paper/plastic cups
- ❏ Wooden ruler (with holes pre-drilled)
- ❏ Clothespins (with pre-drilled holes)
- ❏ Pipe-cleaners for counter balance
- ❏ Film canisters to be used as known weight measurement
- ❏ Measurable substance that can be used to weigh against canisters (i.e. dried beans, tiddley-winks, centimeter cubes, etc.)
- ❏ Substances measured in tablespoons/teaspoons: sand, dried rice, flour)

Student provided:

- ❏ Base (family sized soup can or 1/2 gallon juice carton)

Looking for Length

Standard IV:
Students will understand and apply measurement tools and techniques.

Objective 2:
Determine measurements using appropriate tools and formulas.

Intended Learning Outcomes:
2. Become mathematical problem solvers.
3. Reason mathematically.
4. Communicate mathematically.
6. Represent mathematical situations.

Content Connections:
Language Arts; Social Studies

Math Standard IV

Objective 2

Connections

Background Information

Students need to be aware that we use both a standard and a metric system for measuring. They should be familiar with the vocabulary for both systems (e.g., inch, foot, yard, millimeter and centimeter). Students should also have a background knowledge in linear measurement.

Assessment Suggestions

- Use class generated three-point rubric to assess students' work.

Invitation to Learn

Before beginning, ask students to name different events that require continuous measurement, such as football games, construction, pharmacy, and butchers.

Instructional Procedures

1. Start the lesson by reviewing the names used for measurement. Some of these include inches, foot, yard, millimeter, centimeter, and meter. Point out a previously selected larger item from the classroom. Ask students to estimate its length using a different unit of measurement. Compare it to other objects by using statement such as, "It is longer than..." or "It is wider than..."

Materials

- ❑ Equipment for measuring lengths (e.g., ruler, tape measure, yardstick, trundle wheel)
- ❑ Reference materials (e.g., road maps, atlases)
- ❑ *Looking for Lengths in All the Right Places* worksheet
- ❑ *Looking for Lengths in Your Bedroom* worksheet

Optional:

- ❑ String
- ❑ Scissors
- ❑ Markers

2. Which unit of measurement should be use to measure the object? Guide the students toward meters or yards. Discuss why we wouldn't use centimeters or inches to measure the length of a chalkboard. Select two students to measure the object. Have them report their findings to the class. Remind entire class that measuring is not 100% accurate.
3. As a class, create a three or five-point rubric to assess the measurement activity.
4. Have students form groups of three to four. Each group will create appropriate tools for measurement (e.g., crayon, chain of paper clips, a pencil box, string, etc.). Use colored markers to indicate the appropriate lengths.
5. Allow students 20 minutes to complete the *Looking for Lengths in All the Right Places* worksheet. They need to move about the classroom freely and complete their own worksheet. While monitoring students, encourage them to estimate the lengths of certain objects around the room.
6. If time allows, review the information the students have collected. If time is not permitted, review the next class day. Discuss when estimation is appropriate and when more precise measurement is necessary. For example when they are trying to figure out how many steps they will take to walk a certain distance, estimation is appropriate. However, if they are cutting a hole to put a window in their house, an exact measurement is needed.

How can we describe length without using words related to measurement? Explain how in England of Old a foot changed depending on the reigning King.

Would we use inches to measure the distance down to the lunchroom or office (or another distance far from your room)? Would you use feet to measure your hand? Why or why not?

Can you think of other ways to measure items?

What type of careers would use measuring or estimating lengths?

Agree or disagree with the following statements:

- Estimating is not a guess.
- If you can measure, never estimate.
- Exact measurements are always needed.
- The smaller the unit of measurement, the more exact the measurement.

Curriculum Extensions/Adaptations/Integration

- Resource students gather information using either metric or standard form.
- Provide gifted students with only standard units of measurement and challenge them to convert standard to metric.

Resources

Book

How Big is a Foot?, by Rolf Myller; ISBN 0440404959

Web sites

http://www.pueblo.gsa.gov/cic_text/misc/usmetric/tometric-conv.htm

http://www.pueblo.gsa.gov/cic_text/misc/usmetric/frommetric-conv-htm

Family Connections

- Give students the *Looking for Lengths in Your Living Room* worksheet to complete at home. Give them a couple of days to work on this with family members. When the assignment is due, discuss the information and their findings as a class.

Name _____

Looking for Lengths in All the Right Places

Part A: Searching in the classroom

Use the descriptions below to find something that you can estimate as the same length. After estimating, use a tool to measure its exact length and compare.

Description	Item	Estimation	Measurement
Something longer than 50 centimeters			
Something shorter than 100 millimeters			
Something bigger than 15 feet			
An item that is twice your height			
An item that is twice as wide as it is tall			
Something that is about the same length as width			
Find one item. Find a second item that is about twice as long as the first.	1. 2.	1. 2.	1. 2.

Part B: City Search

Using a map, find cities that match the following:

1. Find two cities that are about 200 miles apart.

2. Find the distance between Los Angeles, California, and your city.

3. Find the distance between San Francisco, California, and Baltimore, Maryland.

Name _____

Looking for Lengths in Your Living Room

Use the descriptions below to find something that you can estimate as the same length. After estimating, use a tool to measure its exact length and compare.

Description	Item	Estimation	English/ Standard	Metric Measurement
Something longer than 2 feet				
Something shorter than 2 feet				
Something longer than 2 yards				
Something taller than 4 feet				
An item that is twice as wide as its length				
Something that is half as tall as you				
Something that is three times as long as your hand				

Using your "normal step," how far is it to these places?

Description	Estimation	Number of Steps
Number of steps to the closest bathroom		
Number of steps to the kitchen		
Number of steps to the driveway		

Math Standards IV-1 & 2 Activities

The Circle's Measure

Standard IV:
Students will understand and apply measurement tools and techniques.

Objective 1:
Identify and describe measurable attributes of objects and units of measurement.

Intended Learning Outcomes:
3. Reason mathematically.

Content Connections:
Language Arts VII-2

Math Standard IV

Objective 1

Connections

Background Information

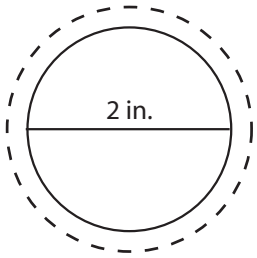
In this lesson, students make a valuable connection between the circumference and diameter of circles. For any given diameter, the circumference of the object is diameter x π. This relationship is often expressed in the formula,

circumference = *pi* x diameter

or

$c = \pi \times d$

Thus, if an object has a diameter of 2 in., the circumference of that object is approximately 6.28 in.



circumference = 2 x π
= 6.28 in.

Pi (π) was probably discovered sometime after people started using the wheel. The people of Mesopotamia (now Iran and Iraq) certainly knew about the ratio of diameter to circumference. The Egyptians knew it, as well. They gave it a value of 3.16. Later, the Babylonians figured it to 3.125. But it was the Greek mathematician Archimedes who really got serious about the ratio. He was the one who figured that the ratio was less than 22/7, but greater than 221/77. But *pi* wasn't called "*pi*" until William Jones, an English mathematician, started referring to the ratio with the Greek letter pi, or "p", in 1706. Even so, *pi* really didn't catch on until the more famous Swiss mathematician, Leonhard Euler, picked up on it in 1737. Thus, *pi* evolved through the contribution of several individuals and cultures.

Before beginning this activity, students should understand how to identify and measure the diameter and circumference of an object. They should also be familiar with metric and standard units of measurement.

Research Basis

Beto, R. (2004). Assessment and Accountability: Strategies for Inquiry-Style Discussions. *Teaching Children Mathematics*, *10*(9) pp. 450-454.

"In inquiry-based instruction, students play the lead role while the teacher makes sure that students are listening to one another and building meaning from one another's work…When students work on problems alone, share strategies, then practice the new strategies, they build flexibility from seeing one problem solved in multiple ways; accuracy arises from using these strategies to verify answers and justify solutions."

Assessment Suggestions

- Make informal observations while students work in groups. Observe the ways in which they select and measure items. Make sure that they demonstrate a sound understanding of the concepts.
- Use the *Teacher Assessment Sheet* to keep a record of each student's level of understanding. This rubric focuses on concepts of reasoning, and is derived from the intended learning outcome for this activity.
- *Saving Sir Cumference* worksheets.

Invitation to Learn

Arrange students in groups of three to four. Introduce *Sir Cumference and the Dragon of Pi*. Have students listen carefully to the story as you begin reading (pages 1-12 only).

Instructional Procedures

1. Abruptly stop the book and display *The Circle's Measure (*overhead) for the class to read. Tell the students that instead of continuing the book, it is their job to solve the riddle and save Sir Cumference before he is slain by the knights.
2. Carefully reveal the Erlenmeyer flasks and place one flask near each group of students. Tell the students not to touch the flasks.

   Note: The students will be curious to know what is in each flask. Allow them to remain curious and reassure them that they will soon learn more about the flasks.

3. Give a *Saving Sir Cumference* worksheet to each student. Students complete Part A individually (Think). Encourage students to share their ideas and thoughts with their group (Pair). Allow 2-3 students to discuss their ideas with the class (Share).

   Note: This time dedicated to class discussion is critical. If a student does not bring up circumference and diameter, help them make this connection.

4. After the students understand that circumference and diameter are the two measurements involved in this problem, open *Sir Cumference and the Dragon of Pi* and tell the students that we will now read to find out what the boy, Radius, has done (pages 14-18 only).

5. By now the students may understand that there is a connection between diameter and circumference and that the answer is very close to 3. If this is not generally understood, hold a brief discussion before moving on.

6. Experiment with various objects to find the true answer to the problem. Direct the students' attention to Part B of the worksheet. Model how to measure the circumference and diameter of an object *in centimeters* and record the data in a table. (String or yarn may be used to wrap around circular objects and then held against a measuring tape.) Give several objects to each group of students. Students may work in groups to make the measurements, but should record the data individually.

7. Circulate around the room as groups work on the tables, taking notes on the various strategies students are using. When a group finishes, direct them to begin Part C of the assignment.

   Note: You could save some time by doing Part C as a class. Create a coordinate grid on a large piece of grid paper prior to the start of the activity. Have each group plot their data points on the class graph.

8. When most students have completed Part C, regain the attention of the class and discuss their findings. Guide the discussion by asking the following questions:

   a. By looking at the data, can you see any relationships? (The graphs should be very close to a straight line, which indicates that there is a direct/linear relationship between diameter and circumference.)

Materials

For teacher:

- ❑ *Sir Cumference and the Dragon of Pi: A Math Adventure*
- ❑ *The Circle's Measure* overhead
- ❑ *Saving Sir Cumference* overhead
- ❑ Small bottle of bleach

For each group:

- ❑ Several circular objects (e.g., soda cans, records, CD's, wastebaskets, paper plates, coins, etc.) You may ask students to bring objects from home.
- ❑ 1 recipe Dragon's Breath: 1 Erlenmeyer flask filled with 100 ml of water and 2 drops of green food coloring, sealed with a rubber stopper. Attach a copy of "The Circle's Measure" to each flask.

  Note: If flasks are not available, you may use sealed jars or bottles.
- ❑ Eyedropper

For each student:

- ❑ *Saving Sir Cumference* worksheet
- ❑ Measuring tape (metric)
- ❑ Pencil
- ❑ String or yarn
- ❑ Centimeter grid paper

b. How much bigger is the circumference than the diameter? (About three times bigger.)

9. Encourage the students to find a more exact number. After the students have identified a more exact number (3.1 or 3.2) you may introduce the symbol π (*pi*) to represent the ratio of the circumference to diameter of a circle.

$$\pi \approx \frac{22}{7} \approx 3.14$$

Note: For our purposes, 3.14 is a close enough approximation of *pi*, however, for the curious student, the value of π to nine decimal places is 3.141592654. This is still an approximate of the number whose decimal expansion has no end.

10. Have students test their claim by finding the diameter and circumference of a new object. Then complete Part D.

Optional: After students have solved the problem, tell them that they have found the dose required to change Sir Cumference back into a knight. Give an eyedropper to each group. Have one student from each group come to the front of the room to collect three drops of the potion (bleach). Allow each group to add 3 or 3.14 drops of bleach to their flask. The color of the water will gradually change from green to blue.

11. Reward the students by reading the remainder of the book.

Curriculum Extensions/Adaptations/Integration

- Challenge students to write their own mathematical poem or riddle using the words *circumference, pi,* and *diameter*.
- Hold a *pi* competition. Challenge students to memorize as many digits of *pi* as possible. One week later, ask students to write down the number *pi* as accurately as possible. The student(s) with the number written down correctly and with the most digits wins!
- National Pi Day is March 14th (3.14). March 14th is also Albert Einstein's birthday.
- Hand out five index cards to each student or group of students. Write the words *circumference, radius, diameter, area,* and *volume* on the board. Ask the students to write one word on the top of each card. Encourage students to use a thesaurus or other reference material to write synonyms, definitions, and examples of each word on the back of the card. Students then arrange the cards in a manner that makes sense to them. (The students may

arrange alphabetically, from least to greatest, or cluster the cards in groups.) Have several groups present and justify their arrangements.

Resources

Books

Fractals, Googols and Other Mathematical Tales, by Theoni Pappas; ISBN 0933174896

Sir Cumference and the First Round Table: A Math Adventure, by Cindy Neuschwander and Wayne Geehan; ISBN 1570911606

Sir Cumference and the Dragon of Pi: A Math Adventure, by Cindy Neuschwander and Wayne Geehan; ISBN 1570911649

Sir Cumference and the Great Knight of Angleland: A Math Adventure, by Cindy Neuschwander and Wayne Geehan; ISBN 157091169X

How Big Is a Foot?, by Rolf Myller; ISBN 0440404959

Web sites

http://www.facade.com/legacy/amiinpi/
Enter your birth date into this Web site and see where the same number sequence occurs in π.

http://www.ballandclaw.com/upi/pi.50000.html
Pi to 50,000 digits

Family Connections

- Encourage students to test their family members' knowledge of *pi*, diameter, and circumference the next time they eat pizza or pie.

Teacher Assessment Sheet

As you observe students, add their name into the appropriate box below.

1	**2**	**3**	**4**	**5**
• *does not generalize from one situation to another* • *has difficulty explaining the activity* • *records answers to problems without checking for validity* • *does not see patterns or relationships* • *has difficulty getting correct answers and justifying answers*	• *can generalize to a few situations* • *can explain the procedures of the activity* • *rarely checks for validity or requires reminders to do so* • *has difficulty recognizing patterns and relationships* • *sometimes struggles to get correct answers and justify them*	• *can generalize to many situations* • *can explain the procedures and his/her reasoning* • *often checks for validity* • *recognizes patterns and relationships with guidance from others* • *gets correct answers but has difficulty justifying them*	• *can generalize to most situations* • *can explain the procedures and his/her reasoning* • *almost always checks for validity* • *independently recognizes patterns and relationships* • *gets correct answers and can justify them*	• *makes generalizations to all situations* • *can explain procedures and his/her reasoning* • *always checks for validity of results* • *independently recognizes patterns and relationships* • *gets correct answers and can justify them clearly to other students*

The Circle's Measure Labels

Copy onto cardstock, cut out, and attach to Erlenmeyer flasks.

The Circle's Measure

Measure the middle and circle around,
Divide so a number can be found.
Every circle, great and small—
The number is the same for all.
It's also the dose, so be clever,
Or a dragon he will stay…
forever...

The Circle's Measure

Measure the middle and circle around,
Divide so a number can be found.
Every circle, great and small—
The number is the same for all.
It's also the dose, so be clever,
Or a dragon he will stay…
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The Circle's Measure

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The number is the same for all.
It's also the dose, so be clever,
Or a dragon he will stay…
forever...

Name _____

Saving Sir Cumference

Help Radius and Lady Di of Ameter save Sir Cumference! The answer to this problem lies in the poem. Can you solve the riddle?

Part A

Carefully read the poem, *The Circle's Measure*. Then answer the following questions:

1. What do you think is meant by "measure the middle and circle around?"

2. According to this poem, what two numbers should you "divide?"

The Circle's Measure

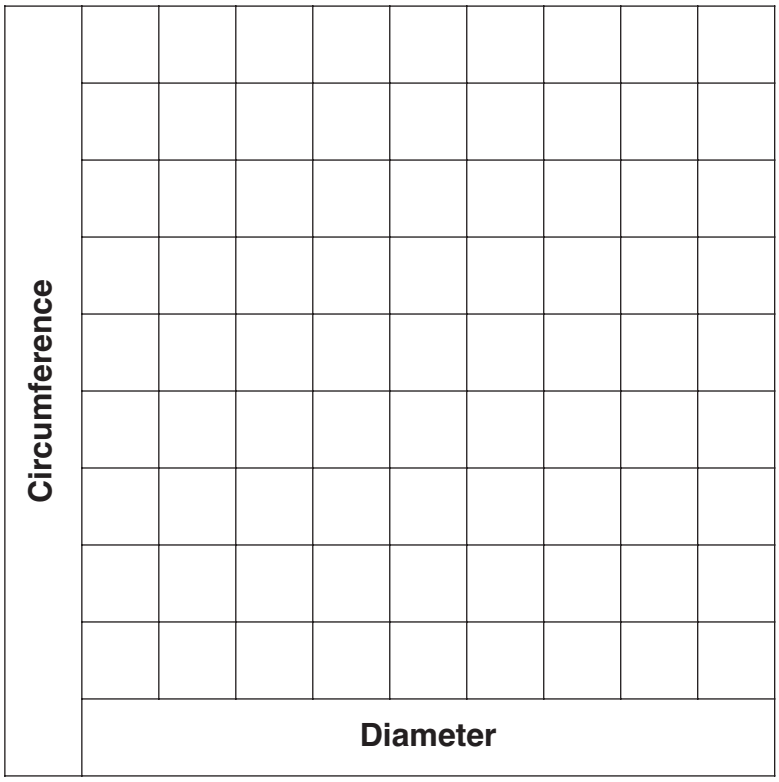
Measure the middle and
circle around,
Divide so a number can be
found.
Every circle, great and
small—
The number is the same
for all.
It's also the dose, so be
clever,
Or a dragon he will stay…
forever...

Part B

Use a measuring tape to find the diameter and circumference of several circular objects. Discuss with your group how to record your data. Create a table below.

Part C

Use a piece of centimeter grid paper to make a coordinate graph like the one below. Use the horizontal axis for diameter and the vertical axis for circumference. Plot the data points for each object your group measured on the graph.



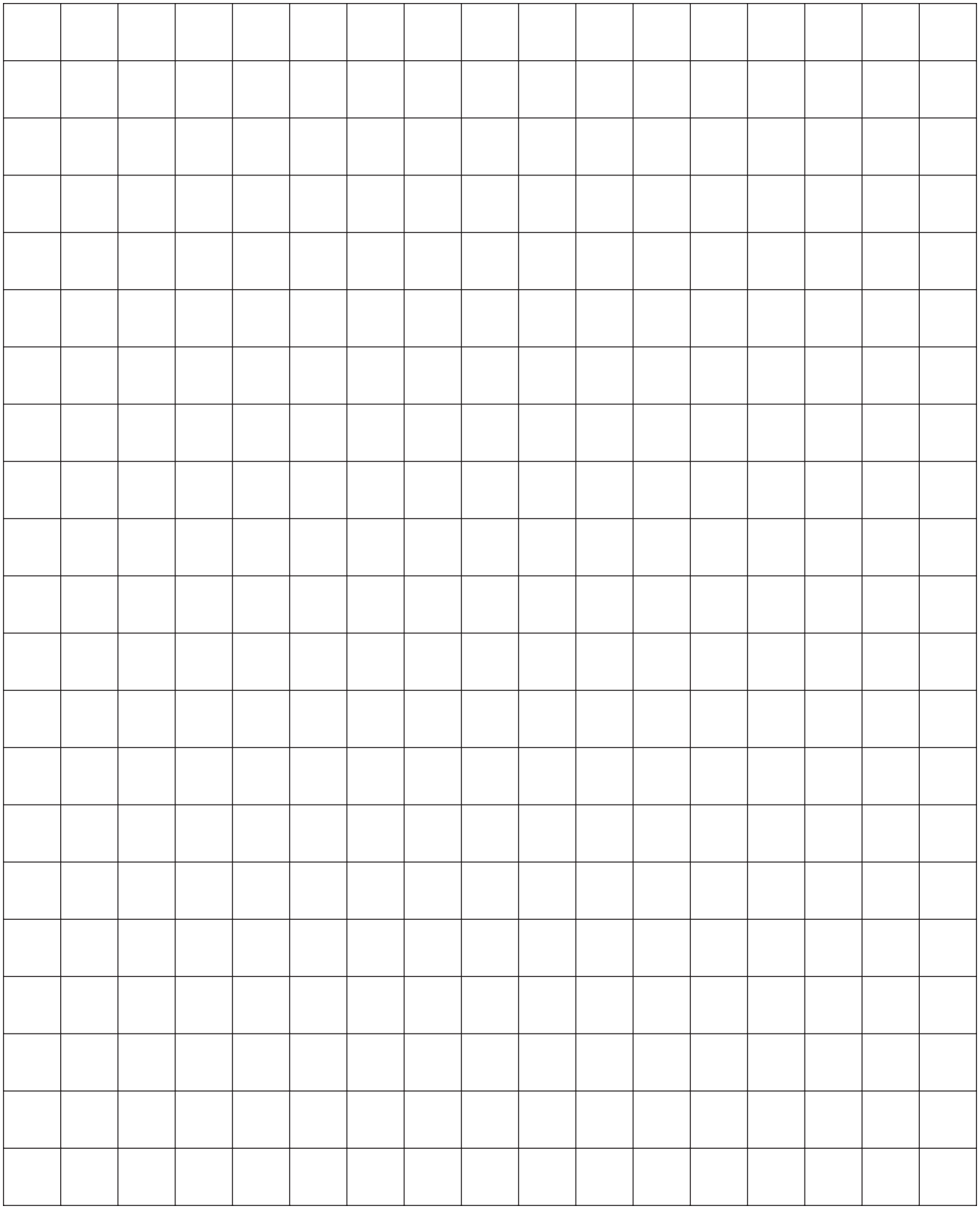
Part D

Study your table and your graph. Look for patterns and relationships that allow you to predict the circumference using the diameter. Test your ideas on other circular objects. Once you think you have found a pattern, answer the questions below:

1. What is the relationship between the diameter and the circumference of a circle?

2. What dose should Radius give Sir Cumference? How do you know this is the correct dose?

Centimeter Grid



Tomb Robbers

Math Standard IV

Objective 2

Connections

Standard IV:
Students will understand and apply measurement tools and techniques.

Objective 2:
Determine measurements using appropriate tools and formulas.

Intended Learning Outcomes:
3. Reason mathematically.

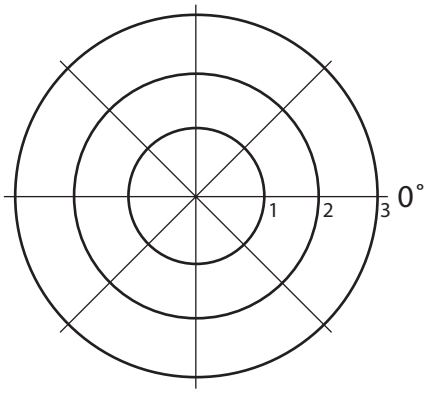
Content Connections:

Background Information

The ancient Babylonians established a system for measuring angles in degrees. They set the measure of an angle that completely surrounds a point at 360˚. This number was likely chosen because the Babylonians used a number system based on 60, or because 360 has many factors.

When looking at angles, students may struggle to appropriately measure angles because they are based on the number 360. The benchmark strategy is a useful strategy for students to use when dealing with these abstract concepts. This strategy teaches students useful benchmarks, or reference points, to use when measuring objects. Some common angle benchmarks are 45˚, 90˚, 180˚, and 360˚.

We often talk of benchmark angles. Although students may have difficulty relating to specific angle measures, many students understand what a 180˚-turn or a 360˚-turn is in the context of snowboarding. Another common example of angles is seen in the marks on a clock.



In this activity, students identify benchmark angles as they are embedded into a circle grid like the one shown here.

Circular grids are similar to coordinate grids. As with coordinate grids, circular grids are used to identify the locations of points given by ordered pairs. Astronomers often use circular grids to identify objects in the night sky. To locate points on a circular grid, start at the vertex (center), move out the number of units given by the first coordinate, and then move counterclockwise along that circle the number of degrees indicated by the second coordinate.

Vocabulary terms used in this lesson:

angle—The opening between two straight lines that meet at a vertex, measured in degrees or radians. The sides of an angle are rays that have the vertex as a starting point.

coordinate grid—A two-dimensional system in which the coordinates of a point are its distances from two intersecting, straight lines called axes.

coordinates—An ordered pair of numbers that identify a point on a coordinate plane or grid.

Research Basis

Joram et. al., (2005). Children's Use of the Reference Point Strategy for Measurement Estimation. *Journal for Research in Mathematics Education*, *36*(1), 4-23.

"Mathematics educators frequently recommend that students use strategies for measurement estimation, such as the reference point or benchmark strategy… Relative to students who did not use a reference point, students who used a reference point had more accurate representations of standard units and estimates of length."

Assessment Suggestions

- Give an *Angle Summary* worksheet to each student. Ask them to first record their estimates of the angles, then to check their measures using an angle ruler.

Invitation to Learn

1. Give a *Star Spangled* worksheet to each student. Allow students two minutes to estimate the measure of all of the *angles* on their page. Show students that angle measurements are usually recorded near the vertex of two lines and labeled with a ° sign, but do not discuss which objects have angles and which ones do not—allow the students to discover this as they go along. Start the timer.
2. At the end of the allotted time, have the students put their pencils down and hold a brief discussion on the assignment. Prompt student's thinking with the following questions:
   a. What shapes can you identify and name on this page?
   b. Are there any quadrilaterals on this page (rectangles, parallelograms, etc.)?

Materials

For teacher:

- ❑ Timer

For each student:

- ❑ *Star Spangled* worksheet
- ❑ Angle ruler (or protractor)

c. Which shapes have angles? Which shapes do not have angles? Why?

3. Next, demonstrate how to use an angle ruler. Give each student an angle ruler and allow students four to five minutes to check their estimates.

Instructional Procedures

Materials

For teacher:

- ❑ *Tomb Robbers Game Board 1* overhead
- ❑ *Using Circle Grids* (overhead)
- ❑ Timer

For each student:

- ❑ Angle ruler (or protractor)
- ❑ Pencil
- ❑ *Tomb Robbers Game Board 1*
- ❑ *Tomb Robbers Game Board 2*

1. Place students in pairs for this activity. Students may play against a partner or in teams of two.
2. Display an overhead transparency of the *Tomb Robbers Game Board 1*.
3. Explain that grid systems do not always have to be square. In fact, astronomers use circular grids frequently.
4. Explain the rules of play (see *Using Circle Grids* overhead). Play a brief game as a class.
5. Distribute a *Tomb Robbers Game Board 1* to each team.
6. Allow teams to play the game several times, using both game boards. Encourage each student to keep a journal entry of his/her strategies.
7. Have a class discussion about the game. Ask students to report their winning strategies. Focus the discussion on what they have learned about angles. You may want to create a new circular grid with 20˚-angle intervals (see *Tomb Robbers Game Board 2*) on an overhead and have students tell you how to locate points on it.

Curriculum Extensions/Adaptations/Integration

- Students create a 15˚ benchmark game board. They can use this board to play again or can take them home to play *Four-In-A-Row* with a family member (see family connections).

Resources

Books

What's Your Angle, Pythagoras?, by Julie Ellis and Phyllis Hornung; ISBN 1570911509

Angles Are Easy As Pie, by Robert Froman and Byron Barton; ISBN 069000916X

Key to Geometry: Angles, by Hawley, Suppes, Gearhart, and Rasmussen; ISBN 0913684767

Secret Treasures and Magical Measures: Adventures in Measuring: Time, Temperature, Length, Weight, Volume, Angles, Shapes and Money, by Chris Kensler; ISBN 0743235258

The Greedy Triangle, by Marilyn Burns; ISBN 0590489917

Web sites

http://www.kidport.com/Grade6/Math/MeasureGeo/MeasuringAngles.htm
Estimate various angle measures and then check to see if you are correct.

http://www.math.psu.edu/geom/koltsova/section2.html
An overview of various types of angles and angle measurements.

http://www.exploratorium.edu/baseball/scientificslugger.html
Use this Shockwave demonstration to see how the type of pitch, angle of ball, and speed of bat swing can affect how and where you hit the ball. Can you figure out what you need to get a home run?

http://dsc.discovery.com/convergence/greatpyramid/tombrobber/tombrobber.html
Read about the daily life of a tomb robber.

Family Connections

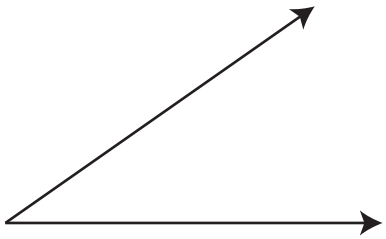
- Use the *Tomb Robbers Game Boards* to play Four-In-A-Row. This game is similar to Tic-Tac-Toe. Before making a move, each player must accurately say the coordinates of a point. They may then place an "X" or "O." over their point. The first player to get four marks in a row wins.

Name _____

Angle Summary

Record your angle estimate. Check your estimate using an angle ruler or other measuring device. Record the actual measure.

1.



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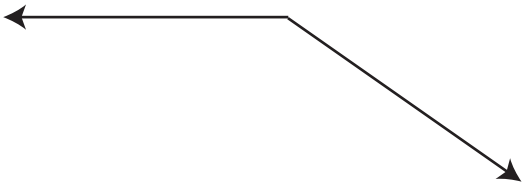
External: Estimate:_____ Actual:_____
Total: Estimate:_____ Actual:_____

2.



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External: Estimate:_____ Actual:_____
Total: Estimate:_____ Actual:_____

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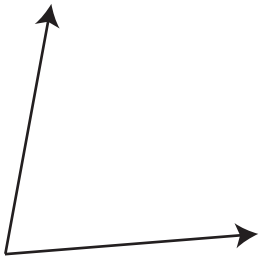

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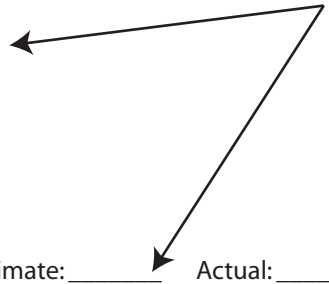
Total: Estimate:_____ Actual:_____

4.



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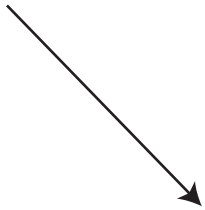
External: Estimate:_____ Actual:_____
Total: Estimate:_____ Actual:_____

5.

Internal: Estimate:_____ Actual:_____
External: Estimate:_____ Actual:_____
Total: Estimate:_____ Actual:_____

6.



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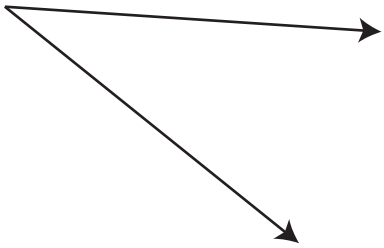
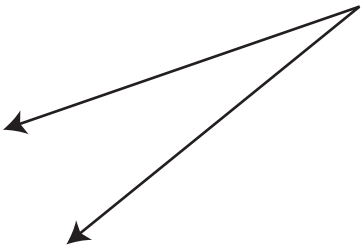
External: Estimate:_____ Actual:_____
Total: Estimate:_____ Actual:_____

7.



Internal: Estimate:_____ Actual:_____
External: Estimate:_____ Actual:_____
Total: Estimate:_____ Actual:_____

8.



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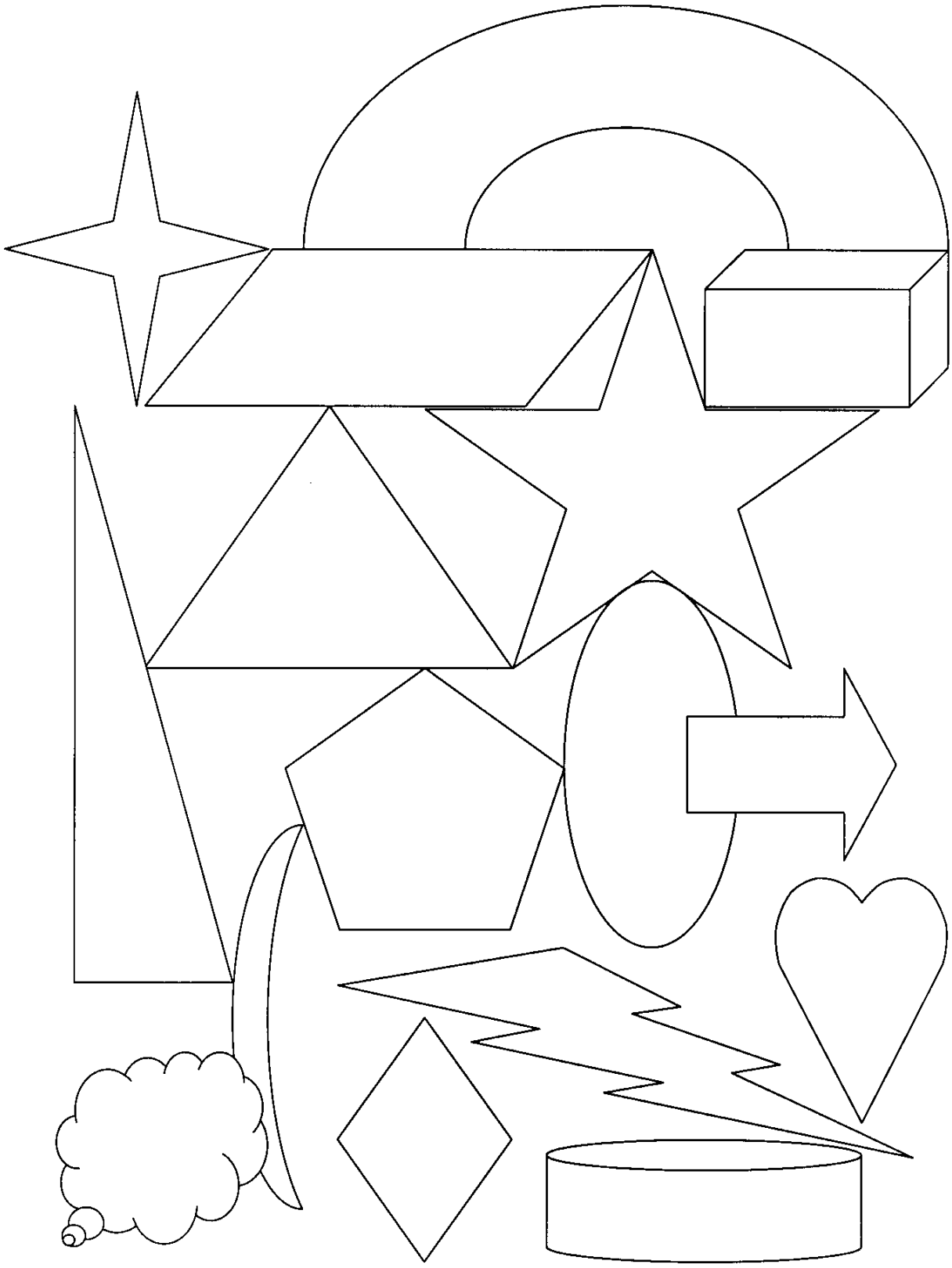
External: Estimate:_____ Actual:_____
Total: Estimate:_____ Actual:_____

Name _____

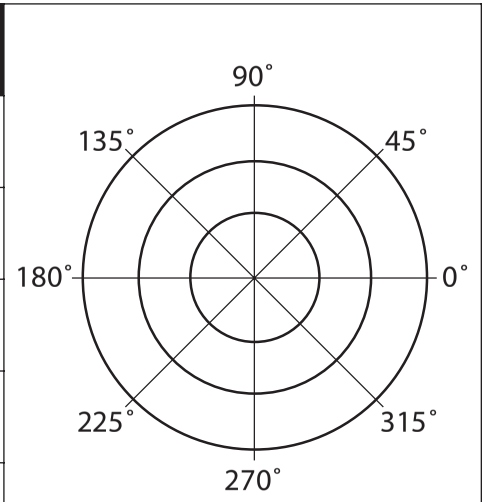
Star Spangled



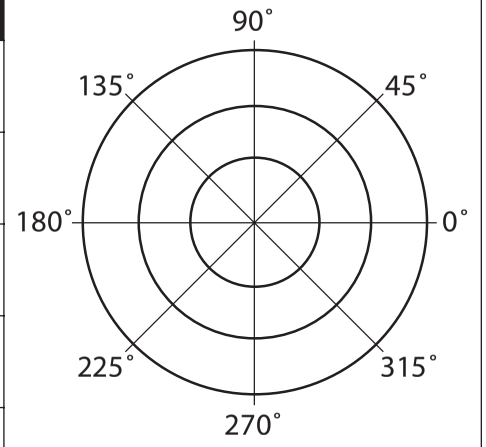
Name _____

Tomb Robbers Game Board 1

My Hidden Treasure		My Partner's Hidden Treasure	
Item	**Coordinates**	(,)	(,)
sarcophagus	(,)	(,)	(,)
scarab	(,)	(,)	(,)
amulet	(,)	(,)	(,)
canopic jars	(,)	(,)	(,)



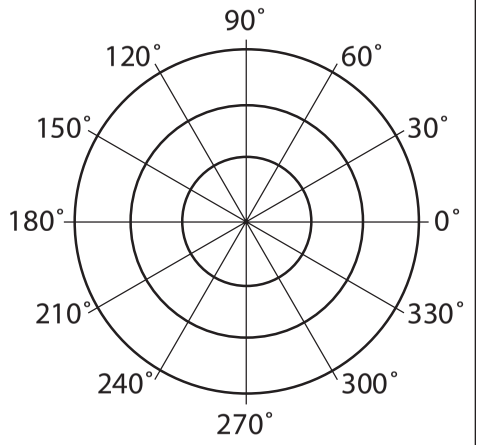
My Hidden Treasure		My Partner's Hidden Treasure	
Item	**Coordinates**	(,)	(,)
sarcophagus	(,)	(,)	(,)
scarab	(,)	(,)	(,)
amulet	(,)	(,)	(,)
canopic jars	(,)	(,)	(,)



Name _____

Tomb Robbers Game Board 2

My Hidden Treasure		My Partner's Hidden Treasure	
Item	**Coordinates**	(,)	(,)
sarcophagus	(,)	(,)	(,)
scarab	(,)	(,)	(,)
amulet	(,)	(,)	(,)
canopic jars	(,)	(,)	(,)



My Hidden Treasure		My Partner's Hidden Treasure	
Item	**Coordinates**	(,)	(,)
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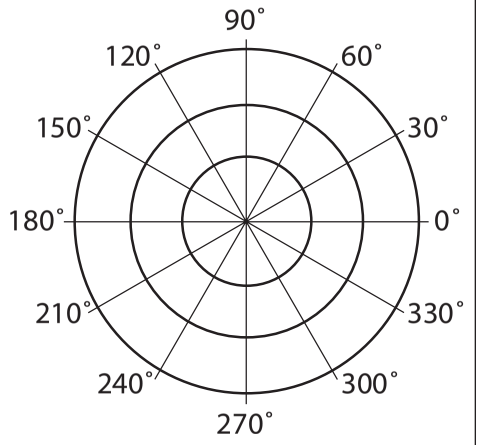
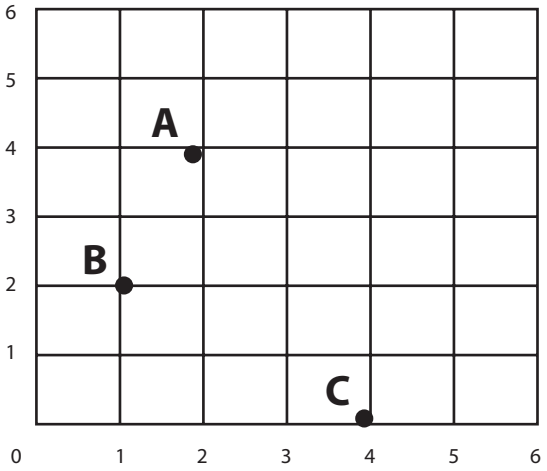
scarab	(,)	(,)	(,)
amulet	(,)	(,)	(,)
canopic jars	(,)	(,)	(,)



Using Circle Grids

You can use a coordinate grid to plot and locate points in ordered pairs. To locate a point, start at (0, 0) and move **over** the number of units given by the first coordinate, and then move **up** the number of units given by the second coordinate.



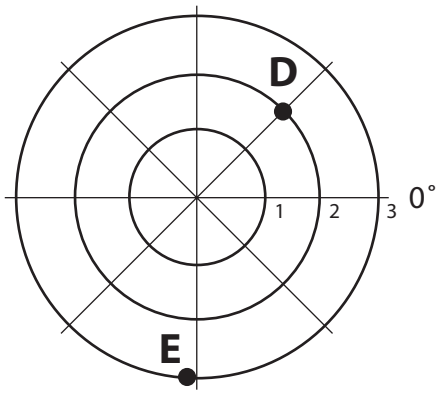
What are the coordinates of the following points?

A = (,)

B = (,)

C = (,)

Circular grids are similar to coordinate grids. Astronomers often use circular grids to identify objects in the night sky. To locate points on a circular grid, start at the vertex and move **out** the number of units given by the first coordinate of an ordered pair. Then move **counterclockwise** along that circle the number of degrees indicated by the second coordinate.



Try to find the circular coordinates of the following points:

D = (,)

E = (,)

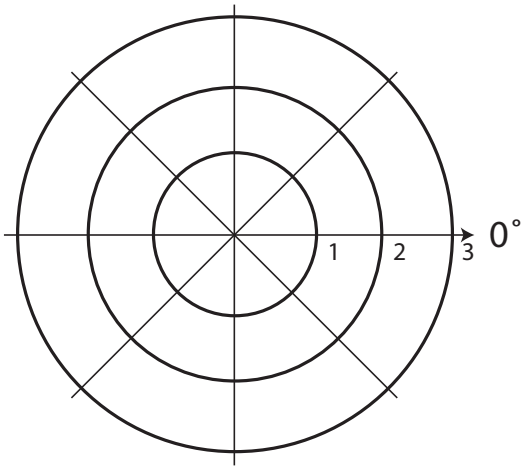
Tomb Robbers Rules & Directions

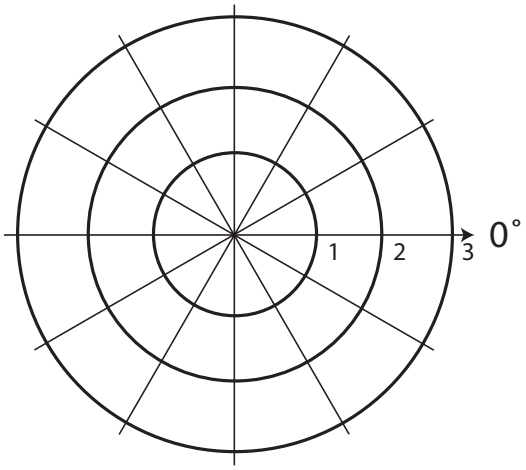
Goal: Find your opponent's treasure before s/he finds yours! The player who finds all four treasure items first wins!

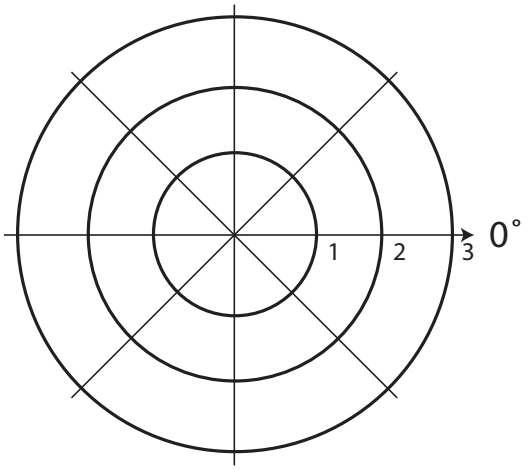
1. Each player begins by secretly hiding his/her treasure in his/her tomb. Record the coordinates of each item in the *My Hidden Treasure* box.
2. Your teacher will tell you which player begins first.
3. To begin, name a coordinate pair you think the treasure is in. Your partner will check his/her game board, and then tell you "Hit" or "Miss." Use the circle grid and *My Partner's Hidden Treasure* box to make notes and record your moves.
4. A player must give correct coordinates before s/he can move. If a player repeats coordinates, s/he loses a turn.

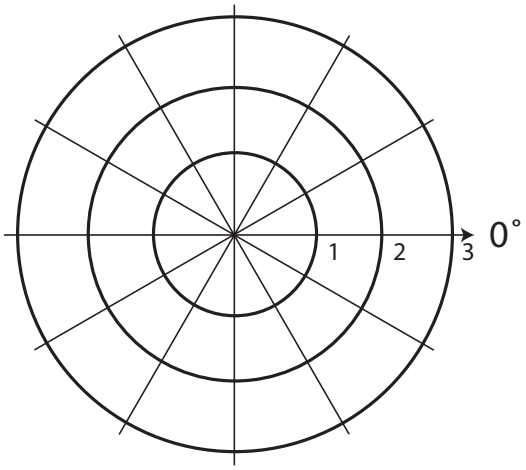
Name _____

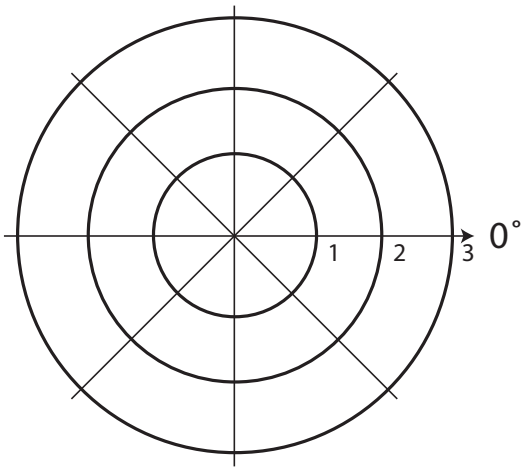
Four-In-A-Row Game Boards

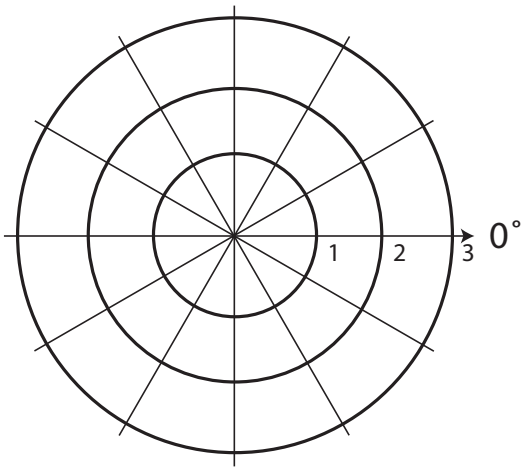












The Northern Lights

Math Standard IV

Objective 2

Connections

Standard IV:
Students will understand and apply measurement tools and techniques.

Objective 2:
Determine measurements using appropriate tools and formulas.

Intended Learning Outcomes:
2. Become mathematical problem solvers.

Content Connections:
Science III-1 & 2

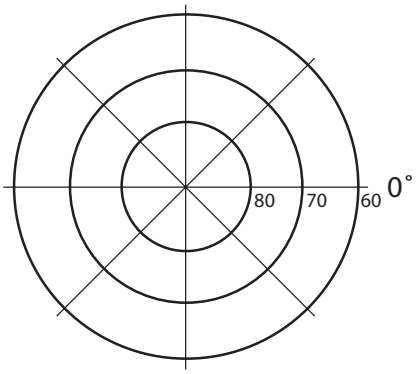
Background Information

Auroras are the beautiful curtains of colored light that are commonly seen in the Arctic and Antarctic regions of Earth. They have a long history of sightings by humans for over 3,000 years. Like lightening and earthquakes, the auroras are natural events.

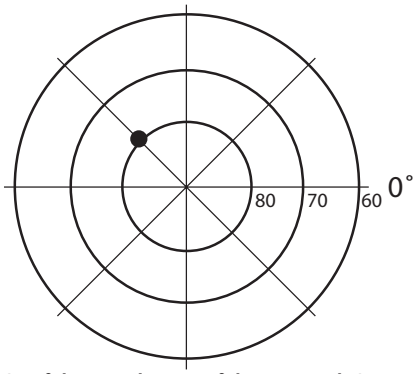
Auroral light is created by interactions between the sun and Earth. The sun is a mass of electrically charged particles (in the form of a gas). The sun is so hot that it's outer layers blow away in the form of solar wind. It takes an average of three days for this wind to reach Earth. In general, Earth's protective atmosphere and magnetic field protect the planet from this solar wind. Instead of penetrating our atmosphere, particles from the sun collect around Earth and gather in a cavity called the magnetosphere.

Energized electrons from the sun collide with oxygen and nitrogen in Earth's atmosphere, producing colorful arrays in Earth's magnetosphere. The different colors of the auroras are created depending on the molecules and altitude of collision. A yellow-green color is the result of an oxygen collision at 100 km. Red auroras occurs at 300 km. Blue light results from ionized nitrogen molecules, and a purplish-red color is the result of neutralized nitrogen molecules.

It is easy to graph the auroral zone using angles and a geographic circular grid of the Northern Hemisphere. This activity is a great follow-up to "Tomb Robbers." Prior to this activity, students should be familiar with benchmark angles and the circular grid system. They should be able to plot points on a circular grid and estimate and draw angles with minimal error.



In this activity, students use a circular coordinate grid to plot zones of auroral activity.



(80˚ latitude, 135˚ longitude)

This grid system is different than a coordinate grid because it is circular. Astronomers often use circular grids to identify objects in the night sky. To locate points on a circular grid, start at the vertex and then move out to the latitude given by the first coordinate of an ordered pair, then move counterclockwise along that circle the number of degrees indicated by the second coordinate.

Vocabulary terms used in this lesson:

angle—The opening between two straight lines that meet at a vertex, measured in degrees.

coordinate grid—A two-dimensional system in which the coordinates of a point are its distances from two intersecting, straight lines called axes.

coordinates—An ordered pair of numbers that identify a point on a coordinate plane or grid.

latitude—A geographic coordinate measured from the equator with positive values going north and negative values going south.

longitude—A geographic coordinate measured from the Prime Meridian (0˚ longitude) with positive values going east and negative values going west.

Research Basis

Joram et. al., (2005). Children's Use of the Reference Point Strategy for Measurement Estimation. *Journal for Research in Mathematics Education*, 36(1), 4-23.

"Mathematics educators frequently recommend that students use strategies for measurement estimation, such as the reference point or benchmark strategy… Relative to students who did not use a reference point, students who used a reference point had more accurate representations of standard units and estimates of length."

Assessment Suggestions

- Have students complete the *Southern Lights* worksheet.

Invitation to Learn

- Display several photos of the Northern Lights for the students. Use some of the following questions to guide a brief class discussion:
  - Do you recognize anything in these photos?
  - Has anyone ever seen the Northern Lights?
  - Where would you go to see the Northern Lights?
  - How are these colors created?
  - What is the connection between the Northern Lights and mathematics?

Instructional Procedures

Materials

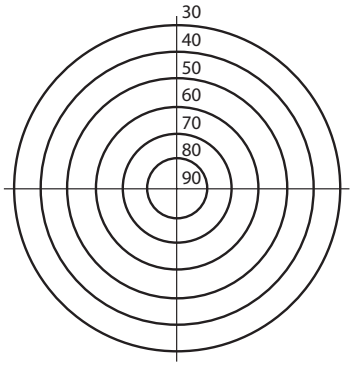
For teacher:

- ❑ Photos of the Northern Lights

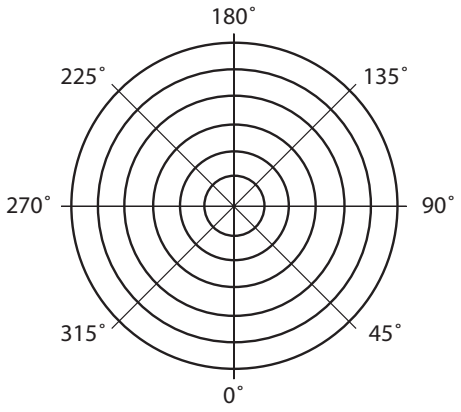
For each student:

- ❑ Angle ruler (or protractor)
- ❑ Colored pencils
- ❑ *Where to Find An Aurora* worksheet
- ❑ *Southern Lights* worksheet
- ❑ Atlas

1. Have students work alone or in pairs for this activity.
2. Distribute a *Where to Find an Aurora* worksheet to each student.
3. Have students label the latitude lines as follows:



4. Next have students estimate and label the unmarked longitude lines.



5. Have students plot the points onto the geographic circular grid for the outer ring. The points are identified as ordered pairs (longitude, latitude).

   Note: If students are doing this activity after completing "Tomb Robbers," you will want to point out that 0˚ begins at the bottom of the coordinate grid and angles move from this point in a counterclockwise direction.

6. Have students connect the points in the outer ring, then plot the points in the inner ring.
7. Using the scale 1 cm = 1,400 km, have students measure the approximate distance (width) of the ring. To have students find the range of widths, help them take measurements of both, the shortest and longest distances between the inner and outer rings.
8. Have students color in the ring with their favorite auroral colors.
9. Using an atlas and the student's *Where to Find An Aurora* worksheet, hold a class discussion on the following questions:
   - Where would you travel in North America to see an aurora?
   - Where is the center of the auroral oval located?
   - How far is the center from the North Pole?
   - What is the range in widths of the auroral oval (in kilometers)?
   - If you were located at (205˚, 65˚), where would you look in the sky to see an aurora?
   - If you were located at (290˚, 60˚), where would you look in the sky to see an aurora?

Curriculum Extensions/Adaptations/Integration

- Go to the following Web site to study Earth's magnetic field with a soda bottle magnetometer.
  http://www.image.gsfc.nasa.gov/poetry/workbook/magnet.html
- Research the possibility of auroras on other planets/moons. Do auroras only occur where there is an atmosphere?

Resources

Web sites

http://www.iww.is/art/shs/pages/thumbs.html
Click on these amazing pictures of the aurora borealis and read explanations of them in the captions.

http://virtual.finland.fi/finfo/english/aurora_borealis.html
Information on the folklore and origins of these cools celestial lights. Includes a video of the phenomenon.

http://vathena.arc.nasa.gov/curric/space/aurora/
Facts, pictures, and a movie of the Northern Lights.

http://www.exploratorium.edu/learning_studio/auroras/
Learn all about the Northern Lights, what causes them, and where they can be found in this site from the Exploratorium.

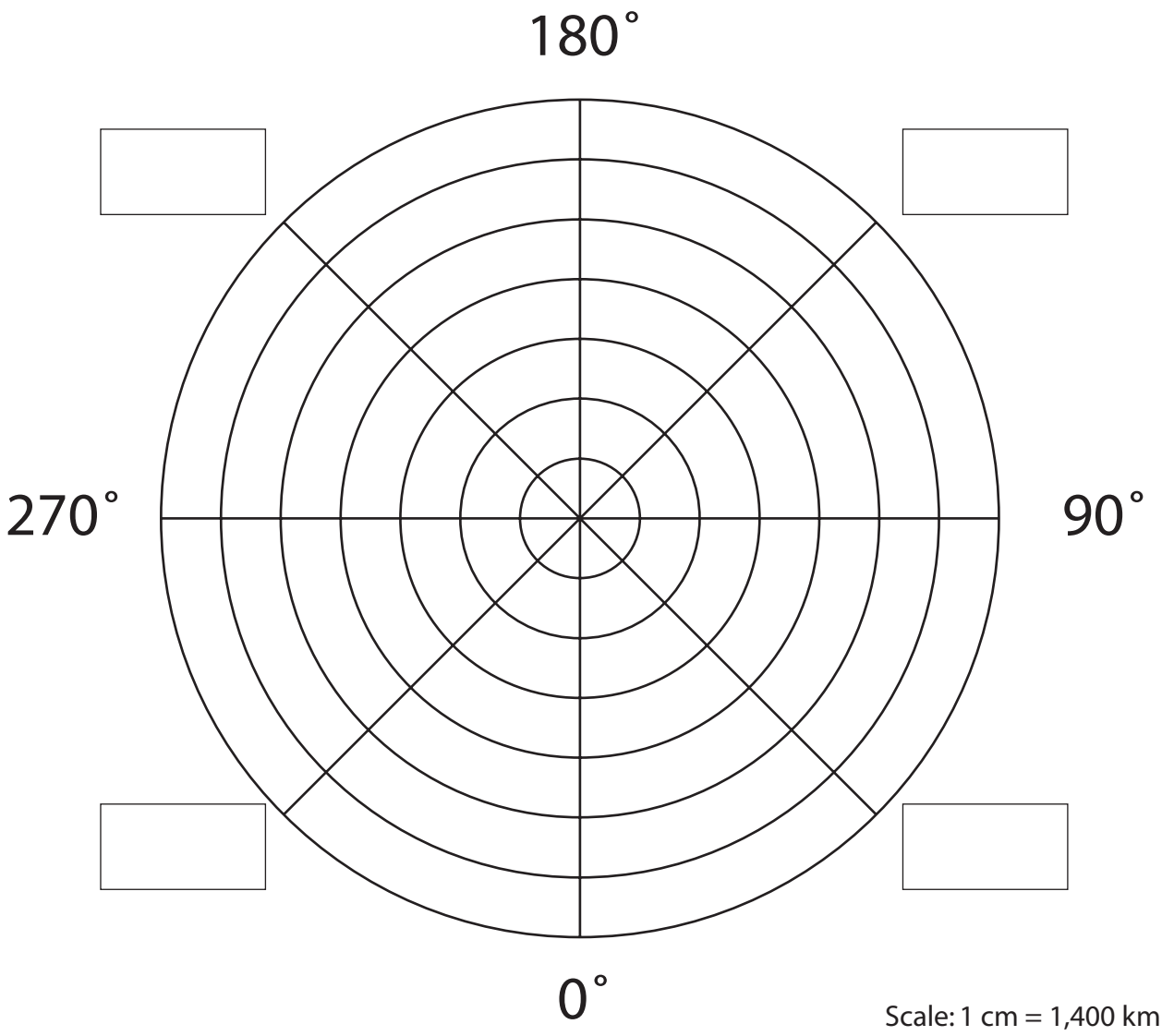
http://www.geo.mtu.edu/weather/aurora/images/space
Pictures of the Northern Lights taken from space.

http://www.northern-lights.no/
This amazing resource has information on the science behind the aurora borealis, explanations for the lights throughout history, descriptions of different patterns, and lots more, including some amazing photos.

Name _____

Where to Find An Aurora

Outer Ring of Auroral Oval				Inner Ring of Auroral Oval			
Point 1	(90, 65)	Point 7	(0, 60)	Point 1	(90, 78)	Point 7	(0, 75)
Point 2	(135, 64)	Point 8	(320, 63)	Point 2	(135, 72)	Point 8	(320, 72)
Point 3	(180, 60)	Point 9	(315, 60)	Point 3	(180, 70)	Point 9	(315, 70)
Point 4	(225, 55)	Point 10	(300, 60)	Point 4	(225, 67)	Point 10	(300, 67)
Point 5	(270, 50)	Point 11	(245, 50)	Point 5	(270, 65)	Point 11	(245, 62)
Point 6	(45, 63)	Point 12	(200, 58)	Point 6	(45, 67)	Point 12	(200, 70)



Scale: 1 cm = 1,400 km

Approximate Width:

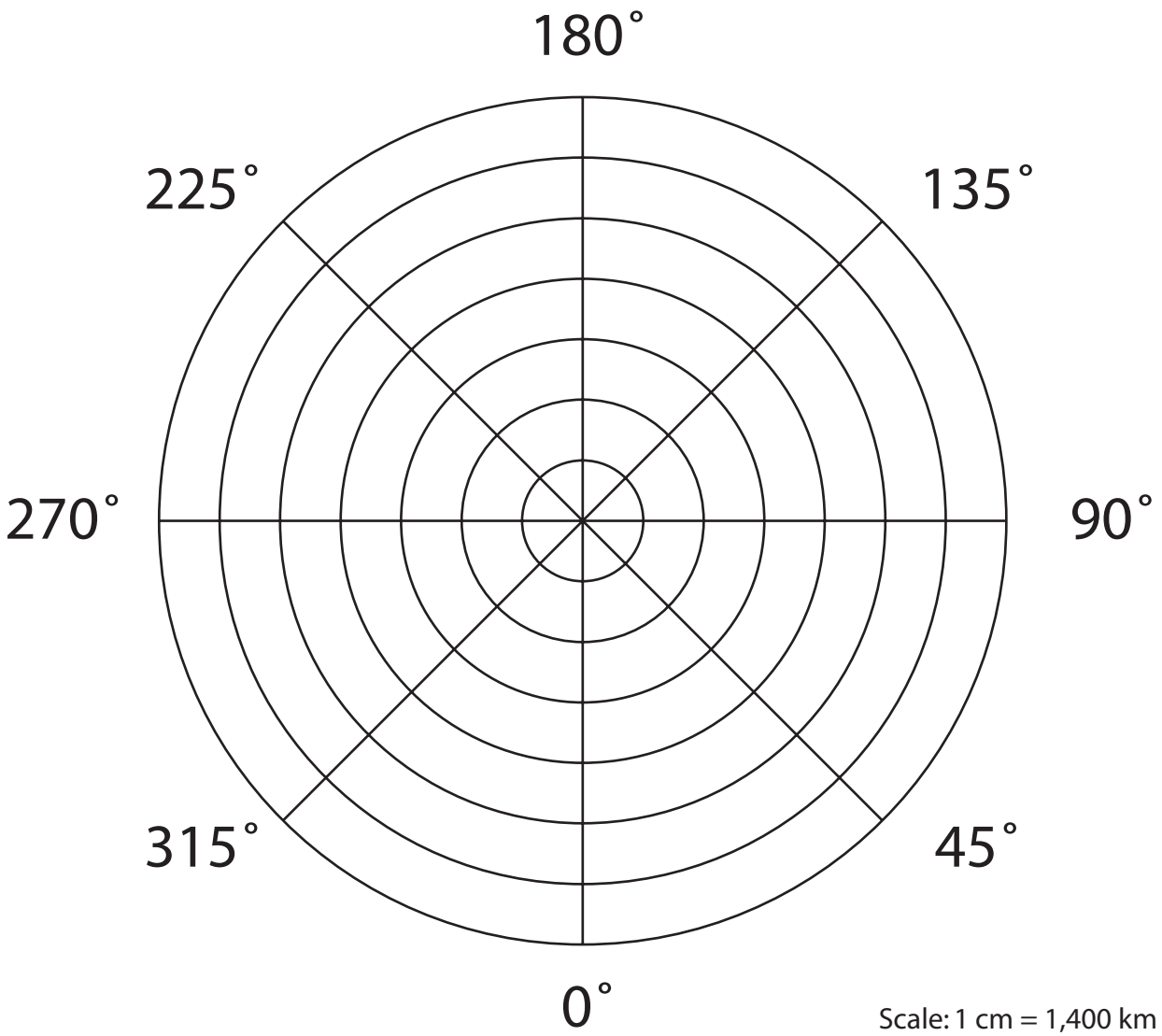
Shortest Distance: _____ km Longest Distance: _____ km Range: _____ km

Name _____

Southern Lights

Auroras are also visible in the southern hemisphere. Use the diagram below to find and correctly label the coordinates for missing points.

Outer Ring of Auroral Oval				Inner Ring of Auroral Oval			
Point 1	(225, 45)	Point 7	(135, 62)	Point 1	(225, 65)	Point 7	(135, 67)
Point 2	(45, 57)	Point 8	(67, 55)	Point 2	(45, 70)	Point 8	(67, 65)
Point 3	(270, 52)	Point 9	(180, 52)	Point 3	(270, 62)	Point 9	(180, 66)
Point 4	(350, 60)	Point 10	(157, 60)	Point 4	(350, 69)	Point 10	(157, 70)
Point 5	(112, 50)	Point 11	(247, 50)	Point 5	(112, 62)	Point 11	(247, 65)
Point 6	(315, 57)	Point 12	(292, 55)	Point 6	(315, 70)	Point 12	(292, 67)



Approximate Width:

Shortest Distance: _____ km Longest Distance: _____ km Range: _____ km

Spacing Out the Solar System

Standard IV:
Students will understand and apply measurement tools and techniques.

Objective 2:
Determine measurements using appropriate tools and formulas.

Intended Learning Outcomes:
5. Make mathematical connections.

Content Connections:
Science III-1

Math Standard IV

Objective 2

Connections

Background Information

This activity encourages students to apply their problem-solving skills to measurement as they measure circular objects and make connections to the planets. Measuring circular objects can sometimes be challenging. One way to measure the circumference of an object is to use a piece of yarn or string. Measure around the object using the yarn. Then lay the yarn against a ruler to find the correct distance. Once the circumference is found, the diameter can be found by dividing the circumference by π. Likewise, if the diameter of an object is known, the circumference can be found by using the formula: $c = \pi \times d$.

The interrelationship between circumference and diameter allows us to make valuable conjectures in science. Astronomers measure the diameter of distant objects in space and then use the $c = \pi \times d$ formula to estimate that object's circumference and architects must use the relationship of circumference and diameter whenever creating circular objects.

Vocabulary terms used in this lesson:

circumference—The *perimeter* of a circle.

diameter—A *chord* that goes through the center of a circle.

metric system—A system of measurement based on tens. The basic unit of length is the meter. The basic unit of mass is the gram. The basic unit of *capacity* is the liter.

pi—The ratio of the *circumference* of any circle to its *diameter*, approximately equal to 3.14.

Assessment Suggestions

- Circulate around the room and make informal assessments while the students work. Make sure you aren't moving too quickly through this activity and that the students understand why this activity requires so many steps. You may wish to take notes on specific misconceptions or questions that arise. You can address these questions at the end of the lesson.
- *It's All Relative* worksheets.
- You may choose to assign one of the curriculum extensions as an assessment piece.

Invitation to Learn

Hold up an object 1 cm in diameter. Show the students that the diameter of this object is 1 cm and explain that it represents Mercury. Have a quick scavenger hunt to find objects that represent Jupiter, Pluto, and Earth. Students search the room and record objects in their journal.

Answers: Jupiter: 28.6 cm

Pluto: 0.5 cm

Earth: 2.5 cm

Instructional Procedures

Materials

For each group:

- ❑ *Planet Cutouts*
- ❑ Measuring devices (metric rulers, measuring tape, string or yarn, etc.)

For each student:

- ❑ *It's All Relative 1* worksheet
- ❑ Calculator

1. Divide the class into nine groups for this activity.
2. Give the *It's All Relative 1* worksheet to each student.
3. Display a packet of *Planet Cutouts*. In each packet are cutouts for the nine planets, but they are not labeled. Each planet has a line drawn through the center of it (the diameter) and we know (from our worksheets) the circumference of each planet. *Is there any way we can figure out what planet each circle corresponds to?* Allow the students to discuss their strategies in their groups. Then hand out one set of cutouts and several measurement supplies to each group and encourage them to match each cutout to the correct planets.
4. Circle around the room as students work on this problem. Make sure that students understand the connection between diameter and circumference ($c = \pi \times d$) and that they are using metric units of measurement.

5. Once groups have made the connections, have them complete the blank column on the *It's All Relative 1* worksheet. After all groups have matched the circles to the planets, go around the room and have groups quickly present the strategies they used while you check for understanding.

Curriculum Extensions/Adaptations/Integration

- Assign a planet to each group. Have each student record their assigned planet on the top of their *It's All Relative 2* worksheet.

  Tell the students that they will now use their planet to construct a class-sized scale model of the solar system.

- Display the sun (helium balloon). Ask the students the following questions:
  - Is the sun accurately drawn to scale? (No)
  - What are some of the problems with the size of this sun? (It's too small. A sun accurately drawn to scale would fit more than 1,000,000 Earth's inside it. It's three-dimensional and our planets are only two-dimensional. A scale model of the sun would have a diameter of 270 cm.)
  - What is true about the sun? (It's yellow in appearance. It contains helium. It is the center of our solar system., etc.)
- Once you have resolved the sun-scale issue, tell the students that we need to figure out how far each planet is from the sun.
- Direct the student's attention to the *It's All Relative 2* worksheet. Tell each group to find his/her planet and practice reading the actual distance from the sun (e.g., "Jupiter is seven hundred seventy eight million, four hundred twelve thousand, ten kilometers from the sun."). This is a good opportunity to teach students not to use the word "and" when reading whole numbers.
- Ask students to convert the distance from the sun to a form of scientific notation using a base of 10^6. Once students have completed the measurement for their planet, encourage them to do the same for the other planets.

  Note: What the students are doing here is not technically referred to as scientific notation. True scientific notation would be written with whole numbers between 0 and 10.
- As a class, divide the prefix of each scientific notation by 30 to find the number of toilet paper sheets from the sun.

Materials

For teacher:

- ❑ Yellow helium balloon on a string

For each group:

- ❑ Roll of toilet paper
- ❑ Scissors
- ❑ Tape

For each student:

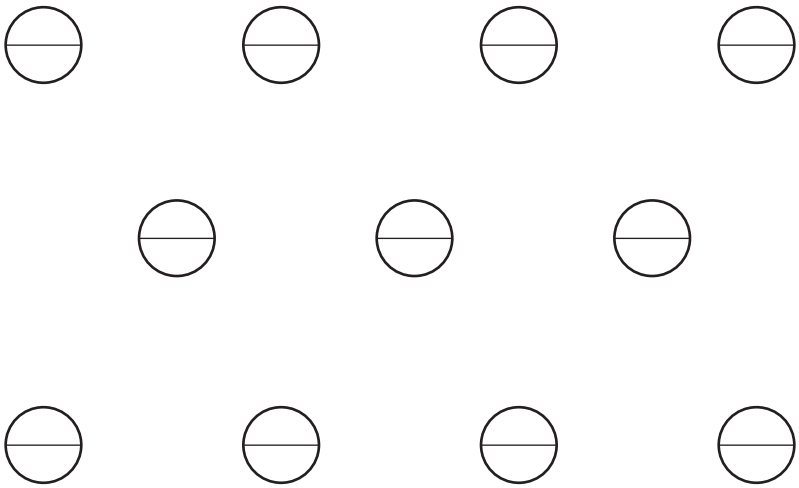
- ❑ *It's All Relative 2* worksheet

- Hand out a roll of toilet paper to each group. Model how to count and cut sheets. You may also want to model how to make estimates for partial sheets. Encourage students to be very accurate in their measurements by using various estimation strategies.
- Find a large open area outside. Place the sun at the center of the model. Have groups stand around the sun and roll their toilet paper outward. Students tape their planet cutouts to the end of each toilet paper roll.
- After the model is constructed, students write a reflective journal piece about what they learned from this activity.

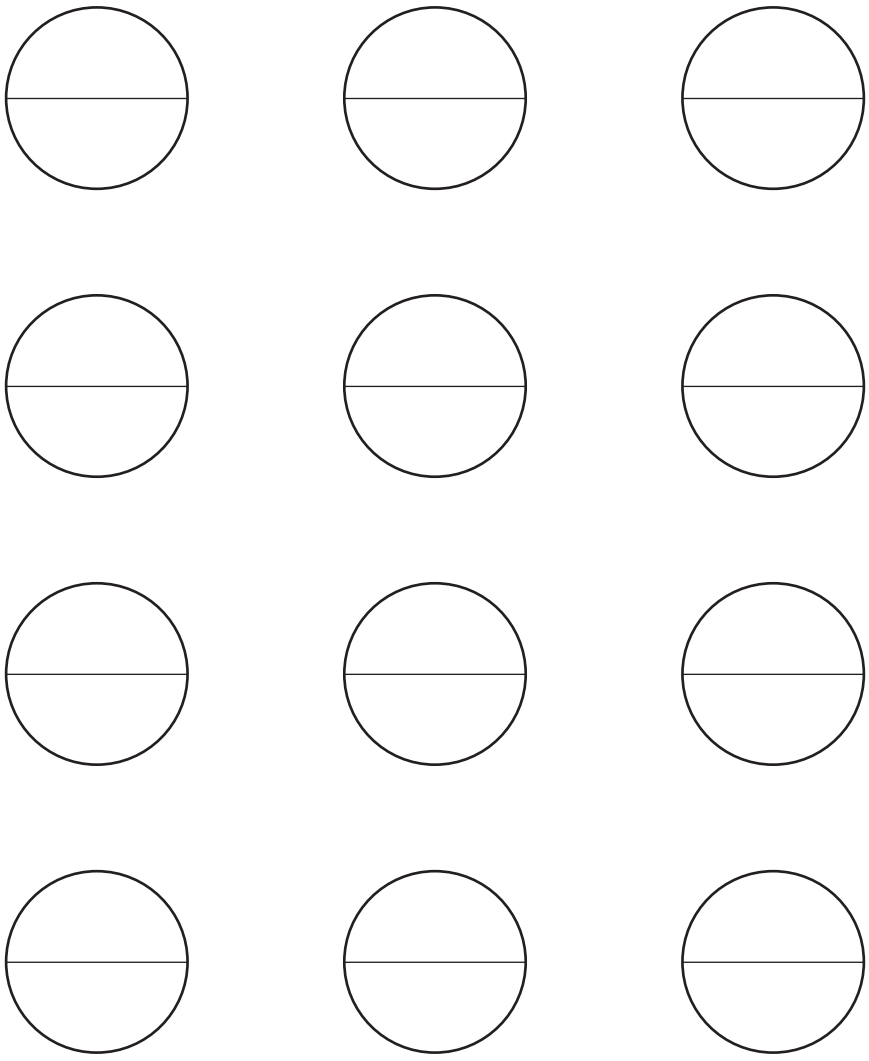
Family Connections

- Students create a scale object for the planetoids Quaoar and Sedna at home.
- Create a scale model of the sun using yellow chart paper and art supplies at home.

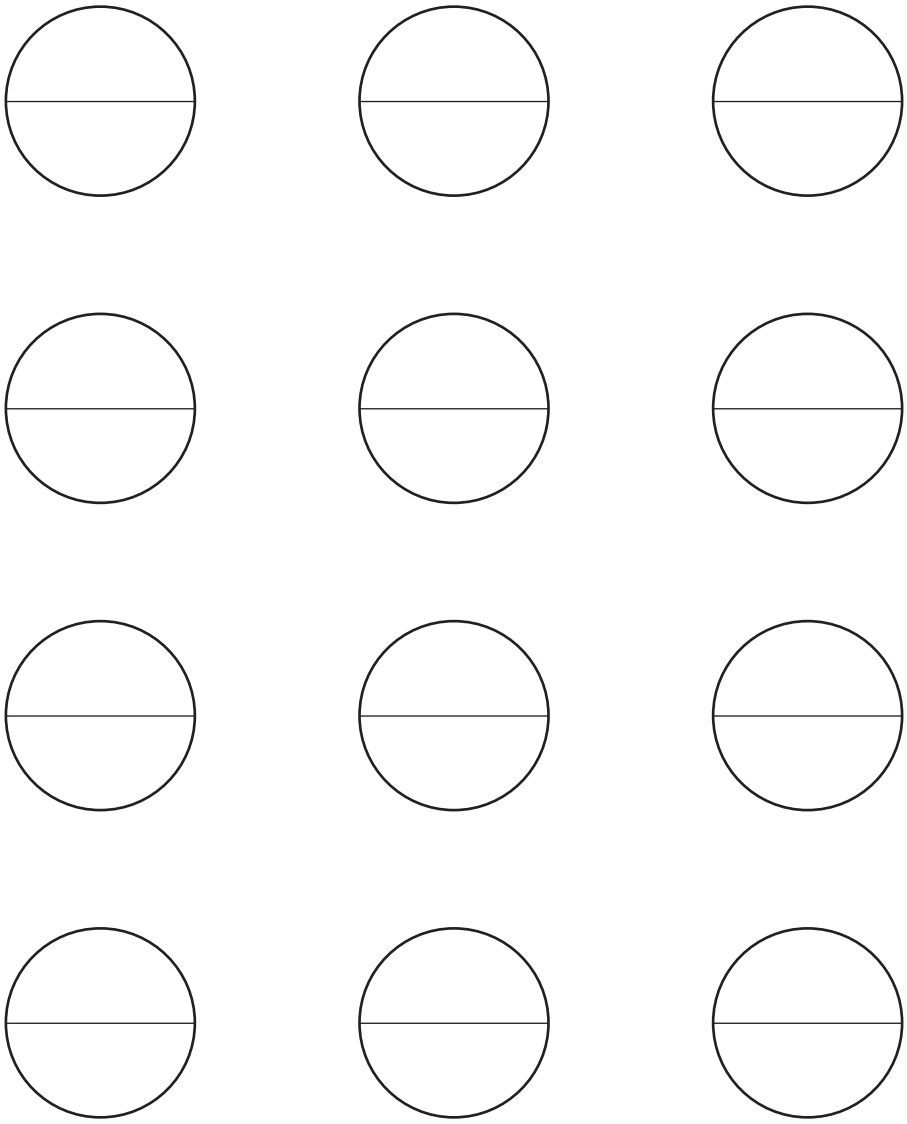
Planet Cutouts—Mercury



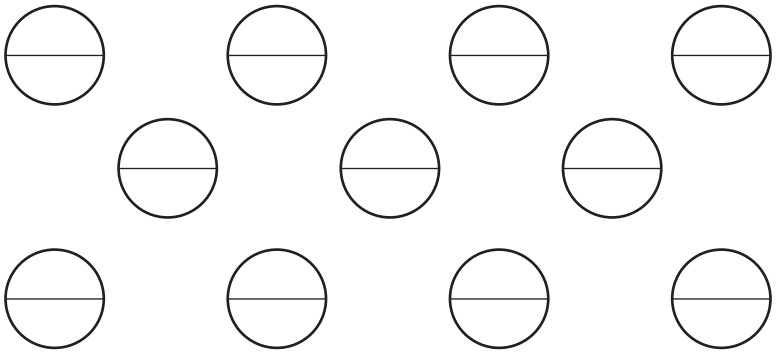
Planet Cutouts–Venus



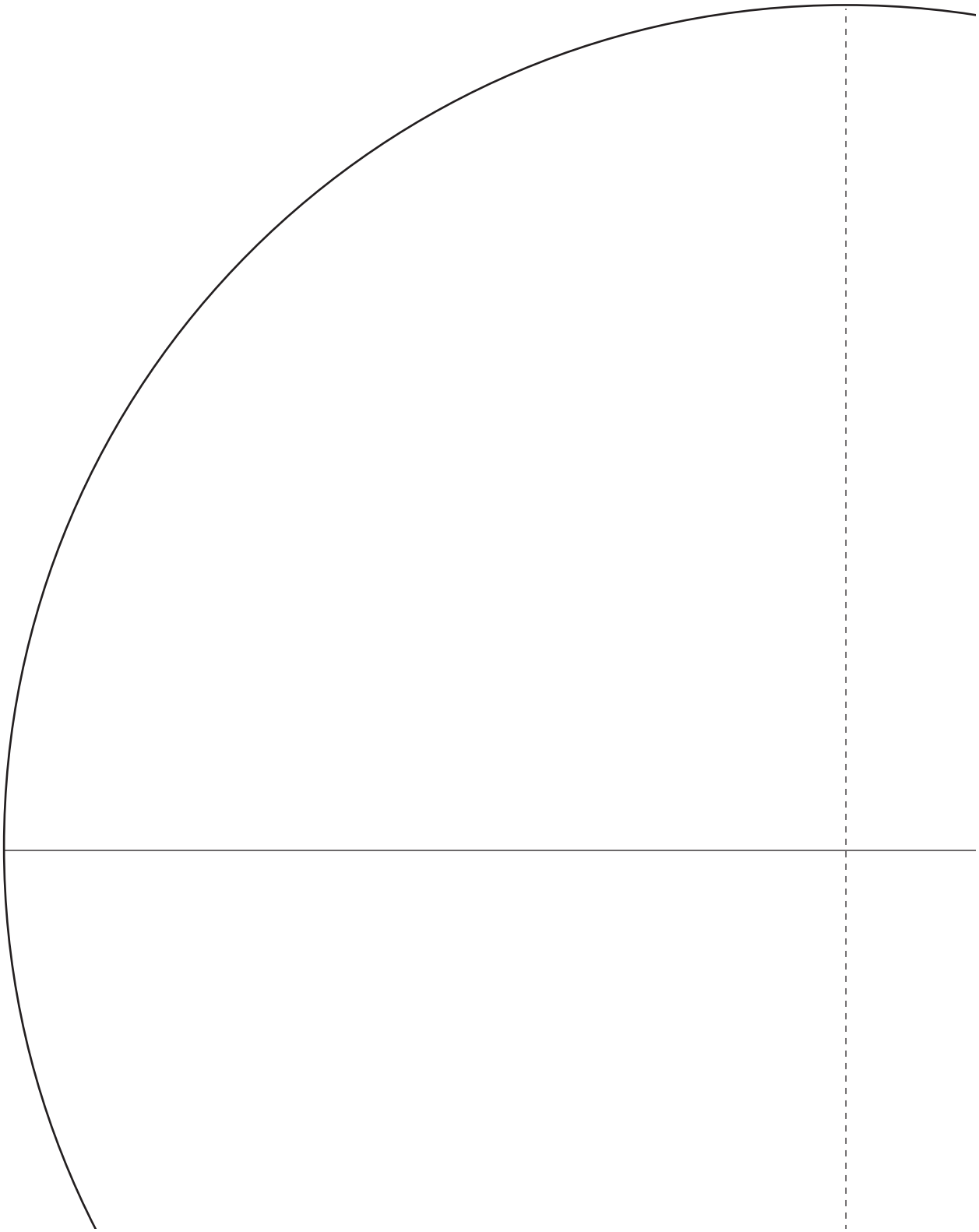
Planet Cutouts—Earth



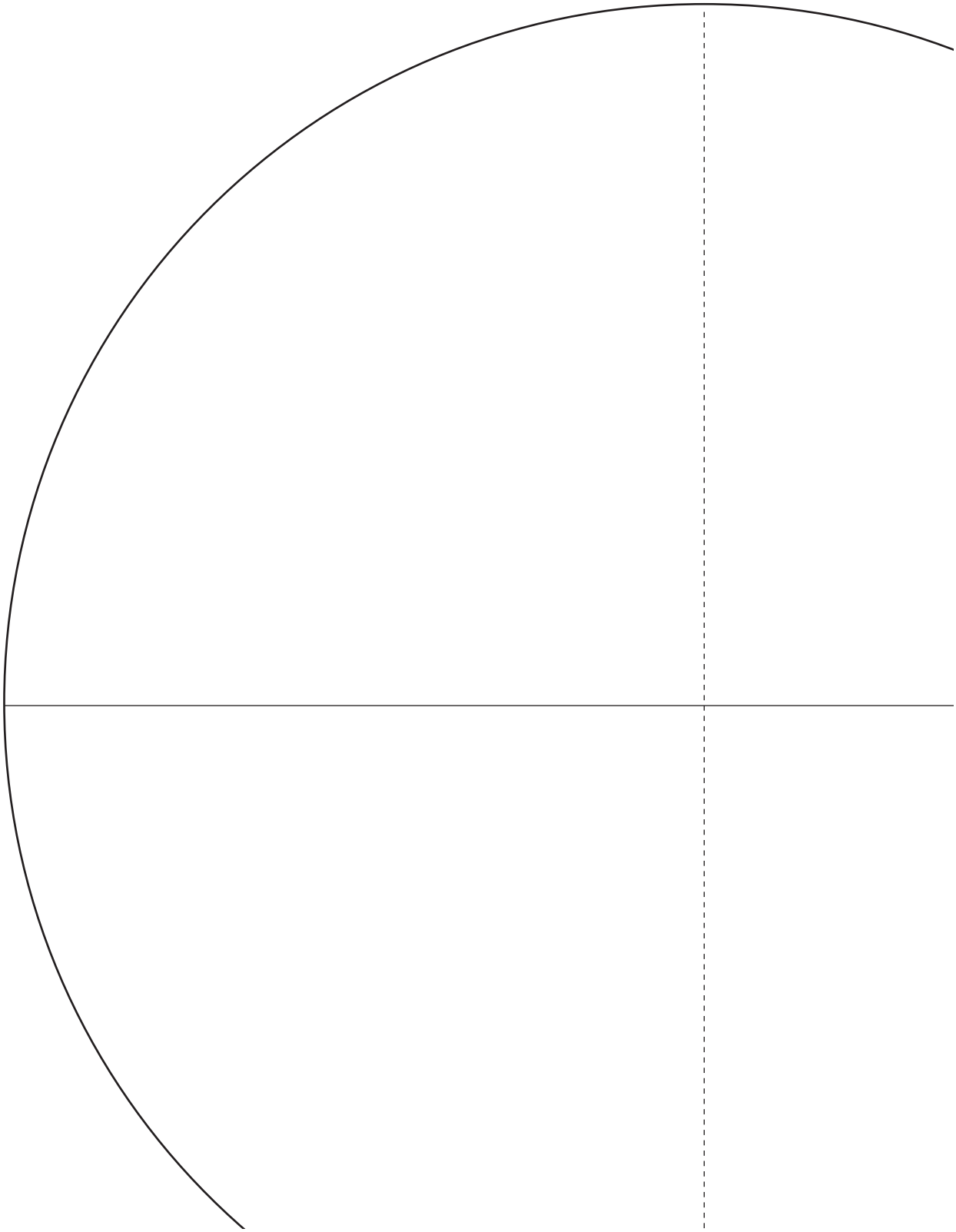
Planet Cutouts—Mars



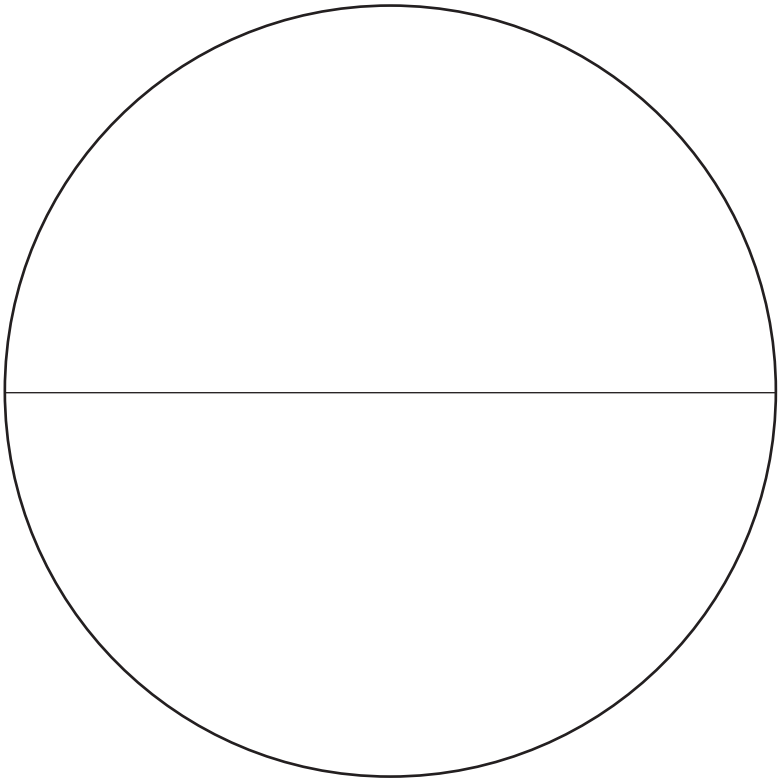
Planet Cutouts—Jupiter

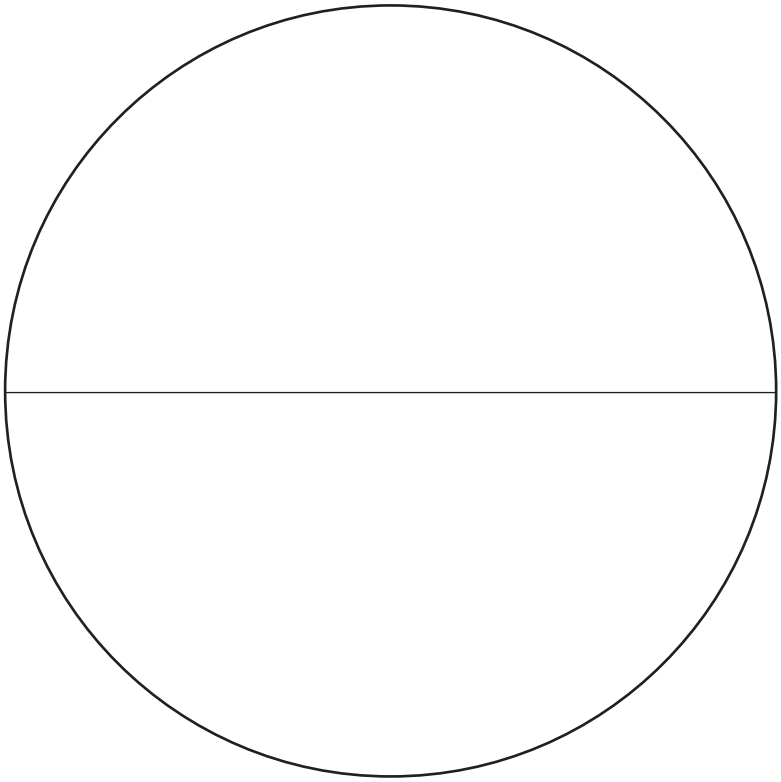


Planet Cutouts–Saturn

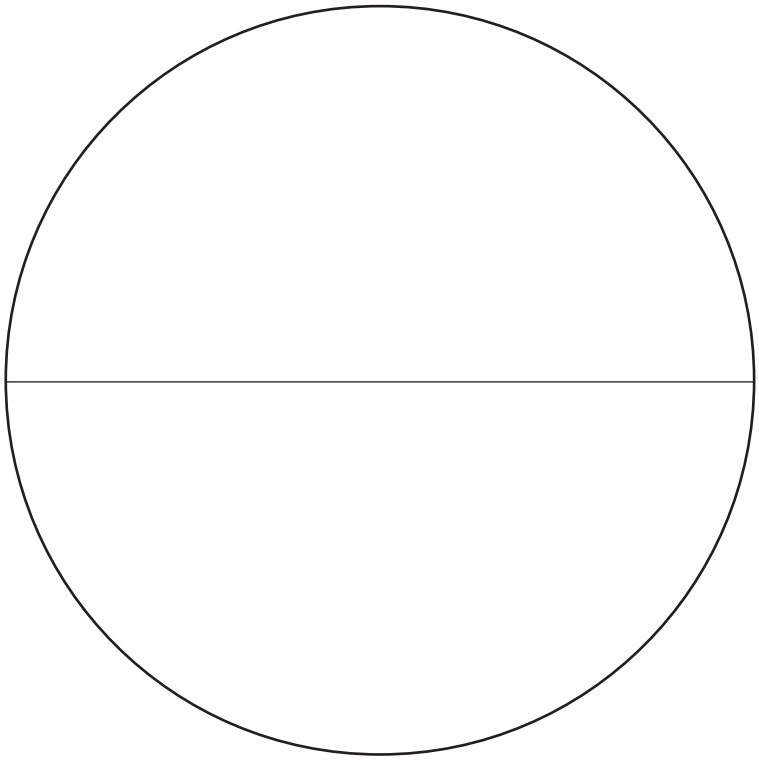


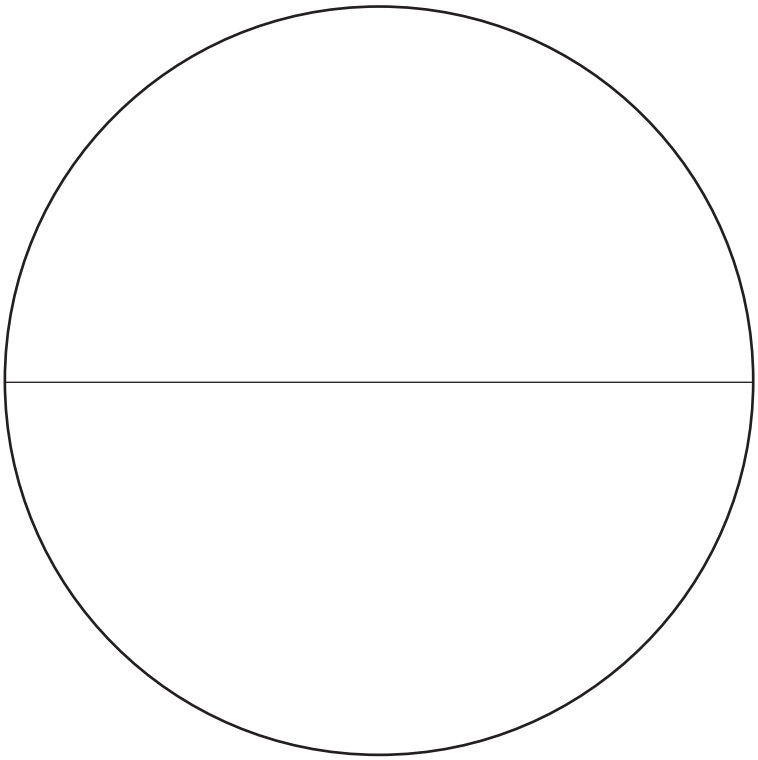
Planet Cutouts—Uranus



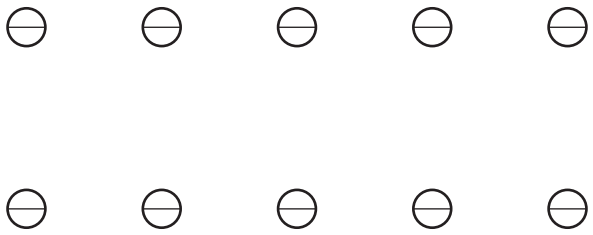


Planet Cutouts–Neptune





Planet Cutouts—Pluto



Name _____

It's All Relative 1

Planet	Actual Diameter (km)	Scale Diameter (cm)	x π	Circumference (cm)
Mercury	4,879		x π	3.1
Venus	12,104		x π	7.5
Earth	12,756		x π	7.9
Mars	6,794		x π	4.1
Jupiter	142,984		x π	89.8
Saturn	120,536		x π	75.7
Uranus	51,118		x π	32.0
Neptune	49,528		x π	31.1
Pluto	2,360		x π	1.57

Explain (in writing) the strategy you used to find the correct planet.

Name _____

Planet: _____

It's All Relative 2

Planet	Approximate Distance to Sun (km)	Scientific Notation	Divide by 30	TP Sheets to Sun
Mercury	57,909,175		÷ 30	
Venus	108,208,930		÷ 30	
Earth	149,597,890		÷ 30	
Mars	227,936,640		÷ 30	
Jupiter	778,412,010		÷ 30	
Saturn	1,426,725,400		÷ 30	
Uranus	2,870,972,200		÷ 30	
Neptune	4,498,252,900		÷ 30	
Pluto	5,905,376,200		÷ 30	

It's All Relative 1
Teacher Key

Planet	Actual Diameter (km)	Scale Diameter (cm)	x π	Circumference (cm)
Mercury	4,879	1.0	x π	3.1
Venus	12,104	2.4	x π	7.5
Earth	12,756	2.5	x π	7.9
Mars	6,794	1.3	x π	4.1
Jupiter	142,984	28.6	x π	89.8
Saturn	120,536	24.1	x π	75.7
Uranus	51,118	10.2	x π	32.0
Neptune	49,528	9.9	x π	31.1
Pluto	2,360	0.5	x π	1.57

Explain (in writing) the strategy you used to find the correct planet.

It's All Relative 2
Teacher Key

Planet	Approximate Distance to Sun (km)	Scientific Notation	Divide by 30	TP Sheets to Sun
Mercury	57,909,175	58×10^6	÷ 30	1.9
Venus	108,208,930	108×10^6	÷ 30	3.6
Earth	149,597,890	150×10^6	÷ 30	5
Mars	227,936,640	228×10^6	÷ 30	7.6
Jupiter	778,412,010	778×10^6	÷ 30	25.9
Saturn	1,426,725,400	$1{,}427 \times 10^6$	÷ 30	47.6
Uranus	2,870,972,200	$2{,}871 \times 10^6$	÷ 30	95.7
Neptune	4,498,252,900	$4{,}498 \times 10^6$	÷ 30	149.9
Pluto	5,905,376,200	$5{,}905 \times 10^6$	÷ 30	196.8

Appendix

Name _____

Number Notation Table

number	numeral	scientific notation
one		
ten		
hundred		
thousand		
ten thousand		
hundred thousand		
million		
ten million		
hundred million		
billion		
ten billion		
hundred billion		
trillion		

Describe any patterns you see in the table:

Name _____

Planet Table

Complete the following table:

Planet	Miles from the sun	Miles from the sun in scientific notation
Mercury	35,000,000	
Venus		6.5×10^7
Earth		9.3×10^7
Mars	137,000,000	
Jupiter	467,000,000	
Saturn	850,000,000	
Uranus		1.7×10^9
Neptune		2.7×10^9
Pluto	3,500,000,000	

Number Cards

0	1
2	3
4	5

6	7
8	9
.	,

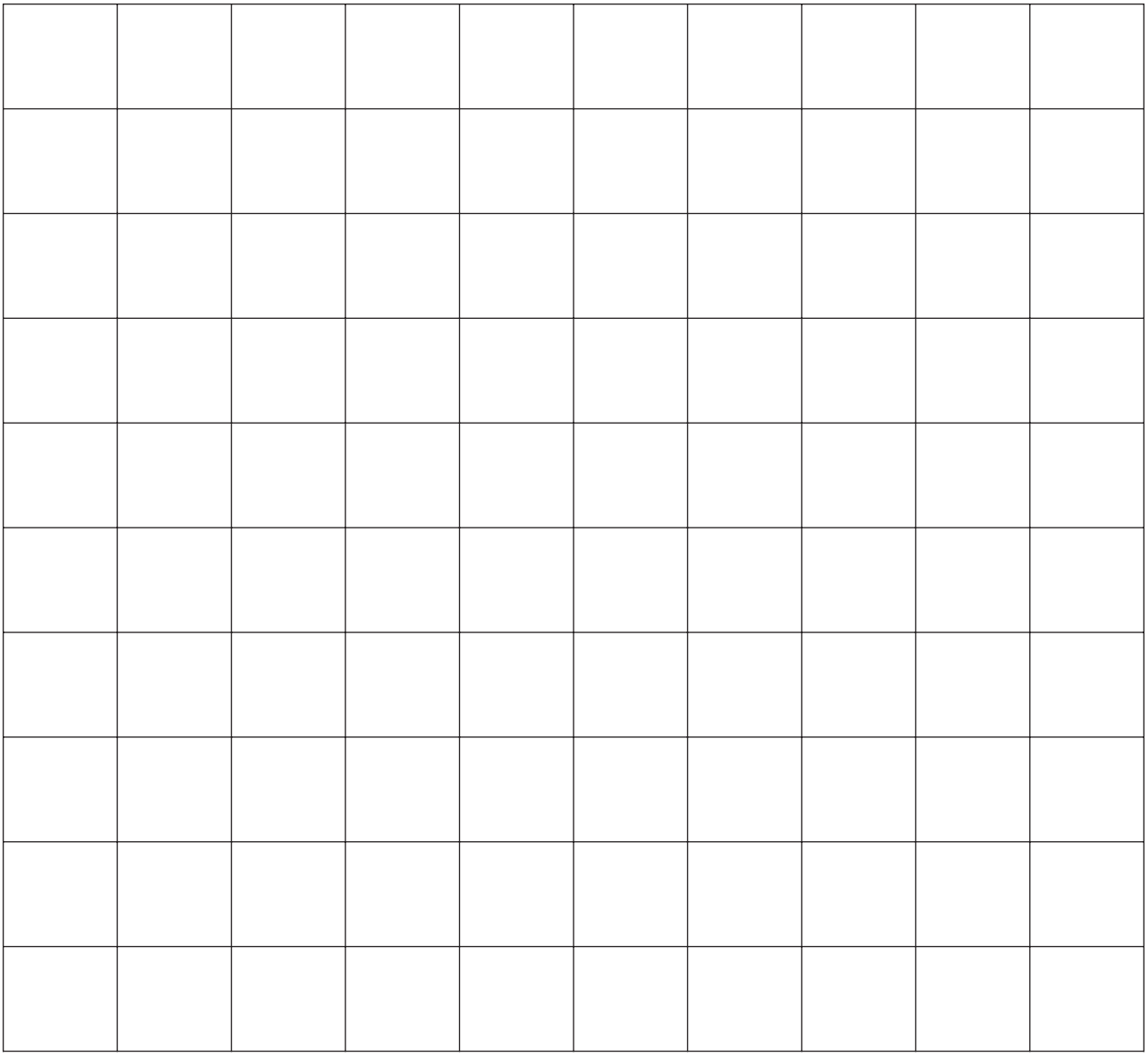
Name _____

Sieve of Eratosthenes

1. Put your finger on number 2. Count by twos and color the top left corner of each number square yellow.
2. Put your finger on number 3. Count by threes and color the top right corner of each number square orange.
3. Put your finger on number 5. Count by fives and color the bottom left corner of each number square green.
4. Put your finger on number 7. Count by sevens and color the bottom right corner of each number square blue.
5. Color the remaining boxes red.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Manipulative Master



Name _____

Prove or Disprove

Prove or disprove these statements:

1. Only three of the numbers below are prime numbers.

   123 635 587 247 198 233 267 307 7101

2. There are 25 prime numbers between 1 and 100 and 25 prime numbers between 100 and 200.

3. Each of the whole numbers between 1 and 50 can be written as a product of two or more prime numbers.

   Example: 20 = 2 x 2 x 5

4. Every number from 1 to 100 can be written as the sum of two prime numbers.

   Example: 18 = 7 + 11

Create three more prove or disprove statements of your own.

_____

_____

_____

_____

_____

_____

_____

_____

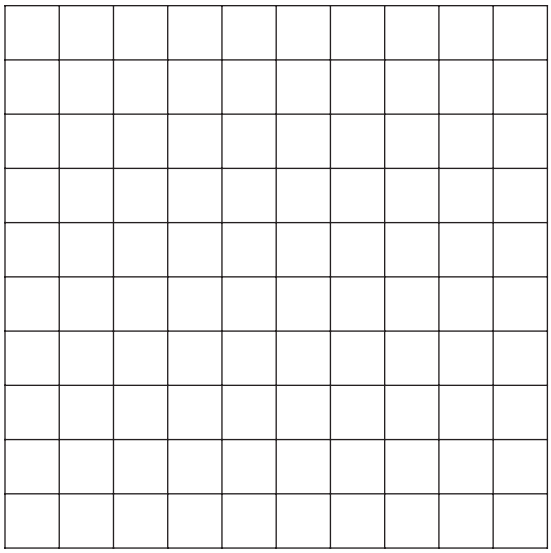
_____

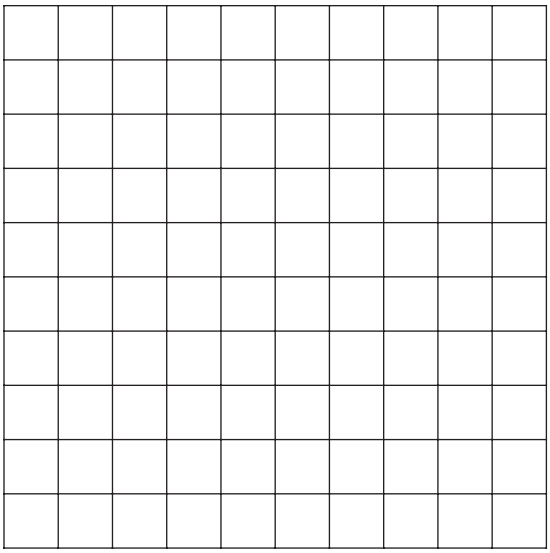
_____

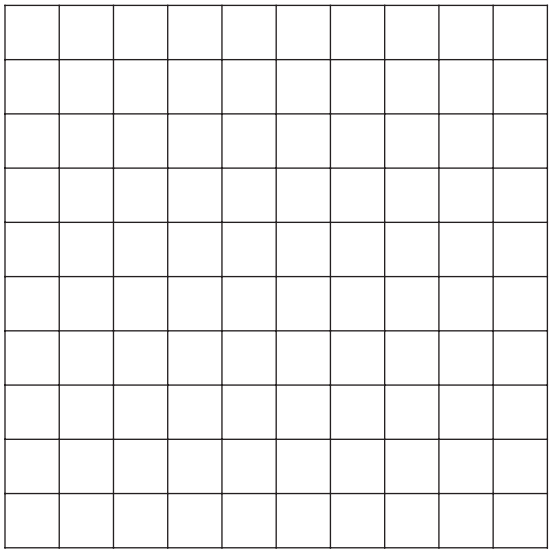
_____

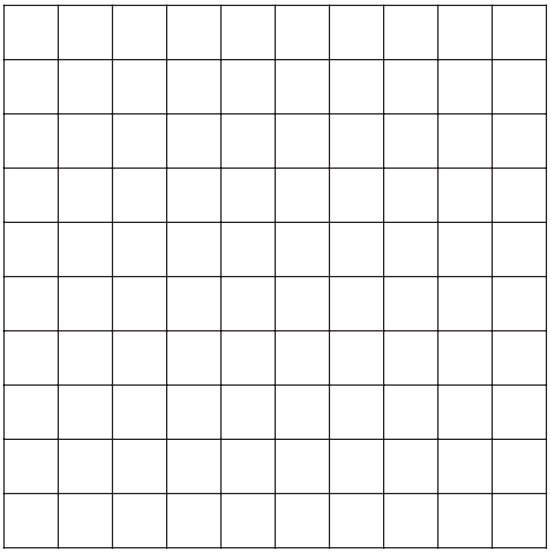
Name _____

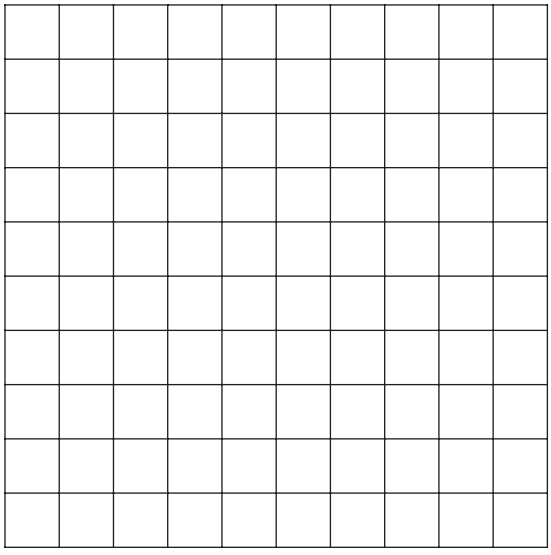
Base 10

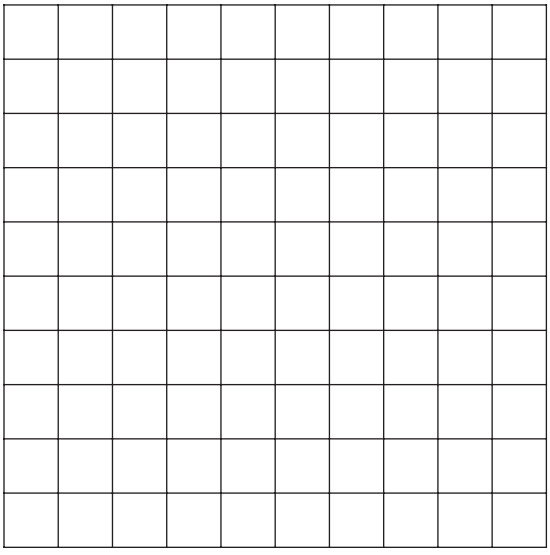






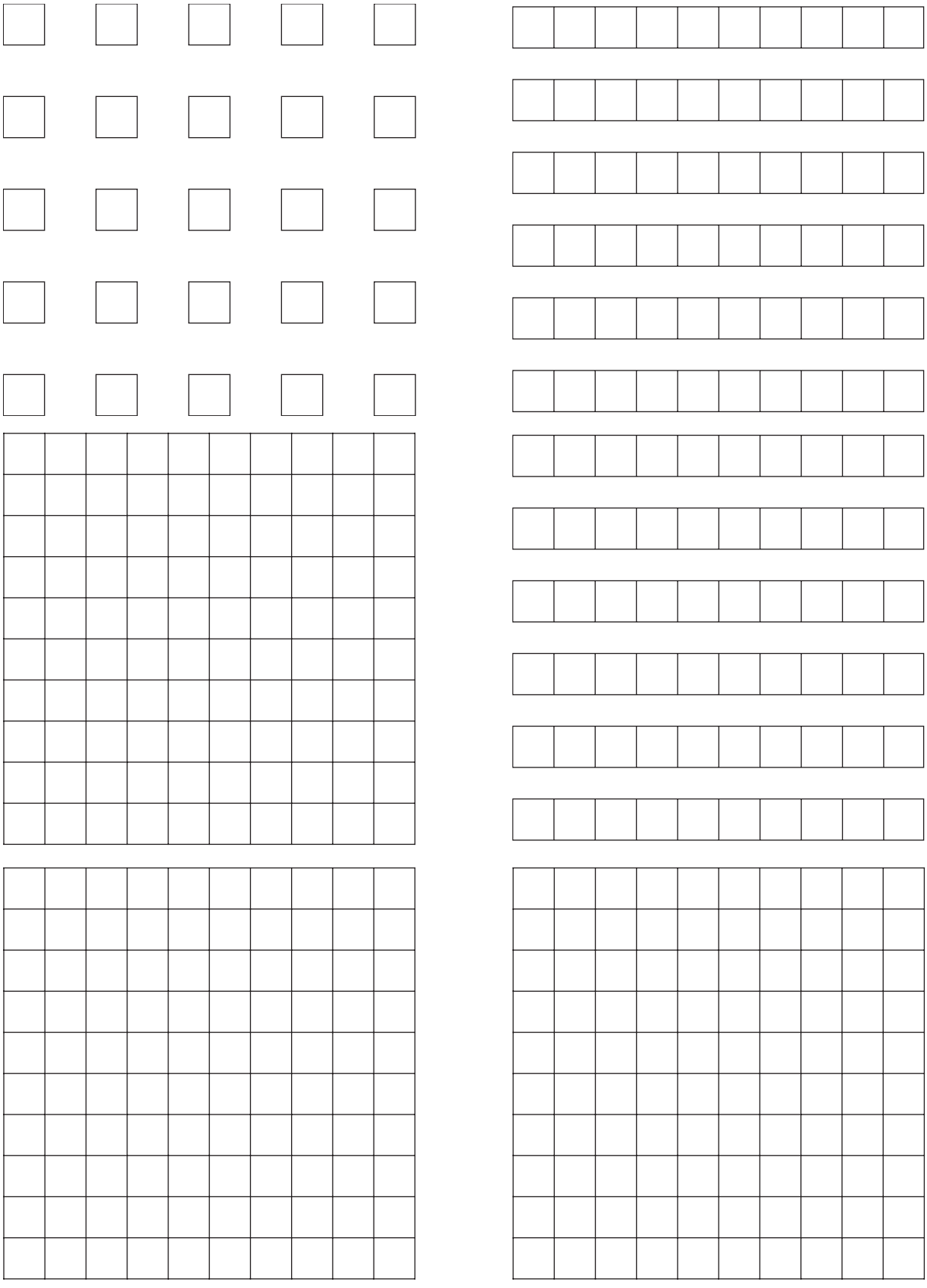




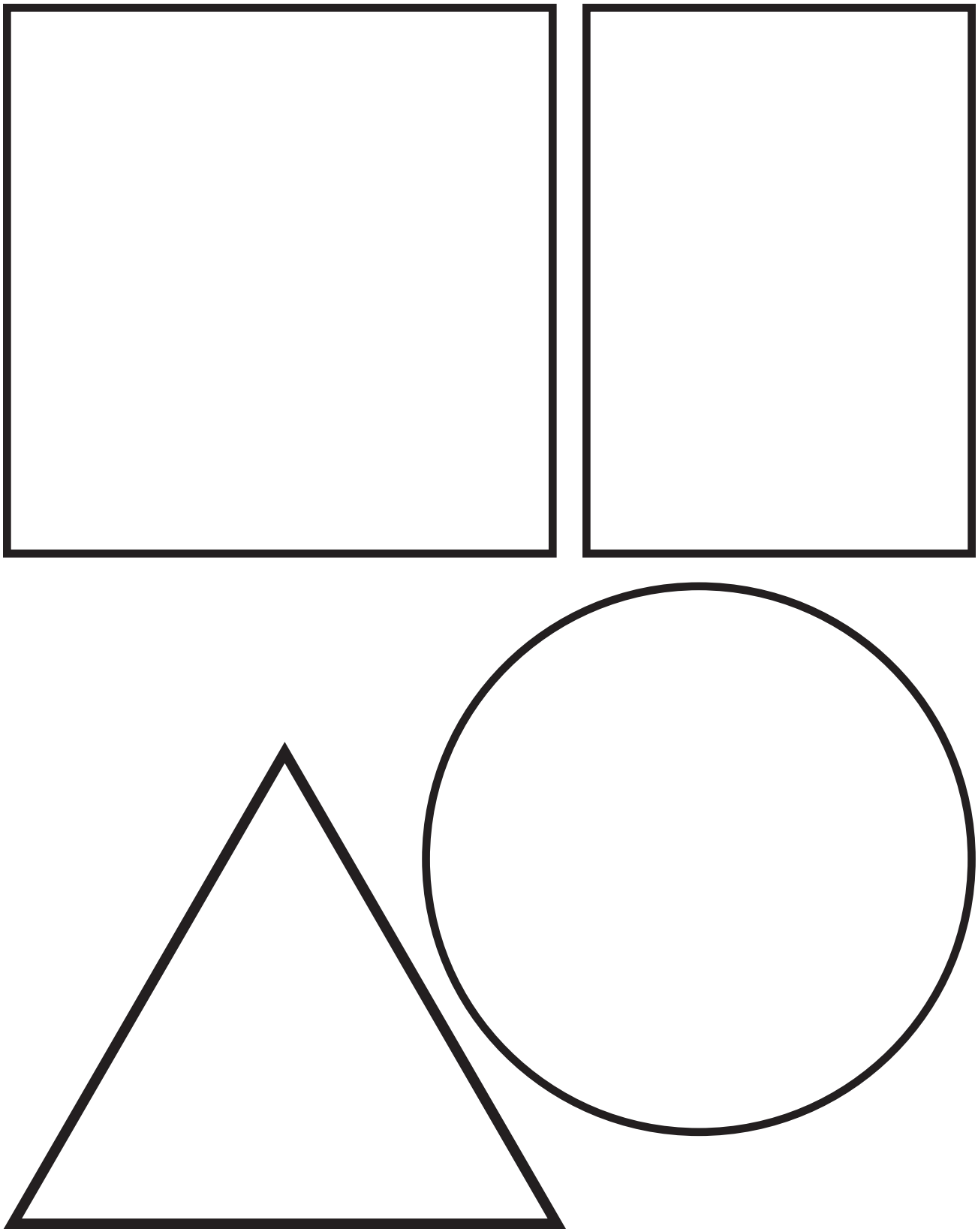


Name _____

Base 10 Blocks



Geometric Shapes



Number Cards

1/10	0.1	10%
1/5	0.2	20%
1/4	0.25	25%

3/10	0.3	30%
2/5	0.4	40%
1/2	0.5	50%

3/5	0.6	60%
7/10	0.7	70%
3/4	0.75	75%

4/5	**0.8**	**80%**
9/10	**0.9**	**90%**
1	**1.0**	**100%**

3/20	0.15	15%
7/20	.35	35%
9/20	.45	45%

11/20	**0.55**	**55%**
13/20	**0.65**	**65%**
17/20	**0.85**	**85%**

19/20	0.95	95%
1/3	0.3	$33\frac{1}{3}\%$
2/3	0.6	$66\frac{2}{3}\%$

1/8	0.125	12.5%
3/8	0.375	37.5%
5/8	0.625	62.5%

7/8	0.875	87.5%
6/15	0.4	40%
4/8	0.5	50%

Name _____

Which Salary is the Best?

You want to buy a go-cart for $1000 and need to find a job to raise the money. You found job openings to mow lawns with two different companies that have different pay scales. One company, Lawns Are Us, will double your salary each day. You will earn $1 the first day, $2 the second day, $4 the third day, $8 the fourth day, and so on. The second company, Smith Lawn Care will increase your salary by $4 each day. You will make $4 the first day, $8 the second day, $12 the third day, $16 the fourth day, and so on. Which company will help you reach the $1,000 needed to buy a go-cart the fastest?

Lawns Are Us			**Smith Lawn Care**		
Day	**Daily Salary (Dollars)**	**Total Earnings (Dollars)**	**Day**	**Daily Salary (Dollars)**	**Total Earnings (Dollars)**
1	1		1	4	
2	2		2	8	
3	4		3	12	
4	8		4	16	
5	16		5	20	
6			6		
7			7		
8			8		
9			9		
10			10		
11			11		
12			12		
13			13		
14			14		
15			15		
16			16		
17			17		
18			18		
19			19		
20			20		
21			21		
22			22		
23			23		
24			24		
25			25		
26			26		
*n*th			*n*th		

Name _____

Patterns in Measurement

Length of Side (cm)	Perimeter (cm)	Area (cm^2)	Volume (cm^3)
1			
2			
3			
4			
5			
6			
7			
8			

1. What pattern do you notice in the perimeter results? Write a rule for the pattern.

   _____

   _____

   _____

2. What pattern do you notice in the area results? Write a rule for the pattern.

   _____

   _____

   _____

3. What pattern do you notice in the volume results? Write a rule for the pattern.

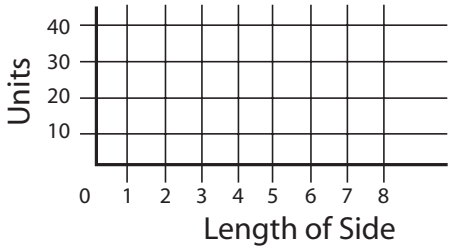
_____

_____

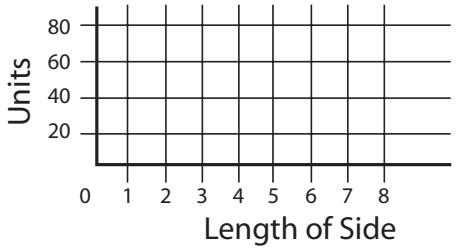
_____

Use the information from the table to draw the following graphs.

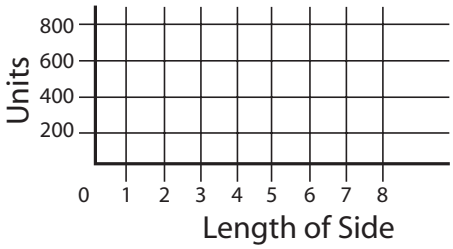
Perimeter



Area



Volume



4. Describe the shape of the perimeter graph.

_____

_____

_____

5. Describe the shape of the area graph.

_____

_____

_____

6. Describe the shape of the volume graph.

_____

_____

_____

Name _____

Balance or Tilt?

1. Find the values of the expressions on each side of the scale. Is the scale balanced or is it tilted? If the scale does not balance, write which side will tilt down. Explain your answer.

x = 5

_____

_____

s = 4

_____

_____

n = 4

s = 1

_____

_____

2. Find the variables that balance the scales.

s = _____

n = _____

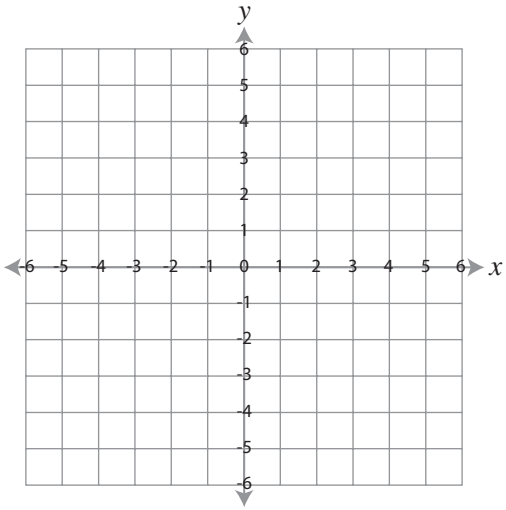
f = _____

x = _____

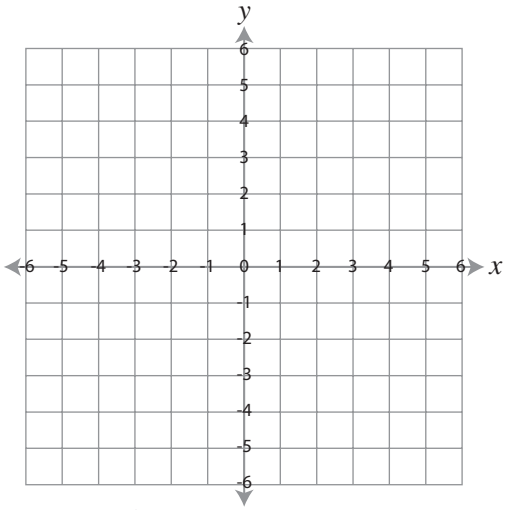
Name _____

Transforming Triangles

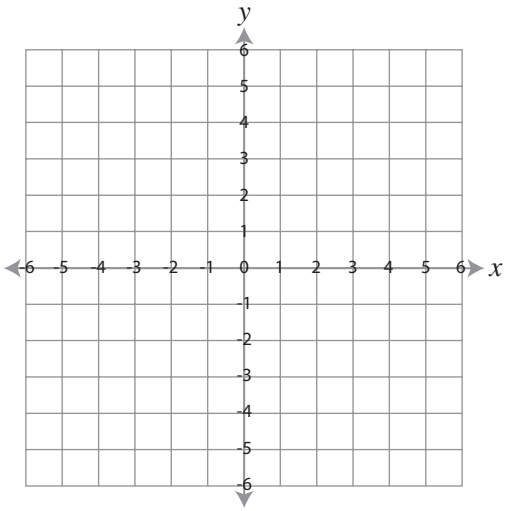
Grid 1—Translate (Slide)



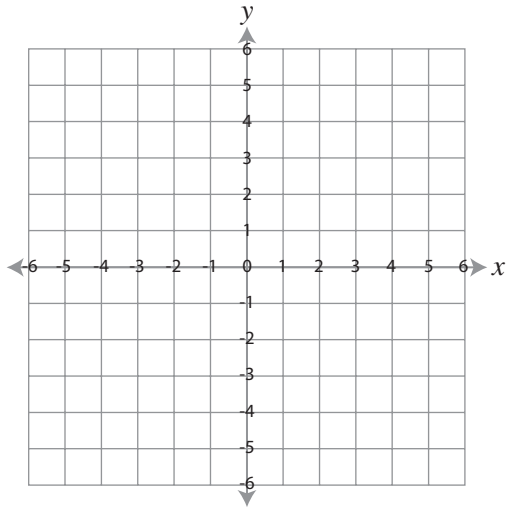
Grid 2—Reflect (Flip) across the x-axis



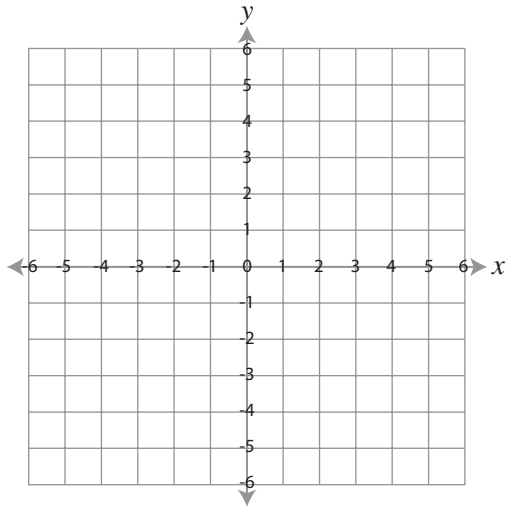
Grid 3—Reflect (FlipP across the y-axis



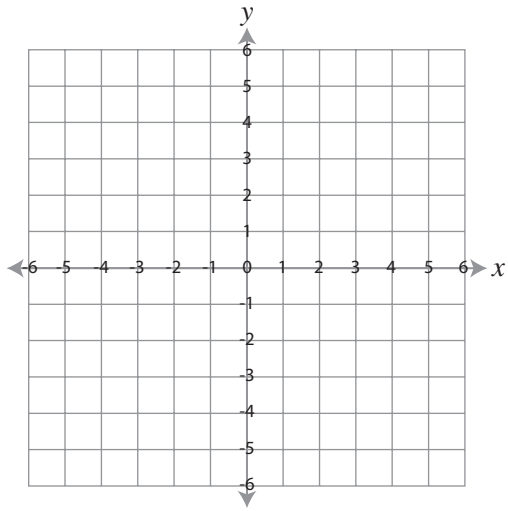
Grid 4—Translate (Slide)



Grid 5—Translate (Slide)

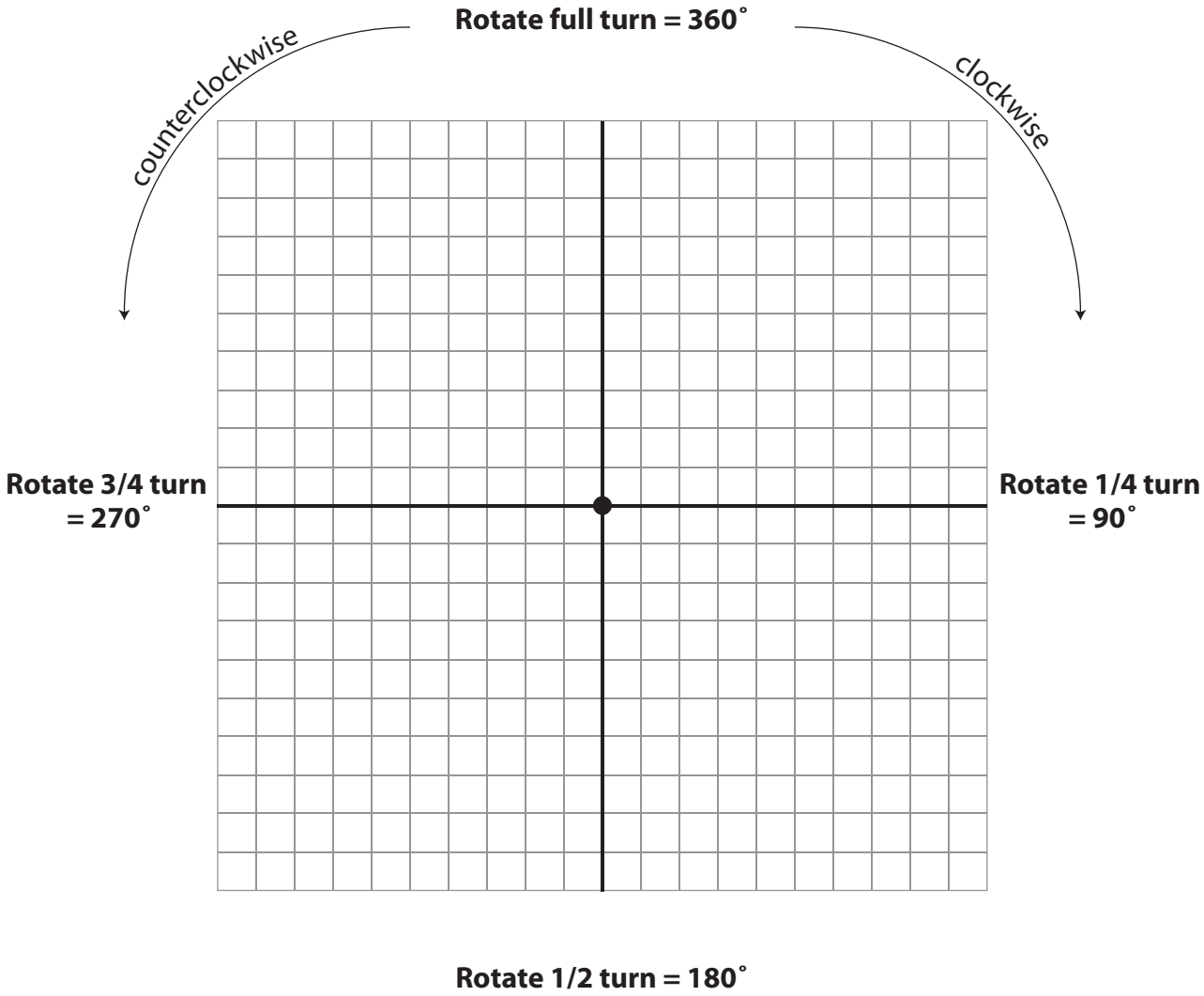


Grid 6—Reflect (Flip) across the x-axis



Name _____

Rotating Triangles



Name _____

Let's Get Weighing

1. What type of fruit did you bring? _____
2. Estimate what percent of the fruit is edible. _____
3. Estimate what percent of the fruit is nonedible. _____
4. Do your two estimates equal 100? _____

   If they do not, adjust your estimates so they equal 100.

Scale Type	Try #1		Try #2		Try #2		Avg. Weight	
	Metric	Standard	Metric	Standard	Metric	Standard	Metric	Standard

What is the cost?

1. What was the cost of your fruit per pound? _____
2. How much did your piece of fruit cost? _____
3. How much did the edible portion cost? _____
4. How much did the nonedible portion cost? _____

Name _____

What Weighs More?

1. Which weighs more—
   A cup of uncooked rice or a cup of uncooked beans?

2. Which weighs more—
   A cup of liquid milk or a cup of powdered milk?

3. Which weighs more—
   A cup of sugar or a cup of flour?

4. Which weighs more—
   A cup of sand or a cup of powdered sugar?

5. Which weighs more—
   A cup of granulated sugar or a cup of brown sugar?

6. Which weighs more—
   A cup of salt or a cup of shredded newspaper?

Bonus Questions

1. Which weighs more—
   A cup of skim milk or a cup of whole milk?

2. Which weighs more—
   A cup of water or a cup of dehydrated water?

Name _____

Looking for Lengths in All the Right Places

Part A: Searching in the classroom

Use the descriptions below to find something that you can estimate as the same length. After estimating, use a tool to measure its exact length and compare.

Description	Item	Estimation	Measurement
Something longer than 50 centimeters			
Something shorter than 100 millimeters			
Something bigger than 15 feet			
An item that is twice your height			
An item that is twice as wide as it is tall			
Something that is about the same length as width			
Find one item. Find a second item that is about twice as long as the first.	1. 2.	1. 2.	1. 2.

Part B: City Search

Using a map, find cities that match the following:

1. Find two cities that are about 200 miles apart.

2. Find the distance between Los Angeles, California, and your city.

3. Find the distance between San Francisco, California, and Baltimore, Maryland.

Name _____

Saving Sir Cumference

Help Radius and Lady Di of Ameter save Sir Cumference! The answer to this problem lies in the poem. Can you solve the riddle?

Part A

Carefully read the poem, *The Circle's Measure*. Then answer the following questions:

1. What do you think is meant by "measure the middle and circle around?"

2. According to this poem, what two numbers should you "divide?"

The Circle's Measure

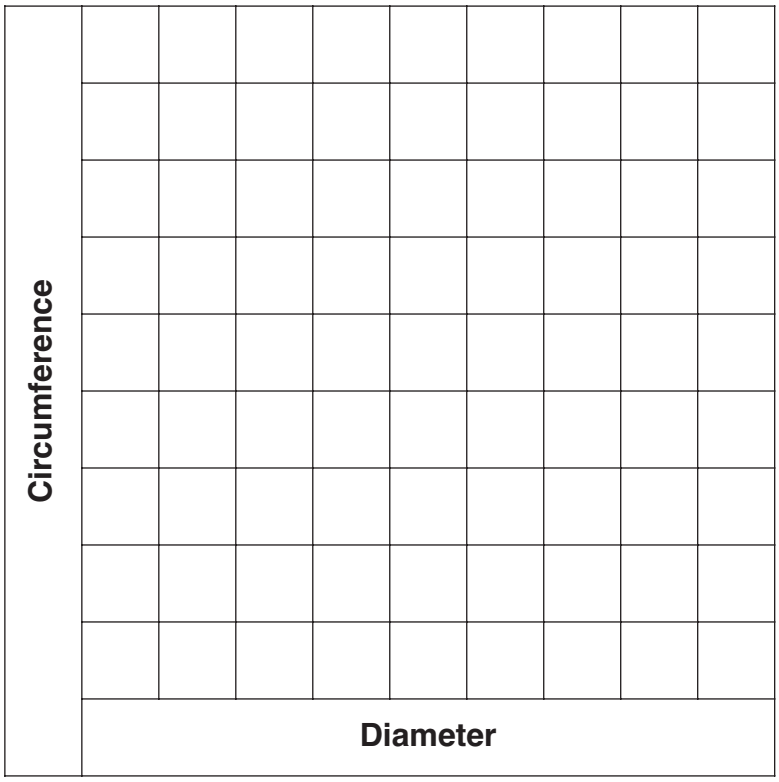
Measure the middle and
circle around,
Divide so a number can be
found.
Every circle, great and
small—
The number is the same
for all.
It's also the dose, so be
clever,
Or a dragon he will stay…
forever...

Part B

Use a measuring tape to find the diameter and circumference of several circular objects. Discuss with your group how to record your data. Create a table below.

Part C

Use a piece of centimeter grid paper to make a coordinate graph like the one below. Use the horizontal axis for diameter and the vertical axis for circumference. Plot the data points for each object your group measured on the graph.



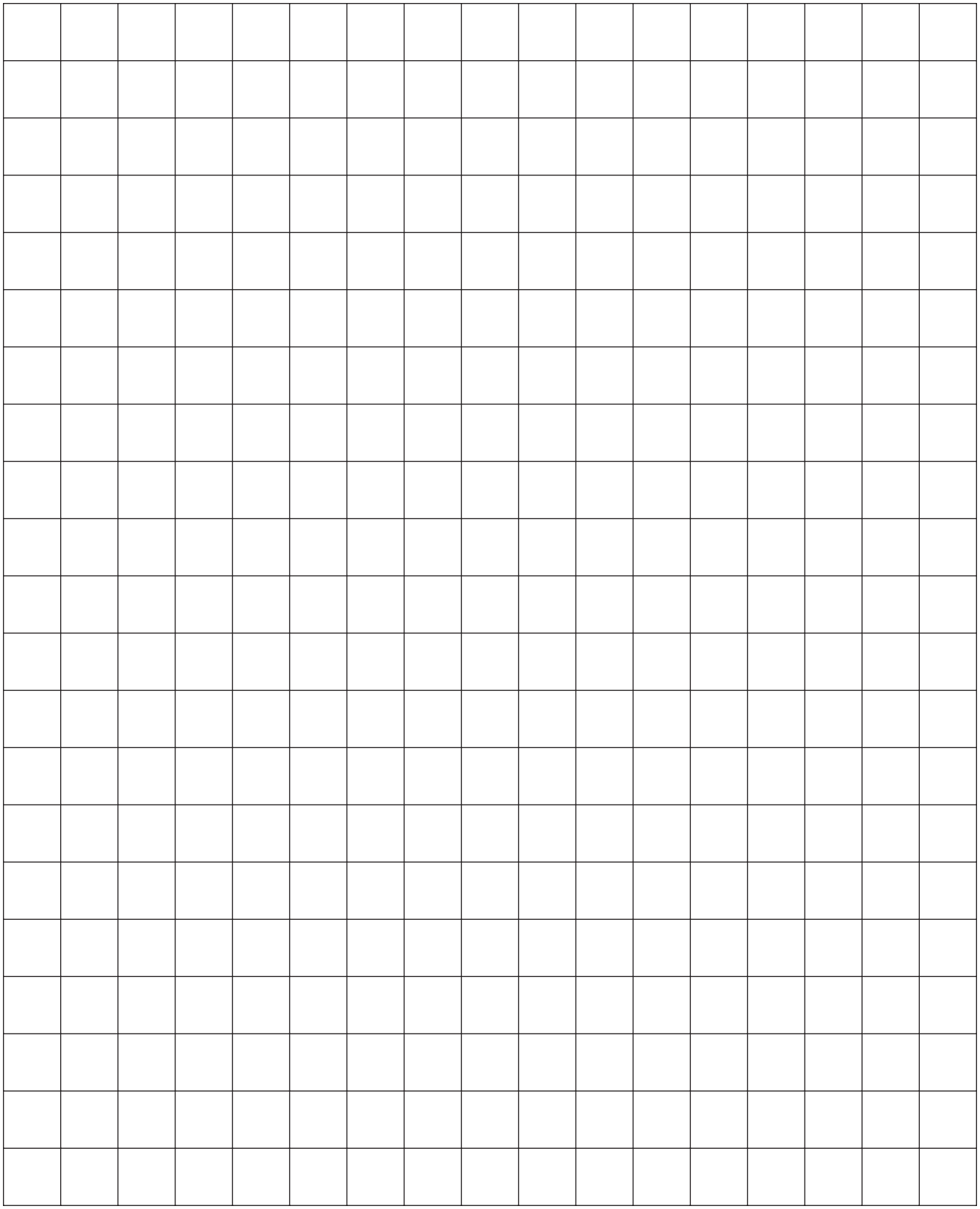
Part D

Study your table and your graph. Look for patterns and relationships that allow you to predict the circumference using the diameter. Test your ideas on other circular objects. Once you think you have found a pattern, answer the questions below:

1. What is the relationship between the diameter and the circumference of a circle?

2. What dose should Radius give Sir Cumference? How do you know this is the correct dose?

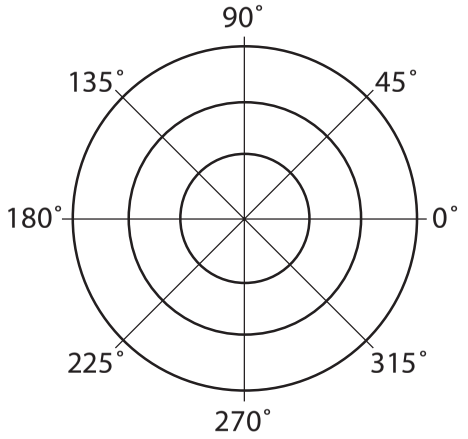
Centimeter Grid



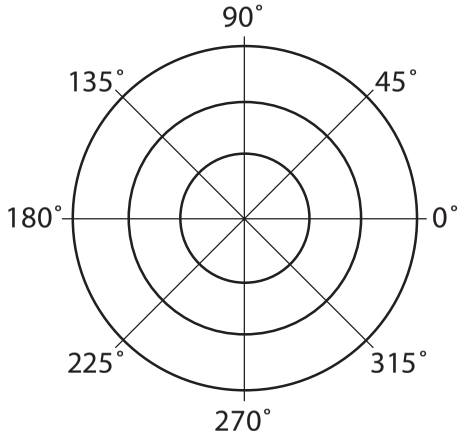
Name _____

Tomb Robbers Game Board 1

My Hidden Treasure		My Partner's Hidden Treasure	
Item	**Coordinates**	(,)	(,)
sarcophagus	(,)	(,)	(,)
scarab	(,)	(,)	(,)
amulet	(,)	(,)	(,)
canopic jars	(,)	(,)	(,)



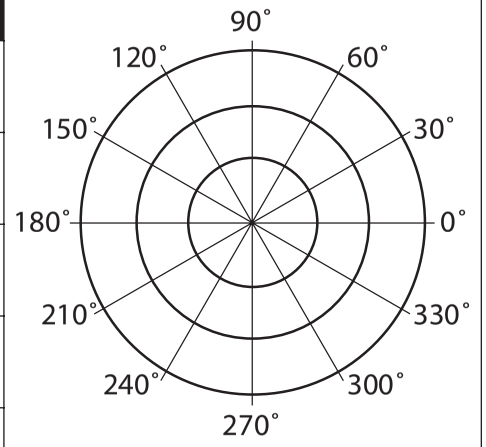
My Hidden Treasure		My Partner's Hidden Treasure	
Item	**Coordinates**	(,)	(,)
sarcophagus	(,)	(,)	(,)
scarab	(,)	(,)	(,)
amulet	(,)	(,)	(,)
canopic jars	(,)	(,)	(,)



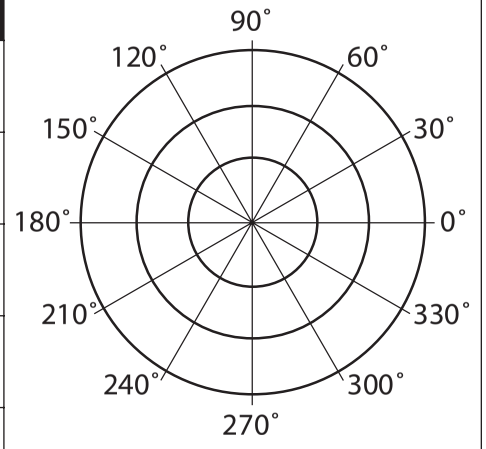
Name _____

Tomb Robbers Game Board 2

My Hidden Treasure		My Partner's Hidden Treasure	
Item	**Coordinates**	(,)	(,)
sarcophagus	(,)	(,)	(,)
scarab	(,)	(,)	(,)
amulet	(,)	(,)	(,)
canopic jars	(,)	(,)	(,)

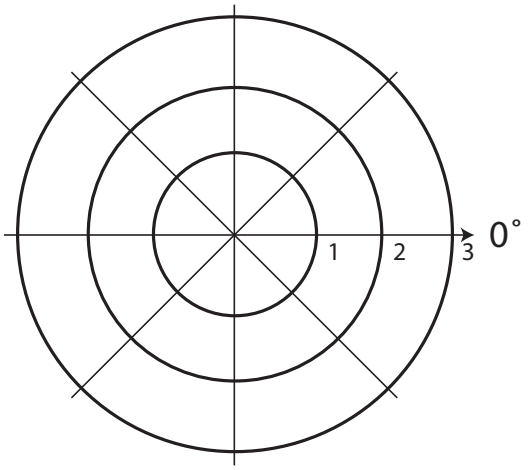


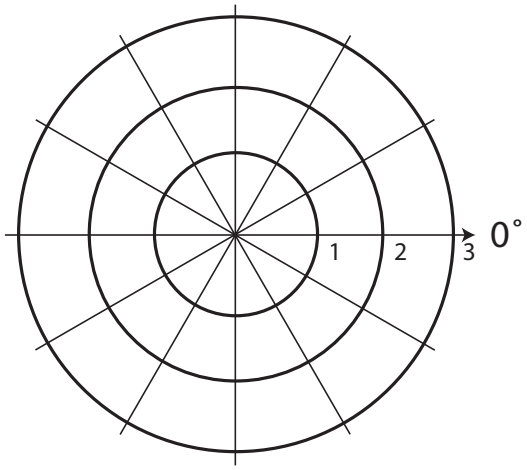
My Hidden Treasure		My Partner's Hidden Treasure	
Item	**Coordinates**	(,)	(,)
sarcophagus	(,)	(,)	(,)
scarab	(,)	(,)	(,)
amulet	(,)	(,)	(,)
canopic jars	(,)	(,)	(,)

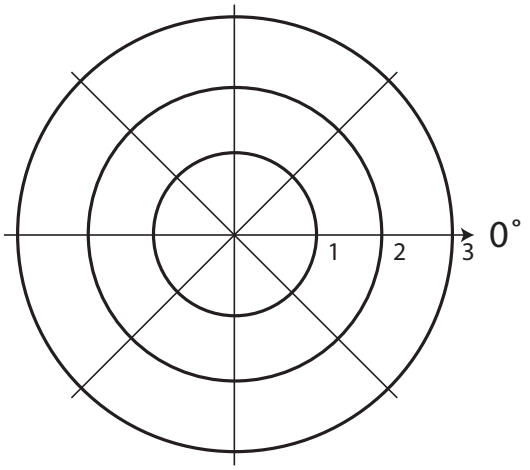


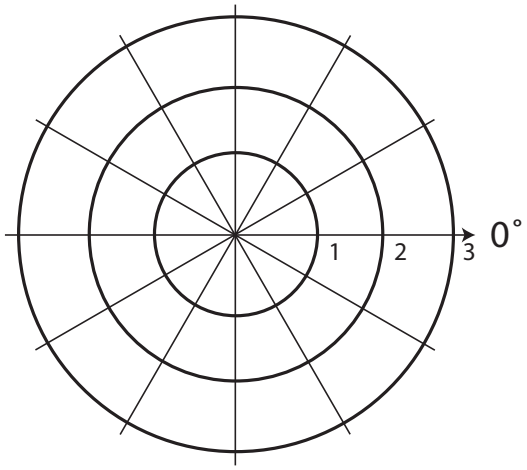
Name _____

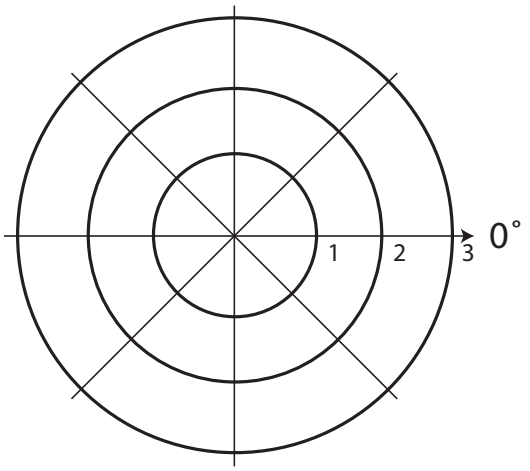
Four-In-A-Row Game Boards

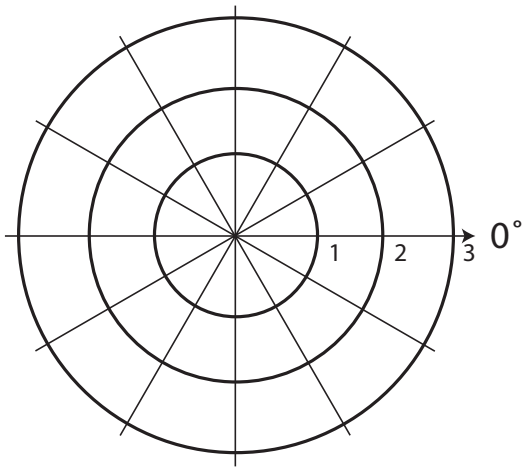








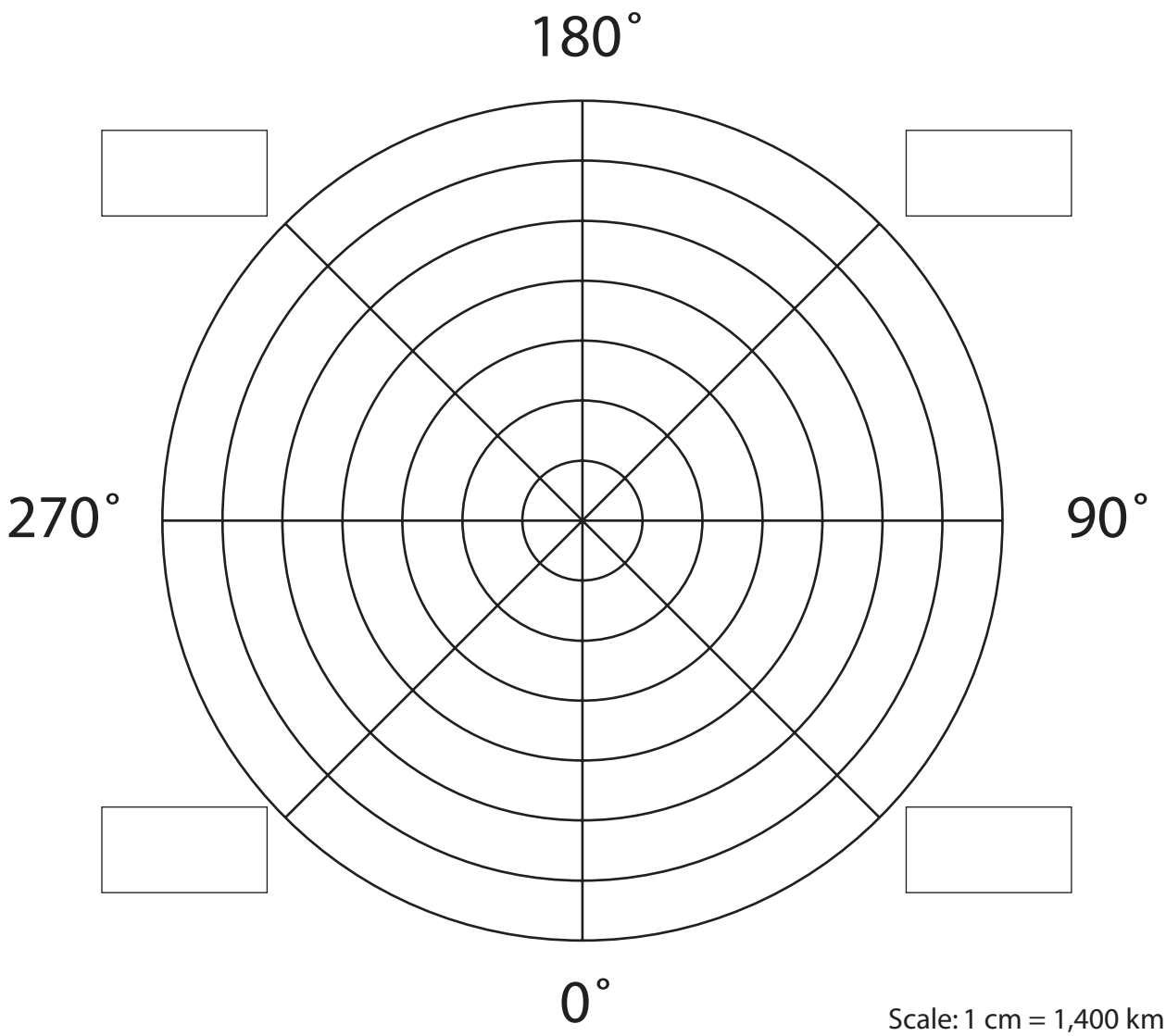




Name _____

Where to Find An Aurora

Outer Ring of Auroral Oval				Inner Ring of Auroral Oval			
Point 1	(90, 65)	Point 7	(0, 60)	Point 1	(90, 78)	Point 7	(0, 75)
Point 2	(135, 64)	Point 8	(320, 63)	Point 2	(135, 72)	Point 8	(320, 72)
Point 3	(180, 60)	Point 9	(315, 60)	Point 3	(180, 70)	Point 9	(315, 70)
Point 4	(225, 55)	Point 10	(300, 60)	Point 4	(225, 67)	Point 10	(300, 67)
Point 5	(270, 50)	Point 11	(245, 50)	Point 5	(270, 65)	Point 11	(245, 62)
Point 6	(45, 63)	Point 12	(200, 58)	Point 6	(45, 67)	Point 12	(200, 70)



Scale: 1 cm = 1,400 km

Approximate Width:

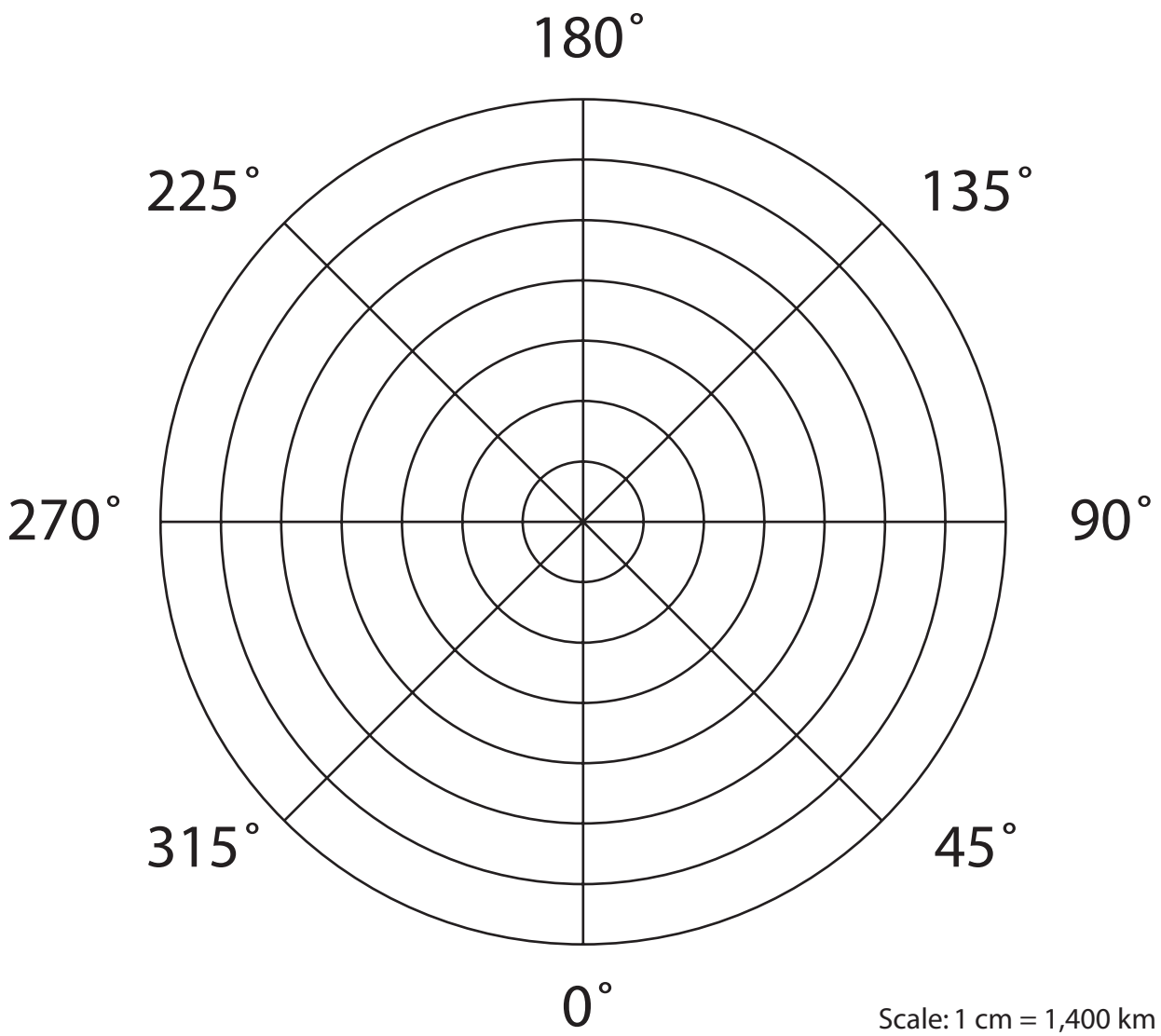
Shortest Distance: _____ km Longest Distance: _____ km Range: _____ km

Name _____

Southern Lights

Auroras are also visible in the southern hemisphere. Use the diagram below to find and correctly label the coordinates for missing points.

Outer Ring of Auroral Oval				Inner Ring of Auroral Oval			
Point 1	(225, 45)	Point 7	(135, 62)	Point 1	(225, 65)	Point 7	(135, 67)
Point 2	(45, 57)	Point 8	(67, 55)	Point 2	(45, 70)	Point 8	(67, 65)
Point 3	(270, 52)	Point 9	(180, 52)	Point 3	(270, 62)	Point 9	(180, 66)
Point 4	(350, 60)	Point 10	(157, 60)	Point 4	(350, 69)	Point 10	(157, 70)
Point 5	(112, 50)	Point 11	(247, 50)	Point 5	(112, 62)	Point 11	(247, 65)
Point 6	(315, 57)	Point 12	(292, 55)	Point 6	(315, 70)	Point 12	(292, 67)



Approximate Width:

Shortest Distance: _____ km Longest Distance: _____ km Range: _____ km

Name _____

It's All Relative 1

Planet	Actual Diameter (km)	Scale Diameter (cm)	x π	Circumference (cm)
Mercury	4,879		x π	3.1
Venus	12,104		x π	7.5
Earth	12,756		x π	7.9
Mars	6,794		x π	4.1
Jupiter	142,984		x π	89.8
Saturn	120,536		x π	75.7
Uranus	51,118		x π	32.0
Neptune	49,528		x π	31.1
Pluto	2,360		x π	1.57

Explain (in writing) the strategy you used to find the correct planet.

Name _____

Planet: _____

It's All Relative 2

Planet	Approximate Distance to Sun (km)	Scientific Notation	Divide by 30	TP Sheets to Sun
Mercury	57,909,175		÷ 30	
Venus	108,208,930		÷ 30	
Earth	149,597,890		÷ 30	
Mars	227,936,640		÷ 30	
Jupiter	778,412,010		÷ 30	
Saturn	1,426,725,400		÷ 30	
Uranus	2,870,972,200		÷ 30	
Neptune	4,498,252,900		÷ 30	
Pluto	5,905,376,200		÷ 30	