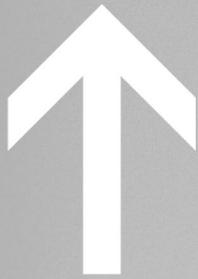


ELEMENTARY CORE ACADEMY

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GRADE



ELEMENTARY CORE ACADEMY

ELEMENTARY CORE ACADEMY

TEACHERS OF UTAH, LOCAL SCHOOL DISTRICTS,
UTAH STATE OFFICE OF EDUCATION, & UTAH STATE UNIVERSITY

a professional
development
resource



UtahState
UNIVERSITY

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Utah State Office of Education (USOE)
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 State Mathematics Education Coordination Committee (SMECC)
 Special Education Services Unit (USOE)
 WestEd Eisenhower Regional Consortium

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Steven O. Laing, Ed.D. State Superintendent of Public Instruction
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Dear Core Academy Teachers:

Your involvement in the Core Academy represents a significant investment by you, your school, and district in educational excellence for the students of Utah.

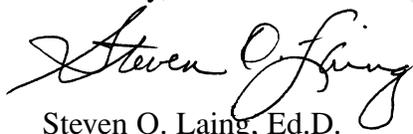
I commend you for your dedication and willingness to engage in meaningful professional growth. Efforts by teachers and administrators to develop, provide, and participate in high quality professional development programs must continue if we desire quality learning experiences for all children.

As the needs of students change, it is critical that educators adjust to meet those needs. Teachers should continue to gain expertise in the collection and use of accurate data and analysis of each student's level of achievement. This investment in accountability will empower teachers, parents, and others educators to be more effective.

Exemplary models of instruction, practical application, and collegial support must be an integral part of all professional development. Embedding sound instructional methods that specifically align to the state Core Curriculum will equip teachers with the skills and tools to meet the needs of Utah students.

It is my belief that educators care deeply about their students and work hard to create successful experiences in the classroom. Despite some challenges facing our schools, dedicated and professional educators make profound differences each day.

Sincerely,



Steven O. Laing, Ed.D.
State Superintendent of Public Instruction

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Major funding for the Academy comes from the following sources:

State Funds:

- Utah State Office of Education
 - Staff Development Funds
 - Special Education Services Unit

- Federal Funds: ESEA Title II

- WestED Eisenhower Regional Consortium

District Funds:

Various sources including Quality Teacher Block, Federal ESEA Title II, and District Professional Development Funds

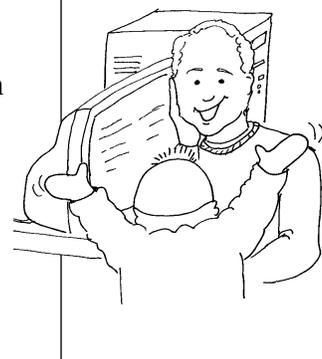
School Funds:

- Trust land, ESEA Title II, and other school funds
- Utah State Office of Education Special Education Services

The state and district funds are allocations from the state legislature. ESEA is part of the "No Child Left Behind" funding that comes to Utah.

Additionally, numerous school districts, individual schools, and principals in Utah have sponsored teachers to attend the Academy. Other educational groups such as the Utah Division of Water Resources, National Energy Foundation, Utah Energy Office, and the Utah Mining Association have assisted in the development and delivery of resources in the Academy.

Most important is the thousands of teachers who take time from their summer to attend these professional development workshops. It is these teachers who make this program possible.



Goals of the Elementary CORE Academy

Overall

The purpose of the Elementary CORE Academy is to create high quality teacher instruction and improve student achievement through the delivery of professional development opportunities and experiences for teachers across Utah.

The Academy will provide elementary teachers in Utah with:

1. Models of exemplary and innovative instructional strategies, tools, and resources to meet newly adopted Core Curriculum standards, objectives, and indicators.
2. Practical models and diverse methods of meeting the learning needs of all children, with instruction implementation aligned to the Core Curriculum.
3. Meaningful opportunities for collaboration, self-reflection, and peer discussion specific to innovative and effective instructional techniques, materials, teaching strategies, and professional practices in order to improve classroom instruction.

Learning a limited set of facts will no longer prepare a student for real experiences encountered in today's world. It is imperative that educators have continued opportunities to obtain instructional skills and strategies that provide methods of meeting the needs of all students. Participants of the Academy experience will be better equipped to meet the challenges faced in today's classrooms.

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***Sixth Grade
Mathematics
Core Curriculum***

Utah Elementary Mathematics Core Curriculum

Introduction

Most students enter school confident in their own abilities; they are curious and eager to learn more. They make sense of the world by reasoning and problem solving. Young students are active, resourceful individuals who construct, modify, and integrate ideas by interacting with the physical world as well as with peers and adults. They learn by doing, collaborating, and sharing their ideas. Students' abilities to communicate through language, pictures, sound, movement, and other symbolic means develop rapidly during these years.

Young students are building beliefs about what mathematics is, about what it means to know and do mathematics, and about themselves as mathematical learners. Mathematics instruction needs to include more than short-term learning of rote procedures. Students must use technology and other mathematical tools, such as manipulative materials, to develop conceptual understanding and solve problems as they do mathematics. Students, as mathematicians, learn best with hands-on, active experiences throughout the instruction of the mathematics curriculum.

Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, the term mathematical proficiency has been chosen to capture what it means to learn mathematics successfully. Mathematical proficiency has five strands: computing (carrying out mathematical procedures flexibly, accurately, efficiently, and appropriately), understanding (comprehending mathematical concepts, operations, and relations), applying (ability to formulate, represent, and solve mathematical problems), reasoning (using logic to explain and justify a solution to a problem), and engaging (seeing mathematics as sensible, useful, and doable, and being able to do the work).

The most important observation about the five strands of mathematical proficiency is that they are interwoven and interdependent. This observation has implications for how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve that goal. At any given moment during a mathematics lesson or unit, one or two strands might be emphasized. But all the strands must eventually be addressed so that the links among them are strengthened. The integrated and balanced development of all five strands of mathematical proficiency should guide

the teaching and learning of school mathematics. Instruction should not be based on extreme positions that students learn solely by internalizing what a teacher or book says or solely by inventing mathematics on their own.

The Elementary Mathematics Core describes what students should know and be able to do at the end of each of the K-6 grade levels. It was developed, critiqued, and revised by a community of Utah mathematics teachers, university mathematics educators, State Office of Education specialists, mathematicians, and an advisory committee representing a wide variety of people from the community. The Core reflects the current philosophy of mathematics education that is expressed in national documents developed by the National Council of the Teachers of Mathematics, the American Association for the Advancement of Science, and the National Research Council. This Mathematics Core has the endorsement of the Utah Council of Teachers of Mathematics Association. The Core reflects high standards of achievement in mathematics for all students.

Organization of the Elementary Mathematics Core

The Core is designed to help teachers organize and deliver instruction.

- The INTENDED LEARNING OUTCOMES (ILOs) describe the goals for mathematical skills and attitudes. They are found at the beginning of each grade level, are an integral part of the Core, and should be included as part of instruction.
- A STANDARD is a broad statement of what students are expected to understand. Several Objectives are listed under each Standard.
- An OBJECTIVE is a more focused description of what students need to know and be able to do at the completion of instruction. If students have mastered the Objectives associated with a given Standard, they have mastered that Standard at that grade level. Several Indicators are described for each Objective.
- An INDICATOR is a measurable or observable student action that enables one to assess whether a student has mastered a particular Objective. Indicators are not meant to be classroom activities, but they can help guide classroom instruction.

Guidelines Used in Developing the Elementary Mathematics Core

The Core is:

Consistent With the Nature of Learning

The main intent of mathematics instruction is for students to value and use mathematics as a process to understand the world. The Core is designed to produce an integrated set of Intended Learning Outcomes for students.

Coherent

The Core has been designed so that, wherever possible, the ideas taught within a particular grade level have a logical and natural connection with each other and with those of earlier grades. Efforts have also been made to select topics and skills that integrate well with one another and with other subject areas appropriate to grade level. In addition, there is an upward articulation of mathematical concepts, skills, and content. This spiraling is intended to prepare students to understand and use more complex mathematical concepts and skills as they advance through the learning process.

Developmentally Appropriate

The Core takes into account the psychological and social readiness of students. It builds from concrete experiences to more abstract understandings. The Core focuses on providing experiences with concepts that students can explore and understand in depth to build the foundation for future mathematical learning experiences.

Reflective of Successful Teaching Practices

Learning through play, movement, and adventure is critical to the early development of the mind and body. The Core emphasizes student exploration. The Intended Learning Outcomes are central in each standard. The Core is designed to encourage instruction with students working in cooperative groups. Instruction should include recognition of the role of mathematics in the classroom, school, and community.

Comprehensive

The Elementary Mathematics Core does not cover all topics that have traditionally been in the elementary mathematics curriculum; however, it provides a comprehensive background in mathematics. By emphasizing depth rather than breadth, the Core seeks to empower students rather than intimidate them with a collection of isolated and eminently forgettable facts. Teachers are free to add

The Core is:

- **Consistent**
- **Coherent**
- **Developmentally Appropriate**
- **Reflective of Successful Teaching Practices**
- **Comprehensive**
- **Feasible**
- **Useful and Relevant**
- **Reliant Upon Effective Assessment Practices**

related concepts and skills, but they are expected to teach all the standards and objectives specified in the Core for their grade level.

Feasible

Teachers and others who are familiar with Utah students, classrooms, teachers, and schools have designed the Core. It can be taught with easily obtained resources and materials. A Teacher Handbook is also available for teachers and has sample lessons on each topic for each grade level. The Teacher Handbook is a document that will grow as teachers add exemplary lessons aligned with the new Core.

Useful and Relevant

This curriculum relates directly to student needs and interests. Relevance of mathematics to other endeavors enables students to transfer skills gained from mathematics instruction into their other school subjects and into their lives outside the classroom.

Reliant Upon Effective Assessment Practices

Student achievement of the standards and objectives in this Core is best assessed using a variety of assessment instruments. Performance tests are particularly appropriate to evaluate student mastery of mathematical processes and problem-solving skills. Teachers should use a variety of classroom assessment approaches in conjunction with standard assessment instruments to inform instruction. Sample test items, keyed to each Core Standard, may be located on the “Utah Mathematics Home Page” at <http://www.usoe.k12.ut.us/curr/math>. Observation of students engaged in instructional activities is highly recommended as a way to assess students’ skills as well as attitudes toward learning. The nature of the questions posed by students provides important evidence of their understanding of mathematics.

Engaging

In the early grades, children are forming attitudes and habits for learning. It is important that instruction maximizes students’ potential and gives them understanding of the intertwined nature of learning. Effective elementary mathematics instruction engages students actively in enjoyable learning experiences. Instruction should be as thrilling an experience for a child as seeing a rainbow, growing a flower, or describing a toad. In a world of rapidly expanding knowledge and technology, all students must gain the skills they will need to understand and function responsibly and successfully in the world. The Core provides skills in a context that enables students to experience the joy of learning.

Intended Learning Outcomes for Sixth Grade Mathematics

The main intent of mathematics instruction is for students to value and use mathematics and reasoning skills to investigate and understand the world.

The Intended Learning Outcomes (ILOs) describe the skills and attitudes students should learn as a result of mathematics instruction. They are an essential part of the Mathematics Core Curriculum and provide teachers with a standard for evaluation of student learning in mathematics. Significant mathematics understanding occurs when teachers incorporate ILOs in planning mathematics instruction.

By the end of sixth grade students will be able to:

1. Demonstrate a positive learning attitude toward mathematics.

- a. Display a sense of curiosity about numbers and patterns.
- b. Pose mathematical questions about objects, events, and processes.
- c. Demonstrate persistence in completing tasks.
- d. Apply prior knowledge and processes to construct new knowledge.
- e. Maintain an open and questioning mind toward new ideas and alternative points of view.

2. Become mathematical problem solvers.

- a. Determine the approach, materials, and strategies to be used in setting up a problem.
- b. Model problem situations in a variety of ways.
- c. Develop understanding of new mathematical concepts and vocabulary by answering questions such as: What made you think that? Did anyone think of this in a different way? Where have we seen a problem like this before?
- d. Construct and use concrete, pictorial, symbolic, and graphical models to represent problem situations.
- e. Know when to select and how to use grade-appropriate mathematical tools and methods as a natural and routine part of the problem-solving process.
- f. Build new mathematical knowledge through problem solving.
- g. Solve problems in both mathematical and everyday contexts.
- h. Recognize that there may be multiple ways to solve a

- **Demonstrate a positive learning attitude toward mathematics**
- **Become mathematical problem solvers**
- **Reason mathematically**
- **Communicate mathematically**
- **Make mathematical connections**
- **Represent mathematical situations**

problem.

- i. Persevere in developing alternative problem-solving strategies if initially selected approaches do not work.

3. Reason mathematically.

- a. Draw logical conclusions and make generalizations.
- b. Determine the approach, materials, and strategies to be used in solving problems.
- c. Use models, known facts, and relationships to explain reasoning.
- d. Make precise calculations and check the validity of the results in the context of the problem.
- e. Make conjectures based on observation and information and test mathematical conjectures and arguments.
- f. Follow and construct logical arguments and judge their validity.
- g. Analyze mathematical situations by recognizing and using patterns and relationships.
- h. Justify answers and solution processes.

4. Communicate mathematically.

- a. Represent mathematical ideas with objects, pictures, and symbols.
- b. Express mathematical ideas to peers, teachers, and others through oral and written language.
- c. Engage in mathematical discussions through brainstorming, asking questions, and sharing strategies for solving problems.
- d. Explain mathematical work and justify reasoning and conclusions.
- e. Analyze, evaluate, and explain mathematical arguments and conclusions presented by others.

5. Make mathematical connections.

- a. Use one mathematical idea to extend understanding of another.
- b. Recognize the role of mathematics in the classroom, school, and community.
- c. Explore problems and describe and confirm results using various representations.

- d. Recognize the connections between mathematics and other content areas and apply mathematical thinking and problem solving in those areas.

6. Represent mathematical situations.

- a. Create and use representations to organize and communicate mathematical ideas.
- b. Represent mathematical concepts using concrete, pictorial, and symbolic models.

Sixth Grade Math Standards

Standard I:
Students will acquire number sense and perform operations with rational numbers.

Standard I: Students will acquire number sense and perform operations with rational numbers.

Objective 1: Represent whole numbers and decimals in a variety of ways.

- a. Change *whole numbers* with *exponents* to *standard form* (e.g., $24 = 16$) and recognize that $100 = 1$.
- b. Read and write *numerals* from thousandths to one billion.
- c. Write a whole number to 999,999 in *expanded form* using *exponents* (e.g., $876,539 = 8 \times 10^5 + 7 \times 10^4 + 6 \times 10^3 + 5 \times 10^2 + 3 \times 10^1 + 9 \times 10^0$).
- d. Express numbers in *scientific notation* using positive powers of ten.
- e. Classify whole numbers to 100 as *prime*, *composite*, or *neither*.
- f. Determine the *prime factorization* for a whole number up to 50.

Objective 2: Identify relationships among whole numbers, fractions (rational numbers), decimals, and percents.

- a. Find the *greatest common factor* and *least common multiple* for two numbers using a variety of methods (e.g., list of multiples, prime factorization).
- b. Compare and order *rational numbers*, including mixed fractions, using a variety of methods and symbols.
- c. Locate positive rational numbers on a number line.
- d. Convert common fractions, decimals, and percents from one form to another (e.g., $\frac{3}{4} = 0.75 = 75\%$).

Objective 3: Model and illustrate meanings of operations and describe how they relate.

- a. Represent division of a multi-digit *dividend* by two-digit *divisors*, including decimals, using models, pictures, and symbols.
- b. Model addition, subtraction, multiplication, and division of fractions and decimals in a variety of ways (e.g., objects, a number line).

- c. Apply *rules of divisibility*.
- d. Select or write a number sentence that can be used to solve a multi-step problem and write a word problem when given a two-step expression or equation.

Objective 4: Use fractions and percents to communicate parts of the whole.

- a. Divide regions, sets of objects, and *line segments* into equal parts using a variety of models and illustrations.
- b. Name and write a fraction to represent a portion of a unit whole for halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and sixteenths.
- c. Write a fraction or ratio in simplest form.
- d. Name equivalent forms for fractions (halves, thirds, fourths, fifths, tenths), ratios, percents, and decimals, including repeating or terminating decimals.
- e. Relate percents less than 1% or greater than 100% to equivalent fractions, decimals, *whole numbers*, and mixed numbers.

Objective 5: Solve problems using the four operations with whole numbers, decimals, and fractions.

- a. Determine when it is appropriate to use estimation, mental math strategies, paper and pencil, or a calculator.
- b. Use estimation strategies to determine whether results obtained using a calculator are reasonable.
- c. Multiply up to a three-digit *factor* by a one- or two-digit factor including decimals.
- d. Divide up to a four-digit *dividend* by a one- or two-digit *divisor* including decimals.
- e. Add and subtract decimals to the thousandths place (e.g., $34.567+3.45$; $65.3-5.987$).
- f. Add, subtract, multiply, and divide fractions and mixed numbers.
- g. Solve problems using ratios and proportions.
- h. Simplify *expressions* with *exponents*, using the *order of operations*.

Objective 6: Model, illustrate, and perform the operations of addition and subtraction of integers.

- a. Recognize that the sum of an *integer* and its opposite is zero.
- b. Model addition and subtraction of integers using manipulatives and a number line.
- c. Add and subtract integers.

Standard II: Students will use patterns, relations, and functions to represent and analyze mathematical situations using algebraic symbols.

Objective 1: Recognize, analyze, and use multiple representations of patterns and functions and describe their attributes.

- a. Analyze patterns on graphs and tables and write a generalization to predict how the patterns will continue.
- b. Create tables and graphs to represent given patterns and algebraic *expressions*.
- c. Write an algebraic expression from a graph or a table of values.
- d. Draw a graph from a table of values or to represent an equation.

Objective 2: Represent, solve, and analyze mathematical situations using algebraic symbols.

- a. Recognize that a number in front of a variable indicates multiplication (e.g., $3y$ means 3 times the quantity y).
- b. Solve two-step equations involving *whole numbers* and a single variable (e.g., $3x+4=19$).
- c. Recognize that “ \approx ” indicates a relationship in which the quantities on each side are approximately of equal value (e.g., $\Pi \approx 3.14$).
- d. Recognize that an *exponent* can be represented in the following ways: 4^3 or 4^3 .
- e. Evaluate *expressions* and formulas, substituting given values for the variables (e.g., $2x+4$; $x=2$; therefore, $2(2)+4=8$).
- f. Recognize that if the *product* is zero, then one or more factors equal zero (i.e., if $a \cdot b=0$ then either $a=0$ or $b=0$ or a and $b=0$).

Standard II:
Students will use patterns, relations, and functions to represent and analyze mathematical situations using algebraic symbols.

Standard III:
Students will use spatial and logical reasoning to recognize, describe, and identify geometric shapes and principles.

Standard III: Students will use spatial and logical reasoning to recognize, describe, and identify geometric shapes and principles.

Objective 1: Identify and analyze characteristics and properties of geometric shapes.

- a. Identify the *midpoint* of a line *segment*.
- b. Identify *concave* and *convex polygons*.
- c. Identify the center, *radius*, *diameter*, and *circumference* of a circle.
- d. Identify the number of *faces*, *edges*, and *vertices* of *prisms* and *pyramids*.

Objective 2: Specify locations and describe spatial relationships using coordinate geometry.

- a. Graph points defined by ordered pairs in all four *quadrants*.
- b. Write the ordered pair for a point in any quadrant.

Objective 3: Visualize and identify geometric shapes after applying transformations.

- a. *Turn (rotate)* a shape around a point and identify the location of the new vertices.
- b. *Slide (translate)* a polygon either horizontally or vertically on a coordinate grid and identify the location of the new vertices.
- c. *Flip (reflect)* a shape across either the x- or y-axis and identify the location of the new vertices.

Standard IV: Students will understand and apply measurement tools and techniques.

Objective 1: Identify and describe measurable attributes of objects and units of measurement.

- a. Compare a meter to a yard, a liter to a quart, and a kilometer to a mile.
- b. Identify π as the ratio of the *circumference* to *diameter* of a circle.
- c. Explain how the size of the unit used in measuring affects the precision.
- d. Estimate length, volume, weight, and area using *metric* and *customary* units.

Objective 2: Determine measurements using appropriate tools and formulas.

- a. Measure length to the nearest one-sixteenth of an inch and to the nearest millimeter.
- b. Estimate and measure an angle to the nearest degree.
- c. Calculate the *circumference* of a circle using a given formula.
- d. Calculate *elapsed time* across a.m. and p.m. time periods.
- e. Calculate the *areas* of triangles, rectangles, and *parallelograms* using given formulas.
- f. Calculate the *surface area* and *volume* of right, rectangular prisms using given formulas.

**Standard IV:
Students will
understand and
apply
measurement tools
and techniques.**

Standard V: Students will collect, analyze, and draw conclusions from data and apply basic concepts of probability.

Objective 1: Design investigations to reach conclusions using statistical methods to make inferences based on data.

- a. Design investigations to answer questions by collecting and organizing data in a variety of ways (e.g., bar graphs, line graphs, frequency tables, stem and leaf plots).
- b. Collect, compare, and display data using an appropriate format (i.e., bar graphs, line graphs, *line plots*, circle graphs, scatter plots).
- c. Compare two similar sets of data on the same graph and compare two graphs representing the same set of data.
- d. Recognize that changing the scale influences the appearance of a display of data.
- e. Develop and evaluate inferences and predictions based on data.

**Standard V:
Students will collect,
analyze, and draw
conclusions from
data and apply basic
concepts of
probability.**

Objective 2: Apply basic concepts of probability.

- a. Write the results of a probability experiment as a fraction, ratio, or percent between zero and one.
- b. Compare experimental results with anticipated results (e.g., experimental: 7 out of 10 tails; whereas, anticipated 5 out of 10 tails).
- c. Compare individual, small group, and large group results for a probability experiment.

Starter Activities

Starter Activity—Bridging the Gap

Standard II

Students will use patterns and relations to represent and analyze mathematical situations using algebraic symbols.

Objective 2

Represent, solve, and analyze mathematical situations using algebraic symbols.

Intended Learning Outcomes

2. Become mathematical problem solvers.
4. Communicate mathematically.
6. Represent mathematical situations.

Standard II

Objective 2

Connections

Background Information

Students should have some skills in recording numerical relations as simple equations.

Invitation to Learn

Materials and instructions are distributed to the tables before participants arrive. Upon arriving, participants will immediately begin working on task.

Instructional Procedures

1. Typed out on instruction card (one for each participant):
2. How many of you have ever wanted to be part of a construction company that is responsible for building houses? Well today is your lucky day! You are going to be responsible for building houses on three islands. However, you are fortunate to have bridges that connect the islands to one another.
3. On your blackline master you will notice the architect has written on each bridge, that connects the two islands, the total number of houses on the two islands. The architect has also indicated the total number of houses to be built on the three islands. Your job is to figure out how many houses go on each island. You may use the cm cubes on your table to represent the houses.

Materials

- Instruction card (one for each participant)
- Blackline masters “Bridging the Gap” (one for each participant)
- cm cubes

Curriculum Integration

Math/Science—The skills developed in the activity will facilitate the modeling of patterns and functions and the drawing of conclusions about them.

Possible Extensions/Adaptations

Use four islands to increase the level of problem solving.

Assessment Suggestion

Observe the strategies the students use to solve the problems and the methods used to record the solutions. Assess the extent to which the students use symbols or variables to represent each problem.

Additional Resources

Navigating through Algebra from NCTM

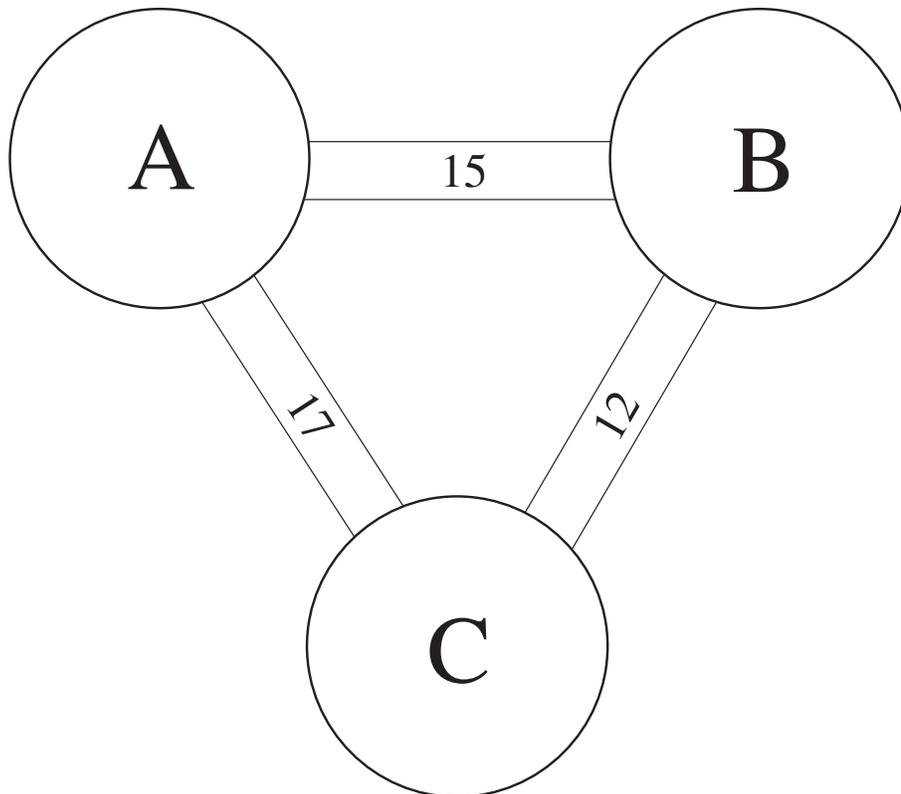
Groundworks Algebra Puzzles and Problems, Creative Publications

Homework & Family Connections

As a homework assignment, send a problem home to be completed by the family.

Bridging the Gap

Exercise
#1



Total Number of Houses = 22

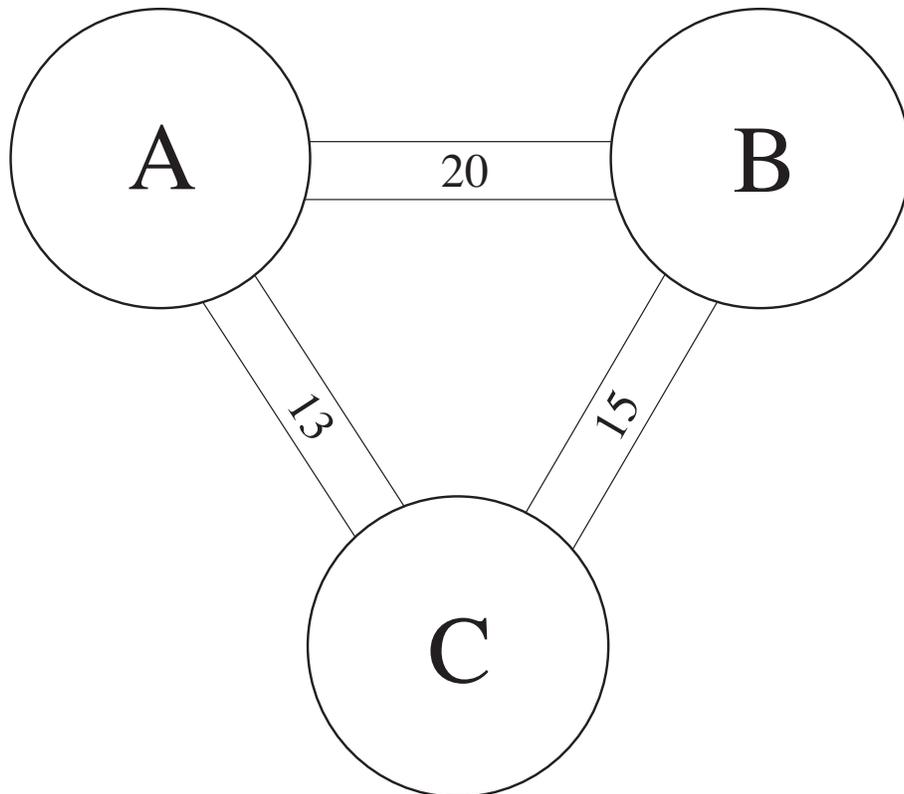
A = _____

B = _____

C = _____

Bridging the Gap

Exercise
#2



Total Number of Houses = 24

A = _____

B = _____

C = _____

Starter Activity–Eyewitness

Standard II

Objective 2

Connections

Standard II Students will use patterns and relations to represent and analyze mathematical situations using algebraic symbols.
Objective 2 Represent, solve, and analyze mathematical situations using algebraic symbols.
Intended Learning Outcome 2. Become mathematical problem solvers.

Background Information

Students should have some skills recording information with variables for the unknown.

Invitation to Learn

Materials and instructions are distributed to the tables before participants arrive. Upon arriving, participants will immediately begin working on task.

Instructional Procedures

Typed out on instruction card:

Have you ever been the eyewitness for a crime scene? When a crime is committed, what is the role of the eyewitness and the detective? Today you will be working with a partner. One of you will be the eyewitness and the other will be the detective.

1. The detective gives the eyewitness one minute to read the story problem. When time is up, the eyewitness turns the story problem over so he or she can't see it.
2. The detective then asks, "What's this about?"
3. The eyewitness then tells the basic story.
4. The detective then asks the eyewitness to give all the information he or she remembers. The detective asks questions to "learn the story and get the facts."
5. After relating all that can be remembered, the eyewitness is given one more opportunity to have an "instant replay" and read the problem one more time. Again, the detective times the eyewitness for one minute, then turns the problem over again.
6. The detective then asks for any missed details.

Materials

One for each participant

- Instruction card
- Two story problems

7. Following the eyewitness interrogation, the detective writes an equation to represent the case with the help of the witness.

8. If time permits, switch roles and use the second story problem.

Remember: No matter what type of story problem you are solving, you must be a “detective.”

Curriculum Integration

Math/Science—Across the curriculum, students need to stop periodically when they are reading to summarize, clarify, predict and question.

Possible Extensions/Adaptations

When students are working on story problems, it is very useful for them to have a highlighter to “highlight” the important details needed to solve the problem. Repeated analysis of story problems enables students to differentiate relevant from irrelevant information.

Assessment Suggestion

Circulate around the room listening to how well the eyewitness can summarize the given information. Also, noting the ability of the detective to record the information as an algebraic or numerical equation.

As a quiz, have all students become the eyewitness and individually record their information as an equation to successfully solve the problems.

Additional Resources

Math Detective books by Terri Husted

Classroom math textbooks

Have students write story problems to be solved by classmates

www.edhelper.com

Homework & Family Connections

When students are reading at home, it’s good for them to stop throughout the text and summarize the information that has been covered.

Eyewitness

Instructions

Have you ever been the eyewitness for a crime scene? When a crime is committed, what is the role of the eyewitness and the detective? Today you will be working with a partner. One of you will be the eyewitness and the other will be the detective.

1. The detective gives the eyewitness one minute to read the story problem. When time is up, the eyewitness turns the story problem over so he or she can't see it.
2. The detective then asks, "What's this about?"
3. The eyewitness then tells the basic story.
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5. After relating all that can be remembered, the eyewitness is given one more opportunity to have an "instant replay" and read the problem one more time. Again, the detective times the eyewitness for one minute, then turns the problem over again.
6. The detective then asks for any missed details.
7. Following the eyewitness interrogation, the detective writes an equation to represent the case with the help of the witness.
8. If time permits, switch roles and use the second story problem.

Remember: No matter what type of story problem you are solving, you must be a "detective."

Story Problem #1 for Eyewitness:

Jazz Payroll

At approximately 1:00 a.m. on March 25, 2003, the Jazz payroll office was broken into and ransacked. After an investigation all payroll checks were located except the check for John Stockton. The clerk knows that Karl Malone earns 20% more money per game than John Stockton. The clerk computed that Malone earned \$120,000 per game. How much money does John Stockton earn per game?

Story Problem #2 for Eyewitness:

Stick-to-it

Johnny bought a box of soccer stickers to use on his project report on soccer. There were two thousand, one hundred sixty stickers in the box, but he could not use some of them. One-ninth of them were stuck so tightly together that he could not detach them from each other. Eighty-three of them were blank. One-fifth of them had no glue. How many were left that he could actually use?



Story Problem #1 for Eyewitness:

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ANSWER: John Stockton earns \$100,000/game

Story Problem #2 for Eyewitness:

Stick-to-it

Johnny bought a box of soccer stickers to use on his project report on soccer. There were two thousand, one hundred sixty stickers in the box, but he could not use some of them. One-ninth of them were stuck so tightly together that he could not detach them from each other. Eighty-three of them were blank. One-fifth of them had no glue. How many were left that he could actually use?

ANSWER: 1,405 soccer stickers could actually be used

Starter Activity—All in a Name

Standard V

Students will use concepts of probability and collect, analyze, and draw conclusions from data.

Objective 1

(5th) Formulate and answer questions using statistical methods to analyze data.

(6th) Design investigations to reach conclusions using statistical methods to analyze data.

Intended Learning Outcomes

3. Reason mathematically.
6. Represent mathematical situations.

Standard

V

Objective

1

Connections

Background Information

Students should be familiar with the terms, “mean,” “median,” “range,” and “mode.” A quick review before beginning the activity may be needed.

Invitation to Learn

Materials and instructions are distributed to the tables before participants arrive. Upon arriving, participants will immediately begin working on task.

Instructional Procedures

Typed out on instruction card:

We all know how important our name is. It identifies who we are. It is so important that today we are going to collect data using our names.

1. Quickly walk around the room and record name data for ten participants including yourself. This will give you a chance to meet others in the room and introduce yourself. Have each participant record his or her first and last name in the appropriate columns. Returning to your seat, record the total number of letters for each name in the last column.
2. Using the data collected in your third column, make a quick graph (bar, line, etc.). Describe your data to another participant, in terms of clumps or bunches, gaps or holes, and bumps.
3. Using the same data collected in your third column, find the: mean, range, mode, and median. Compare these findings to your graph. Do you see any patterns?

Materials

One for each participant:

- Instruction card
- Blackline master “It’s all in a Name”
- Blackline master of graph paper

4. Discuss with another participant:
 - a. Why/When is it useful to find the mean, range, mode, and median of data?
 - b. If more names are added to your data set, predict how your measures and graph will change. Explain why.

Curriculum Integration

Math/Science—Students can determine the mean, range, median, and mode of data collected from a science experiment. Analysis of the measures will then give the students valuable information to make informative conclusions.

Possible Extensions/Adaptations

A meaningful activity to use at the beginning of the school year to learn the names of your students. Also, after students have successfully analyzed the data from their group, then collect and compare the data using the whole class.

Assessment Suggestion

Observe students while they are working together to organize collected data. Quiz the class on given data. See if the student is able to organize the data to determine the mean, range, median, and mode.

Additional Resource

Connect to *NCTM Standards 2000* (Creative Publications)

Homework & Family Connections

Collect data from your family (names, age, height, etc.), and organize the data and determine the mean, range, median, and mode.

All in a Name

Instructions

We all know how important our name is. It identifies who we are. It is so important that today we are going to collect data using our names.

1. Quickly walk around the room and record name data for ten participants including yourself. This will give you a chance to meet others in the room and introduce yourself. Have each participant record his or her first and last name in the appropriate columns. Returning to your seat, record the total number of letters for each name in the last column.
2. Using the data collected in your third column, make a quick graph (bar, line, etc.). Describe your data to another participant, in terms of clumps or bunches, gaps or holes, and bumps.
3. Using the same data collected in your third column, find the mean, range, mode, and median. Compare these findings to your graph. Do you see any patterns?
4. Discuss with another participant:
 - a. Why/When is it useful to find the mean, range, mode, and median of data?
 - b. If more names are added to your data set, predict how your measures and graph will change. Explain why.



All in a Name

Recording Sheet

<i>First Name</i>	<i>Last Name</i>	<i>Total # of Letters</i>

MEAN:	
RANGE:	
MODE:	
MEDIAN:	

Mean: the sum of all the numbers divided by the number of items you're adding.

Range: the difference between the highest and lowest numbers.

Mode: the number that appears most often (there isn't always a mode, but sometimes there's more than one!).

Median: the middle number.

Starter Activity—Roped Into Quadrilaterals

Standard III

Objective 1

Connections

Standard III

Students will use spatial reasoning to recognize and describe geometric shapes and principles.

Objective 1

Identify and analyze characteristics and properties of geometric shapes.

Intended Learning Outcomes

3. Reason mathematically.
4. Communicate mathematically.

Background Information

Students should be familiar with the common attributes of quadrilaterals such as parallelograms, trapezoids, rectangles, rhombi, and squares. If the students are not familiar with Venn diagrams, explain that the quadrilaterals that possess the characteristics for both circles should be placed in the intersection of the two rings. For example, if one circle is labeled “right angles” and the other “congruent sides,” a square would be placed in the intersection of the two rings.

Invitation to Learn

Materials and instructions are distributed to the tables before participants arrive. Upon arriving, participants will immediately begin working on task.

Instructional Procedures

Typed out on instruction card (one for each table):

1. Assign members of your group (table) to do the following:
 - a. Cut out the quadrilateral pieces.
 - b. Tie the ends of **each** piece of yarn to make three circles.
 - c. Cut out the Task Activity Quadrilateral Labels.
2. The object of the activity is to place the quadrilateral pieces appropriately in your circles (yarn) according to the labels. You may need to overlap the circles to form intersections. The number of labels determines the number of circles used.
3. Participants progress through the labels, placing quadrilateral pieces according to common attributes.

Materials

One set for each table:

- 3 pieces of yarn (18 in.)
- Blackline master “Quadrilaterals”
- Blackline master “Quadrilateral Labels”
- scissors

Curriculum Integration

Math/Science—Give the students an opportunity to write and defend their reasoning.

Possible Extensions/Adaptations

Have students make up their own labels and then challenge a partner to use them to create quadrilateral circles. Also, students can make their own “Mystery Circles” for their classmates to label.

Assessment Suggestion

During the activity, circulate among the groups and ask the students to defend their placement of different pieces.

Give each student a “Mystery Circle” hand-out to label. In their writing journals, have them explain and defend their reasoning for their labels.

Additional Resource

Activity adapted from NCTM “Navigating through Geometry in Grades 3-5”

Homework & Family Connections

Identify, label, and group quadrilaterals found “hidden” throughout your house and in magazines. Make a collage of your findings.

ROPED INTO QUADRILATERALS

Instructions

1. Assign members of your group (table) to do the following:
 - a. Cut out the quadrilateral pieces.
 - b. Tie the ends of **each** piece of yarn to make three circles.
 - c. Cut out the Task Activity Quadrilateral Labels.
2. The object of the activity is to place the quadrilateral pieces appropriately in your circles (yarn) according to the labels. You may need to overlap the circles to form intersections. The number of labels determines the number of circles used.
3. Progress through the labels, placing quadrilateral pieces according to common attributes.



Quadrilateral Labels

Task Activity #1:

At least one right angle	No right angles
--------------------------	-----------------

Task Activity #2:

No congruent sides	Congruent sides
--------------------	-----------------

Task Activity #3:

At least one obtuse angle	At least one acute angle
---------------------------	--------------------------

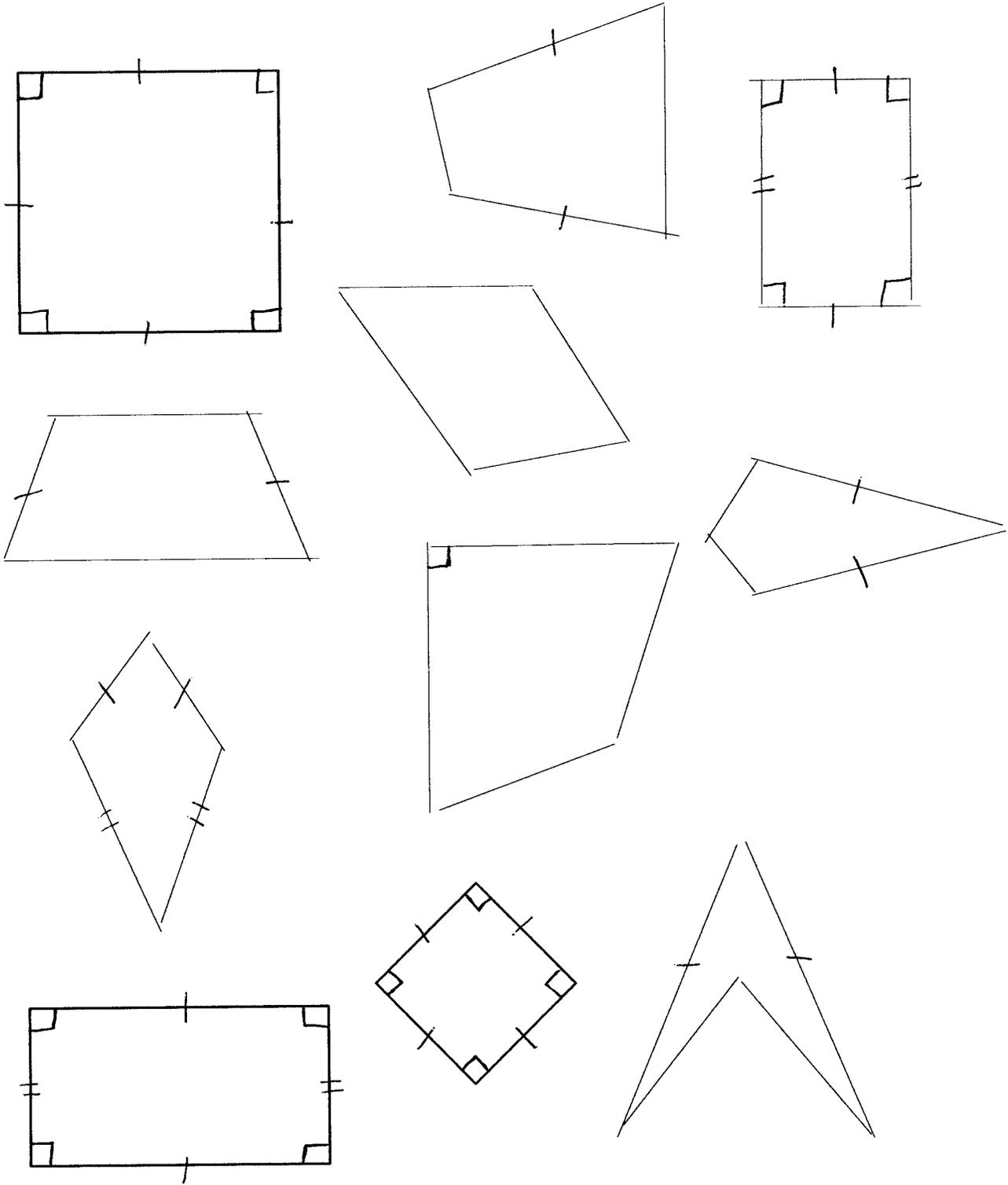
Task Activity #4:

Rectangles	Squares	Rhombi
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Task Activity #5:

At least one acute angle	At least one pair of parallel sides
--------------------------	-------------------------------------

Quadrilaterals



Starter Activity–The “Right” Place

Standard I

Student will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.

Objective 5

Solve problems using the four operations with whole numbers, decimals, and fractions.

Intended Learning Outcomes

2. Become mathematical problem solvers.
3. Reason mathematically.

Standard

I

Objective

5

Connections

Background Information

Reinforce there is not always just one right answer. Multiple answers are accepted in many problem solving activities. The important thing is for students to “stick with” it long enough to have success.

Invitation to Learn

Materials and instructions are distributed to the tables before participants arrive. Upon arriving, participants will immediately begin working on task.

Instructional Procedures

Follow directions on the Blackline master The Right Place

Curriculum Integration

Math/Science—Great way to reinforce the importance of problem solving throughout the curriculum. Multiple processes and ways to analyze, interpret and present information.

Possible Extensions/Adaptations/Integration

You may allow a number to be used more than once. The use of cards or number tiles may be used instead of writing the numbers.

Assessment Suggestion

Observations while students are working on problems. Observing their ability to reason through the process and communicate with one another about their thinking, and the student’s ability to “stick with” the problem and come up with multiple solutions.

Materials

One for each participant:

- Blackline master of The Right Place
- Scissors

Additional Resources

Problem Solving with Numbers (Grades 3-6), Exclusive Educational Products

Arithme Twists, Creative Publications

Homework & Family Connections

Great way for parents to help their children with basic facts, instead of “drill and kill.”

The "Right" Place

Use the 1-6 to complete the equations. Each number can only be used once in an equation.

$$\square \times \square = \square + \square + \square$$

$$(\square - \square) \times \square = \square$$

$$\square \times \square \times \square (\square - \square) = \square \square$$

Place the numbers 0-8 to complete the equation.

Hint: The three-digit number is >500.

$$\begin{array}{r} \square \square \square \\ \times \quad \square \\ \hline \square \square \square \square \end{array}$$

Cut apart:

1
2
3
4
5
6
7
8
9
0

Randomly select 4 of your numbers. Place them in the equation below so that:

1. the product will be a 3-digit number
2. the product is between 750 and 2,500
3. it is an even product
4. the answer is close to 6,000

$$\begin{array}{r} \square \square \\ \times \square \square \\ \hline \end{array}$$

Starter Ideas

Starter Ideas

Starter Ideas

Inquiry

Helping students work together to make sense of mathematics:

“What do others think about what _____ said?”

“Can you explain it in your own words?”

“Do you agree? Disagree?”

“What is alike and what is different about your method of solution and hers?”

“Does anyone have the same answer but a different way to explain it?”

“Would you ask the rest of the class that question?”

“Do you understand what they are saying?”

“Can you convince the rest of us that that makes sense?”



Helping students to rely more on themselves to determine whether something is mathematically correct:

“Why do you think that?”

“Why is that true?”

“How did you reach that conclusion?”

“Does that make sense?”

“Can you make a model to show that?”





Helping students learn to reason mathematically:

“Does that always work?”

“Is that true for all cases?”

“Can you predict the next one in the pattern?”

“What about the tenth one in the pattern?”

“Can you give me an example?”

“Can you think of a counterexample?”

“How did you reach that conclusion?”

“How could you prove that?”





Helping students to connect mathematics, its ideas, and its applications:

“How does this relate to...?”

“What ideas that we have learned before were useful in solving this problem?”

“Have we ever solved a problem like this one before?”

“What uses of mathematics did you find in the newspaper last night?”

“Can you give me an example of...?”



Inquiry Activity—Using Questioning Strategies

Standard Multiple
Objective Develop understanding of new mathematical concepts and vocabulary by answering questions.
Intended Learning Outcomes 2. Become mathematical problem solvers.

Standard

Objective

Connections

Background Information

One of the most important aspects of teaching is monitoring student understanding so that it is possible to provide feedback to students, allowing them to correct and refine their understanding of what is being taught. Mathematics teachers are beginning to use a wide variety of techniques to accomplish this, from multiple-choice tests assessing skills to rubrics assessing the quality of problem solving and mathematical reasoning. However, too often we forget one of the most useful and efficient ways to assess student learning: asking students probing questions that challenge and engage their thinking. A powerful learning experience can often fail to reach its desired goals because the students were not given the opportunity to explore and refine their understanding. When employed by a skilled teacher, questions will encourage students to work together to understand mathematics, to rely on themselves to determine if their thinking is correct, to conjecture, invent, and solve problems, and to see how mathematics connects to the real world and other branches of mathematics.

NCTM describes the shift towards better assessment practice in this way:

A shift toward judging the progress of each student’s attainment of mathematical power, and away from assessing students’ knowledge of specific facts and isolated skills.

A shift toward communicating with students about their performance in a continuous, comprehensive manner, and away from simply indicating whether or not answers are correct.

A shift toward students learning to assess their own progress, and away from teachers and external agencies as the sole judges of progress. (NCTM Assessment)

To make this shift occur in classrooms teachers need to carefully plan questions to engage student’s thinking, listen to students to determine if there are any misconceptions in their thinking, ask students to clarify and justify their ideas orally, and monitor when to clarify concepts.

Introduce the two dice tossing games: the sum game and the product game:

<i>Sum Game</i>	<i>Product Game</i>
<p>Two Players:</p> <ol style="list-style-type: none"> 1. Choose one player to be “even” and the other to be “odd.” 2. Throw two dice. 3. Add the numbers on the two faces. 4. If the sum is even, the even player gets 1 point. 5. If the sum is odd, the odd player gets 1 point. 	<p>Two Players:</p> <ol style="list-style-type: none"> 1. Choose one player to be “even” and the other to be “odd.” 2. Throw two dice. 3. Multiply the numbers on the two faces. 4. If the sum is even, the even player gets 1 point. 5. If the sum is odd, the odd player gets 1 point.

Ask the participants if the games are fair? Lead a discussion in which good questioning practices are used. Ask the kinds of questions that will lead to deep understanding of what it means for a probability situation to be fair, and what happens when we add and multiply even and odd numbers.

Brainstorm reasons why teachers would ask such questions of their students. Write these on chart paper.

Instructional Procedures

Materials

- ❑ *A Thinking Approach to Computation* Video by Creative Publications
- ❑ Questions and question categories already cut and placed in envelopes. One envelope for every 2 par
- ❑ A pair of dice for every 2 participants
- ❑ A transparency of the rules for the Sum/Product Game

1. Examine categories looking specifically for the following:
 - **Helping students work together to make sense of mathematics**
 - **Helping students to rely more on themselves to determine whether something is mathematically correct**
 - **Helping students learn to reason mathematically**
 - **Helping students to connect mathematics, its ideas, and its applications**
2. Participants work as partners to organize the questions that are provided into these four categories.
3. View the video “A Thinking Approach to Computation,” Creative Publications, Part 2. This is an excellent example of the type of questioning practice described in the NCTM Professional Standards.

Curriculum Integration

This same questioning strategy can be used with any content area.

Sum Game

Two Players:

1. Choose one player to be “even” and the other to be “odd.”
2. Throw two dice.
3. Add the numbers on the two faces.
4. If the sum is even, the even player gets 1 point.
5. If the sum is odd, the odd player gets 1 point.

Product Game

Two Players:

1. Choose one player to be “even” and the other to be “odd.”
2. Throw two dice.
3. Multiply the numbers on the two faces.
4. If the sum is even, the even player gets 1 point.
5. If the sum is odd, the odd player gets 1 point.

What are characteristics of effective homework in mathematics?

Research and Best Practice

Daily, children hurry home from school and arrive to face the obligatory question from parents, “What did you learn in school today?” They return to school the following day, and their teacher asks, “Do you have your homework assignment?” Perhaps a better question would be “What did you learn at home?” The home should be a place to extend mathematics learning.

Student learning in mathematics should always focus on understanding the set of skills and knowledge needed to investigate the world. Homework must emphasize developing students’ mathematics skills to solve problems, which will help them understand the world. These mathematics skills are described as “process skills” in the *NCTM Principles and Standards for School Mathematics* and “habits of mind” in *Benchmarks for Science Literacy*. Mathematics educators and mathematicians agree that knowing mathematics is more than being able to recall facts. Research indicates that individuals with expertise in mathematics understand mathematics concepts, how to apply them to challenging, non-routine real-life problem-solving situations, and how to learn from their own problem-solving efforts.

Homework assignments provide the opportunity for students to do long term projects that require multiple levels of understanding. Students take ownership when they spend weeks following stock prices in the newspaper, paying close attention to favorites, predicting industry trends, interviewing traders, or perhaps even participating in an investment club. Watching TV and timing commercial breaks one night may be interesting, but when students keep data over a few weeks — timing commercials in different types of programs, making charts, and drawing graphs — their learning will go beyond the curriculum.

Homework time is an opportunity for students to reflect on learning and synthesize their mathematics understandings. Well-designed homework can bring parents and other adults into a student’s community of mathematics learners. Assignments should include students discussing their learning with others. This can be done through student learning teams, parent involvement, or the teacher using e-mail to have discussion groups. Mathematics is in every aspect of life. Teachers should take advantage of the opportunity to provide students with authentic learning opportunities at home.

- **The home should be a place to extend mathematics learning.**

Excerpted from
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Classroom Implications

The value placed on various aspects of mathematics learning can be seen in the allocation of instructional time in class and by the nature of homework assigned. Teachers who value problem-solving skills will provide time in class to develop students' ability to solve problems and then will assign homework that uses these skills in new settings. What goes on in class should match the homework assigned.

Mathematics homework should not be schoolwork done at home. The home provides a unique opportunity for students to gain mathematics understanding by solving mathematics problems. Placing the major emphasis on basic skills and drill for skill development in mathematics is somewhat of a waste of student, parent, and teacher time and effort.

Teaching for understanding requires carefully designed tasks. Homework assignments should have clear criteria and/or written rubrics that describe expectations and establish student goals. The teacher must be certain that students have access to the materials and resources they will need to complete the assignment.

It is important for students to do their best, and for teachers to examine student work. Less is often more when it comes to homework. A product that has been refined by the student results in more effective learning than a large volume of work completed with little thought. The quality of student work is often determined by the standards a teacher sets on the assignment, time spent reviewing the expectations, and suggestions for improvements. A homework assignment should be a major event in student learning. Selling students on the importance of an assignment as a learning event is important: their ownership will determine the depth and breadth of their learning.

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Problem Solving

Problem solving is an important skill for students (Barba 1991). The abilities to think, reason, create, and find solutions to problems are evidence that a student is on the path of effective learning. Problem solving is the process of reaching a goal when a direct path to that goal is temporarily blocked. Students must recognize that the process is multi-faceted and involves questions about the ways we think as well as the ways we learn.

Successful problem solvers share many of the same qualities. Students who are successful problem solvers tend to:

- Use prediction during their problem solving and find support for those predictions.
- Eliminate gaps in their knowledge.
- Rely on information learned from a variety of experiences.
- Make fewer mistakes reading or interpreting data or important information.
- Rely on note taking and journals rather than memory.
- Express less doubt and confusion and embody fewer misconceptions.

Students who become skilled at practicing problem solving techniques tend to:

- Grasp a problem quickly and can generalize information rapidly and easily.
- Switch easily from one solution method to another.
- Strive for an “elegant solution” where possible.
- Remember relationships in a problem and principles of solutions.
- Reflect on their own problem solving strategies and are capable of reversing their train of thought.

Students who are successful problem solvers also use facts, rules, skills, and strategies that are called heuristics. “Heuristic” means steps, or a sequences of steps, that are used to organize thinking and solve problems. Some effective heuristics for problem solving strategies are listed below.

- Study the solution process
- Make a figure, graph, drawing, table, chart, or equation
- Check the solution

- **Problem solving is the process of reaching a goal when a direct path to that goal is temporarily blocked.**



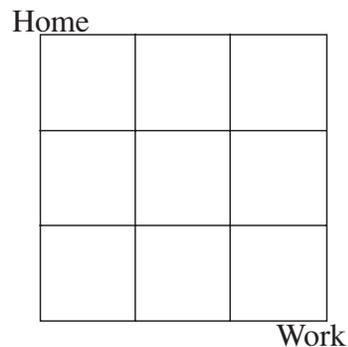
- Identify information
- Solve similar, but simpler problems
- Restate the problem
- Look for patterns
- Work backwards

Problem Solving Activities

1. “No Repeats!” (Standard 3, objective 2, indicator c – 5th Grade)

I will be working every day this summer. Going the same way can get boring! I’d like to try a different 6-block route each day.

How many days can I go to work without repeating a route?



(There are 20 possible paths.)

2. “Multiplying and Dividing with Fractions” (6th Grade – Standard I: objective 3: indicator b, Standard I: objective 5: indicator f)

Draw a diagram for each word problem. Then determine which, if any, of the four number sentences matches your diagram. You may choose more than one number sentence for each. Explain the connections between the problem, your diagram, and the math.

- A. Maurine has 6 cups of sugar. Each batch of cookies requires $\frac{1}{2}$ cup of sugar. How many batches of cookies can she make?
- B. Maurine has 6 batches of cookies. She plans to share them equally with her friend, Janelle. How many batches will each of them get?
- C. Maurine has to travel 6 miles. When she walks, she covers $\frac{1}{2}$ mile in an hour. How many hours will it take her?
- D. Maurine has to travel 6 miles. She plans to walk halfway and run halfway. How far will she run?

I. $6 \div 2$	II. $6 \div \frac{1}{2}$	III. 6×2	IV. $6 \times \frac{1}{2}$
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Have participants share their drawings for each problem on the board and explain the connections between their drawings and the number sentences.

Even though this is a 6th grade task, it helps 5th grade teachers understand the meaning of the operations and the relationship between them!

3. “*Funny Stuff*” (5th Grade – Standard II: objective 1: indicator b, 6th Grade – Standard II: objective 1: indicator a)

Mom went shopping at Funny Stuff. She tried to pay for a video game marked 2 \diamond with \$2.00, but the clerk said \$2.00 wasn’t enough. She was stumped until the clerk handed her this chart.

How much did the 2 \diamond games cost in dollars?

$$1 \diamond = \$6$$

$$3 \diamond = \$12$$

$$6 \diamond = \$21$$

$$10 \diamond = \$33$$

$$17 \diamond = \$54$$

Principles and Standards for School Mathematics

- *build new mathematical knowledge*
- *solve problems that arise*
- *apply and adapt*
- *monitor and reflect*

Introduction

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- *build new mathematical knowledge* through problem solving
- *solve problems that arise* in mathematics and in other contexts
- *apply and adapt* a variety of appropriate strategies to solve problems
- *monitor and reflect* on the process of mathematical problem solving

Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge. Through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort, and should then be encouraged to reflect on their thinking.

By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages.

Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program. Problem solving in mathematics should involve all the five content areas described in these Standards. The contexts of the problems can vary from familiar experiences involving students' lives or the school day to applications involving the sciences or the world of work. Good problems will integrate multiple topics and will involve significant mathematics

Item 1: Build new mathematical knowledge through problem solving

How can problem solving help students learn mathematics? Good problems give students the chance to solidify and extend what they know and, when well chosen, can stimulate mathematics learning. With young children, most mathematical concepts can be introduced through problems that come from their worlds.

For example, suppose second graders wanted to find out whether there are more boys or girls in the four second-grade classes. To solve this problem, they would need to learn how to gather information, record data, and accurately add several numbers at a time.

In the middle grades, the concept of proportion might be introduced through an investigation in which students are given recipes for punch that call for different amounts of water and juice and are asked to determine which is “fruitier.” Since no two recipes yield the same amount of juice, this problem is difficult for students who do not have an understanding of proportion. As various ideas are tried, with good questioning and guidance by a teacher, students eventually converge on using proportions. In high school, many areas of the curriculum can be introduced through problems from mathematical or applications contexts.

Problem solving can and should be used to help students develop fluency with specific skills. For example, consider the following problem, which is adapted from the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989, p. 24):

“I have pennies, dimes, and nickels in my pocket. If I take three coins out of my pocket, how much money could I have taken?”

Knowledge is needed to solve this problem—knowledge of the value of pennies, dimes, and nickels and also some understanding of addition. Working on this problem offers good practice in addition skills. But the important mathematical goal of this problem—helping students to think systematically about possibilities and to organize and record their thinking—need not wait until students can add fluently.

The teacher’s role in choosing worthwhile problems and mathematical tasks is crucial. By analyzing and adapting a problem, anticipating the mathematical ideas that can be brought out by working on the problem, and anticipating students’ questions, teachers can decide if particular problems will help to further their mathematical goals for the class. There are many, many problems that are interesting and fun but that may not lead to the development of the mathematical ideas that are important for a class at a particular time. Choosing problems wisely and using and adapting problems from instructional materials is a difficult part of teaching mathematics.

Item 2: Solve problems that arise in mathematics and in other contexts

People who see the world mathematically are said to have a “mathematical disposition.” Good problem solvers naturally tend to analyze situations carefully in mathematical terms and pose problems based on situations they see. They first consider simple cases before trying something more complicated, yet they will readily consider a more sophisticated analysis.

For example, a task for middle-grades students presents data about two ambulance companies and asks which company is more reliable (Balanced Assessment for the Mathematics Curriculum 1999a). A quick answer found by looking at the average time customers had to wait for each company turns out to be misleading. A more careful mathematical analysis involving plotting response times versus time of day reveals a different solution. In this task, a disposition to analyze more deeply leads to a more complete understanding of the situation and a correct solution. Throughout the grades, teachers can help build this disposition by asking questions that help students find the mathematics in their worlds and experiences and by encouraging students to persist with interesting but challenging problems.

Posing problems comes naturally to young children: I wonder how long it would take to count to a million? How many soda cans would it take to fill the school building? Teachers and parents can foster this inclination by helping students make mathematical problems from their worlds. Teachers play an important role in the development of students’ dispositions by creating and maintaining classroom environments, from pre-kindergarten on, in which students are encouraged to explore, take risks, share failures and successes, and question one another. In such supportive environments, students develop confidence in their abilities and a willingness to engage in and explore problems, and they will be more likely to pose problems and to persist with challenging problems.

Item 3: Apply and adapt a variety of appropriate strategies to solve problems

Of the many descriptions of strategies, some of the best known can be found in the work of Pólya (1957). Frequently cited strategies include:

- using diagrams
- looking for patterns
- listing all possibilities
- trying special values or cases

- working backward
- guessing and checking
- creating an equivalent problem
- creating a simpler problem

An obvious question is, How should these strategies be taught? Should they receive explicit attention, and how should they be integrated with the mathematics curriculum?

As with any other component of the mathematical tool kit, strategies must receive instructional attention if students are expected to learn them. In the lower grades, teachers can help children express, categorize, and compare their strategies. Opportunities to use strategies must be embedded naturally in the curriculum across the content areas.

By the time students reach the middle grades, they should be skilled at recognizing when various strategies are appropriate to use and should be capable of deciding when and how to use them. By high school, students should have access to a wide range of strategies, be able to decide which one to use, and be able to adapt and invent strategies.

Young children's earliest experiences with mathematics come through solving problems. Different strategies are necessary as students experience a wider variety of problems. Students must become aware of these strategies as the need for them arises, and as they are modeled during classroom activities, the teacher should encourage students to take note of them.

For example, after a student has shared a solution and how it was obtained, the teacher may identify the strategy by saying, "It sounds like you made an organized list to find the solution. Did anyone solve the problem a different way?" This verbalization helps develop common language and representations and helps other students understand what the first student was doing. Such discussion also suggests that no strategy is learned once and for all; strategies are learned over time, are applied in particular contexts, and become more refined, elaborate, and flexible as they are used in increasingly complex problem situations.

Item 4: Monitor and reflect on the process of mathematical problem solving

Effective problem solvers constantly monitor and adjust what they are doing. They make sure they understand the problem. If a problem is written down, they read it carefully; if it is told to them orally, they ask questions until they understand it. Effective problem solvers plan frequently. They periodically take stock of their progress to see whether

they seem to be on the right track. If they decide they are not making progress, they stop to consider alternatives and do not hesitate to take a completely different approach. Research (Garofalo and Lester 1985; Schoenfeld 1987) indicates that students' failures are often due not to a lack of mathematical knowledge but to the ineffective use of what they do know.

Good problem solvers become aware of what they are doing and frequently monitor, or self-assess, their progress or adjust their strategies as they encounter and solve problems (Bransford et al. 1999). Such reflective skills (called metacognition) are much more likely to develop in a classroom environment that supports them. Teachers play an important role in helping to enable the development of these reflective habits of mind by asking questions such as:

- “Before we go on, are we sure we understand this?”
- “What are our options?”
- “Do we have a plan?”
- “Are we making progress or should we reconsider what we are doing?”
- “Why do we think this is true?”

Such questions help students get in the habit of checking their understanding as they go along. This habit should begin in the lowest grades. As teachers maintain an environment in which the development of understanding is consistently monitored through reflection, students are more likely to learn to take responsibility for reflecting on their work and make the adjustments necessary when solving problems.

Inquiry Notes

Inquiry Notes

Inquiry Notes

Inquiry Notes

Assessment Strategies

100 Yard Dash

Juan and Maria race for 100 yards. Maria wins by ten yards. Juan demands a second race, but this time Maria spots Juan a handicap – she begins ten yards behind the starting line. Assuming both run the same speed as before, who will win? Why?

Draw a diagram.

Explain your reasoning.



Evaluating and Designing a Rubric

Student Task

Make a graph to illustrate how many hours you spend during a typical school day doing different things. These things might be sleeping, eating, school, homework, playing sports, scouts, dance, playing with friends, playing games, watching TV, etc.

Your task is to collect, organize, and display your data. A chart or table may be helpful because you must in some way show evidence of data organization. You must also choose the best type of graph to represent this data (bar, circle, etc.). Please show any calculations that you make, and write an explanation for why you chose the graph you did and how you made your graph.

<i>Student Rubric</i>				
	1 Point	2 Points	3 Points	4 Points
Organization of information	Information about time spent very disorganized	Some data organization evident, but not carried through	Data well-organized	In addition, the data is clearly presented
Graph	Graph chosen is inappropriate to the topic or very poorly executed	Graph chosen is adequate, but execution is poor	Appropriate form of graph and adequate information	In addition, the graph is very accurately and neatly presented
Calculations	Major errors in calculations	A number of errors in calculations	Very few errors in calculations	No calculation errors
Explanation	Explanation very muddled	Explanation difficult to follow	Explanation clear enough to follow	The explanation is clear and displays comprehensive understanding of the relative merits of different types of graphs

A few questions to consider as you evaluate the rubric:

- Is this a “good” rubric for the student task given? Why or why not?
- What changes should be made?
- Are the descriptions well stated and not ambiguous?
- Are there definite differences among the points?

Write any changes you would make on the rubric—use your State Core to see what expectations for student achievement are.

Pizza Party

Ms. Williams' 5th grade class is having a pizza party. They are trying to decide which pizzeria has the cheapest price. The local pizzerias and their prices are listed below.

Pizza Prices

Pizza to Go	\$ 8.50	8 slices per pizza
Rosa's Pizza	\$10.50	10 slices per pizza
Pizza with Pizazz	\$ 6.25	6 slices per pizza

There are 30 students in Ms. Williams' class. Each person (including Ms. Williams) will eat two slices of pizza. All the pizza slices from each pizzeria are the same size. Where should you buy the pizza to get the best deal? Why?

Write a brief description of how you arrived at your decision. Provide your work (any calculations you made), a picture, or a diagram to support your thinking.

Activity–Process Categories

Standard

Objective

Connections

<p>Standard II: Students will develop a sense of self in relation to families and community.</p> <hr/> <p>Objective 1: Describe factors that influence relationships with family and friends.</p> <hr/> <p>Indicator C: Describe how children change over time.</p> <hr/> <p>Process Skills: Symbolization, prediction, classification</p> <hr/> <p>Intended Learning Outcomes: Demonstrate a positive learning attitude.</p>
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Background Information

Process categories refer to the mental abilities that are most likely required to answer a given question. Therefore, in addition to measuring content classified by standard, objective, and indicator, we can also measure the “process” used to answer a question as well. For the Utah CRT, we have employed the same “process” abilities as the NAEP assessments in Mathematics. These process standards are described below.

1. **Procedural:** Procedural questions are questions can be viewed as a student’s “*knowing how.*” Below are verbs that would generally associate with procedural questions.
 - a. Compute
 - b. Solve
 - c. Evaluate
 - d. Show
 - e. Round
 - f. Order

2. **Conceptual:** Conceptual questions are questions that can be viewed as a student’s “*knowing that,*” or “*knowing about.*” Below are verbs that would generally associate with conceptual questions.
 - a. Identify
 - b. Recognize
 - c. Classify
 - d. Compare
 - e. Contrast
 - f. Interpret

3. **Problem Solving:** Problem solving questions require both procedural and conceptual skills as noted above in 1 and 2. Problem solving questions typically require the following.
 - a. Integration of discrete skill sets
 - b. Extensions of existing knowledge
 - c. Formulations of new strategy
 - d. Investigations of new or unique approaches
 - e. Reflections about methodologies used

These process standards are not wholly discrete. Often problems may fall in a gray area that surrounds these categories. Some problems will not fit only one process. In many instances, the background knowledge of the students may affect where a question could be classified. However, it is still important that questions be classified as accurately as possible within the aforementioned categories.

Background Information for Sample Test Score Grid

Question Analysis

It is important to remember that this analysis will only suggest potential problems or that questions are working just fine. Ultimately, the teacher as a professional must make the decision on whether or not a question is functioning as intended or requires edits or rejection.

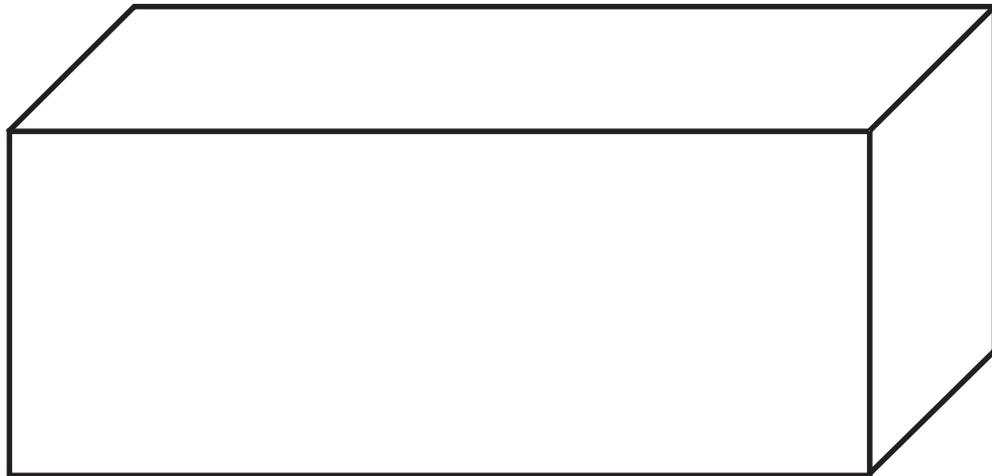
- **Question 1:** 100% of the students answered it correctly. It may be working just fine. However, you might consider if it is too easy for what you are trying to measure.
- **Question 2:** This question seems to be working fine. Those students who missed it also did poorly on the test overall, suggesting that it is their lack of skill that caused the wrong answer. However, it has the potential of being too easy.
- **Question 3:** This question appears to be functioning perfectly. The best students answered correctly, the poorest students did not.
- **Question 4:** This question, like question #2 also appears to be functioning well.
- **Question 5:** This question does not appear to be working well at all. It seems to be a fifty/fifty chance of whether or not a student will answer correctly.
- **Question 6:** This question like #2 and #4 also seems to be working fine.

- **Question 7:** Unfortunately, this question like #5 seems to have a random level of performance surrounding it.
- **Question 8:** This question does not seem to be working at all and should probably be rejected. It is clear that the very best students are having extreme difficulty with it but that the students who are doing poorly overall are getting it correct. This question is probably functioning as some sort of “trick question” that is causing the best students to over-think and proceed with a set of false assumptions about what is being asked.
- **Question 9:** This question is probably fine. However the low percentage might suggest that it is written at a level that may be too difficult.
- **Question 10:** This question is quite likely too difficult as noted by the fact that not even the best students achieved a correct response.

REMEMBER: The above analysis with the accompanying spreadsheet is for illustration purposes only. Teachers should have as their objective to place the majority, if not all, of their students at or near the top of the performance ladder. As such, they will want their test analyses for almost all items to look like Questions #1 and #2.

Sample Math Test for Math Core Academy

1. Compute: $3\frac{1}{2} \times 4\frac{3}{4}$
2. Suppose that Billy ate $\frac{3}{8}$ of a pizza. What do the 3 and the 8 represent?
3. Draw a model that shows the product of 9 and 8.
4. If b is 125% of a , then how does b compare to a ?
5. Use the picture below to answer a, b, and c.



- a. How many cubic feet of water can be stored in a tank that measures 8 feet x 4 feet x 2 feet?
- b. How many gallons of water will the above tank hold if 1 cubic foot of water is equal to 7.5 gallons?
- c. How many hours will it take to empty the water tank above if it is being drained at a rate of 2 pints per minute?

SAMPLE TEST SCORE GRID FOR MATH CORE ACADEMY

QUESTION NUMBER	1	2	3	4	5	6	7	8	9	10	TOTAL
POINTS POSSIBLE	1	1	1	1	1	1	1	1	1	1	%
BILLY	1	1	1	1	1	1	0	0	1	0	70%
SALLY	1	1	1	1	0	1	1	0	1	0	70%
MARK	1	1	1	1	1	1	0	0	1	0	70%
BRETT	1	1	1	1	0	0	1	0	1	0	60%
LISA	1	1	1	1	1	1	0	0	0	0	60%
JOHN	1	1	1	1	0	1	1	0	0	0	60%
SUSAN	1	0	1	1	1	1	0	0	1	0	60%
VAN	1	1	1	1	0	1	1	0	0	0	60%
YVONNE	1	1	1	0	1	1	0	0	1	0	60%
HANS	1	1	1	1	0	1	1	0	0	0	60%
FRANCIS	1	1	0	1	1	0	0	1	1	0	60%
STEPHANIE	1	1	0	1	0	1	1	1	0	0	60%
MILHAUS	1	1	0	1	1	0	0	1	0	0	50%
WILLIAM	1	1	0	1	0	0	1	1	0	0	50%
BECKY	1	1	0	0	1	0	0	1	0	0	40%
RALPH	1	1	0	0	0	0	1	1	0	0	40%
RHETT	1	0	0	0	1	0	0	1	0	0	30%
SCARLETT	1	0	0	0	0	0	1	1	0	0	30%
ISHMAEL	1	0	0	0	1	0	0	1	0	0	30%
CHARLES	1	0	0	0	0	0	1	1	0	0	30%
QUESTION %	100%	75%	50%	65%	50%	50%	50%	50%	35%	0%	

QUESTION ANALYSIS

- | | |
|-----|------|
| 1.) | 6.) |
| 2.) | 7.) |
| 3.) | 8.) |
| 4.) | 9.) |
| 5.) | 10.) |

MATHEMATICS 5 ITEM DEVELOPMENT SCHEMATIC

STANDARD	OBJECTIVE	INDICATOR	PROCEDURAL	CONCEPTUAL	PROBLEM SOLVING
I	1	a			
		b			
		c			
		d			
	2	a			
		b			
		c			
		d			
	3	a			
		b			
		c			
		d			
		e			
		f			
		g			
	4	a			
		b			
		c			
		d			
		e			
		f			
	5	a			
		b			
		c			
d					
e					
f					
g					
6	a				
II	1	a			
	2	b			
		a			
		b			
		c			
		d			
e					
III	1	f			
		a			
		b			
		c			
		d			
		e			
		g			
	2	a			
		b			
		c			
	3	a			
		b			
c					

IV	1	a			
		b			
		c			
		d			
		e			
	2	a			
		b			
		c			
		d			
		e			
V	1	a			
		b			
		c			
		d			
		e			
	2	a			
		b			
		c			

MATHEMATICS 6 ITEM DEVELOPMENT SCHEMATIC

STANDARD	OBJECTIVE	INDICATOR	PROCEDURAL	CONCEPTUAL	PROBLEM SOLVING	
I	1	a				
		b				
		c				
		d				
		e				
		f				
	2		a			
			b			
			c			
			d			
	3		a			
			b			
			c			
			d			
	4		a			
			b			
			c			
			d			
			e			
	5		a			
			b			
			c			
			d			
			e			
f						
g						
h						
6		a				
		b				
		c				
II	1	a				
		b				
		c				
		d				
	2		a			
			b			
			c			
			d			
			e			
			f			
III	1	a				
		b				
		c				
		d				
	2		a			
			b			
	3		a			
			b			
			c			

IV	1	a			
		b			
		c			
		d			
	2	a			
		b			
		c			
		d			
V	1	e			
		f			
		a			
	2	b			
		c			
		d			

Misconceptions

Misconceptions

“Sometimes we learn false concepts, commonly called ‘misconceptions.’ These data can enter into the processing of new information and interfere with the correct interpretation of new information or experience ...Students must confront the inconsistencies that lie between their mental notions and the new information before they can deal productively with the new ideas.” (Tolman, *Discovering Elem. Science: Method, Content, and Problem-Solving Activities*, 3rd edition, Allyn & Bacon, 2002, p.26).

Note: If some of the bricks used in building a house were improperly cured, the inferior bricks become part of the house. Covering them over with stucco will not strengthen that part of the wall much. To correct the flaw, the weak part of the wall must be identified, the bad bricks removed and then replaced with good bricks. Since poorly cured bricks look the same as properly cured ones, identifying and removing the weak elements can be difficult. Similarly, it is necessary but difficult to identify and remove misconceptions. However, they must be removed, because like cancers, they continue to erode the formation of correct concepts.

“Every time we communicate, new concepts compete with the preconceived ideas of our listeners. All students hold these ideas, but they are unaware of their private theories.”

From the video “A Private Universe: Misconceptions that Block Learning.”

Produced at the Harvard-Smithsonian Center for Astrophysics.

Distributed by: The Astronomical Society of the Pacific
390 Ashton Ave.
San Francisco, CA 94112

Examples of common misconceptions:

- Every time we communicate, new concepts compete with the preconceived ideas of our listeners. All students hold these ideas, but they are unaware of their private theories.

Misconceptions Notes

Journaling Ideas

Journaling

Background Information

Science and math concepts have been compared to a brick building where, course by course, each brick provides the cement foundation for the information that is to follow. The foundation of radioactivity research was the life work of the great French physicists, Pierre and Marie Curie. Madame Curie's carefully preserved journals minutely detail step-by-step the process of their work which has become the basis for innumerable advancements in science.

Purdue University research focused on using children's journals as a tool for teaching, learning, and assessing science. The findings showed that journals were valuable in teaching communication and literacy techniques during science lessons. Journals also provided documentation that students had a better understanding of scientific concepts and had begun an inquiry-based self-learning process.

It is vital to introduce student journals at the elementary school level to create the habit of journaling as a lifelong learning tool. Both formal and informal journaling models have been found to be equally effective. Journaling experiences are described in *Jordan School District Science TRB3* as:

1. Drawing pictures and labeling them
2. Writing short paragraphs about observed changes and interactions
3. Noting reactions to different stimuli
4. Making predictions
5. Recording observations
6. Analyzing reasoning
7. Keeping measurements
8. Making comparisons

Journaling Examples

from *Dinah Zike's Big Book of Books and Activities*

Layered Look Book p. 70-78

Pop-Up Book p. 112-115 *Remember the rule—Always cut on a fold, and Never glue on a fold.

Top Tab Book p. 80-83

Materials

- scissors
- paper
- glue sticks
- markers
- staplers and staples

Pyramid Book p. 38-41

Circle Book p. 64-65

Additional Resources

Jordan School District Science TRB3

Dinah Zike's Big Book of Projects by Dinah Zike (M. Ed.)

Big Book of Books and Activities by Dinah Zike (M. Ed.)

Journaling Notes

Journaling Notes

Standard I
Activities

Activity—Dividing Fractions: Another Perspective

Standard I

Students will acquire number sense and perform operations with rational numbers.

Objective 5

Solve problems using the four operations with whole numbers, decimals, and fractions.

Intended Learning Outcomes

6. Represent mathematical situations.

Standard
I
Objective
5

Connections

Background Information

When dividing fractions, students will have a better understanding if they think of how many times the divisor will “fit” into the dividend. The reason we use reciprocals when dividing fractions is to make the divisor one.

Invitation to Learn

Ask the question, “What does it mean to divide 8 by 4?” Have students draw a representation of what it means. Encourage the students to share their representation. Ask students to then draw a model to show what $3/4$ divided by $1/4$ means.

Instructional Procedures

The students will typically look at the above division problem as dividing 8 into 4 parts. Help them to see that you can look at the problem in another way; how many fours are in 8. This works the same for $3/4$ divided by $1/4$. How many $1/4$'s are in $3/4$?

1. Give each student a fraction pie. Instruct the students to layout before them the disk equivalent to $1/2$. Ask the students to demonstrate what $1/2$ divided by $1/4$ would be. (How many $1/4$ are in $1/2$). Have the students do five or six similar problems in which the first number is bigger. Be sure that they are very comfortable with this way of thinking before moving on to the next step. They should always use the language of how many dividends are in the divisor.
2. Next have the students lay out a disk equivalent to $1/4$. Ask them to demonstrate $1/4$ divided by $1/8$. Then ask them what they think $1/4$ divided by $1/2$ would be? Help them to see that there is only $1/2$ of the $1/2$ in $1/4$. $1/4$ divided by $1/2 = 1/2$. Do similar problems until the students are comfortable with the concept. Help them to see that when dividing fractions, sometimes the quotients are larger than the dividend or the divisors.

Materials

- Fraction pies
- Rulers
- Paper and pencil

3. Next give each student a ruler. Present to the students $5 \frac{1}{2}$ divided by $\frac{1}{2}$. Ask the students to demonstrate the answer using their rules. (How many $\frac{1}{2}$ inches are in $5 \frac{1}{2}$ inches). Instruct them to estimate, then find the exact answer. Guide them through several examples of dividing mixed numbers.
4. Give the students several written problems and have them estimate what they think the answers would be. Working in groups, and then having the groups report on their procedures would develop opportunities for more math talk.
5. Ask the students to develop story problems for several division of fraction expressions. Encourage the students to share their different examples.
6. Ask the students if they would like to learn a short cut for dividing the fractions. Demonstrate the following two ways of representing $\frac{1}{4}$ divided by $\frac{1}{2}$:

$$\frac{\frac{1}{4}}{\frac{1}{2}} \quad \frac{1}{2} \overline{) \frac{1}{4}}$$

7. Explain, "It is difficult to think about dividing by $\frac{1}{2}$. But, it is easy to divide by 1. Is there a way that we can change the $\frac{1}{2}$ to a 1?" Guide the students to the understanding that the divisor can be multiplied by $\frac{2}{1}$, its reciprocal. Help them to see that they must multiply the dividend and the divisor by $\frac{2}{1}$ to get the correct answer.

$$\frac{\frac{1}{4} \times \frac{2}{1}}{\frac{1}{2} \times \frac{2}{1}} = \frac{\frac{1}{4} \times \frac{2}{1}}{1} \quad \frac{1}{2} \times \frac{2}{1} \overline{) \frac{1}{4} \times \frac{2}{1}} = 1 \overline{) \frac{1}{4} \times \frac{2}{1}}$$

The shortcut is to multiply the dividend by the reciprocal of the divisor. Have the students record in their journal the meaning and methods of fraction division.

Curriculum Integration

This perspective of division works very well with decimals also.

Possible Extensions/Adaptations

The following are several real world situations in which division of decimals is used. Have the students explain to one another how they would solve these situations. Then have them write equations for their procedures.

You need \$69.99 to purchase a new video game. You decided to sell boxes of chocolates to kids at your school to earn the money. You buy 15 $\frac{1}{2}$ lbs of chocolate for \$ 35.00. The clerk suggested that you package the chocolates in either $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$ or $\frac{3}{4}$ lb boxes. Determine how many boxes of chocolates you could fill for each of the fractions. Select which size you think would sell the best. How much would you charge for each box of chocolates? Be sure that you make enough to pay for the chocolate and your game. Do you think students would buy the chocolates for the price you are asking?

You decide to make sock bags for your friends for Christmas. You find in the closet 5 $\frac{1}{4}$ yards of cloth. You need $\frac{3}{4}$ of a yard to make one bag. How many bags could you make?

You get a job installing tires on cars in an assembly line. You get paid \$5.00 for each set of tires you install. It takes you $\frac{1}{4}$ of an hour to put on a set of tires. If you work for 8 $\frac{1}{2}$ hours, how much would you expect to make?

Assessment Suggestion

As students respond and explain their response to you and fellow students, assess their understanding. Their journal entry would be another opportunity for assessment.

Additional Resource

<http://mathforum.org>

Homework & Family Connections

Have the students use manipulatives to explain to their parents or siblings what division of fractions means.

Assign students to measure the amount of food in a box (cereal, rice, noodles etc.). Have them determine how many batches they could make if a recipe calls for $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, or $\frac{3}{4}$ cups of their selected food. Have them write a math sentence using division of fractions to represent what they did.

Assign students to measure the length of their bedroom. Then have them determine how many $\frac{3}{4}$ foot tiles they would need to lie tiles across the length of the room. Have them write a math sentence using division of fractions to represent what they did. They could also find how many $1\frac{1}{2}$ foot dressers they could line up, or how many $\frac{1}{3}$ foot candy bars, or $3\frac{1}{2}$ ft wide beds, etc.

Activity—Investing Money

Standard I

Students will acquire number sense and perform operations with rational numbers.

Objective 5

Solve problems using the four operations with whole numbers, decimals, and fractions.

Intended Learning Outcomes

5. Make mathematical connections.

Standard
I
Objective
5
Connections

Information

Stocks are shares of ownership in a company. A company sells shares to get money to operate their business. People and companies buy stocks with the hope that they can sell them in the future for a profit. The price of shares is published on the internet and in newspapers. Stock histories can be researched on the internet.

Invitation to Learn

Ask the following questions: “How many of you are hoping to buy a car when you graduate from High School? What kind would you like to buy? How much would it cost? Imagine you receive \$3,000 today. How would you save the money? What are some of the options you have?”

Instructional Procedures

Explain to the students that they are going to invest and track their pretend \$3,000 over a period of time. They will pretend to invest \$1,000 each in stocks, \$1,000 in a saving account, and \$1,000 in some type of “metal.”

1. Begin by explaining stocks and their purpose. Describe some of the things that can cause stock prices to rise and fall. Have students pick at least three stocks from the newspaper or the internet. If you have computer access, have them research the stock history. Encourage them to explain why they selected their companies. Demonstrate how to calculate the purchase of their stocks. Their goal is to spend as much as possible of the \$1,000.
2. Explain that some people prefer to buy metals. Have them make some suggestions about where would be good places to store the metals they purchase. Have the students pick a metal to buy and calculate how many pounds of the metal they can purchase for \$1,000.

Materials

- Internet access or the market sections of newspapers
- Tally sheets
- Graph paper

3. Explain that other people prefer to keep their money in banks. This is a very secure way of saving money. Another advantage is the easy availability of the money. Then explain that the banks pay you interest for the use of your money. Demonstrate how saving accounts accrue interest. Have the students pick a bank in which to deposit \$1,000.
4. Have the students answer the following questions in their journals:

Which of the three investments do you think will earn you the most money? Which investment do you think is the most convenient? Which investment is the most secure? If you were actually investing \$3,000 dollars, how would you invest it? Explain how to calculate your purchase price for a stock.
5. Throughout the year, have the students periodically calculate and graph their earnings.

Curriculum Integration

This lesson would fulfill several requirements of economic study in Social Studies.

Possible Extension/Adaptations

Students could do more extensive research about the companies they selected. Encourage the students to find reasons why their companies have been successful/unsuccessful. Invite stockbrokers, investment managers, and bankers as guest speakers. You could also have the students calculate interest on loans and credit cards.

Assessment Suggestion

Assessment would be the accuracy of the calculations made in purchasing and selling investments. Their ability to estimate can be observed as they calculate how much they can buy of the different stocks.

Additional Resources

www.finance.yahoo
www.japersonalfinance.com

Homework & Family Connections

Students could be encouraged to discuss with their parents family investment strategies and values.

Name _____

Investment Tally Sheet

Stock Market			
<i>Stock</i>	<i>Price</i>	<i>Number of Shares</i>	<i>Total Cost</i>

Metals			
<i>Type of Metal</i>	<i>Price</i>	<i>Number of Pounds</i>	<i>Total Cost</i>

Savings Accounts			
<i>Bank</i>	<i>Interest Rate</i>	<i>Deposit</i>	<i>Total</i>

Activity–Cyclops

Standard

I

Objective

5

Connections

Standard I

Students will acquire number sense and perform operations with rational numbers.

Objective 5

Solve problems using the four operations with whole numbers, decimals, and fractions.

Intended Learning Outcomes

4. Communicate mathematically.

Background Information

The story of Odysseus and the Cyclops is found in Homer’s book *Odysseus*. There are many different versions and translations of the Greek myth. In the story, Odysseus and his crew of sailors, looking for food and water, dock in the land of the Cyclops’. One Cyclops, Polyphemus, captures Odysseus and his men. Polyphemus takes the men home to eat. Odysseus get Polyphemus drunk and pokes out his eye. Odysseus and his men escape by clinging to the undersides of the sheep as they go out the door.

Invitation to Learn

Read the story of Odysseus and the Cyclops to the students. Ask them how big they think the Cyclops was.

Instructional Procedures

1. Divide the students into groups. Instruct the students to come up with a group estimate of how tall Cyclops would have been. Have them explain their reasoning to others.
2. Next instruct the students to select a Cyclops’ possession (comb, toothbrush, spoon, ring, pencil, etc.) and determine what the size of the object would be. Have them write a mathematical sentence representing their reasoning. They should then cut out their object from butcher paper and hang them on the wall.
3. The students will notice that the objects are of different sizes. Ask them why they think they are so different. Help them to determine that it depended on the size they chose for Cyclops and the size of the object they used for comparison.

Materials

- Story of Odysseus and the Cyclops
- Rulers
- Paper and Pencils
- Butcher Paper
- Scissors

Curriculum Integration

Greek Mythology

Possible Extension/Adaptations

The students can determine the size of other Cyclops' possessions.

This same teaching format works well to help students determine sizes and distances in making space models. (If the Earth were the size of a basketball, what object would be the size of the moon?)

In the book, *Cry Uncle* (chapter 12), the older boy divides their bedroom using the ratio of their ages. Read and discuss this to the students. Have the students pretend they must share a room with an older brother or sister. Have them determine what fraction of their rooms they would have, using the older brother's methods from the story.

Assessment Suggestion

Evaluate the students' explanations of how they developed the math equation to determine the size of their objects.

Additional Resources

www.mythweb.com/odyssey (Cyclops story)

www.bulfinch.org/fables (Cyclops story)

<http://web.ukonline.co.uk> (Cyclops story)

Cry Uncle by Mary Jane Auch

Homework & Family Connections

Assign students to find the ratio of their height to the height of their parents or a sibling. Using this ratio, have them determine what different sizes of clothing, beds, bedrooms, and other objects would be if made to this scale.

Activity—Dishes for a Penny

Standard

I

Objective

1

Connections

Standard I

Students will acquire number sense and perform operations with rational numbers.

Objective 1

Represent whole numbers and decimals in a variety of ways.

Intended Learning Outcomes

5. Make mathematical connections.

Background Information

To evaluate exponents, most calculators use a caret key (^) or an x^y . To evaluate 5^6 , press 5 (^ or x^y) 6 and (= or enter).

Invitation to Learn

Ask the students if they would be willing to do the dishes at their home for the following pay schedule:

The first night their parents will give them one penny. For each night after, if they continuously do the dishes, their parents will double the amount.

Instructional Procedures

1. Begin the lesson by asking the students to predict how much they think they would earn in the two-week period. Have the students explain how they would calculate the amount. Explain to the students that rather than writing out repeated multiplication of one factor, there is a more efficient way to write it. Explain to them the meaning of the terms base and exponent. Be sure to model for them the correct wording (2 raised to the 10th power or 2 to the tenth power).
2. Using the Dishes Chart, have the students calculate how much money they would receive at the end of the second week. The students are usually comfortable doing up to 2^5 using mental math. After they have calculated 2^5 students will be ready to learn how to use their calculators to evaluate the expressions. The students will be amazed how quickly the values increase.

Materials

- Calculators
- Dishes chart worksheet for each student

Curriculum Integration

This is a good time to explain the meaning of the dimensions used in geometry. Have the students draw a person in the first dimension. The person would just be lines. In geometry, we say lines and measure them in inches, miles, centimeters, etc.

Next have them draw a person in the second dimension. This would be a person that has both width and length, but no depth. Have the students describe what they think it would feel like. Many have read stories about Flat Stanley. He was a two-dimensional person. In geometry, we describe two-dimensional figures as being squares, triangles, quadrilaterals, etc. We label them in², cm², mm², etc.

Then have them draw a person in the third dimension. This drawing should have width, length, and depth or height. In geometry, we describe these dimensional figures as being cubes, spheres, pyramids, etc. We label them in³, cm³, mm³, etc. *Wrinkle in Time* has examples of the dimensions. The children travel in the fourth dimension to a two-dimensional planet.

Possible Extension/Adaptations

Reading the story *King's Checkerboard* is an excellent follow up. Bring a bag of rice and have the students count how many grains of rice the man will receive during the first week.

Another follow up activity is to give each student a sheet of small-squared graph paper. The students color in the number of squares to represent 5¹, 5², 5³, etc. Challenge them to get 5 to the highest power possible on just one sheet.

Assessment Suggestion

Ask the students to determine the value of 4⁴.

Additional Resource

A Wrinkle in Time by Madeleine L'Engle

Homework & Family Connections

The students enjoying going home and trying to make the deal for doing dishes with their parents.

Dishes for a Penny Worksheet

<i>Day</i>	<i>Exponent</i>	<i>Amount</i>	<i>Total</i>
1	2^1	\$ 0.01	\$ 0.01
2	2^2		
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			

Activity—Making Games

Standard I

Students will acquire number sense and perform operations with rational numbers.

Objective 5

Solve problems using the four operations with whole numbers, decimals, and fractions.

Standard
I
Objective
5
Connections

Background Information

Students are typically motivated by games. Games can be a fun and motivational tool for practicing number and operation skills. While playing games, the students will practice skills over and over and think of it only as playing a game. It makes computation practice painless. While playing games, the students receive instant feedback. There is often much math communication as students discuss the reasons why answers are right or wrong. It is important to insist that students use correct math words while playing the games.

Parents can be a great help in playing games. Some teachers have parent volunteers come in for “Math Game Friday” or have them play a game with a small group of students who need extra practice and immediate feedback. The volunteers can also help the teacher assess the needs of the students.

Having the element of “luck” is important. No student should feel stupid because they lost a game. With the element of “luck” the students lose the game because they are unlucky, not because they are stupid. Students especially love it when they are lucky enough to beat the teacher.

Invitation to Learn

Students almost always enjoy playing games. It is important that clear and simple directions are given. If the directions are too complicated, the game loses its appeal for most students.

Instructional Procedures

1. Determine the skill or process to be practiced.
2. Match the skill with a simple or common game.
3. Develop the materials needed.
4. Have the students play the game.

Materials

- Dice or spinners
- Fraction, decimal, percent, ratio cards
- Paper

5. Evaluate and change the game—always be ready when needed to change the rules in the middle of play. Often the students have excellent ideas for changes.

Curriculum Integration

Many developed games can also be adapted to practice vocabulary and skills in geometry, probability, algebra, and problem solving.

Possible Extension/Adaptations

After teaching a new skill or process, challenge the students to come up with a game that will help them practice the skill.

Assessment Suggestion

Assessment is typically instant when playing games. The teacher, volunteer, and other students quickly detect mistakes and can quickly re-teach.

Additional Resources

Mega-Fun Math Games by Dr. Michael
www.funbrain.com
www.coolmath.com

Homework & Family Connections

Games can be taken home to play with parents and siblings. It is an excellent way to involve parents and help them to be aware of what their students are currently studying. It often leads to excellent parent discussions of the use of math in the real world.

FACTOR CAPTURE

Participants

2 players

Procedure

Player One selects a number (crosses it out on the Chart). Player Two then captures the factors of that number (crosses them out on the chart, while saying, “(the factor being crossed out) is a factor of (the number selected by the other player).” Player One’s points equal the number that Player One crossed out. Player Two’s points are the sum of the factors Player Two crossed out.

Example: If Player One selects 12, Player One would receive 12 points. Player Two could cross out the factors of 12 (1, 2, 3, 4, and 6). The sum of these factors is 16. Player Two would receive 16 points. The players keep a running total of their points.

Player Two then selects a number. Player One captures the factors. If the player selects a number for which there are no factors available, then no points are awarded either player and the other player selects the next number. The play continues until only prime numbers are left on the board.

Materials

- Factor Capture Chart
- Paper and Pencil

FACTOR CAPTURE

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

GARBAGE CAN BASKET BALL

Participants

Any number of players

Procedure

Set up the following basketball area by putting the garbage can in the middle. Then make a circle around the basket with the string. Put the card $\frac{5}{6}$ in the circle. Make a bigger circle around this and put the $\frac{3}{4}$ card in the outer circle. Continue for the remaining fractions. Give each player a ball (the crumpled sheet of paper). The players all shoot their balls at the garbage can. A score of one point is give to all who make the basket. All other students get the score of the circle their ball lands in. Have the students “shoot” again. Students are to keep a running total of their scores. You can play for a set time period or until a predetermined score is achieved.

Materials

- Garbage can
- Paper and pencil
- String
- Cards labeled $\frac{5}{6}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$
- Crumpled sheets of paper of different colors

EQUIVALENT FRACTION CAPTURE

Participants

2 -5 players

Procedure

Have the students get into groups of 2-5 players. Have the students stand the cards up in the middle. They sit around the cards. The teacher/parent/student flips over the first of the fraction cards. The students in the circle grab the card in the middle with an equivalent fraction written on it. The player to grab the card gets one point and puts the card back into the middle. The teacher/parent/student flips over the next number.

Materials

- Cards folded in half with the following fractions labeled on each side:
1/2, 1/3, 2/3, 1/4, 3/4,
1/5, 2/5, 3/5, 4/5, 1/6,
5/6
- Fraction cards (The cards need to be fractions that are not simplified)

EQUIVALENT SPOONS

Participants

4 or more players

Procedure:

Have the students sit in a circle and place all of the spoons in the middles of the circle. Pass out all of the cards. The players look at their cards and try to find matching sets of four. When a player has a matching set they *quietly* pick up a spoon. All of the other players then pick up a spoon. The last player to notice the spoons being picked up will not get one. If none of the players have a set, then each player takes one card from their hand and passes it to the right. It helps to have one person calling out “pass” when it is time to pass a card. If you wish to keep score, the player without the spoon receives a letter. The first letter would be an “S,”

then a “P,” etc. spelling Spoons. The cards are then gathered, dealt again, and play begins again. When a player spells Spoons the game is over.

Materials

- Fraction/decimal/percent/ratio Cards
- Spoons (one less than the number of players)

Fraction/Decimal/Percent/Ratio Cards

$\frac{1}{4}$

25%

1:4

0.25

$$\frac{1}{3}$$

33.3%

0.33

1:3

50%

$\frac{1}{2}$

1:2

0.5

$$\frac{3}{4}$$

75%

3:4

0.75

30%

$\frac{3}{10}$

3:10

0.3

$$\frac{1}{8}$$

12.5%

1:8

0.125

20%

$\frac{1}{5}$

1:5

0.2

0.8

4:5

$\frac{4}{5}$

80%

70%

$\frac{7}{10}$

7:10

0.7

Standard II
Activities

Activity—Two-Step Equations

Standard II

Students will use patterns, relations, and functions to represent and analyze mathematical situations using algebraic symbols.

Objective 2

Represent, solve, and analyze mathematical situations using algebraic symbols.

Intended Learning Outcome

6. Represent mathematical situations.

Standard II

Objective 2

Connections

Background Information

Solving two-step equations algebraically has many real world applications. Keeping an equation in balance is a common thread throughout all algebra. Students must learn to automatically perform the same operation with the same number on both sides of the equation.

Linear equations with one variable have only one solution. To find the solution, first simplify by using the properties and the order of operations to rewrite without grouping symbols and to collect like terms. Then isolate the variable by using inverse operations. Addition and subtraction are inverse operations; multiplication and division are inverse operations.

When you solve a two-step equation, whether modeling with tiles or using algebraic properties, you get the variable alone on one side of the equation by reversing the order of operations.

Invitation to Learn

Choose one task from the following list and write the step-by-step instructions on how to accomplish the task:

- making cookies
- mowing the lawn
- building a snowman
- building a fence

Call on students to share their step-by-step instructions. Discuss the importance of completing one step before moving on to the next.

Many math problems need to be handled in a step-by-step method as well.

Instructional Procedures

Materials

- Algebra tiles for each student
- Foam working mat for desk top
- Colored pencils
- One-step and two-step equations worksheets

1. Distribute algebra tiles, mats, and colored pencils accompanying worksheets.
2. Review writing and solving one-step equations using algebra tiles. (Note: one-step equations should have been mastered in the 5th grade. However, since the equation concept is relatively new and requires such abstract thinking, the teacher may want to conduct a very thorough review of one-step equations. Two or three class periods might even be used for review to ensure confidence in these equations.)

3. Demonstrate the following on overhead:

Addition equation $x + 2 = 6$

Subtraction equation $x - 3 = 7$

Multiplication equation $3x = 6$

4. Have students work through the worksheet using their algebra tiles, drawing pictures, and writing the solution to each problem.

Two-Step Equations:

Step 1: Add or subtract the same number from each side of equation

Step 2: Divide the same number (the number beside the variable) from both sides of equation.

5. Demonstrate the following on the overhead with students using tiles. Discuss each step thoroughly.

$$2x + 3 = 5 \qquad 2 + 5x = 12$$

$$3y - 2 = 7 \qquad 3y - 5 = 10$$

6. Have students work through problems on the worksheet using algebra tiles, drawing pictures, and writing the step-by-step process as well as the solution to each equation.

Curriculum Integration

Present some real world problems that require a two-step equation to solve the problem.

Economics example—Carmela wants to buy a digital camera for \$249. She has \$24 and is saving \$15 each week. Solve the equation $\$15w + \$24 = \$249$ to find how many weeks she will take to save enough to buy the camera. (answer: 15 weeks)

Nutrition example—A soccer player wants to eat 800 calories at a meal that includes a roast beef sandwich and potato chips. The sandwich has 570 calories and the potato chips have 23 calories each. Solve the equation $570 + 23p = 800$ to find the number of potato chips the soccer player can eat. (answer: 10 chips)

Assessment Suggestion

Give students 2 or 3 two-step equations and ask them to draw algebra tiles to represent the equations, showing the steps involved. Then find the solution to the equations.

Additional Resources

www.etacuisenaire.com

The Algebra Lab: Middle School, 1990 Creative Publications, 1300
Villa Street, Mountain View, CA 94041

Middle Grades Math, Course 1, Prentice Hall, 2001

Homework & Family Connection

A family vacation to Disneyland will cost \$2,000. Your family has already saved \$450 toward the trip and they are saving \$300 every month. How many months will you have to save before you can go on your family vacation? Write an equation to solve the problem. Since the solution is not a whole number, discuss with your family about how to round the answer. Should you round up or down?

Addition and Subtraction Equation Review

Use algebra tiles to solve the following equations, then use the space provided to draw the model of the algebra tiles for each equation.

$y - 3 = 6$ number added to each side _____ solution $y =$ _____	$2 = x - 1$ number added to each side _____ solution $x =$ _____
$5 = y - 4$ number added to each side _____ solution $y =$ _____	$1 + x = 7$ number subtracted from each side _____ solution $x =$ _____
$3 + d = 5$ number subtracted from each side _____ solution $d =$ _____	$2 + t = 8$ number subtracted from each side _____ solution $t =$ _____

Name _____

Multiplication Equation Review

Use algebra tiles to solve the following equations, then use the space provided to draw the model of the algebra tiles for each equation.

$3x = 6$ divide each side into _____ groups. solution $x =$ _____	$10 = 2y$ divide each side into _____ groups. solution $y =$ _____
$4t = 8$ divide each side into _____ groups. solution $t =$ _____	$6 = 2n$ divide each side into _____ groups. solution $n =$ _____
$3 = 3s$ divide each side into _____ groups. solution $s =$ _____	$5x = 10$ divide each side into _____ groups. solution $x =$ _____

$7 = 2k + 1$ <p>Step 1: Subtract _____ from each side.</p> <p>Step 2: Divide each side into _____ groups.</p> <p>Solution: $k =$ _____</p>	$4r - 3 = 5$ <p>Step 1: Add _____ to each side.</p> <p>Step 2: Divide each side into _____ groups.</p> <p>Solution: $r =$ _____</p>
$11 = 4p - 5$ <p>Step 1: Add _____ to each side.</p> <p>Step 2: Divide each side into _____ groups.</p> <p>Solution: $p =$ _____</p>	$5x - 6 = 9$ <p>Step 1: Add _____ to each side.</p> <p>Step 2: Divide each side into _____ groups.</p> <p>Solution: $x =$ _____</p>

Activity—Evaluating Expressions Using Algebra Tiles

Standard II

Objective 2

Connections

Standard II

Students will use patterns, relations, and functions to represent and analyze mathematical situations using algebraic symbols.

Objective 2

Represent, solve, and analyze mathematical situations using algebraic symbols.

Intended Learning Outcomes

5. Make mathematical connections.
6. Represent mathematical situations.

Background Information

Evaluating expressions is a concept used throughout all of algebra. The student is required to replace a variable (unknown) with a given value and then evaluate (calculate) the answer. There are many real-world situations in which this is used, (e.g., If I order 3 CD's at \$17 apiece and add shipping and handling charges, what will my final cost be?) We also come upon formulas that require a value to take the place of an unknown variable. Students must follow the order of operations in order to correctly evaluate algebraic expressions.

Using algebra tiles to introduce the evaluation of expressions helps students to have a visual image of what is happening. They are actually replacing the variable with a tile that represents a number. This should make the transition to paper and pencil evaluations easier to understand.

Invitation to Learn

Discuss with students the fact that some jobs pay the worker a certain amount per hour. For instance, babysitting may pay \$2 or \$3 per hour. Other jobs pay per piece. Lawn mowing may pay \$10 or \$15 per lawn. In both situations, there is always an unknown involved. In babysitting, one must know the number of hours worked and in lawn mowing, one must know the number of lawns mowed. As soon as we know that, we can calculate how much money we have earned.

Instructional Procedures

1. Using the algebra tiles, demonstrate a variable expression and have students model it with their tiles, (e.g., $3h$ to show \$3 per hour of babysitting). Have a student choose the number of hours they babysat (2 hours) and demonstrate how each variable (h) is replaced with 2 “ones.” They will see that 6 ones or \$6 is the amount earned. Use the algebra tiles to manipulate the same

Materials

- Algebra tiles
- Foam craft mat for each student
- Colored pencils

situation with lawn mowing at \$10 per lawn. These are very simple problems and most will immediately see the connection between the problem and the algebra tiles.

- Continue working with algebra tiles making the problems increasingly more difficult.

$$2x + 1, \text{ where } x = 3$$

$$5b - 1, \text{ where } b = 2$$

$$4 + 2t, \text{ where } t = 4$$

Using 2 variables:

$$2x + 3y, \text{ where } x = 3 \text{ and } y = 4$$

$$3g - h, \text{ where } g = 2 \text{ and } h = 5$$

- While using the algebra tiles for each problem, make sure students write down the expression and solve it with pencil and paper so that the transition becomes easier.

Extensions and Adaptations

Introduce some formulas and have students evaluate the formulas.

For instance:

$$\mathbf{A = l \times w}$$
 (Area = length times width), where $l = 8$ and $w = 6$.

$$\mathbf{P = 2l + 2w}$$
 (perimeter = 2 times length + 2 times width), where $l = 3$ and $w = 2$.

$$\mathbf{d = rt}$$
 (distance = rate times time), where $r = 60$ mph and $t = 2$ hours.

$$\mathbf{T = (n \div 4) + 37}$$
 (Temperature ($^{\circ}\text{F}$) = number of times a cricket chirps in one minute divided by 4 plus 37), where $n = 100$.

$$\mathbf{a = h \div n}$$
 (batting average = number of hits \div number of times at bat), where $h = 11$ and $n = 40$.

Curriculum Integration

Meteorology—The temperature formula above ($T = (n/4) + 37$) is fun for students when they learn that crickets actually chirp more times per minute as the temperature rises. There are also formulas to convert from $^{\circ}\text{C}$ to $^{\circ}\text{F}$. $^{\circ}\text{F} = 1.8 \times ^{\circ}\text{C} + 32$, or $^{\circ}\text{C} = 1.8(^{\circ}\text{F} - 32)$. Many countries use the Celsius scale rather than the Fahrenheit scale for measuring temperature.

Assessment Suggestion

Have students evaluate the expression $2a + 3b$ for $a = 2$ and $b = 5$, showing each step. Have them exchange their work with a partner. Next to each step, the partner writes what operation was performed.

Additional Resources

www.etacuisenaire.com

The Algebra Lab: Middle School (Creative Publications)

Prentice Hall Middle Grades Math Tools for Success, Course 1, 2001.

Homework & Family Connections

Give the students the formula $s = 3f - 24$ (shoe size = 3 times foot length in inches minus 24), which is used to calculate men's shoe size. Assign them to calculate the shoe size of 3 men in their family or among their friends by measuring the length of the man's foot in inches and calculating the formula.

Activity–Patterns

Standard II

Students will use patterns, relations, and functions, to represent and analyze mathematical situations using algebraic symbols.

Objective 1

Recognize, analyze, and use multiple representations of patterns and functions and describe their attributes.

Intended Learning Outcomes

6. Represent mathematical situations.

Standard II

Objective 1

Connections

Background Information

Many students readily take to finding and continuing patterns. Patterns are found in a sequence of numbers when there is a common difference between the terms in the sequences. Patterns are also observed in many architectural works (buildings). These patterns are more geometric in nature.

This activity furthers the concept of patterns by creating tables for the patterns and then writing a rule about the pattern. The table (sometimes called ‘table of values,’ ‘input/output table,’ or ‘function table’) becomes the “X and Y coordinates,” and can later be graphed on the coordinate plane.

Consistency is a basic rule in algebra, so consistent patterns can often explain why algebra works the way it does.

Invitation to Learn

Give each student 2 containers of centimeter cubes (one has 20 cubes—10 of one color and 10 of another and the other has 30 cubes—10 each of 3 different colors). Using their foam mats with the line down the middle, have them build a structure on the left with the 20 cubes and another structure on the right with the 30 cubes. Observe and comment on patterns that are being used in the building of the structures.

Instructional Procedures

1. Guide the students through the first couple of patterns from the overhead transparency, completing the table on the answer sheet. Guide them through the rule portion of the answer sheet. The rule needs to relate the first column of the table to the second column of the table (how a change in “x” affects “y”). They should be encouraged to be explicit in writing the rule.

Materials

- Overhead transparency of patterns to be built
- Answer sheet for each student
- Centimeter cubes for each student
- Containers for cubes

2. Working in small groups, they can complete each table and rule from the patterns provided. The patterns should get progressively more difficult.
3. After the worksheets are completed, students can be taught that the left side of the table represents the “X coordinate,” and the right column represents the “Y coordinate.” Then, knowing the X and Y coordinates, they can graph the points on a coordinate plane.
4. Guide students in writing an equation from each table. The “rule” will help in establishing the equation.
5. Assign students to write in their journal to explain the connection between a table of values and a coordinate plane.
6. This activity will probably take more than one class period in order to make full understanding and connections between patterns, tables, graphs, and equations.

Curriculum Integration

Geometry—You can draw diagonals from one vertex inside a convex polygon to create triangles. As the number of sides of a polygon increases, the number of triangles you can make also increases. This can be recorded on a function table using “number of sides” on the left side of the table and “number of triangles” on the right side.

Physical Education—A table can be created relating the points scored as a function of baskets made (basketball), field goals kicked (football), or touchdowns completed (football).

Science— A table can be created showing the time it takes a person (animal) to travel to a certain location and the distance the person traveled. The speed would need to be constant. The information could be graphed to show the linear relationship that exists between time and distance.

Possible Extensions/Adaptations

Have students use a graphing calculator to enter the data from a table of values. The calculator will create the table, create a graph, and allow for predictions to be made.

Assessment Suggestion

Tell students that they have just been hired at a job that pays \$5 an hour. Assign them to work with a partner to make a Table of Values with “Hours Worked” on the left and “Wages Earned” on the right. Also have them construct a graph from the table and write an equation that relates to the table. Ask questions such as “How many hours must you work to earn \$40?” and “If you receive a paycheck in the amount of \$150, how many hours did you work during that pay period?”

Additional Resources

Algebra Thinking, First Experiences (Creative Publications)

Middle Grades Math, Course 1 (Prentice Hall)

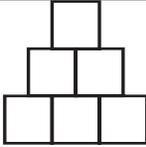
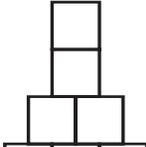
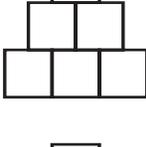
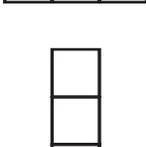
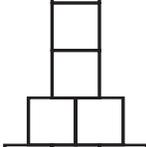
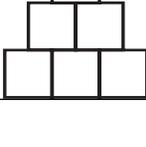
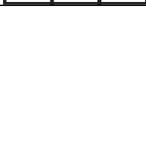
Get It Together (EQUALS, Lawrence Hall of Science,)

Homework & Family Connections

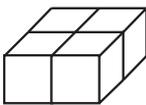
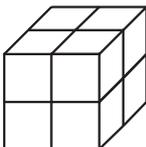
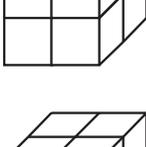
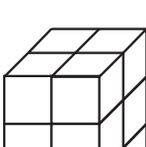
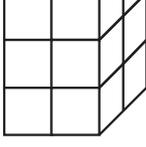
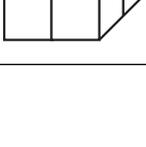
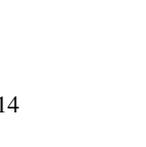
Assign students to check with parent(s) for permission to use a recent long-distance telephone bill. Then make a table with “time spent talking” (minutes) on the left and “cost of call” on the right. Make a graph of the information.

Pattern Building

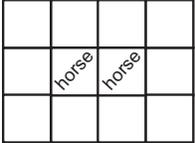
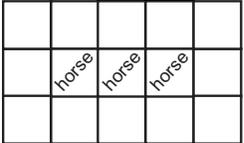
Castles and Towers

	# cubes in tower	# of cubes in all	Rule:
	1	7	
	2	8	
	3	_____	
	4	_____	
	5	_____	
	10	_____	
	20	_____	
	30	_____	

Apartment Buildings

	# of floors	# of cubes in all	Rule:
	1	4	
	2	8	
	3	_____	
	4	_____	
	5	_____	
	10	_____	
	20	_____	
	30	_____	

Horse Corrals

	# of floors	# of cubes in all	Rule:
	1	4	
	2	8	
	3	—	
	4	—	
	5	—	
	10	—	
	20	—	
	30	—	

Standard III
Activities

Activity—The Shape Shifter

Standard III

Students will use spatial and logical reasoning to recognize, describe, and identify geometric shapes and principles.

Objective 1

Identify and analyze characteristics and properties of geometric shapes.

Intended Learning Outcomes:

3. Reason mathematically.
4. Communicate mathematically.

Standard III

Objective 1

Connections

Background Information

This activity provides students with the opportunity to explore a surprising number of geometric concepts through folding and unfolding a circular piece of paper. Each student will need a circle in order to participate fully in the activity as the teacher guides the students through a variety of folds. The activity is presented as a set of instructions for making the folds and is accompanied by a set of questions that the teacher directs the students to explore along with each fold. The activity is presented here in a direct-instruction format but can easily be adapted for small group or individualized learning. Asking meaningful questions is a key strategy in this particular activity.

This activity can be used in a variety of ways within a geometry unit. It is presented here as an opening activity to a geometry unit. Used in this way, it invites students to engage in geometric thinking and helps the teacher to determine what students might already know about geometry. Alternatively, this activity could serve as a fun way to review concepts studied in a geometry unit. It is important to keep in mind that students must be given adequate time to explore and make connections between models, concepts, and the related terminology.

Invitation to Learn

Prepare a set of pictures of objects that are circular or bring in actual objects (such as coins, clock, Frisbee, etc.). Present them one at a time and ask students to find an attribute that the objects have in common.

Instructional Procedures

1. Students should easily identify the circle shape as being the common attribute among the displayed objects. Let the students know that they will be working with circles to explore a lot of geometry.

Materials

- Paper circles
- Scissors
- Handouts (for teacher)
- Pencil
- Pictures of circular objects
- Diagrams of geometric concepts created from folding

2. Distribute paper circles and scissors to all students.
3. Have students cut out their circle so that the dark line is saved.
4. Guide students through an exploration of geometric concepts by modeling a sequence of folds, posing questions, encouraging participants to make and test conjectures, and relating the concepts to the real world (see suggested folds and questions handout to guide this exploration).
5. After the folding, questioning, conjecturing, and reasoning has been completed, have students get with a partner or small group of 3-4 to recall and write down as many of the geometry concepts as they can recall that were generated through the folding activity. Give only about 90 seconds for this task.
6. Have small groups read off and share how many terms they found.
7. Point out that geometry is an area of mathematics that has quite a few terms and symbols. They will be learning more about many of these terms.
8. Closing: Pull out sketches/diagrams of a variety of geometric concepts that students explored through the paper folding. Upon seeing the picture, see if the students can identify the concept.

Curriculum Integration

Math / Real World / Art—Have students go on a “shape hunt” to identify and find examples of geometric shapes in the real world. Students could make a three-column table in which they write the name of the geometric concept in the first column, put a sketch of the real world object in the second column, and write the name of the object in the third column.

Students could observe geometry in art through looking at art prints that are available in many schools.

Students could begin creating their own geometry glossary that includes names of concepts, diagrams/examples of concepts, and symbols related to concepts.

Possible Extensions/Adaptations

Adaptations

Although this activity is presented as one complete activity, the teacher may decide to take only a few steps at a time in order to thoroughly discuss the new mathematical concepts, terms, and relationships connected to each fold. In presenting the activity this way,

the teacher may want to prepare a handout with circles on it so that students could sketch the concepts they folded.

Another variation the teacher might want to consider is to still present all of the folds in their entirety with only initial questions the first time around in order to show the broad scope of geometry concepts. The teacher then could follow with more specific questions related to each fold at later times.

Most students should be able to handle the folding. One modification a teacher might make, though, is to have each student check with a partner after each fold is made to make sure they agree with each other.

Extension

Have the students explore with their circles on their own to see what new shapes they can find.

Assessment Suggestion

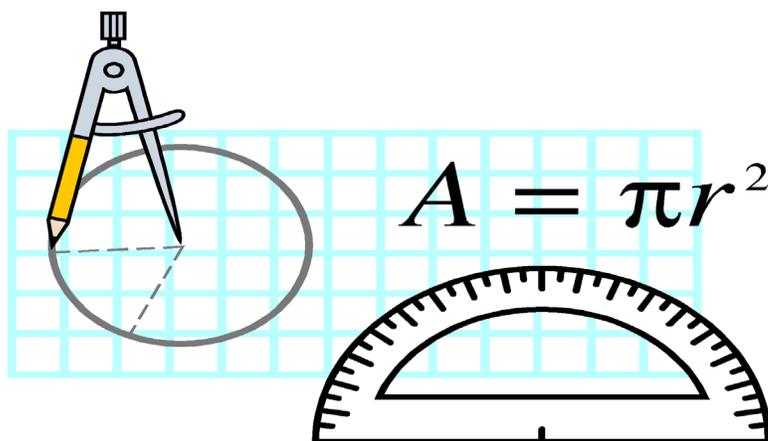
Observation and listening as students explore and provide reasons to support their conjectures. Identification of terms at the closure could be done as a paper-and-pencil check. The shape hunt could also serve as a written assessment. A geometry journal could serve as an ongoing place to record responses to teacher prompts, questions, or student inquiries related to geometry.

Additional Resource

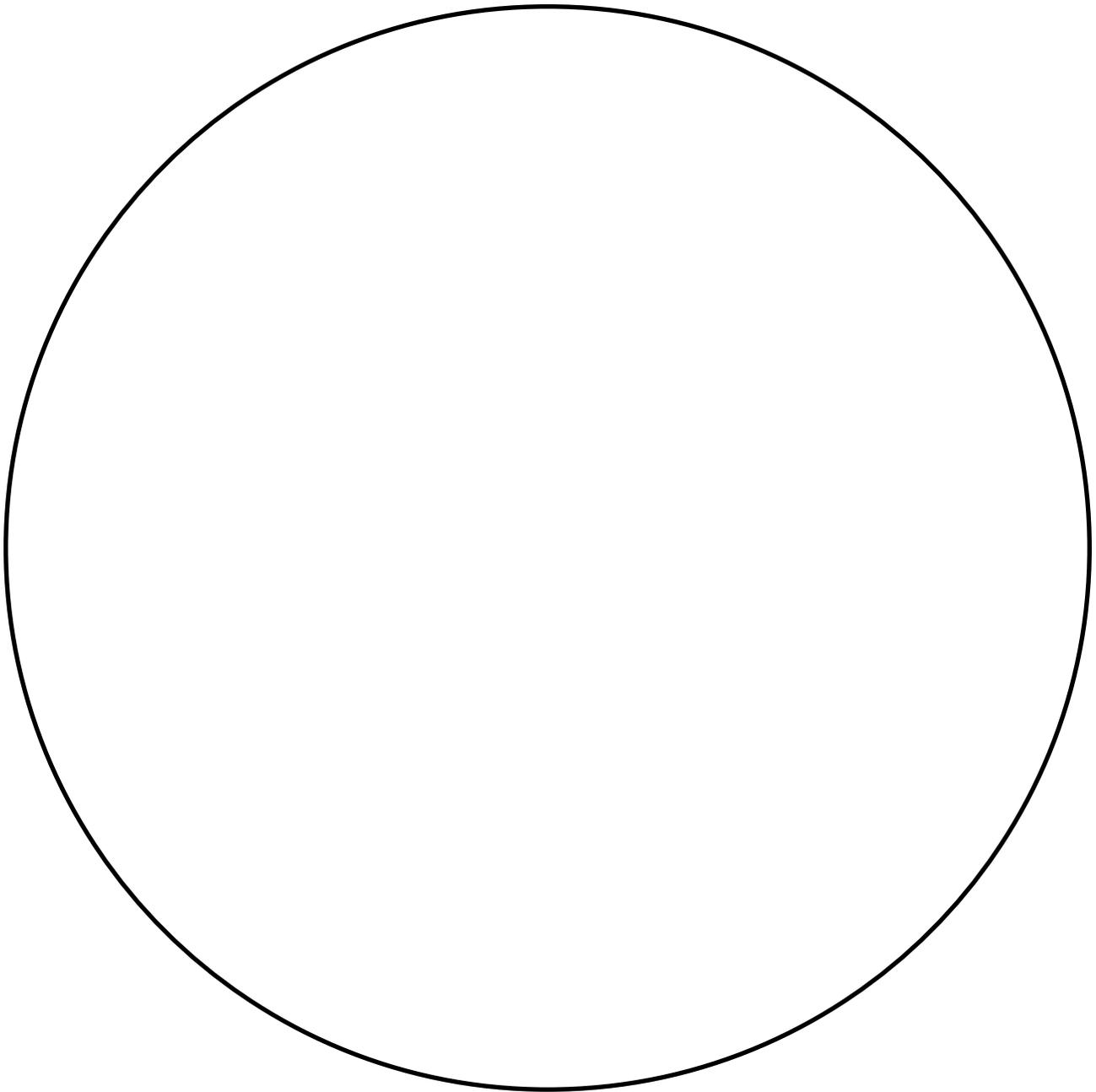
The Amazing Circle (AIMS Education Foundation)

Homework & Family Connections

Have students take the shape hunt activity home and invite their family members to participate in the search.

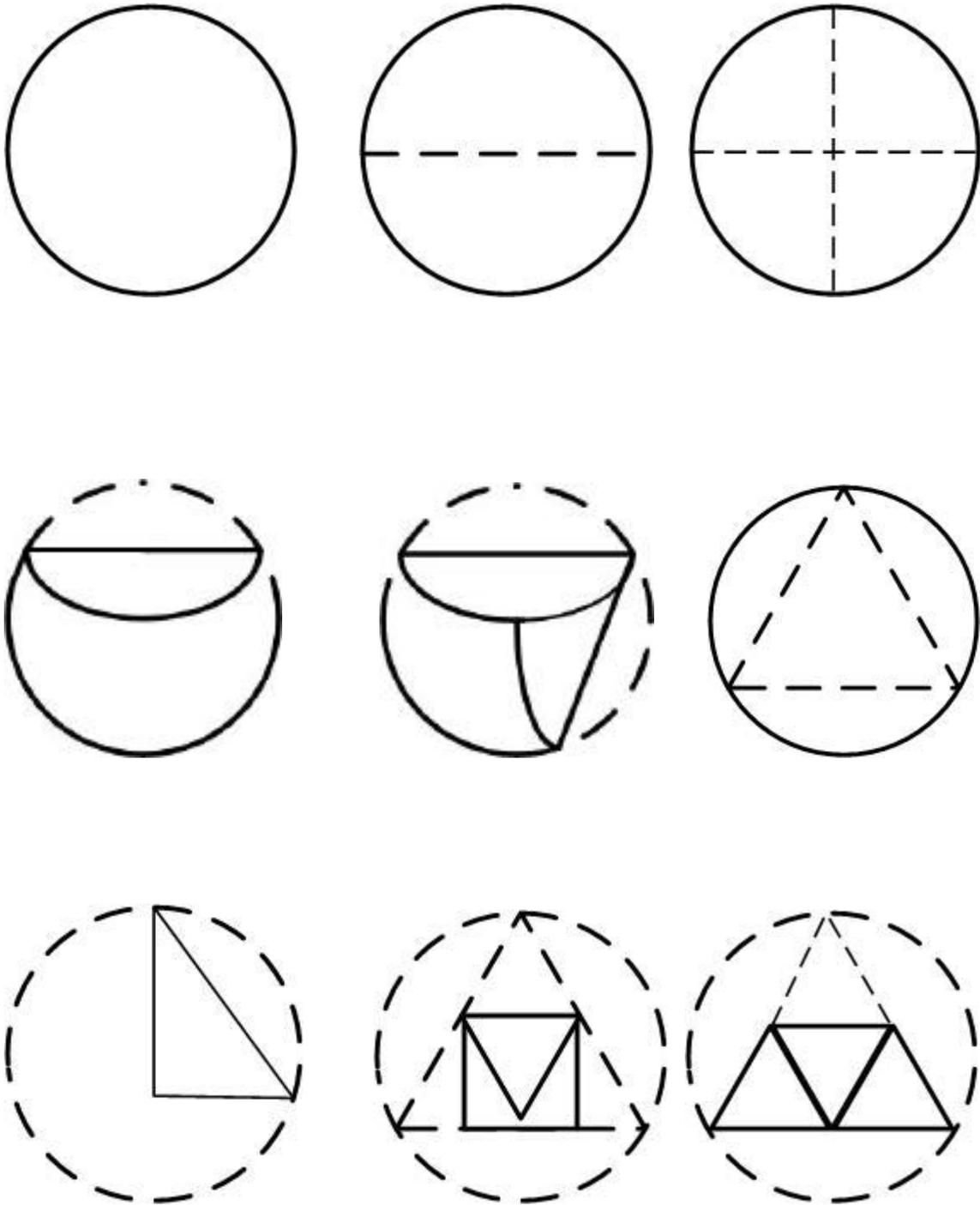


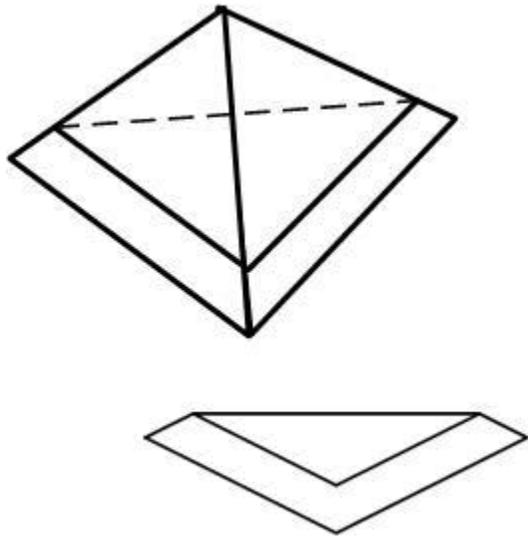
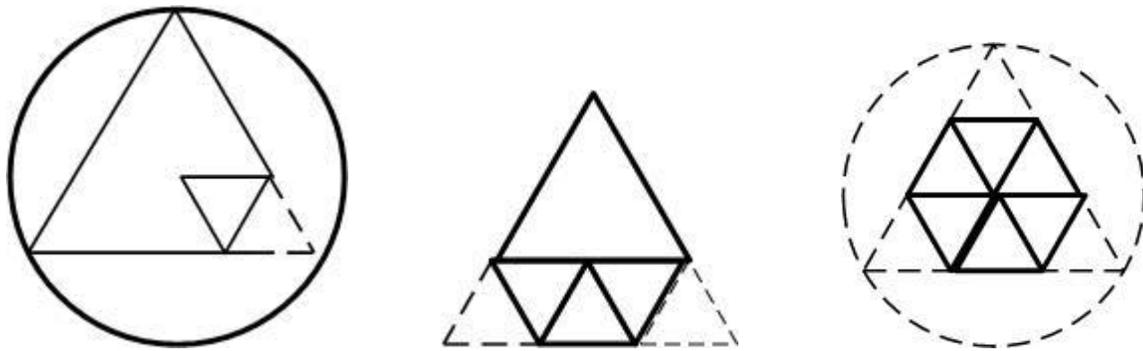
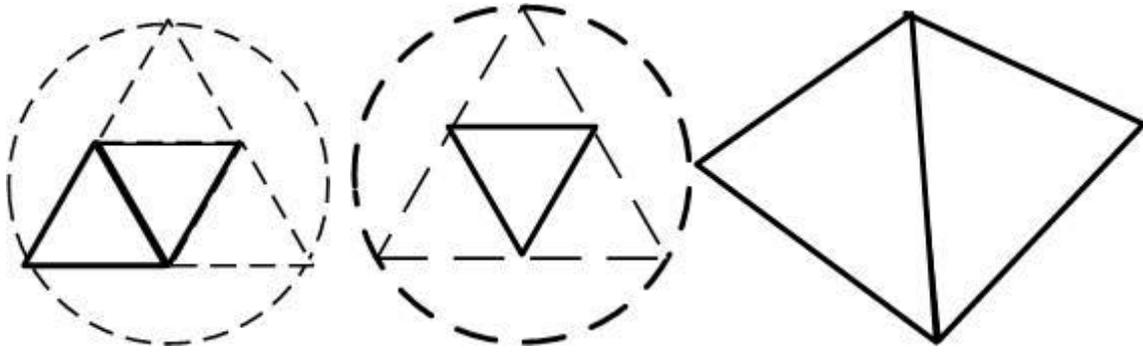
The Amazing Circle



Cut along the outer edge of your circle so that the dark line remains.

Circle Folding





Activity—Pattern Block Polygons

Standard III

Students will use spatial and logical reasoning to recognize, describe, and identify geometric shapes and principles.

Objective 1

Identify and analyze characteristics and properties of geometric shapes.

Intended Learning Outcomes

2. Become mathematical problem solvers.
6. Represent mathematical situations.

Standard III

Objective 1

Connections

Background Information

This activity provides students with the opportunity to construct geometric shapes in a small group, exploratory setting. Students use pattern blocks to create two different 3-sided, 4-sided, 5-sided, 6-sided, 7-sided, 8-sided, 9-sided, and 10-sided polygons. Each polygon construction should be made up of at least two pattern blocks pieces (although the pattern blocks do not need to be different). In addition, sides of the pattern blocks should match up (as will be demonstrated in the session). Students record their constructions, utilizing a choice of methods that will be introduced (cutouts, stamping, tracing).

This particular activity allows students to realize that shapes are made up of other shapes. Once the constructions are complete, the activity serves as a knowledge base that the students will refer to in later activities to learn about the following concepts: convex polygon, concave polygon, symmetry, interior angles of a polygon, sum of the interior angles of a polygon, number of vertices (sides, angles) of specific polygons.

Invitation to Learn

Prepare a set of words with the same prefix as several of the polygons the students will construct, (e.g., trio, and triplets for triangle, octave, octopus, and octet for octagon, etc.) Prepare sets for at least: triangle, quadrilateral, pentagon, hexagon, octagon, and decagon. Divide the class into 6 groups. Have them use a dictionary to look up each word and write down a brief definition. Then have them observe what all of the words in their list have in common. Have groups share findings to reinforce the names of the polygons.

Instructional Procedures

Materials

- pattern blocks
- overhead pattern blocks
- recording handout
- paper
- pattern block stamps
- ink stamp pads (yellow, orange, red, blue, green, purple) or
- pattern block cutouts or pattern block stickers
- glue sticks
- colored markers
- pencil

1. Present the invitation to learn activity and have groups share their findings.
2. Describe the goal of the task—to construct two different polygons out of pattern blocks for each of the types of polygons, from triangles through decagons, according to the following two conditions: 1) pattern block sides must match up, and 2) at least two pattern blocks must be used for the construction. Constructions should be recorded.
3. Model how to record the constructions. One of the following methods may be suggested: tracing, pattern block stamps, or pattern block cutouts. Remind students to label the type of polygons they construct and to limit the number of sets of polygons they put on a page (room will be needed to record new information later).
4. Have students work in small groups/learning centers to construct the polygons and record their findings.
5. Closing: Have students volunteer to share one of their constructions by building it on the overhead with overhead pattern block manipulatives. Have students in the class identify the type of polygon it is.

Curriculum Integration

Math / Real World / Art—Connection to art through viewing pieces of art and observing a variety of geometric concepts (e.g., parallel lines, similar shapes, symmetry). The shape building can also be compared to puzzles and tilings.

Possible Extensions/Adaptations

Once the polygons have been constructed, they serve as a rich knowledge base from which the students can explore other geometry concepts. For example, after the teacher introduces the concepts of convex polygons and concave polygons, students can go back and label each of the polygons they constructed as either convex or concave. Similarly, the students can use a mirror to test for line symmetry. If they find symmetry, they can draw dashed lines to indicate where the line of symmetry is.

Ask students about the strategies that they discovered when they constructed the polygons. For example, some students might come up with a strategy for certain polygons to get one more side by putting a triangle where there was a straight side before.

Assessment Suggestions

Observation and questioning are good informal strategies for this activity. The teacher can observe and ask individual questions while the students are working in their groups, as well as when the students share their findings at the end of the lesson. The completed record of the polygon constructions is a good formal assessment.

A possible journaling prompt is “In your journals, write about the experience you had building the shapes. Include thoughts about which ones were more difficult for you (e.g., ones with an odd number of sides or an even number of sides) and any strategies you used.”

Additional Resources

There are a variety of internet sites that can be used as interactive manipulatives. Students can use the interactive manipulatives to record the designs they construct with the concrete manipulatives. This kind of experience provides opportunities for students to use visual and spatial reasoning in rotating, flipping, and translating the shapes.

Homework & Family Connections

Students can repeat or continue the activity at home with help from family members. They might be challenged to construct shapes using the fewest possible number of pattern block pieces.

Constructing Polygons with Pattern Blocks

Use your pattern blocks to create two different 3-sided, 4-sided, . . . , 10-sided two dimensional figures. Each figure should be made up of at least two pattern blocks (but the pattern blocks do not have to be different). Sides should match up, as discussed in class. Record the diagram for each of your findings using construction paper cutouts, stamping, tracing, or a combination of the three. Label each figure according to whether it is 3-sided, 4-sided, . . . and so on. Do not put more than two sets of shapes on a paper.

3-Sided Shapes

4-Sided Shapes

5-Sided Shapes

6-Sided Shapes

7-Sided Shapes

8-Sided Shapes

9-Sided Shapes

10-Sided Shapes

Activity—Measuring Pattern Block Angles

Standard III

Objective 1

Connections

Standard III

Students will use spatial and logical reasoning to recognize, describe, and identify geometric shapes and principles.

Objective 1

Identify, and analyze characteristics and properties of geometric shapes.

Intended Learning Outcomes

3. Reason mathematically.
5. Make mathematical connections.

Background Information

This activity is designed to provide exploration of angle types, polygons, and angle measurement through a nonstandard measurement strategy. Participants work as partners to figure out how many degrees are in each of the different angles of each of the different pattern blocks. They use hinged mirrors along with the concept that a full rotation is 360 degrees to logically determine the measure of each pattern block angle. Although the mirror is not necessary to find the measures of the angles, it adds a lot of interest and reinforces some mathematical ideas.

Most of the students will easily recognize that the orange pattern block, being a square, has angles that are all 90 degrees. This is a good pattern block to model the measuring process with since, although the measuring method is likely to be new, the result they find is what they expect. Students can then explore the other angles of the other pattern blocks as partners. There is one pattern block angle that will not work with this strategy. The teacher can foreshadow that there is one tricky angle that will not work with the mirror and the students must come up with their own strategy based on what they already know about the other angles.

Invitation to Learn

Review the concept of a full rotation (360 degrees) and half rotation (180 degrees) in the context of where students relate to these terms (e.g., skateboarding, skiing, diving, merry-go-round). Demonstrate “doing” a 360 or have a volunteer from the class demonstrate it.

Instructional Procedures

1. Have students use a right angle referent to estimate the angles in several of the letters of the alphabet to review concepts of right angle, straight angle, obtuse angle, and acute angle.
2. Introduce the hinged mirror task and model the use of the materials. Using the square pattern block as an example, the teacher should gather the students where they can see one of the mirrors in the hinge (several stations may need to be set up). Then, perhaps with student helpers, put one angle of the square pattern block in the hinge and makes sure the mirror is “snug” around the angle. Encourage students to take a look. Have one student build what he or she sees in the mirror using pattern blocks. Have students agree/disagree with what the student builds. Explain how the construction “proves” that the angle placed in the hinge is $360 \text{ degrees}/4$ (because there are 4 copies of the angle to make a complete rotation of 360 degrees). Explain how to record the findings using a directed arrow to show the rotation (see handout for reference).
3. Students work as partners to measure each of the different pattern block angles using the hinged materials, and record their results using one (or more) of the suggested methods.
4. Suggest a hint if students get stuck on the “trick” angle (the large angle of the tan pattern block). For instance, point out that although the mirror does not directly help them, they might be able to build a rotation without the mirror using other pattern block angles for which they already know the measurement.
5. Facilitate a discussion session in order for participants to share findings and draw conclusions about the angle measurements of the pattern blocks.
6. Encourage students to share their strategies for determining the angle measure of the “mystery” angle.
7. Lead a closing discussion to summarize the findings.

Materials

- pattern blocks
- hinged mirrors
- recording handout
- pattern block stamps
- ink stamp pads (yellow, orange, red, blue, green, or purple)
- pattern block cutouts or pattern block stickers
- glue sticks
- colored markers

Curriculum Integration

Math—Geometry and Measurement/Real World—This lesson connects geometry with a nonstandard (but conceptual) measurement strategy. It can also connect the concept of angles with places the students encounter them in the real world. Using the materials to construct a design in the first quadrant of a coordinate plane makes it possible for students to “see” what the completed design would look if it were flipped across the x axis and the y axis.

Possible Extensions/Adaptations

Students can be encouraged to find as many ways as they can to make 360 degrees using the pattern blocks. They should record the ways and include the angle measure of the pattern blocks they used to verify their solution (see handout).

Students can be challenged to find the measure of other angles in a similar way—with or without the mirrors.

Assessment Suggestion

Observation and questioning as students are exploring with the hinged mirrors and pattern blocks are excellent informal strategies. Make sure the students are constructing what they see with pattern blocks before they try to record since some of the angles get difficult to record without visual help to reinforce what happening.

Additional Resource

A Collection of Math Lessons from Grades 3 through 6 by Marilyn Burns (Math Solutions Publications)

Homework & Family Connections

Students could be asked to use the materials, strategies, and results from this activity to find the measure of a “mystery” angle that the teacher hands out. Students could also be asked to design a tiling pattern in one quadrant and reflect it in another quadrant with the use of the mirrors.

Name _____

Alphabet Angles: Estimating Acute, Right, and Obtuse with a “Right Angle Referent”

This task is designed to provide you with the opportunity to mentally categorize angles as **acute**, **right**, or **obtuse**. Once you make your mental estimate, measure with an informal measuring tool—a right angle—to verify your mental conjectures.

Procedures:

1. Mentally assess each angle in each alphabet letter and categorize it as acute, right, or obtuse.
2. Measure the angle with the right angle to check your conjecture.
3. Shade the interior of all acute angles in 1 color.
4. Shade the interior angles of all obtuse angles in another color.
5. Label right angles with a right angle symbol.

A E F H I

K L M N

T V X Y Z

Exploring Pattern Block Angles with Hinged Mirrors

This task is designed to provide exploration of angles of polygons. You will need to figure out how many degrees are in **each** of the **different** angles of the pattern blocks. Use the hinged mirrors and what you know about angles to help you. Record your findings on the sheets as follows:

1. Record what you see using the hinged mirrors by drawing (or stamping or pasting). It will help tremendously to first build what you see using the pattern blocks.
2. Draw a directed circle to show the complete rotation you see, when it is present.
3. Then, figure out, in degrees, the measure of the pattern block angle you are investigating.

For example, when we looked at the square pattern block, we positioned the block and mirrors as in figure 1. Then we used the blocks to build what we saw, as in figure 2. We used this figure to help us determine the number of degrees in the angle of the pattern block angle under investigation.

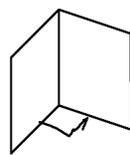


Figure 1

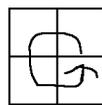


Figure 2

$$360^\circ \div 4 = 90^\circ$$

Note: You will not be able to directly measure one of the pattern block angles using this strategy. Use what you know about angles at this point, along with what you know about other pattern block angles, to create a solution for this “problem” angle.

GREEN TRIANGLE

ORANGE SQUARE

BLUE RHOMBUS

(Note: There are two different angles in this polygon! You will need to find the measure of each.)

YELLOW HEXAGON

RED TRAPEZOID

(Note: There are two different angles in this polygon! You will need to find the measure of each.)

TAN RHOMBUS

(Note: There are two different angles in this polygon! You will need to find the measure of each.)

Activity—Patterns in the Sums of Polygon Angles

Standard III

Objective 1

Connections

Standard III

Students will use spatial and logical reasoning to recognize, describe, and identify geometric shapes and principles.

Objective 1

Identify, and analyze characteristics and properties of geometric shapes.

Intended Learning Outcomes

2. Become mathematical problem solvers.
5. Make mathematical connections.

Background Information

This two-part lesson connects the patterns students observe about the sum of the interior angles of a polygon to an algebraic formula for determining the sum of the angles of any polygon. The lessons build on what the students found out in previous activities. Depending on the ability of the class, either part (or ideally both parts) may be used to develop the conclusion that each time a side is added, 180 degrees is added to the sum of the interior angles of the polygon.

Invitation to Learn

Have students recall what they learned through participating in Measuring Pattern Block Angles with Hinged Mirrors Activity. List the findings on the board because they will be useful in completing Part 1 of this activity.

Part 1: Analyzing Interior Angles of Polygons

Instructional Procedures:

1. Have students recall what they learned from Measuring Pattern Block Angles with Hinged Mirrors Activity.
2. Introduce the new task. Students are to revisit Pattern Block Polygons Activity and use the findings from Measuring Pattern Block Angles with Hinged Mirrors Activity to determine the sum of the interior angles of the polygons they constructed in Pattern Block Polygons Activity.
3. Model using the information about the individual pattern block angles to mark the polygon angles in Pattern Block Polygons Activity and then to record each vertex angle on the new recording sheet (see handout as reference).

4. Have students work to complete the task, showing their work on Pattern Block Polygons Activity and recording findings on the recording handout.
5. Students should share their findings and compare with the rest of the group.
6. Encourage students to generalize their findings to determine an algebraic formula to describe the geometric patterns they see.
7. Closing discussion.

Part 2: Tearing Polygon Angles

Instructional Procedures:

1. Introduce the task, pass out a triangle to each student, and instruct participants to label the angles of the triangle as “1,” “2,” and “3.”
2. Demonstrate how to tear the interior angles of the triangle and place each angle around a point on the handout. It is important that students actually tear the angles off. Cutting the angles off will result in a small triangle and it will be difficult for the students to keep track of which angle of the small triangle was the vertex angle of their original triangle.
3. Instruct students to tape their angles around the indicated points on the handout.
4. Have students work on their own to discover relationships among the interior angles of the remaining shapes using the same “tearing” and angle placement strategy.
5. Closing discussion.

Curriculum Integration

Math: Geometry and Algebra—Have students use a table to record information they gather from the task. Students can analyze the data to determine patterns and work towards finding an algebraic formula to represent what is happening geometrically with the sums of the interior angles of the polygons. Students might explore a variety of polygons to determine which ones can tessellate a plane.

Possible Extensions/Adaptations

Provide students with a protractor to actually measure the angles of a variety of triangles, quadrilaterals, pentagons, and hexagons. Have students test their conjectures on irregular polygons, as well as concave polygons.

Materials—Part 1

- Results of Pattern Block Polygons Activity
- Results of Measuring Pattern Block Angles with Hinged Mirrors Activity
- Recording handout
- Colored markers
- Pencil

Materials—Part 2

- Recording handout
- Tape
- Assorted triangles, quadrilaterals, pentagons, and hexagons
(each shape should be made from a different color paper)

Assessment Suggestion

Ask students to summarize in their journal what they found through the investigation. Given the measures of the all but one of the angles of a polygon, have students identify the last angle.

Additional Resource

Geometry Teacher's Activities Kit: Ready-to-Use Lesson & Worksheets for Grades 6-12 by Judith Muschla & Gary Muschla
(The Center for Applied Research in Education)

Homework & Family Connections

Have students create a collection of polygons (triangles, quadrilaterals, pentagons) and determine which ones tessellate. Have students identify angles that rotate around a point, and then sketch them and try to determine the angle measurements based on logical reasoning. For example, students might find an intersection of angles in a sidewalk and try to estimate the angles based on logical reasoning and the findings of the activities.

Name _____

Exploring Interior Angles of Polygons Using Pattern Block Angles

For this task, you will further explore the polygon shapes you constructed in a previous lesson. Go back to find the measure of the interior angles of each of the shapes you created. To do this, you will need to do the following:

1. Write the measure of each interior angle next to the vertex in each of the polygons from the previous lesson.
2. Once you have labeled each interior angle with its measure, find the sum of the interior angles of each of the shapes you created.
3. In the following chart, record how you calculated the sum of the interior angles for each shape by making a number sentence using the angles measures of the polygon.
4. Once you have completed the chart, look for patterns. What do you observe? Use the back of the sheet to explain.

<i>Number of Sides</i>	<i>Sum of Interior Angles of Each Shape</i>
3 (triangles)	Shape 1: Shape 2:
4 (quadrilaterals)	Shape 1: Shape 2:
5 (pentagons)	Shape 1: Shape 2:
6 (hexagons)	Shape 1: Shape 2:
7 (heptagons)	Shape 1: Shape 2:
8 (octagons)	Shape 1: Shape 2:
9 (nonagons)	Shape 1: Shape 2:
10 (decagons)	Shape 1: Shape 2:

Tearing Polygon Angles

1. Number the three corners of a triangle from 1 to 3. Then tear off the three corners of the triangle. Next, position the angles carefully around the point on line M, as illustrated by the instructor. Make sure to place one side of the first angle so that it is actually on line M. Notice that the vertex of each angle touches the point on line M and that the angles are placed **adjacent** to each other so that they share a side but do not overlap and do not leave gaps between the angle sides. Once all three angles have been placed and taped around the point, make observations. What do you observe? Compare your results with the class members around you. Record your observations in writing.



2. Repeat part 1 for any other triangle. What do you observe?



3. Now, label the four corners of a quadrilateral. As you did with the triangles, take care to position the angles around the point on line M. Notice that the vertex of each angle touches the point on line M and that the angles are placed adjacent to each other so that they do not overlap and do not leave gaps between them. What do you observe? Compare your results with the class members around you. Record your observations in writing.



4. Repeat what you did in part 3 with another quadrilateral. Once again, write about your observations.



5. Make a conjecture about what you think would happen if you placed torn angles from a pentagon around a given point. How could you test your conjecture? Test your conjecture and report your findings.

----- • ----- line M

6. Complete the following chart with the information you obtained by tearing polygon angles from triangles, quadrilaterals, and hexagons. Do you see a pattern? Use your observations to make a conjecture about the sum of the interior angles of a hexagon. Go ahead and test your conjecture.

<i>Number of Sides</i>	<i>Polygon Type</i>	<i>Number of Interior Angles</i>	<i>Sum of Interior Angles</i>
	Triangle		
	Quadrilateral		
	Pentagon		
	Hexagon		

Activity—Constructing Tangrams

Standard III

Students will use spatial and logical reasoning to recognize, describe, and identify geometric shapes and principles.

Objective 2

Specify locations and describe spatial relationships using coordinate geometry.

Objective 3

Visualize and identify geometric shapes after applying transformations.

Intended Learning Outcomes

3. Reason mathematically.
5. Make mathematical connections.

Standard III

Objectives 2 & 3

Connections

Background Information

This activity provides a connection between geometry and algebra by looking at geometric concepts in a coordinate plane. In addition, it draws upon an ancient Chinese legend and culture to add interest to the context of the lesson.

As they construct the tangrams, students are required to graph points defined by ordered pairs in the four quadrants and write the ordered pair for a given point in any one of the four quadrants. They are also required to identify midpoints of line segments. Once the tans are constructed, students can explore a variety of other mathematical concepts related to geometry, measurement, and fractions (e.g., transformations in a plane and area of geometric shapes).

The main lesson itself provides meaningful learning opportunities, but the possible extensions enrich the depth of the mathematical learning by connecting the geometry to concepts of measurement, fractions, decimals, and percents.

Invitation to Learn

Share the story of the tangram. According to legend, the tangram puzzle originated in China. The legend tells of Tan, a Chinese nobleman, who wished to present the emperor with a gift of an exquisite square tile. Unfortunately, he dropped the tile on his journey and broke it into pieces. When he opened the bag he was carrying it in, he found that the tile broke into exactly 7 pieces—5 right triangles (2 small, 2 large, and 1 medium), a square, and a parallelogram. Tan tried to put the pieces back together to reconstruct the square tile, but he could not. What he found, however, was that he could make many other shapes with the seven pieces. He decided that the broken pieces made an even grander gift than his original square tile.

Although it is not known exactly when tangrams came about, we know that they became quite popular in the United States and Europe during the nineteenth century and have remained popular among geometry enthusiasts today.

Instructional Procedures

Materials

- Graph paper
- Colored markers
- Ruler
- Extra set of tangrams
- Overhead set of tangrams
- Poster-size graph paper
- Scissors
- Envelopes
- Pencil

1. Explain that the tangram consists of seven special geometric shapes. Each student will construct a set of tangrams through paper folding and cutting. Then each student will construct a set of tangrams on a coordinate system.
2. Explain the instructions and distribute materials for the activity (see handout for reference).
3. Allow time for students to construct a set of tangrams following the directions given on the handout.
4. Once students have constructed tangrams, they will use their tangram set, along with a second set handed out by the teacher, to answer a variety of extension questions related to measurement, fractions, and transformations. Divide the students into small groups and give each group a different task to complete. Have the groups share findings from the task they were asked to explore.
5. Lead a closing discussion.

Curriculum Integration

Math/Puzzles; Geometry and other Math; Literature—The tangrams the students construct can be used to explore a variety of mathematical concepts: fractions, decimals, percents, area, and transformational geometry. There is a wonderful book called *Grandfather Tang's Story* that tells about a man sharing a story of a variety of animals. As each animal is introduced, Grandfather Tang rearranges the tangram pieces to illustrate the next animal in the story.

Possible Extensions/Adaptations

Extensions:

- Let the original tangram be equal to one whole unit. Find the fractional part of the whole represented by each of the tans.
- The same task can be repeated by having the students find the decimal representation and/or percentage each tan represents of the whole.

- Orient the tans on a coordinate plane and direct students to transform the shape by translation along a line (perpendicular or parallel to an axis), rotation around a point (such as the vertex of the shape), or reflecting about an axis.
- Explore line symmetry in more detail using single mirrors (see handout for line symmetry in alphabet letters).

Adaptation:

The original activity can be done without using the coordinate plane.

Exploring the topic of tessellations in more detail would be a rich opportunity here. Students can look at tessellation concepts in art (e.g., MC Escher) as well as in the real world (e.g., honeycomb structures).

Assessment Suggestion

Observe as students work with the instructions for constructing the tangram. The completed tangram and completed worksheet can serve as formal assessment strategies. Students could create a kite design using reflection symmetry and record their design in a coordinate plane. Students could then construct their design, connecting it to art.

Additional Resources

Kaleidoscope Math by Joe Kennedy and Diane Thomas (Creative Publications)

Twenty Thinking Questions for Pattern Blocks (Grades 6-8) (Creative Publications), *Geometry and Fractions with Tangrams Learning Resources (Grades 3-6)* by Barbara Bando Irvin

Homework & Family Connections

Have students take their tangrams home and create additional shapes.

Constructing a Tangram on a Coordinate Plane

The pieces of a tangram can be used to create many interesting figures. They can also be used to analyze relationships among each other. Follow the given set of instructions to make your own tangram.

1. On a piece of graph paper, label the x-axis from -16 to 16 and the y-axis from -16 to 16 .
2. Plot and label the following points: $A(-16, 16)$, $B(16, 16)$, $C(16, -16)$, and $D(-16, -16)$.
3. Use a ruler to construct segments AB , BC , CD , and DA .
4. Draw segment BD .
5. Find the midpoint of AF . Label it E .
6. What are the coordinates of point E ?

7. Find the midpoint of AD . Label it F .
8. What are the coordinates of point F ?

9. Draw EF .
10. Find the midpoint of EF . Label it G .
11. What are the coordinates of point G ?

12. Draw CG .
13. Find the intersection of GC and BD . Label it H .
14. What are the coordinates of point H ?

15. Find the midpoint of HD and label it J .
16. What are the coordinates of point J ?

17. Draw FJ .
18. Find the midpoint of HB and label it K .
19. What are the coordinates of point K ?

20. Draw GK .

You should now have seven pieces constructed. Each piece is called a tan. Take the envelope provided by your teacher to check the tans on your coordinate plane. Use the shapes to create other interesting figures.

Activity—Pyramids and Prisms: Discovering Euler’s Formula

Standard III

Students will use spatial and logical reasoning to recognize, describe, and identify geometric shapes and principles.

Objective 3

Identify, and analyze characteristics and properties of geometric shapes.

Intended Learning Outcomes

3. Reason mathematically.
6. Represent mathematical situations.

Standard III

Objective 3

Connections

Background Information

Students will relate their understandings of two-dimensional shapes to three-dimensional shapes. Students will construct a variety of polyhedra, in particular, the five Platonic solids. Using the three-dimensional constructions, students will analyze the models to observe relationships among the number of vertices, the number of edges, and the number of faces. Students will develop a formula relating the number of vertices, edges, and faces of a polyhedron and test the formula by analyzing other polyhedra.

Invitation to Learn

Introduce this task by recalling the tetrahedron students created during the Shape Shifter Activity. Have a display of jumbo size pyramids, prisms, cones, and cylinders to use as demonstration.

Instructional Procedures

1. Ask students what they can recall about the set of shapes on display (e.g., names of the parts—faces, edges, vertices, bases, apex, what the shapes remind them of in real life—ice cream cone, pyramids, etc.).
2. Hold up two shapes up (e.g., the cone and the pyramid) and ask how the shapes are the same and how they are different. Do the same with another pair.
3. Hold up three shapes for the students to observe (e.g., the cone, the square pyramid, and a cube). Ask which shape does not belong in the group and why. Encourage multiple responses, for instance, “The cube does not belong because it does not come to a point (apex)” or “The cone does not belong because it has a round base.”

Materials

- Patterns or nets for constructing a variety of polyhedra (on card stock)
- Scissors
- Glue or stapler
- Pencil
- Colored pencils
- Sample large prisms, pyramids, cylinders, and cones
- Recording sheet

4. Demonstrate how to construct one or more of the shapes using nets printed on cardstock or straws.
5. Provide patterns for participants to construct their own 3-D shapes.
6. Using the models that the students construct, have the students analyze the solids to find out the number of faces, vertices, and edges of each of the solids. They will keep track of the information on a handout.
7. Have students share patterns. This will lead to uncovering the Euler's Formula.
8. Lead a closing discussion.

Curriculum Integration

Math/Real World/Art—A fun connection with shapes and the real world is to have the students envision and then sketch what certain shapes would look like from a variety of perspectives. For example, ask what a shape might look like from a “bird’s-eye” view, from a street level view, and from a “foot print” view. This is very much like envisioning what the shapes look like in 2-D from a variety of perspectives. Another possibility is to have students sketch shapes and relate to shading and casting shadows. Geometric shapes are nicely related to the study of how certain crystals and minerals grow.

Possible Extensions/Adaptations

Students could construct a variety of shapes and decorate them for holiday ornaments. Because shapes can be constructed in a variety of ways, it is valuable to have students use multiple construction techniques. If students use toothpicks for edges and little round balls of clay for vertices, the shapes can then be dipped in a tub of soap bubbles and students can make conjectures about how well the soap bubbles will cling.

Assessment Suggestion

Observe and listen as students explore and provide reasons to support their conjectures. Assess performance on students' handout. Playing a game of “Win, Lose, or Draw” with these and other geometry terms can be a fun and insightful way to check for understanding. Students can also create a variety of “Geometry Poems” using some of the following poetry types: Cinquain, Diamante, Acrostic, or Haiku.

Additional Resource

The Amazing Circle by AIMS Education Foundation.

Homework & Family Connections

Begin a rock collection of crystals that reflect a variety of geometric structures. Have students build additional geometric structures at home out of toothpicks and clay and have fun dipping them in dish soap. Have families take a neighborhood walk to look for geometry in architecture and other sights. The student could take a camera and the family could take pictures of sights in the neighborhood, which reflect any of the geometry concepts that have been studied.

Standard IV
Activities

Activity–Metric Rhyme

Standard IV

Students will understand and apply measurement tools and techniques.

Objective 1

Identify and describe measurable attributes of objects and the units of measurement.

Intended Learning Outcomes

5. Make mathematical connections.

Standard IV

Objective 1

Connections

Background Information

Meter refers to distance. Liter refers to volume. Gram refers to weight.

In this lesson, the students will learn a chant that combines vocabulary with actions to help all learning modalities.

The United States uses an inch-pound system of measurement derived (in England) from older measurement units originating in the 1200's. This system is also known as the English system or customary system. The United States is the only large country using the customary system, though two smaller countries use it as well. People in other countries use the Metric system. The Metric system was created in France in the 1790's.

Interesting Fact

The ancient Mediterranean systems used body measurements for linear measurements. Weight units were determined by how much a human or animal could carry.

Invitation to Learn

Who knows how big a millimeter is? Is a kilometer bigger or smaller than a mile? (smaller). Did you know that the United States is only one of three countries in the world that use customary units of measure? What do the rest use?

Instructional Procedures

1. Have the students sit at their desk and look at their metersticks. Have them measure the width of their finger. How big is it? What do the lines on your meterstick represent?
2. Have them measure the very tip of their pencil on the meterstick. What measurement do they come up with?

Materials

- Meterstick
- Crayon
- Pencil

3. Have them measure across their desk, and then the width of the classroom door. What measurements do they come up with?
4. If they take their pencil tip, how many dots can they put side by side across the width of their fingertip?
5. How many fingertips span the length of a brand new, unused crayon?
6. About how many brand new, unused crayons, will go across the width of the door?

Once the students gather all this information, talk to them about the fact that they will be using estimates to create a picture in their head. We know that everyone's fingertip isn't exactly the same size, but we could find the average, and this would be our "about the same size" measurement.

7. Teach the Metric Rhyme:

"I say millimeter, you say pencil tip." Students hold up pencil tip.

"I say centimeter, you say fingertip." Students hold up fingertip.

"I say decimeter, you say brand new crayon." Students hold up crayon.

"I say meter, you say width of door." Students hold hands apart about the width of a door.

"I say kilometer, you say ten minute walk." Students make a walking motion with their arms.

Curriculum Integration

Math/Science—Let them know that most science projects will only be accepted if the student has used metric measurements. Why do they think this is required?

Possible Extensions/Adaptations

Have them convert measurements. Convert millimeters to meters, etc.

Assessment Suggestion

Hold up objects and ask the students what that object should be measured in.

Watch as they do their metric rhyme, have them play Simon Says with it. The last one standing gets to be Simon.

Additional Resources

www.ask.com (Type in any question, and it will list answers, and the research backing it up).

Homework & Family Connections

How many tools for measurement can you find around your house?
Examples could be:

- Measuring spoons and cups—cooking
- Rain gage—measures rain
- Thermometer—measures temperature
- Bathroom scales—weight
- Electricity, water, and gas meters
- Watches and clocks—time
- Barometer—atmospheric pressure
- Yardstick—measuring a yard
- Fingers—pinch, as in pinch of salt
- Clock—seconds, minutes, hours

Activity—Trundle Wheel

Standard IV

Students will understand and apply measurement tools and techniques.

Objective 2

Determine measurements using appropriate tools and formulas.

Intended Learning Outcomes

- Reason mathematically

Standard IV

Objective 2

Connections

Background Information

This lesson will take three to five days to complete all parts, including extensions at the end of the lesson. Students should have knowledge of scientific variables, such as speed, length of legs, or wind velocity and be able to convert meters and decimeters into kilometers. They should also be able to convert the customary and metric systems.

Invitation to Learn

Ask the students, “If each one of us were to walk for ten minutes, would we walk the same distance? What are some of the things that might cause a difference in the distance we walk?” (speed, length of legs, or wind velocity).

Instructional Procedures

- Take the students outside and establish a starting point. Have them estimate how far they could walk in one minute and in ten minutes. Have team members discuss this, place their markers, and explain their reasoning to the class.
- Have the students place their trundle wheel on the ground and, as a class, walk for one minute. Have students compare their distance to their first estimate, and allow teams to change their second estimate based on their first reading.
- Continue walking the remaining nine minutes. After each minute, have the students stop and record the distance they went. At the end they should compare the total distance they walked to a kilometer.
- Go back to the classroom and graph the information. Have the students find the mean, median, mode, and range for the distances recorded after each minute. Take the mean and times it by ten to see how close this distance is to a kilometer.

Materials

- Trundle wheel
- Recording sheet

Curriculum Integration

Math/Science; Health—Have the students mark the outside field for either one mile or one kilometer. Over a five-week period, see if they can increase their distance in the same amount of time. A final activity would be to have a 5k race or a relay.

Science—Have students test different variables that may make them run faster or slower (arms out to sides, different shoes, breathing through their nose, etc.).

Possible Extensions/Adaptations

Have the students convert the kilometers to miles. There is a conversion web site at www.texloc.com that lets the students put their conversions in. Then have the students figure out how many miles per hour they were walking.

Assessment Suggestion

Have students write in journals during the last 5-10 minutes of each lesson. Have them include what has been easy, and what has been hard for them. Follow up with having markers placed outside, and have them measure to the most accurate centimeter, millimeter, decimeter, etc. Have them convert these measurements.

Additional Resource

www.askjeeves.com (will give conversion and measurement links)

Homework & Family Connections

Have students check out trundle wheels to take home and measure the following things:

- The distance around their block.
- The length or width of their bedroom.
- How far one mile is from their home. (Where would they end up?) This one should be done with parent supervision.

Trundle Wheel Recording Sheet

	Estimated Distance	Actual Distance
One Minute		
Two Minutes		
Three Minutes		
Four Minutes		
Five Minutes		
Six Minutes		
Seven Minutes		
Eight Minutes		
Nine Minutes		
Ten Minutes		
Total Distance:		

Using your information, figure out the following:

Miles Per Minute Average _____

Mode _____

Is there an outlier? _____

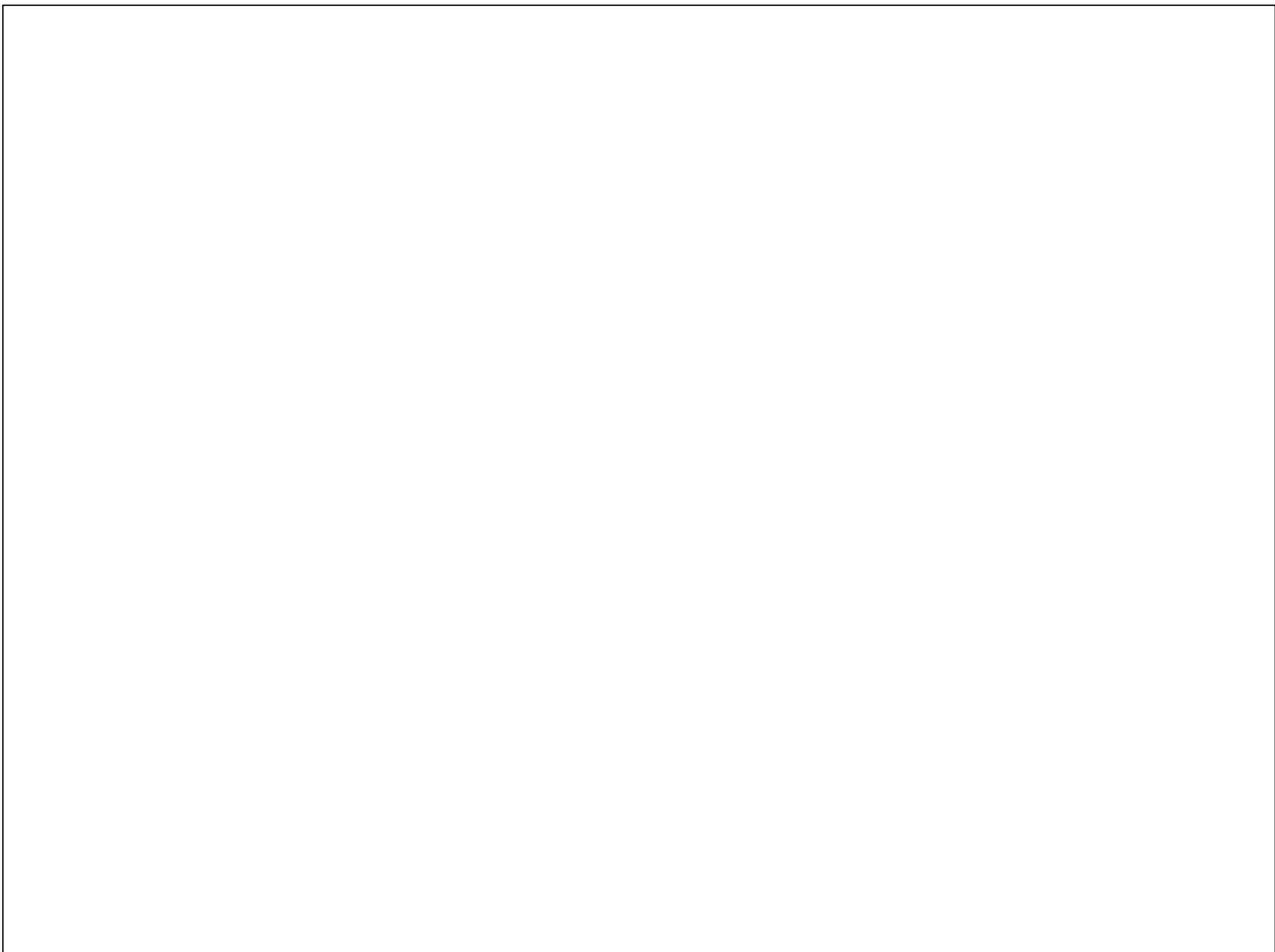
Range _____

Trundle Wheel

How Far Can I Walk in Ten Minutes?

How to Make a Trundle Wheel

1. Take a long stick, at least three feet long, measuring at least 2" x 1/4".
2. Drill a hole in the bottom of the stick about 2" up from the bottom.
3. Get a large, sturdy circle made from any firm material (cardboard, plastic, wood, etc.). The circle is used as a variable, and is meant to be taken on and off, and the size is meant to be exchanged for further variables and testing.
4. Complete putting the trundle wheel together using the following diagram.



Activity—Pi

Standard IV

Students will understand and apply measurement tools and techniques.

Objective 1

Identify and describe measurable attributes of objects and the units of measurement.

Intended Learning Outcomes

4. Communicate mathematically.

Standard IV

Objective 1

Connections

Background Information

Pi is the distance around the outside of an object (the circumference) divided by the distance across the middle (the diameter). 3.14 is actually a rounded number. Pi actually goes on, and on, and on.... The notation of pi was introduced by Euler in 1737. Archimedes was the first to show that $\text{Area} = \pi r^2$.

Invitation to Learn

Read *Sir Cumference and Knights of the Round Table*. This book will help remind students of the vocabulary used during the next lesson. It is also a good reinforcement to access previous knowledge. Have metric measuring tapes ready for the students to use. Students think that the symbol pi, is some magical symbol that is associated with numbers invented by a brilliant mathematician. During this activity, they will see that they too can come up with the formula, and the numbers behind it. Mathematicians today are still coming up with computer programs that will calculate the value of pi to the last digit.

Questions to ask students:

- What is the circumference, radius, and diameter of a circle?
- Where did pi or 3.14 come from?
- Can we be as smart as historical mathematicians?
- Can we relive their experience and come up with our own formula?

Instructional Procedures

1. Find ten different items in our classroom that are round.
2. Record the distance around the object (the circumference) and its diameter (the distance across the middle passing through the center).

Materials

- Metersticks
- Round objects found naturally in the environment

3. After having measured the 10 different objects, divide the distance around the object by the diameter.
4. Record answers on recording sheet.
5. Take all of your answers and find the average (add up your ten numbers and divide by ten).
6. What was your average?
7. Have each group tell you the average they came up with. Then find the average in the class.
8. How close to pi is this?
9. What do you think would happen if we had every class in the school do this and found the average of all their numbers?

Curriculum Integration

Math/Science—How will knowing the relationship of diameter, radius, circumference, and pi help us in real life? Has there ever been a time when you couldn't measure all the way around an object? How could we use the formula circumference divided by diameter, if we only knew half of the diameter?

Possible Extensions/Adaptations

Algebra and Number Patterns Connections—Go over the first thirty digits in pi. How many 0's, 1's, 2's, 3's . . . 9's are there? Tally the results. Go over the next twenty digits, doing the same thing. Are there any patterns you can find? What is the mode for this set of data?

Writing Connections—Use the "Interesting Pi Facts" worksheet. After reading "Pi Jokes" have the students create their own pi joke.

Pi day celebration—On March 14th, (3/14) celebrate Pi Day, and have the students eat a piece of pie at exactly 1:59 (Pi=3.14159 . . .).

Assessment Suggestion

As the students are measuring, ask them the following questions: How do I find the diameter of the round object? How will knowing the relationship of diameter, radius, circumference, and pi help us in real life? Has there ever been a time when you couldn't measure all the way around an object? How could we use the formula circumference divided by diameter if we only knew half of the radius?

Rubric:

As you are listening to their responses, use the following rubric to show understanding:

- 1= Is still unsure of what measurements you are asking for, or is unsure how to measure.
- 2= Can show you how to measure the radius, diameter, and circumference, but mixes them up or doesn't measure accurately. Also doesn't use vocabulary terms.
- 3= Can show you how to measure, but is unsure of the reasoning behind it. Can verbalize some of the reasoning, but not all.
- 4= Can verbalize and show the measurements and reasoning behind it.

Additional Resource

www.edhelper.com

(provides additional hands-on activities for discovering pi).

Homework & Family Connections

Have the students go home with their metersticks, and find three different objects at home that they can measure with their metersticks.

Have them use their recording sheet to find the circumference and the diameter. Then have them divide the circumference by the diameter, record the results, and find the average. Have the students bring this information to class the following day, and combine it with the information they have already accumulated. How close are they to pi?

Discovering Pi

<i>Object</i>	<i>Distance Around Objecct</i>	<i>Distance Across Middle</i>	<i>Distance Around Outside/Distance Across Middle</i>
1.			
2.			
3.			
4.			
5.			
6.			
7.			
8.			
9.			
10.			

Interesting Pi Facts

1. If you were to type one billion digits of pi they would stretch from New York City to the middle of Kansas. Find a map and see how many miles this is.
2. You can determine your hat size by measuring the circumference of your head, then divide by pi, and round off to the nearest one-eighth of an inch. Test it and see.
3. The height of an elephant, from his foot to his shoulder, is $2 \times \pi \times$ the diameter of its foot. Is this true for humans? Test it and see.
4. The Babylonians, in 2000 B.C.E., were the first people known to find a value for pi. In which country today would the ancient Babylonians be found?
5. The most accurate fraction for pi is $104348/33215$. It is accurate to 0.00000001056%. What place value does the digit 1 represent?
6. In the first one million digits of pi there are:
 - 99,959 zeros
 - 99,758 ones
 - 100,026 twos
 - 100,229 threes
 - 100,230 fours
 - 100,359 fives
 - 99,548 sixes
 - 99,800 sevens
 - 99,985 eights
 - 100,106 ninesWhat does this information tell you?
7. In the Greek alphabet, the symbol for pi is the sixteenth letter. What is the sixteenth letter in the English alphabet? The Spanish alphabet? German? Russian?
8. On Star Trek, Captain Kirk once asked, “Aren’t there some mathematical problems that simply can’t be solved?”
Spock fries the brains of a computer by asking it to “Compute to the last digit the value of pi.”
9. Pi came from the name Piscine Molitor Patel, also known as Pi Patel.

Pi Jokes

Q: What do you get if you divide the circumference of a jack-o-lantern by its diameter?

A: Pumpkin pi

Q: What do you get when you take a bovine and divide its circumference by its diameter?

A: Cow pi

Q: What do you get when you divide the circumference of a bowl of ice cream by its diameter?

A: Pi a la mode

Q: What do you get when you take the sun and divide its circumference by its diameter?

A: Pi in the sky

Activity—Angle Land and Angle Aerobics

Standard IV

Students will understand and apply measurement tools and techniques.

Objective 2

Determine measurements using appropriate tools and formulas.

Intended Learning Outcomes

2. Become mathematical problem solvers.

**Standard
IV**
**Objective
2**
Connections

Background Information

Why is a circle divided up into 360 degrees? The degree is derived from the Babylonian base 60 numerical system. Hours and minutes are also divided into 60's.

Mathematicians were going to use the number 365 (corresponding to the number of days in a year), but since everyone has at least 5 bad days in a year, they threw them out!

360 degrees was also used because it is easily divisible by many numbers. Students should understand the following vocabulary for this activity: right angle, obtuse angle straight angle, and acute angle.

Invitation to Learn

Ask the students if they have ever heard anyone say, "That car just spun a 360!" Or, "I can do a 180 on my skateboard." Ask them to share what they think this means.

Instructional Procedures

1. Read *Sir Cumference and the Great Knight of Angleland*.
2. Discuss what the medallion could represent.
3. Show a protractor, and explain that it is used to measure degrees in a circle.
4. Go over the following:
 - acute—a positive angle whose measure is less than 90 degrees
 - obtuse—an angle whose measure is greater than 90 degrees
 - straight—an angle whose measure is 180 degrees
 - right—an angle whose measure is exactly 90 degrees.
5. Compare a protractor to an overhead clock and fraction circle.

Materials

- Sir Cumference and the Knights of Angle Land*
- Protractor
- Clock Protractor Overhead
- Fraction Protractor Overhead

6. Have students find examples of these angles as they move around the room.
7. Play angle aerobics as described on the handout.

Curriculum Integration

Math/Science; Health—Increase the speed of Angle Aerobics, then talk about aerobic fitness. Have them measure resting heart rate before they exercise and then their heart rate afterwards.

Science—Have students find angles in nature, and record them. What type of objects occur naturally with angles? (spider webs, tree branches etc.)

Possible Extensions/Adaptations

For physically challenged students, have them point to the direction being spoken instead of moving their bodies.

If you can do this activity outside, you can add action words like, “jump around 180 degrees,” and “spin around 360 degrees.”

Assessment Suggestion

While students are playing angle aerobics, evaluate them visually using the following rubric;

Rubric:

1. The student can move right or left, but confuses the angles.
2. The student can move 90 degrees, 180 degrees, and 360 degrees, but confuses them.
3. The student can move 90 degrees, 180 degrees, and 360 degrees, without mistake.
4. The student can move 90 degrees, 180 degrees, and 360 degrees, and can verbalize what they have learned either in a journal or orally.

Additional Resource

www.math.com (has a moving protractor the students can manipulate, estimate angles, and then check their estimate).

Homework & Family Connections

Have students play angle aerobics at home. Have students find four objects in their home that have acute, obtuse, straight, and right angles. Have them write them down so that they can share them with the class the next day.

Angle Aerobics

1. Have students stand by their desks with their hands to their sides.
2. Ask them to show you what a right angle would look like, using their arms as the lines of the angle.
3. Do the same for acute, obtuse, and straight.
4. Now ask the students what kind of an angle 30 degrees is.
5. Have them show you the answer with their arms.
6. Call out angles and check for understanding by looking at their arms.
5. As a final assessment, have the students close their eyes while showing you the angle measures. This way, if you call out 130 degrees and someone shows you an acute angle, you know that they don't grasp the concept yet.

Angle Aerobics with Full Body Movement

1. Have the students stand facing the front of the room.
2. Ask them how many of them can turn a 360?
3. They should turn completely around and then face the front of the room again.
4. Ask them how many of them can show you what a 180 degree turn would look like.
5. Have them turn 180 degrees to the left or to the right.
6. Show 45 degrees.
7. Have them turn 45 degrees to the left or right.
8. Say the following chant:

“360 left, 180 right, 90 right, 45 left, 45 right, 180 left, 360 left, 90 right.”

All students should end in the same direction.

Which direction did you end facing?

Activity—Cube Models

Standard IV

Students will understand and apply measurement tools and techniques.

Objective 2

Determine measurements using appropriate tools and formulas.

Intended Learning Outcomes

3. Reason mathematically.

Standard IV

Objective 2

Connections

Background Information

Many students have a difficult time understanding the concepts of area and volume. The standard unit of volume in the metric system is the liter.

Invitation to Learn

Show a box that has centimeter grid marks on the outside. Ask the students if they could guess how many multi-link cubes could fit inside. Have them construct multi-link cubes so that it looks like the shape of the box. Ask the students if there is another way to arrange the multi-link cubes in order to get another box that is a different shape. How many different boxes can they find? Have them record their results both pictorially and using numbers (2 x 3 x 5).

Instructional Procedures

1. Following an introduction to area and volume, students will work in groups to build models of square centimeters. This becomes a good cooperative team effort at problem solving.
2. Students are provided with materials, but no initial instruction is given on how to build their models. They do have guidelines which are:
 - a. The multi-link model must contain at least five different size boxes.
 - b. They must recreate their models onto two-centimeter paper.
 - c. They must know how many multi-link cubes it took to build their model.
 - d. They must be able to tell you the dimension of the box.
3. After they have created their models from multi-link cubes, and then re-created them on paper, have the students count the number of squares there are on the outside of their object.

Materials

- Multi-link cubes
- Two-centimeter graph paper
- Colored pencils

4. Tell them that this is the surface area (they may need to undo their paper models and lay them flat in order to count each square without missing any of them).
5. Have them list the dimensions of each of their boxes, as well as the surface area, using the worksheet provided.
6. Are there any patterns or generalizations they can see?
7. Record the findings of the class, on different size boxes, and the surface area that goes along with it.
8. Can any of their generalizations be made into a rule?

Volume = length x width x height

Surface area = the area of all the faces of the object
or $2lh + 2lh + 2wh$

Curriculum Integration

Math/Science—Have students use what they know about volume and surface area to come up with the dimensions of the container that will keep liquid hot the longest. The total volume of the object cannot be greater than 1,000 cubic centimeters.

Possible Extensions/Adaptations

Using other three-dimensional shapes, such as cones and prisms, come up with another model, and find its volume and surface area.

Assessment Suggestion

When the groups have completed their projects, they will send a spokesperson to the front of the room to share with the class what they have built, what it is called, and how it compares to some of the other models built by other groups. They will also tell the class the total volume and surface area of their model.

Additional Resource

Investigations in Number, Data, and Space – Containers and Cubes (Dale Seymour Publications)

Homework & Family Connections

Have the students take their models home and share them with their parents, describing what they have learned about the relationship of surface area and volume. Have parents sign a note stating what they think their child has learned from this project.

Surface Area and Volume

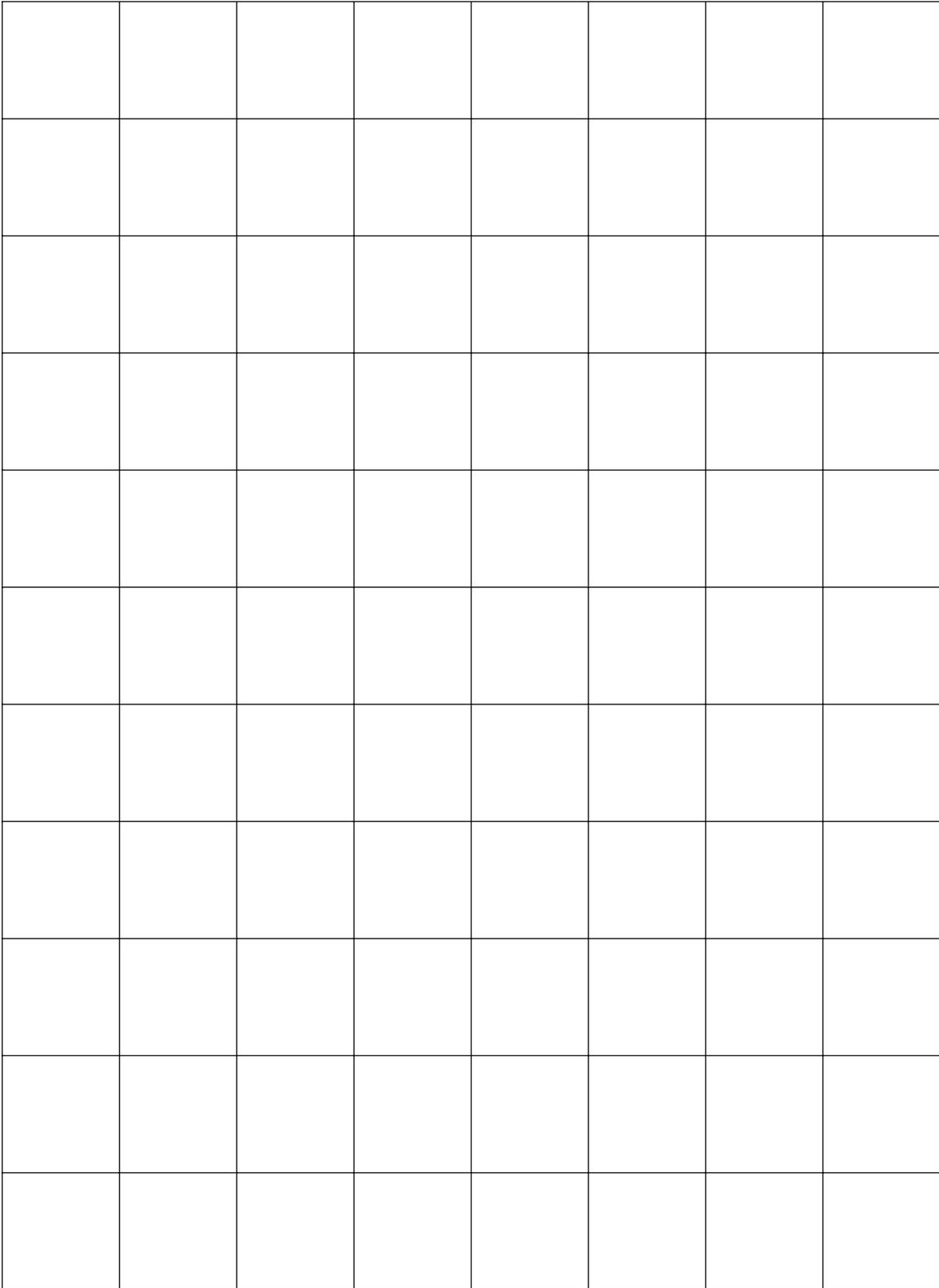
Rectangular prisms have six sides. Please label the dimensions of each of the sides: (Surface Area)

	Dimensions (ex: 3cm x 4cm)	Area (3cm x 4cm = 12cm squared)
Side 1		
Side 2		
Side 3		
Side 4		
Side 5		
Side 6		

How many cubes fit in each box?

	Dimensions (ex: 3cm x 4cm x 2cm)	Volume (3cm x 4cm x 2cm squared)
Box 1		
Box 2		
Box 3		
Box 4		
Box 5		
Box 6		
Box 7		

2 cm Grid Paper



Standard V
Activities

Activity—Which Month First?

Standard V

Students will collect, analyze, and draw conclusions from data and apply the basic concepts of probability.

Objective 1

Design investigations to reach conclusions using statistical methods to make inferences based on data.

Intended Learning Outcomes

4. Communicate mathematically.

Standard

V

Objective

1

Connections

Background Information

Understanding the connection between real world data and its representation on a graph is sometimes difficult for students to understand. Participation in this activity helps students to discover that connection. This is also a great activity to use near the beginning of the school year to help students get to know each other better.

Invitation to Learn

Read aloud *Bart's Amazing Charts* by Dianne Ochiltree.

Instructional Procedures

1. Explain that during each week, classroom privileges are given to students based on the month in which they were born. Inform the class that the rotation will begin with the month in which the most students were born, move to the month in which the next highest number of students were born, and continues in this rotation. Ask the students for ideas on how to record and collect this information.
2. While taking suggestions, guide the students to making a human graph.
3. Place month cards (see materials) on the floor.
4. Have each student stand in line behind the month of their birth.
5. Ask, "How can this information be remembered?" Suggest making a chart if the suggestion is not given.
6. Distribute a sticky note to each student and instruct students to write their name on it.
7. Write the months along the bottom of the board.
8. Let students place their sticky note in its appropriate month column.

Materials

- Laminated cards displaying the twelve months of the year
- Sticky notes (one per student)
- Graph paper (one per student)

9. Lead a class discussion about the rotation and decide on privileges.
10. Attempt to go on with another lesson by erasing the board, then stop and ask students how they can remember the rotation without leaving this up.
11. Guide students to the idea of recording the information on graph paper.
12. Have each student copy the information onto graph paper.

Curriculum Integration

Social Studies—population graphs

Possible Extensions/Adaptations

Birthday Paradox—You want to have a party at which at least two people share the same birthday (month and day). How many people do you have to invite so that the probability of two people sharing the same birthday is more than 0.5%? Remarkably, the probability of two people sharing a birthday is about .5 in a group of just 23 people. To calculate this probability, you have to look at the probability that every one has a *different* birthday. For a group of two people, the probability is extremely high— $364/365$ —that they will have different birthdays. With a group of three, the probability is not as high— $363/365$ —and, since the group of three still contains the group of two, the two probabilities are multiplied. Continue along this track until the probability of everybody in the group having a different birthday drops below .5.

Assessment Suggestion

Have students choose a topic appropriate for investigation whose results can be displayed in a bar graph. Have students collect the data and complete the graph.

Additional Resource

1000 Play Thinks: Puzzles, Paradoxes, Illusions & Games, “Birthday Paradox from Ivan Muscovich” (Workman Publishing)

Homework & Family Connections

Challenge students to find a graph in a magazine or newspaper. Then have them explain what the graph represents to their families.

Month Cards

January

February

March

April

May

June

July

August

September

October

November

December

Activity—Spinning Probability

Standard V

Students will collect, analyze, and draw conclusions from data and apply basic concepts of probability.

Objective 1

Design investigations to reach conclusions using statistical methods to make inferences based on data.

Intended Learning Outcomes

4. Communicate mathematically.

Standard
V
Objective
1
Connections

Background Information

A certain probability can be expressed as a ratio (3 out of 8), as a fraction ($\frac{3}{8}$), as a percentage (3 divided by 8 or 37.5%), or as a decimal (.375). The first number (3) represents the portion of the whole that are your chances, while the other number (8) represents all the possible chances. For instance, if there are four different colored balls in a bag, your chances of drawing out one certain color would be 1 out of 4, $\frac{1}{4}$, 25%, or .25. The “Invitation to Learn” activity models the activity for the cooperative learning groups to later follow. The idea for this activity came from Grognet, Jameson, Franco, & Derrick-Mescua in *Enhancing English Language Learning in Elementary Classrooms, Study Guide* (Delta Publish Company).

Invitation to Learn

Make an overhead spinner by copying the overhead, putting a thumbtack with point upwards through the center of the overhead’s circle, and laying a paperclip over the tack. Display the spinner and tell the class that the spinner determines their prize for a contest. Use think-pair-share to ask the group the following:

1. What are your chances of winning a pencil? Express your chances three ways.
2. What is your chance of winning a ball?
3. What is your chance of winning a yo-yo?
4. Which do you have more chances of winning, a book or a ball? How do you know?
5. Which do you have more chances of winning, a pencil or a ball? Use fractions to prove your answer?

Materials

- Worksheets
- Pencils
- Crayons
- Rulers

Instructional Procedures

1. Pass out the “Student Answer Sheet” to each student, and a “Student Clue Set” to each group.
2. Each person in the group will have one turn to read the clues. The group will discuss together until they can agree on a spinner design to meet the clues. Then each person will draw the spinner on his or her own paper.
3. Each group will pass two of their “Student Answer Sheets” to another group. Then each group will compare their own spinner designs with another group’s spinner designs.

Curriculum Integration

The study of probability is linked to the study of genetics. There are some gambling subjects that could also be discussed.

Possible Extensions

Have each student write a clue set. The four clue sets produced by a group could be passed to another group to solve.

Assessment Suggestion

This activity has a built-in assessment because each student will produce an answer sheet with spinners designed to show their understanding.

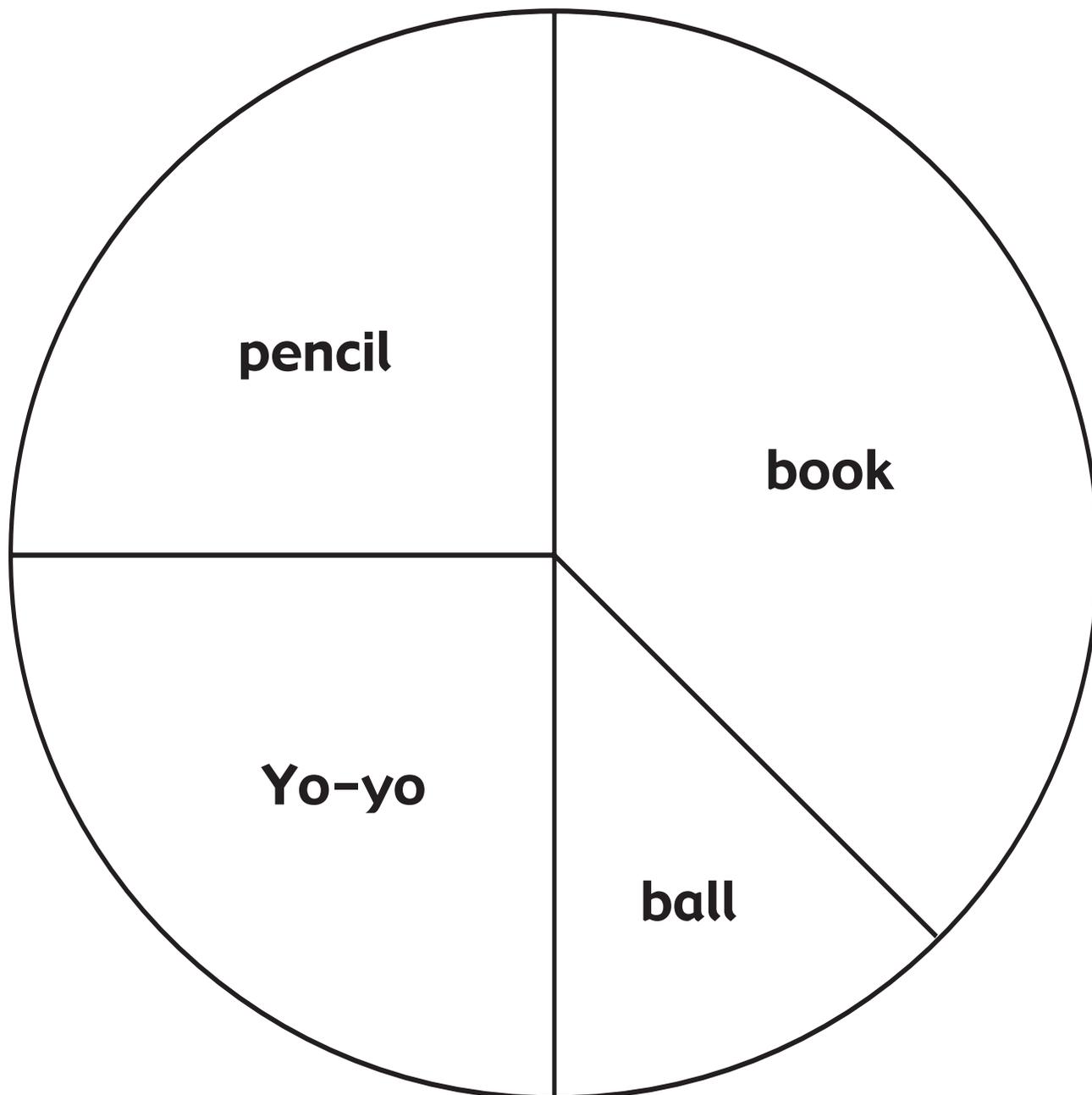
Additional Resource

<http://www.mathresources.ca/spinner.htm> has a randomized spinner.

Homework & Family Connections

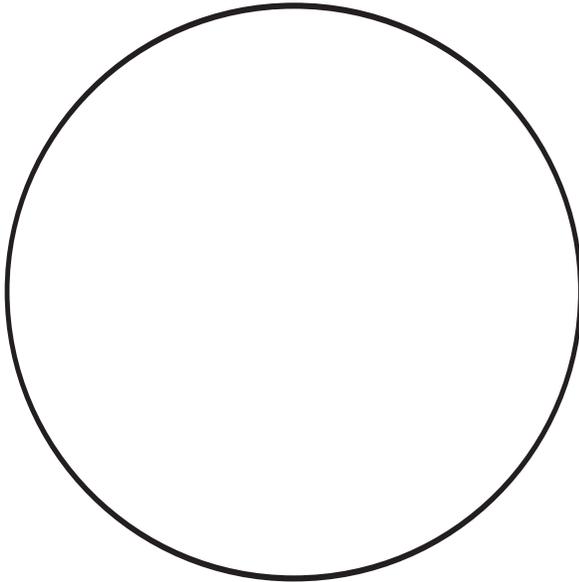
Give each student a spinner as used in the Invitation to Learn activity. Have them try spinning it to see if the odds they predicted are really true.

Spin Your Prize

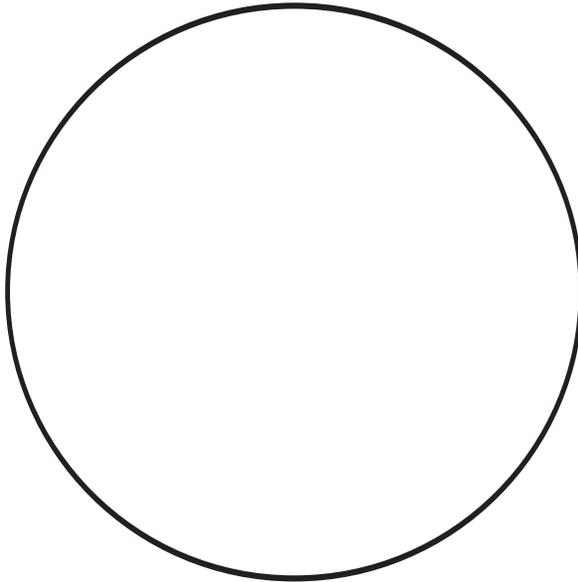


Student Answer Sheet

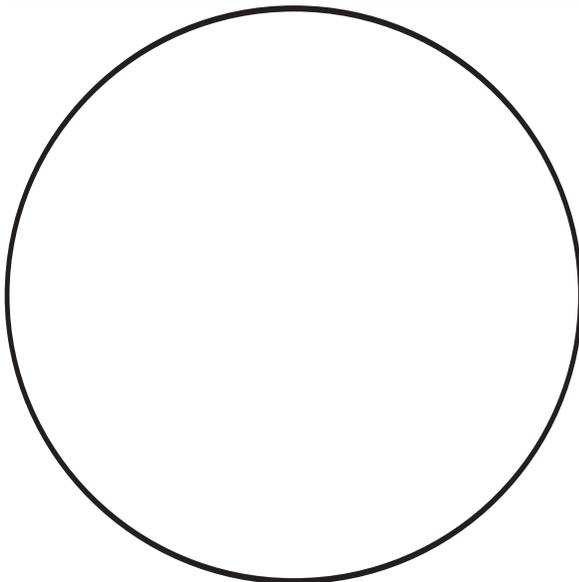
Clue Set #1



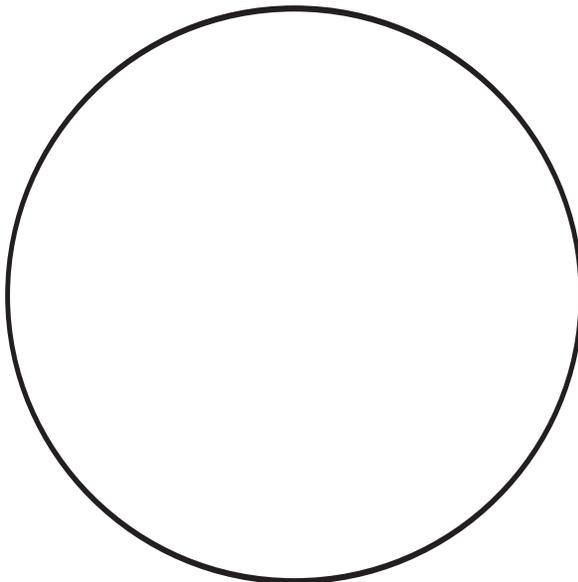
Clue Set #2



Clue Set #3



Clue Set #5



Clue Sets

Clue Set #1

At a department store, you can spin a spinner to win free merchandise. You have the same chances of getting a tie as a ring.

You will probably get a scarf $\frac{3}{8}$ of the time.

You get a free pair of socks about 1 out of 8 spins.

The chances of getting a ring is about 25%.

Clue Set #2

The shoe store has a prize spinner. You are likely to win a pair of socks about 1 out of 4 times.

You will win a pair of shoelaces about 50% of the time.

You will win shoe polish about one half as often as you win a pair of socks.

Your chances of winning a new purse is about $\frac{1}{8}$ of the time.

Clue Set #3

A TV station is giving away tickets to sports events. You are likely to win tickets to gymnastics about 1 out of every 4 spins.

In 100 spins, you are likely to win baseball tickets about 25 times.

You will probably get basketball tickets about $\frac{1}{6}$ of the time.

You are twice as likely to get volleyball tickets as you are to win basketball tickets.

Clue Set #4

At the music store you can get a free CD when you purchase \$25 worth of products. You get a jazz CD $\frac{1}{16}$ of the time.

In 200 spins you are likely to get a rap CD 100 times and a rock CD 50 times.

You have the same chances of getting a country-western CD as you do a jazz CD.

You are twice as likely to win a rock CD as you are to win a blues CD.

Activity—The Dice Game

Standard V

Students will collect, analyze, and draw conclusions from data and apply basic concepts of probability.

Objective 2

Apply basic concepts of probability.

Intended Learning Outcomes

1. Demonstrate a positive learning attitude toward mathematics.

Standard
V
Objective
2
Connections

Background Information

There is a widely misunderstood notion that all numbers (2-12) have an even chance of occurrence with one roll of the dice. This activity helps student to see the results of a series of rolls.

Invitation to Learn

Who can be the first to clear their board? (Games are self-motivating).

Instructional Procedures

1. Pass out numbered graph paper (included).
2. Instruct students to place 20 counters on the squares, one per square beginning next to the numbers, and then transfer their set up to a tracking sheet.
3. Taking turns, each student rolls the dice and removes one disc from the number column that matches their rolled results.
4. The first student to remove all the discs wins.
5. Have students play a series of at least three games, tracking their layout patterns each game.

Curriculum Integration

Heredity

Materials

- Dice (one set of two per group of four students)
- 1 number graph paper per student
- 1 tracking paper per student
- Counters (20 per student - more if using consumables such as Smarties)
- Graph paper (one or more per student)

Possible Extensions/Adaptations

Do even or odd numbers occur more often with a roll of two dice? There are six possibilities of even numbers and only five possibilities of odd numbers. Why then do odd numbers occur more often? Have students create and complete a diagram showing results.

Use spinners instead of dice.

Assessment Suggestion

Ask the question, if you were to place a bet on a single spin of a roulette wheel, on which number would you place all your money and why?

Additional Resource

Addison Wesley Math, Level 6, Guided Problem Solving 12-5 by
Scott Foresman

Homework & Family Connections

Have students challenge their parents/families to play the game.

Tracking Paper

2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									

Activity—Drawing Conclusions

Standard

V

Objective

1

Connections

Standard V

Students will collect, analyze, and draw conclusions from data and apply basic concepts of probability.

Objective 1

Design investigations to reach conclusions using statistical methods to make inference based on data.

Intended Learning Outcomes

2. Become mathematical problem solvers.

Background Information

In the following experiment, it would appear that the chances of a red counter remaining in the box are 50%. However, there are actually three (not two) equally possible states.

1. The initial red counter was drawn, leaving the added red counter.
2. The added red counter was taken, leaving the initial red counter.
3. The added red counter was taken, leaving the blue counter.

Invitation to Learn

The Sock Problem

Instructional Procedures

1. Hold up the box and explain that it contains either a red or a blue counter.
2. Add a red counter, so the box now contains two counters.
3. Ask the question, “If I pull out a red counter, what is the probability that the remaining counter is also red?”
4. Pass out materials to teams.
5. Teams will conduct a series of experiments (at least 10).
6. Teams will determine an appropriate format for displaying results (e.g., bar graphs, line graphs).
7. Have the teams share their results with the class and propose a reason for these results.

Materials

- One box per group of four students
- Three counters (one blue and two red) per group of four students
- Paper
- Pencil

Possible Extensions/Adaptations

Add more counters to the bag. Does it change the odds? How so?

Assessment Suggestion

Have students design and complete a probability problem concerning changing odds.

Additional Resource

1000 Play Thinks: Puzzles, Paradoxes, Illusions & Games by Ivan Muscovich (Workman Publishing).

Homework & Family Connections

Challenge students to conduct the same experiment with their families using materials commonly found at home.

Activity–Coin Tossing

Standard
V

Objectives
1 & 2

Connections

Standard V

Students will collect, analyze, and draw conclusions from data and apply basic concepts of probability. .

Objective 1

Design investigations to reach conclusions using statistical methods to make inferences based on data.

Objective 2

Apply basic concepts of probability.

Intended Learning Outcomes

3. Reason mathematically.

Background Information

Most of us would guess that the probability of flipping heads on a coin are one out of two, or 1/2. The students can readily understand this. However in practice, the result of flipping a coin ten times will not always come up 5 heads and 5 tails. This experiment is designed to help students see that the probability is not the same as reality. As we do the experiment more times, however, the results resemble the probability more closely.

Benford’s law shows that there is a high probability that either heads or tails will come up six or more times in a row when the coin is tossed 200 times. Most fakers will not know this and will not put such an event in their fake results.

Invitation to Learn

Introduce money probability problems.

Instructional Procedures

Materials

for each student:

- A penny
- Experiment chart
- Pencils
- Calculator

1. Place students in cooperative learning groups and pass out materials.
2. Say: “Suppose you ask a friend to flip a coin 200 times and record the outcome. When you are given the results, you want to know whether your friend really flipped the coin all those times or just faked it.” *1000 Play Thinks: Puzzles, Paradoxes, Illusions & Game* by Ivan Mascovich (Workman Publishing)
3. Agree that each player on the team will toss the coin 10 times. Another student will record the results on the enclosed bar graph and post around the room. Color heads red and tails green.

4. Choose one person to roll the dice 50 times. Stop them after each 50 tosses and calculate the most times heads or tails came up in a row. Also calculate how many total times heads and tails came up. Use the chart to do this.
5. Have each person on the team roll 50 times, recording the results as you go. Stop after each 50 rolls and evaluate the results. At the end of 200 throws, have the group calculate the most times heads or tails came up in a row, as well as the total number of times heads and tails came up.
6. Initiate a discussion about how you would know whether the friend had actually thrown the dice 200 times. Ask the group:
7. How many times did head or tails come up 10 times in a row? 9 times in a row, etc.
8. What is the balance between heads and tails like?
9. After how many throws did you notice that heads and tails were about even?
10. Have each group devise a way to tell if the friend had actually thrown the dice 200 times.

Curriculum Integration

Girls and boys are born about equally. How does this experiment relate to the proportion to girls and boys? What are your chances of having either a girl or a boy? How many children would you need to have to ensure equal numbers of boys and girls?

Possible Extensions

How many possible outcomes are there when you toss two coins? What are the chances of getting either a heads or a tails when throwing the dice?

Question: Lauren has 12 coins in her pocket. The probability of her pulling out a penny is $\frac{1}{2}$. How many pennies are in her pocket?

Assessment Suggestion

Jenny claims to have tossed her penny 300 times. The greatest number of either heads or tails in a row is 3. Do you think Jenny actually tossed her coins? Why or why not?

Additional Resource

For a computerized spinner game showing random spins done quickly, see: <http://www.mathresources.ca/spinner.htm>

Homework & Family Connection

Give students an extra sheet to try this with their parents. Offer extra credit in math if they do it and bring it signed by their parents.

Probability Warm-up

1. Stacy has 14 coins in her pocket. The probability of pulling out a penny is $\frac{1}{2}$. How many pennies are in her pocket? _____ Express the probability in three ways:
 - a. Express as a fraction _____
 - b. Express as a decimal _____
 - c. Express as _____ out of _____
2. Steven has 2 nickels in his pocket. The probability of him pulling out a nickel is $\frac{1}{4}$. How many coins are in this pocket? _____ Express the probability of pulling out a nickel in three ways.
 - a. Express as a fraction _____
 - b. Express as a decimal _____
 - c. Express as _____ out of _____

Challenge:

3. Gene has less than 12 nickels, dimes, and quarters in his pocket. The probability of pulling out a nickel or a quarter is $\frac{3}{4}$. The probability of pulling out a dime is $\frac{1}{4}$. How many coins does Gene have in his pocket? _____ How many of each does he have?
Nickels _____ Dimes _____ Quarters _____

Tossing Coins

1. What are your odds of tossing heads when you flip a coin? Express your odds as a fraction. _____ Express it as _____ out of _____. Express it as a percentage: _____%

2. **Predict:** How many heads and how many tails will show up if you flip the coin 10 times? Why?

3. **Test:** Flip the coin ten times. Record you results on the chart below. Color all heads red and tails green.

1	2	3	4	5	6	7	8	9	10
---	---	---	---	---	---	---	---	---	----

Was your prediction accurate? _____ If not, explain what went wrong.

4. Make a bar graph representing the results of your first 10 flips. Do this bar graph on the worksheet provided. Make the number of heads red and the tails green.

5. Compare your results with the rest of the class. Were their results according to your prediction? _____ Next, add the results of the whole class. How do the results compare now?

6. **Experiment:** Now we will begin to see what happens when you flip the coin 200 times. Record all the flips for your group in the order that they happen. Color heads red and tails green. Remember to stop after every 50 and evaluate. Ask how many heads and how many tails? What is the largest number of either heads or tails in a row?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

How many times did heads come up 6 or more times in a row? _____

How many times did tails come up 6 or more times in a row? _____

How many total heads were there? _____ Total tails? _____

Was your prediction correct? _____

How could you tell if your friend really tossed the coin 200 times?

Activity—Exploring Scale

Standard V

Student will collect, analyze, and draw conclusions from data and apply basic concepts of probability.

Objective 1

Design investigations to reach conclusions using statistical methods to make inferences based on data.

Intended Learning Outcomes

6. Represent mathematical situations.

Standard
V
Objective
1
Connections

Background Information

The scale and spread of the graph can make the same data appear differently. If the scale on the y-axis is measured in smaller increments, the graph will appear taller. This makes small differences appear greater. If there is more spread on the x-axis, the line graph will appear to cover more time, and small differences will be harder to detect in a bar graph. In this experiment, each student will map the same data on several graphs. The student will write about how the different graphs change the appearance of the data. They will determine if the changing scale communicates the information differently. The activity will also ask the students to use higher order thinking skills to evaluate their graphs.

Invitations to Learn

To begin the lesson, each student in the learning pair will have a tally graph. The students will be given three minutes to conclude as much information from the graph as they can. They will be encouraged to write their conclusions down so they can remember their conclusions to communicate later during the sharing time. When the thinking time is up, the learning pairs will take turns communicating their conclusions to their learning partner (see Interpreting Tally Graphs worksheet).

Instructional Procedures

Graphs can impart a lot of information. However, sometimes the way the graph is constructed influences our interpretation of the data. Do this experiment in graphing to find out some of the tricks you may encounter in the newspaper or from the government.

1. Use learning pairs.
2. Introduce invitation to learn problem.
3. Pass out the first graphing experience.

Materials

- Worksheets
- Pencils
- Crayons

4. Let pairs work on graphs and discuss how the graph could be interpreted.
5. Let each pair share the results of their thinking with another pair.

Curriculum Integration

On newspaper day, have groups collect the graphs in the newspaper and describe how these graphs could be altered to influence how people interpret the information on the graph.

Possible Extensions/Adaptations

Give students data, and have them determine the best type of graph and the best scale to show the information. Their graph could be done on large paper and used as wall decorations in the room. Let each group explain their thinking to the class before posting the graph.

Assessment Suggestion

Give students an altered graph and have them explain in words how the graph could be improved to show the information more correctly and honestly.

Additional Resource

For graphing activities, see:

<http://illuminations.nctm.org/swr/review.asp?SWR=1742>

Homework & Family Connections

Challenge the students to search their newspapers and magazines at home and bring in an example of a graph. Create a bulletin board to display the graphs. Students could also write their conclusions from a particular graph you supplied in their journals.

Name _____

Interpreting Tally Graphs

Directions: Write your conclusions from each frequency table. Be prepared to share your conclusions with your learning partner.

Learning Partner A:

<i>Students Absent</i>		
Grade	Boys	Girls
6th Grade	////	
7th Grade	//	//
8th Grade		///
9th Grade	//	///

Conclusions:

Learning Partner B:

<i>Volleyball Tournaments Won</i>		
	Won	Total
Team 1	///	3
Team 2	////	4
Team 3	/	1
Team 4	////	4

Conclusions:

Scale Makes a Difference

Suppose you have this data:

Homework Assignments Completed

Henry	4
John	6
Hillary	2
Pamela	10
David	3

Represent this data on this bar graph: Use the same colored crayon for each student.

<i>Homework Assignments Completed</i>					
<i>13</i>					
<i>12</i>					
<i>11</i>					
<i>10</i>					
<i>9</i>					
<i>8</i>					
<i>7</i>					
<i>6</i>					
<i>5</i>					
<i>4</i>					
<i>3</i>					
<i>2</i>					
<i>1</i>					
	<i>Henry</i>	<i>John</i>	<i>Hillary</i>	<i>Pamela</i>	<i>David</i>

Now represent the same data on this graph. Use the same colors for each student.

10					
8					
6					
4					
2					
	Henry	John	Hillary	Pamela	David

How does this look compared to the first graph?

Now put the same data on another graph. Use the same color for each student.

10									
5									
	Henry		John		Hillary		Pamela		David

How does this graph look compared to the first and second graph?

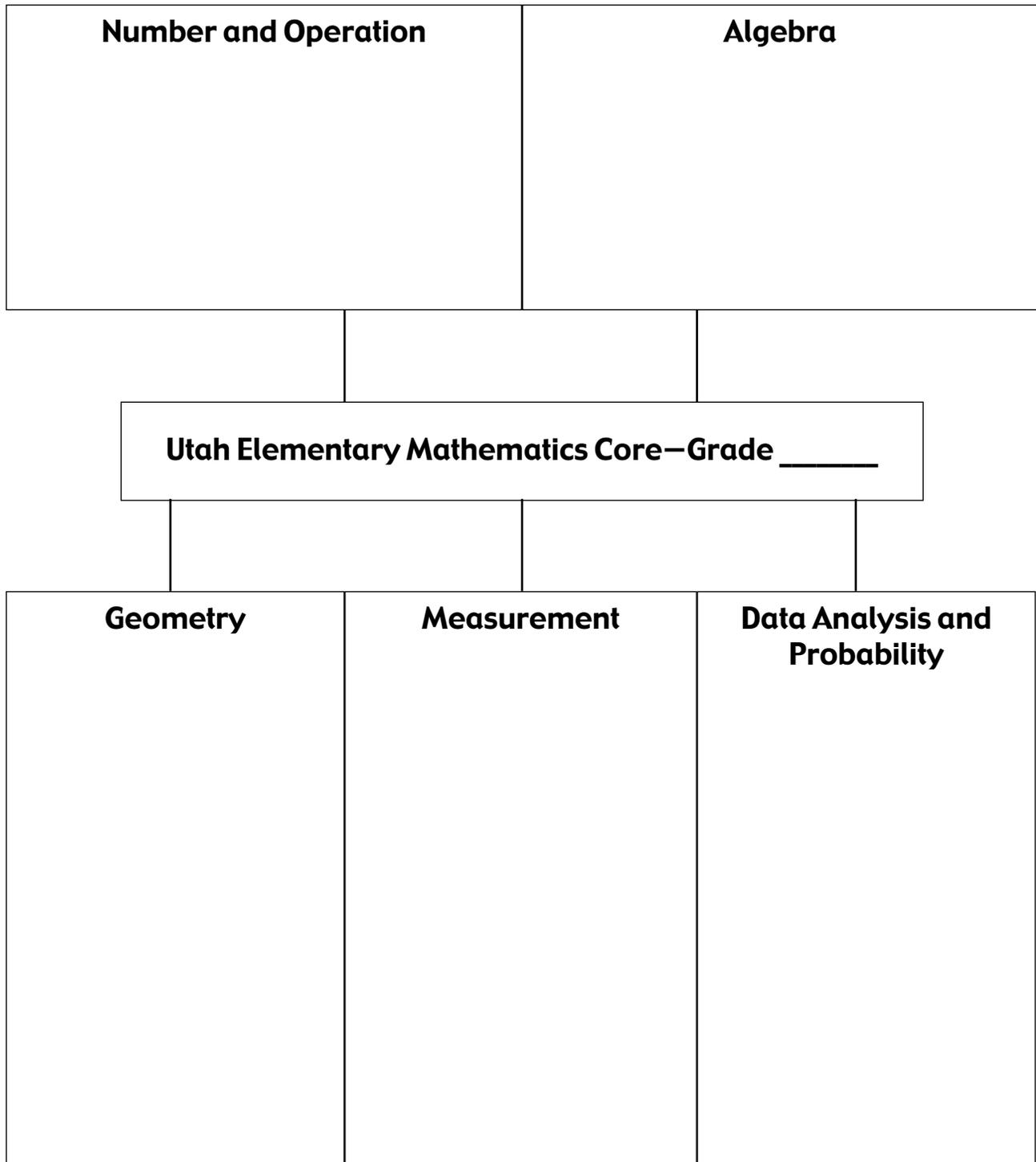
Which graph makes the data look like there is the greatest difference in the homework or the students? _____ Which is the least? _____

Generalization: If you want to emphasize the differences in data, what kind of scale would you use?

Appendix

***What was a typical day in math class like
when you were in school?***

How did it look?	How did it sound?	How did you feel?



ILOs _____

Bridging the Gap

Instructions

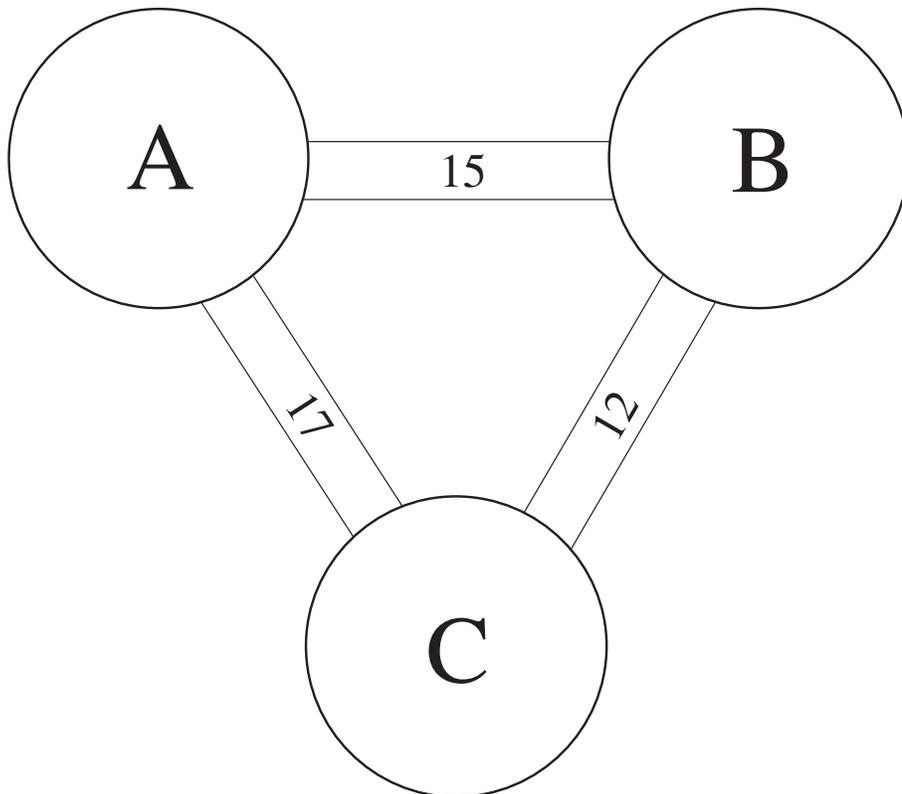
How many of you have ever wanted to be part of a construction company that is responsible for building houses? Well today is your lucky day! You are going to be responsible for building houses on three islands. However, you are fortunate to have bridges that connect the islands to one another.

On your blackline master you will notice the architect has written on each bridge, that connects the two islands, the total number of houses on the two islands. The architect has also indicated the total number of houses to be built on the three islands. Your job is to figure out how many houses go on each island. You may use the cm cubes on your table to represent the houses.



Bridging the Gap

Exercise
#1



Total Number of Houses = 22

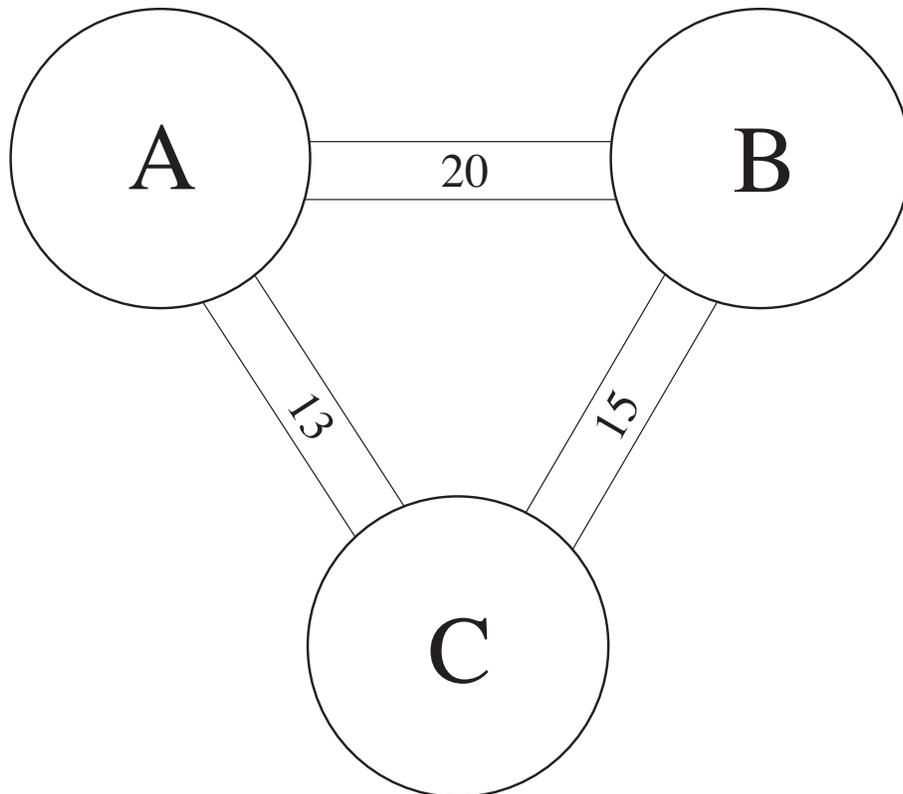
A = _____

B = _____

C = _____

Bridging the Gap

Exercise
#2



Total Number of Houses = 24

A = _____

B = _____

C = _____

Eyewitness

Instructions

Have you ever been the eyewitness for a crime scene? When a crime is committed, what is the role of the eyewitness and the detective? Today you will be working with a partner. One of you will be the eyewitness and the other will be the detective.

1. The detective gives the eyewitness one minute to read the story problem. When time is up, the eyewitness turns the story problem over so he or she can't see it.
2. The detective then asks, "What's this about?"
3. The eyewitness then tells the basic story.
4. The detective then asks the eyewitness to give all the information he or she remembers. The detective asks questions to "learn the story and get the facts."
5. After relating all that can be remembered, the eyewitness is given one more opportunity to have an "instant replay" and read the problem one more time. Again, the detective times the eyewitness for one minute, then turns the problem over again.
6. The detective then asks for any missed details.
7. Following the eyewitness interrogation, the detective writes an equation to represent the case with the help of the witness.
8. If time permits, switch roles and use the second story problem.

Remember: No matter what type of story problem you are solving, you must be a "detective."



Story Problem #1 for Eyewitness:

Jazz Payroll

At approximately 1:00 a.m. on March 25, 2003, the Jazz payroll office was broken into and ransacked. After an investigation all payroll checks were located except the check for John Stockton. The clerk knows that Karl Malone earns 20% more money per game than John Stockton. The clerk computed that Malone earned \$120,000 per game. How much money does John Stockton earn per game?

Story Problem #2 for Eyewitness:

Stick-to-it

Johnny bought a box of soccer stickers to use on his project report on soccer. There were two thousand, one hundred sixty stickers in the box, but he could not use some of them. One-ninth of them were stuck so tightly together that he could not detach them from each other. Eighty-three of them were blank. One-fifth of them had no glue. How many were left that he could actually use?



Story Problem #1 for Eyewitness:

Jazz Payroll

At approximately 1:00 a.m. on March 25, 2003, the Jazz payroll office was broken into and ransacked. After an investigation all payroll checks were located except the check for John Stockton. The clerk knows that Karl Malone earns 20% more money per game than John Stockton. The clerk computed that Malone earned \$120,000 per game. How much money does John Stockton earn per game?

ANSWER: John Stockton earns \$100,000/game

Story Problem #2 for Eyewitness:

Stick-to-it

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ANSWER: 1,405 soccer stickers could actually be used

All in a Name

Instructions

We all know how important our name is. It identifies who we are. It is so important that today we are going to collect data using our names.

1. Quickly walk around the room and record name data for ten participants including yourself. This will give you a chance to meet others in the room and introduce yourself. Have each participant record his or her first and last name in the appropriate columns. Returning to your seat, record the total number of letters for each name in the last column.
2. Using the data collected in your third column, make a quick graph (bar, line, etc.). Describe your data to another participant, in terms of clumps or bunches, gaps or holes, and bumps.
3. Using the same data collected in your third column, find the mean, range, mode, and median. Compare these findings to your graph. Do you see any patterns?
4. Discuss with another participant:
 - a. Why/When is it useful to find the mean, range, mode, and median of data?
 - b. If more names are added to your data set, predict how your measures and graph will change. Explain why.



ROPED INTO QUADRILATERALS

Instructions

1. Assign members of your group (table) to do the following:
 - a. Cut out the quadrilateral pieces.
 - b. Tie the ends of **each** piece of yarn to make three circles.
 - c. Cut out the Task Activity Quadrilateral Labels.
2. The object of the activity is to place the quadrilateral pieces appropriately in your circles (yarn) according to the labels. You may need to overlap the circles to form intersections. The number of labels determines the number of circles used.
3. Progress through the labels, placing quadrilateral pieces according to common attributes.



Quadrilateral Labels

Task Activity #1:

At least one right angle	No right angles
--------------------------	-----------------

Task Activity #2:

No congruent sides	Congruent sides
--------------------	-----------------

Task Activity #3:

At least one obtuse angle	At least one acute angle
---------------------------	--------------------------

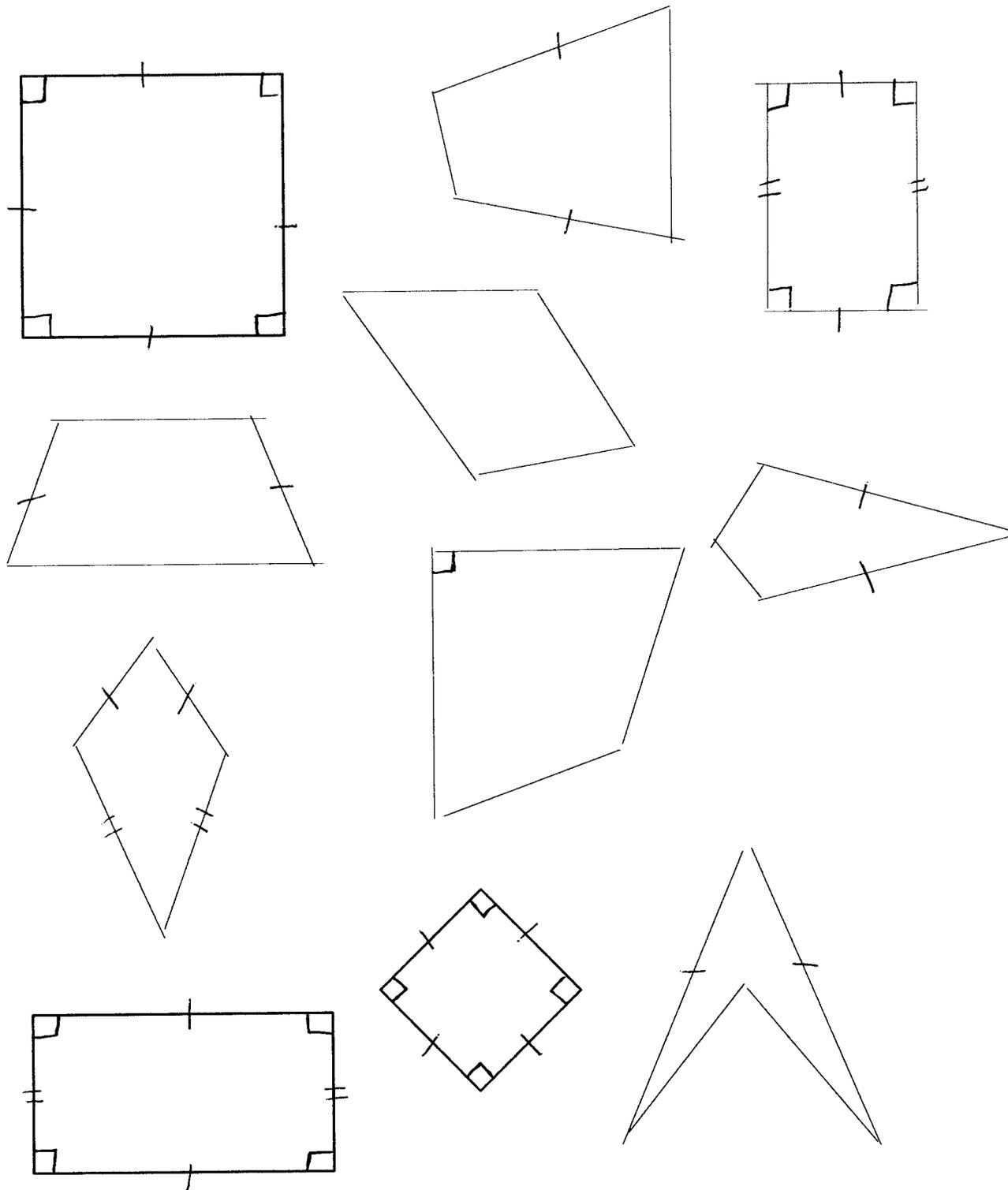
Task Activity #4:

Rectangles	Squares	Rhombi
------------	---------	--------

Task Activity #5:

At least one acute angle	At least one pair of parallel sides
--------------------------	-------------------------------------

Quadrilaterals



The "Right" Place

Use the 1-6 to complete the equations. Each number can only be used once in an equation.

$$\square \times \square = \square + \square + \square$$

$$(\square - \square) \times \square = \square$$

$$\square \times \square \times \square (\square - \square) = \square \square$$

Place the numbers 0-8 to complete the equation.

Hint: The three-digit number is >500.

$$\begin{array}{r}
 \square \square \square \\
 \times \quad \square \\
 \hline
 \square \square \square \square
 \end{array}$$

Cut apart:

1
2
3
4
5
6
7
8
9
0

Randomly select 4 of your numbers. Place them in the equation below so that:

1. the product will be a 3-digit number
2. the product is between 750 and 2,500
3. it is an even product
4. the answer is close to 6,000

$$\begin{array}{r}
 \square \square \\
 \times \square \square \\
 \hline
 \end{array}$$

Evaluating and Designing a Rubric

Student Task

Make a graph to illustrate how many hours you spend during a typical school day doing different things. These things might be sleeping, eating, school, homework, playing sports, scouts, dance, playing with friends, playing games, watching TV, etc.

Your task is to collect, organize, and display your data. A chart or table may be helpful because you must in some way show evidence of data organization. You must also choose the best type of graph to represent this data (bar, circle, etc.). Please show any calculations that you make, and write an explanation for why you chose the graph you did and how you made your graph.

Student Rubric				
	1 Point	2 Points	3 Points	4 Points
Organization of information	Information about time spent very disorganized	Some data organization evident, but not carried through	Data well-organized	In addition, the data is clearly presented
Graph	Graph chosen is inappropriate to the topic or very poorly executed	Graph chosen is adequate, but execution is poor	Appropriate form of graph and adequate information	In addition, the graph is very accurately and neatly presented
Calculations	Major errors in calculations	A number of errors in calculations	Very few errors in calculations	No calculation errors
Explanation	Explanation very muddled	Explanation difficult to follow	Explanation clear enough to follow	The explanation is clear and displays comprehensive understanding of the relative merits of different types of graphs

A few questions to consider as you evaluate the rubric:

- Is this a “good” rubric for the student task given? Why or why not?
- What changes should be made?
- Are the descriptions well stated and not ambiguous?
- Are there definite differences among the points?

Write any changes you would make on the rubric—use your State Core to see what expectations for student achievement are.

Pizza Party

Ms. Williams' 5th grade class is having a pizza party. They are trying to decide which pizzeria has the cheapest price. The local pizzerias and their prices are listed below.

Pizza Prices

Pizza to Go	\$ 8.50	8 slices per pizza
Rosa's Pizza	\$10.50	10 slices per pizza
Pizza with Pizazz	\$ 6.25	6 slices per pizza

There are 30 students in Ms. Williams' class. Each person (including Ms. Williams) will eat two slices of pizza. All the pizza slices from each pizzeria are the same size. Where should you buy the pizza to get the best deal? Why?

Write a brief description of how you arrived at your decision. Provide your work (any calculations you made), a picture, or a diagram to support your thinking.

SAMPLE TEST SCORE GRID FOR MATH CORE ACADEMY

QUESTION NUMBER	1	2	3	4	5	6	7	8	9	10	TOTAL
POINTS POSSIBLE	1	1	1	1	1	1	1	1	1	1	%
BILLY	1	1	1	1	1	1	0	0	1	0	70%
SALLY	1	1	1	1	0	1	1	0	1	0	70%
MARK	1	1	1	1	1	1	0	0	1	0	70%
BRETT	1	1	1	1	0	0	1	0	1	0	60%
LISA	1	1	1	1	1	1	0	0	0	0	60%
JOHN	1	1	1	1	0	1	1	0	0	0	60%
SUSAN	1	0	1	1	1	1	0	0	1	0	60%
VAN	1	1	1	1	0	1	1	0	0	0	60%
YVONNE	1	1	1	0	1	1	0	0	1	0	60%
HANS	1	1	1	1	0	1	1	0	0	0	60%
FRANCIS	1	1	0	1	1	0	0	1	1	0	60%
STEPHANIE	1	1	0	1	0	1	1	1	0	0	60%
MILHAUS	1	1	0	1	1	0	0	1	0	0	50%
WILLIAM	1	1	0	1	0	0	1	1	0	0	50%
BECKY	1	1	0	0	1	0	0	1	0	0	40%
RALPH	1	1	0	0	0	0	1	1	0	0	40%
RHETT	1	0	0	0	1	0	0	1	0	0	30%
SCARLETT	1	0	0	0	0	0	1	1	0	0	30%
ISHMAEL	1	0	0	0	1	0	0	1	0	0	30%
CHARLES	1	0	0	0	0	0	1	1	0	0	30%
QUESTION %	100%	75%	50%	65%	50%	50%	50%	50%	35%	0%	

QUESTION ANALYSIS

- | | |
|-----|------|
| 1.) | 6.) |
| 2.) | 7.) |
| 3.) | 8.) |
| 4.) | 9.) |
| 5.) | 10.) |

Name _____

Investment Tally Sheet

<i>Stock Market</i>			
<i>Stock</i>	<i>Price</i>	<i>Number of Shares</i>	<i>Total Cost</i>

<i>Metals</i>			
<i>Type of Metal</i>	<i>Price</i>	<i>Number of Pounds</i>	<i>Total Cost</i>

<i>Savings Accounts</i>			
<i>Bank</i>	<i>Interest Rate</i>	<i>Deposit</i>	<i>Total</i>

Dishes for a Penny Worksheet

<i>Day</i>	<i>Exponent</i>	<i>Amount</i>	<i>Total</i>
1	2^1	\$ 0.01	\$ 0.01
2	2^2		
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			

FACTOR CAPTURE

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

Fraction/Decimal/Percent/Ratio Cards

$$\frac{1}{4}$$

25%

1:4

0.25

$\frac{1}{3}$	33.3%
0.33	1:3

50%

$\frac{1}{2}$

1:2

0.5

$$\frac{3}{4}$$

75%

3:4

0.75

30%

$\frac{3}{10}$

3:10

0.3

$\frac{1}{8}$

12.5%

1:8

0.125

20%

$\frac{1}{5}$

1:5

0.2

0.8

4:5

$\frac{4}{5}$

80%

70%

$\frac{7}{10}$

7:10

0.7

Name _____

Addition and Subtraction Equation Review

Use algebra tiles to solve the following equations, then use the space provided to draw the model of the algebra tiles for each equation.

$y - 3 = 6$ number added to each side _____ solution $y =$ _____	$2 = x - 1$ number added to each side _____ solution $x =$ _____
$5 = y - 4$ number added to each side _____ solution $y =$ _____	$1 + x = 7$ number subtracted from each side _____ solution $x =$ _____
$3 + d = 5$ number subtracted from each side _____ solution $d =$ _____	$2 + t = 8$ number subtracted from each side _____ solution $t =$ _____

Name _____

Multiplication Equation Review

Use algebra tiles to solve the following equations, then use the space provided to draw the model of the algebra tiles for each equation.

$3x = 6$ divide each side into _____ groups. solution $x =$ _____	$10 = 2y$ divide each side into _____ groups. solution $y =$ _____
$4t = 8$ divide each side into _____ groups. solution $t =$ _____	$6 = 2n$ divide each side into _____ groups. solution $n =$ _____
$3 = 3s$ divide each side into _____ groups. solution $s =$ _____	$5x = 10$ divide each side into _____ groups. solution $x =$ _____

Name _____

Two-Step Equations

Use algebra tiles to solve the following equations, then use the space provided to draw the model of the algebra tiles for each equation.

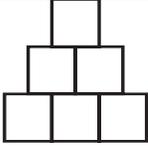
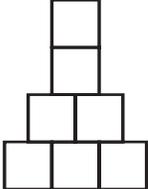
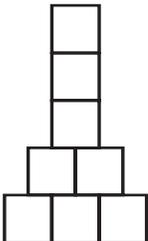
<p style="text-align: center;">$4x + 3 = 11$</p> <p>Step 1: Subtract _____ from each side.</p> <p>Step 2: Divide each side into _____ groups.</p> <p>Solution: $x =$ _____</p>	<p style="text-align: center;">$6y - 1 = 5$</p> <p>Step 1: Add _____ to each side.</p> <p>Step 2: Divide each side into _____ groups.</p> <p>Solution: $y =$ _____</p>
<p style="text-align: center;">$2 + 5n = 12$</p> <p>Step 1: Subtract _____ from each side.</p> <p>Step 2: Divide each side into _____ groups.</p> <p>Solution: $n =$ _____</p>	<p style="text-align: center;">$3t - 1 = 8$</p> <p>Step 1: Add _____ to each side.</p> <p>Step 2: Divide each side into _____ groups.</p> <p>Solution: $t =$ _____</p>

<p style="text-align: center;">$7 = 2k + 1$</p> <p>Step 1: Subtract _____ from each side.</p> <p>Step 2: Divide each side into _____ groups.</p> <p>Solution: $k =$ _____</p>	<p style="text-align: center;">$4r - 3 = 5$</p> <p>Step 1: Add _____ to each side.</p> <p>Step 2: Divide each side into _____ groups.</p> <p>Solution: $r =$ _____</p>
<p style="text-align: center;">$11 = 4p - 5$</p> <p>Step 1: Add _____ to each side.</p> <p>Step 2: Divide each side into _____ groups.</p> <p>Solution: $p =$ _____</p>	<p style="text-align: center;">$5x - 6 = 9$</p> <p>Step 1: Add _____ to each side.</p> <p>Step 2: Divide each side into _____ groups.</p> <p>Solution: $x =$ _____</p>

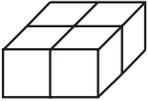
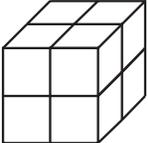
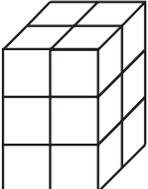
Name _____

Pattern Building

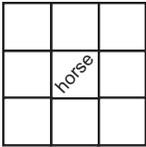
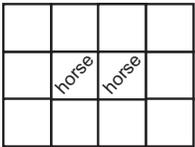
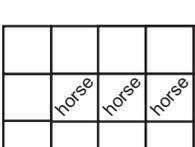
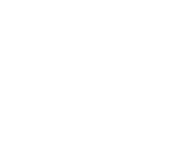
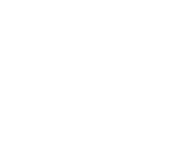
Castles and Towers

	# cubes in tower	# of cubes in all	Rule:
	1	7	
	2	8	
	3	—	
	4	—	
	5	—	
	10	—	
	20	—	
	30	—	

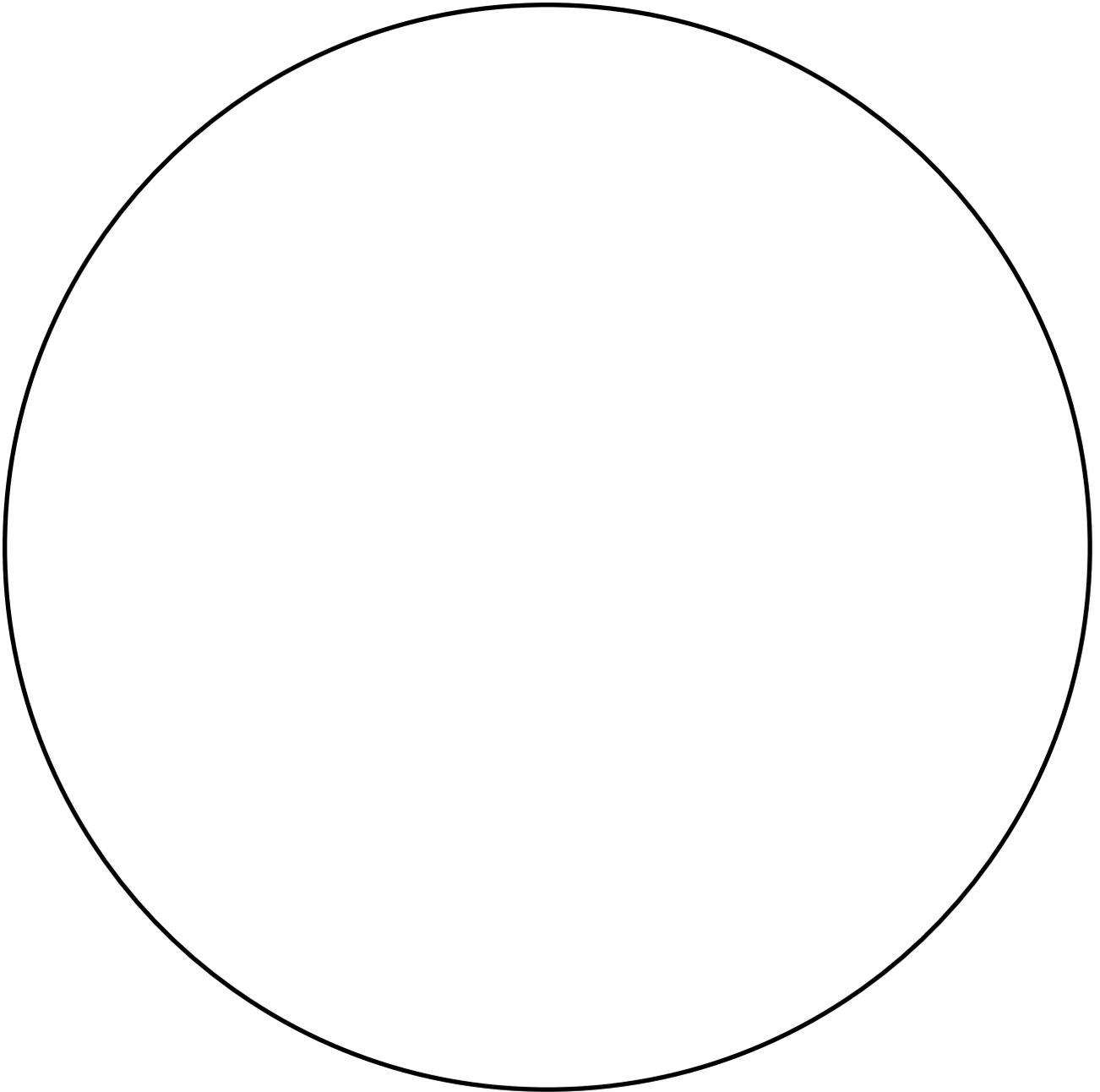
Apartment Buildings

	# of floors	# of cubes in all	Rule:
	1	4	
	2	8	
	3	—	
	4	—	
	5	—	
	10	—	
	20	—	
	30	—	

Horse Corrals

	# of floors	# of cubes in all	Rule:
	1	4	
	2	8	
	3	—	
	4	—	
	5	—	
	10	—	
	20	—	
	30	—	

The Amazing Circle



Cut along the outer edge of your circle so that the dark line remains.

Name _____

Constructing Polygons with Pattern Blocks

Use your pattern blocks to create two different 3-sided, 4-sided, . . . , 10-sided two dimensional figures. Each figure should be made up of at least two pattern blocks (but the pattern blocks do not have to be different). Sides should match up, as discussed in class. Record the diagram for each of your findings using construction paper cutouts, stamping, tracing, or a combination of the three. Label each figure according to whether it is 3-sided, 4-sided, . . . and so on. Do not put more than two sets of shapes on a paper.

3-Sided Shapes

4-Sided Shapes

5-Sided Shapes

6-Sided Shapes

7-Sided Shapes

8-Sided Shapes

9-Sided Shapes

10-Sided Shapes

Name _____

Alphabet Angles: Estimating Acute, Right, and Obtuse with a “Right Angle Referent”

This task is designed to provide you with the opportunity to mentally categorize angles as **acute**, **right**, or **obtuse**. Once you make your mental estimate, measure with an informal measuring tool—a right angle—to verify your mental conjectures.

Procedures:

1. Mentally assess each angle in each alphabet letter and categorize it as acute, right, or obtuse.
2. Measure the angle with the right angle to check your conjecture.
3. Shade the interior of all acute angles in 1 color.
4. Shade the interior angles of all obtuse angles in another color.
5. Label right angles with a right angle symbol.

A E F H I

K L M N

T V X Y Z

Exploring Pattern Block Angles with Hinged Mirrors

This task is designed to provide exploration of angles of polygons. You will need to figure out how many degrees are in **each** of the **different** angles of the pattern blocks. Use the hinged mirrors and what you know about angles to help you. Record your findings on the sheets as follows:

1. Record what you see using the hinged mirrors by drawing (or stamping or pasting). It will help tremendously to first build what you see using the pattern blocks.
2. Draw a directed circle to show the complete rotation you see, when it is present.
3. Then, figure out, in degrees, the measure of the pattern block angle you are investigating.

For example, when we looked at the square pattern block, we positioned the block and mirrors as in figure 1. Then we used the blocks to build what we saw, as in figure 2. We used this figure to help us determine the number of degrees in the angle of the pattern block angle under investigation.

Note: You will not be able to directly measure one of the pattern block angles using this strategy. Use what you know about angles at this point, along with what you know about other pattern block angles, to create a solution for this “problem” angle.

GREEN TRIANGLE

ORANGE SQUARE

BLUE RHOMBUS

(Note: There are two different angles in this polygon! You will need to find the measure of each.)

YELLOW HEXAGON

RED TRAPEZOID

(Note: There are two different angles in this polygon! You will need to find the measure of each.)

TAN RHOMBUS

(Note: There are two different angles in this polygon! You will need to find the measure of each.)

Name _____

Exploring Interior Angles of Polygons Using Pattern Block Angles

For this task, you will further explore the polygon shapes you constructed in a previous lesson. Go back to find the measure of the interior angles of each of the shapes you created. To do this, you will need to do the following:

1. Write the measure of each interior angle next to the vertex in each of the polygons from the previous lesson.
2. Once you have labeled each interior angle with its measure, find the sum of the interior angles of each of the shapes you created.
3. In the following chart, record how you calculated the sum of the interior angles for each shape by making a number sentence using the angles measures of the polygon.
4. Once you have completed the chart, look for patterns. What do you observe? Use the back of the sheet to explain.

<i>Number of Sides</i>	<i>Sum of Interior Angles of Each Shape</i>
3 (triangles)	Shape 1: Shape 2:
4 (quadrilaterals)	Shape 1: Shape 2:
5 (pentagons)	Shape 1: Shape 2:
6 (hexagons)	Shape 1: Shape 2:
7 (heptagons)	Shape 1: Shape 2:
8 (octagons)	Shape 1: Shape 2:
9 (nonagons)	Shape 1: Shape 2:
10 (decagons)	Shape 1: Shape 2:

Name _____

Tearing Polygon Angles

1. Number the three corners of a triangle from 1 to 3. Then tear off the three corners of the triangle. Next, position the angles carefully around the point on line M, as illustrated by the instructor. Make sure to place one side of the first angle so that it is actually on line M. Notice that the vertex of each angle touches the point on line M and that the angles are placed **adjacent** to each other so that they share a side but do not overlap and do not leave gaps between the angle sides. Once all three angles have been placed and taped around the point, make observations. What do you observe? Compare your results with the class members around you. Record your observations in writing.

----- • ----- line M

2. Repeat part 1 for any other triangle. What do you observe?

----- • ----- line M

3. Now, label the four corners of a quadrilateral. As you did with the triangles, take care to position the angles around the point on line M. Notice that the vertex of each angle touches the point on line M and that the angles are placed adjacent to each other so that they do not overlap and do not leave gaps between them. What do you observe? Compare your results with the class members around you. Record your observations in writing.



4. Repeat what you did in part 3 with another quadrilateral. Once again, write about your observations.



5. Make a conjecture about what you think would happen if you placed torn angles from a pentagon around a given point. How could you test your conjecture? Test your conjecture and report your findings.



6. Complete the following chart with the information you obtained by tearing polygon angles from triangles, quadrilaterals, and hexagons. Do you see a pattern? Use your observations to make a conjecture about the sum of the interior angles of a hexagon. Go ahead and test your conjecture.

Name _____

Constructing a Tangram on a Coordinate Plane

The pieces of a tangram can be used to create many interesting figures. They can also be used to analyze relationships among each other. Follow the given set of instructions to make your own tangram.

1. On a piece of graph paper, label the x-axis from -16 to 16 and the y-axis from -16 to 16 .
2. Plot and label the following points: A($-16, 16$), B($16, 16$), C($16, -16$), and D($-16, -16$).
3. Use a ruler to construct segments AB, BC, CD, and DA.
4. Draw segment BD.
5. Find the midpoint of AF. Label it E.
6. What are the coordinates of point E?

7. Find the midpoint of AD. Label it F.
8. What are the coordinates of point F?

9. Draw EF.
10. Find the midpoint of EF. Label it G.
11. What are the coordinates of point G?

12. Draw CG.
13. Find the intersection of GC and BD. Label it H.
14. What are the coordinates of point H?

15. Find the midpoint of HD and label it J.
16. What are the coordinates of point J?

17. Draw FJ.
18. Find the midpoint of HB and label it K.
19. What are the coordinates of point K?

20. Draw GK.

You should now have seven pieces constructed. Each piece is called a tan. Take the envelope provided by your teacher to check the tans on your coordinate plane. Use the shapes to create other interesting figures.

Metric Tape Measure I

Cut each rectangle down the center of the bold lines. Tape rectangles together to create a meter tape. Label the meter tape as shown in the example.



Adapted from Gayle Cloke

Metric Tape Measure I

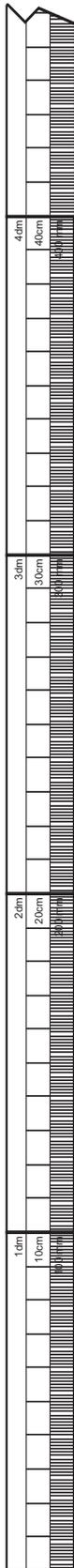
Cut each rectangle down the center of the bold lines. Tape rectangles together to create a meter tape. Label the meter tape as shown in the example.



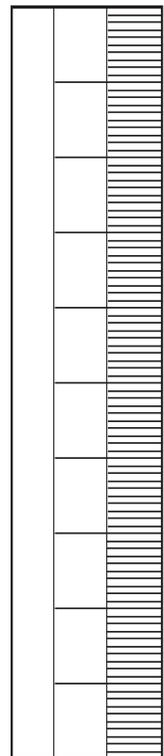
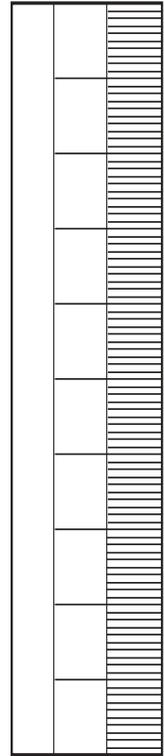
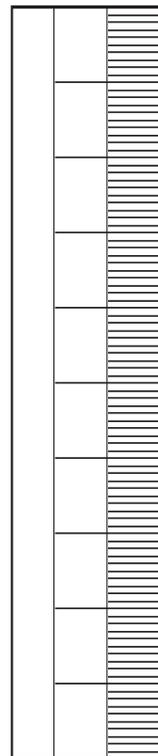
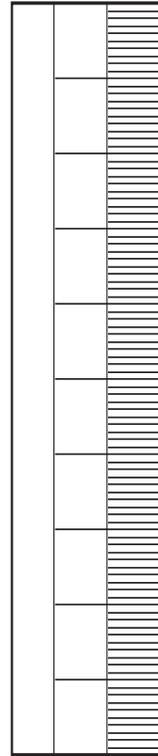
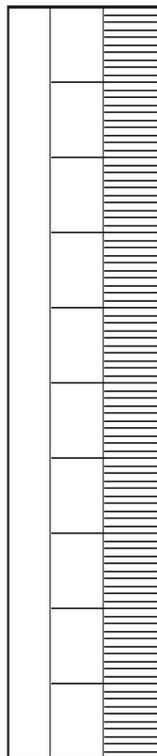
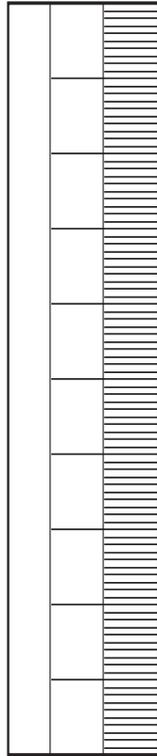
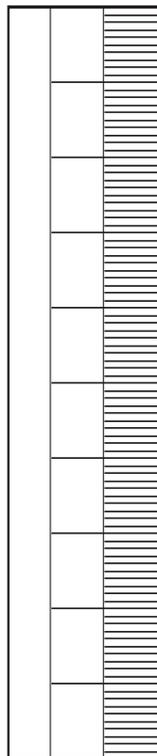
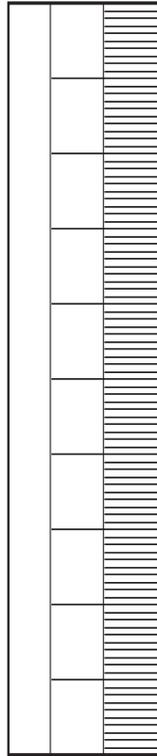
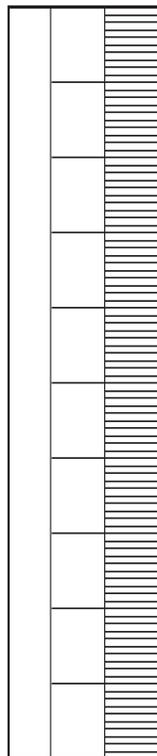
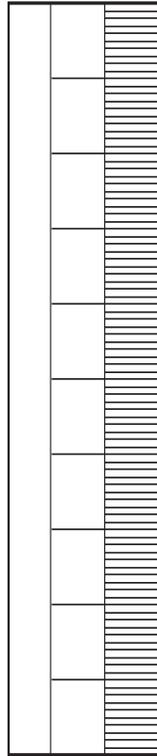
Adapted from Gayle Cloke

Metric Tape Measure 2

Cut each rectangle down the center of the bold lines. Tape rectangles together to create a meter tape. Label the meter tape as shown in the example.

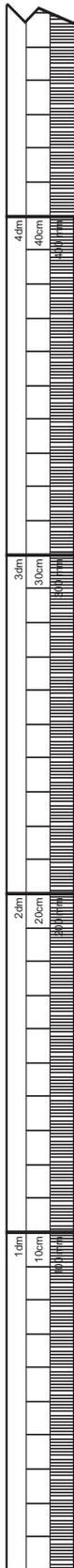


Adapted from Gayle Cloke

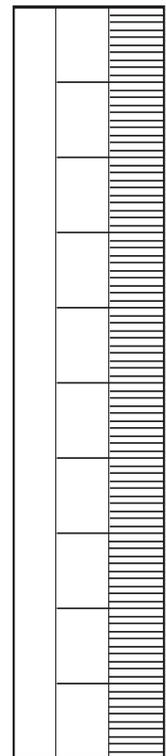
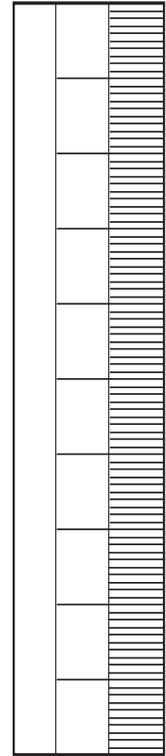
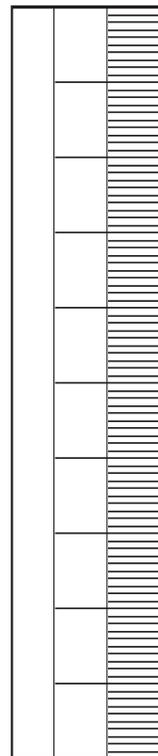
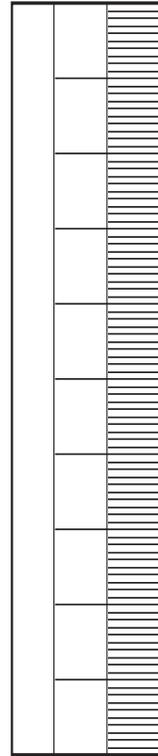
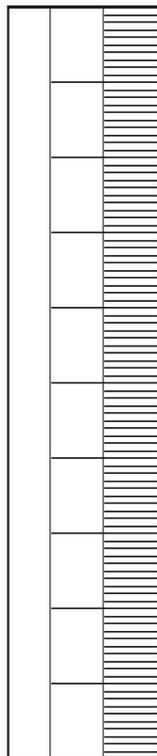
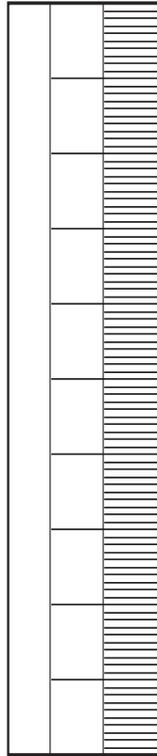
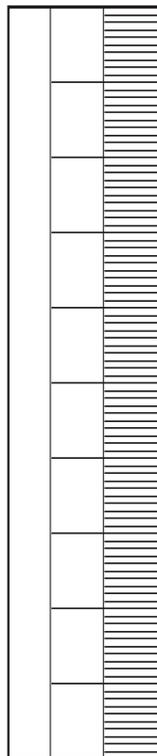
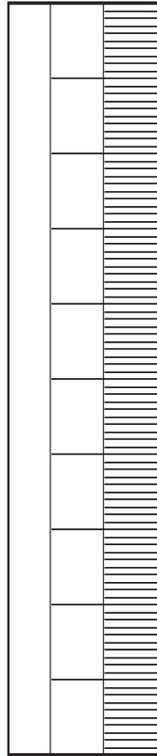
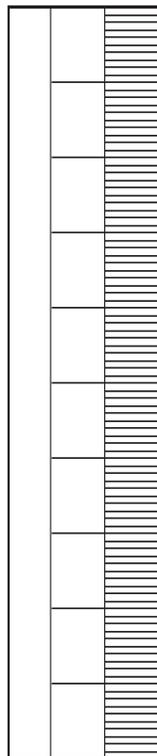
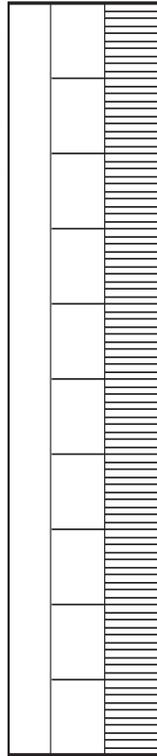


Metric Tape Measure 2

Cut each rectangle down the center of the bold lines. Tape rectangles together to create a meter tape. Label the meter tape as shown in the example.



Adapted from Gayle Cloke



Trundle Wheel Recording Sheet

	Estimated Distance	Actual Distance
One Minute		
Two Minutes		
Three Minutes		
Four Minutes		
Five Minutes		
Six Minutes		
Seven Minutes		
Eight Minutes		
Nine Minutes		
Ten Minutes		
Total Distance:		

Using your information, figure out the following:

Miles Per Minute Average _____

Mode _____

Is there an outlier? _____

Range _____

Discovering Pi

<i>Object</i>	<i>Distance Around Objecct</i>	<i>Distance Across Middle</i>	<i>Distance Around Outside/Distance Across Middle</i>
1.			
2.			
3.			
4.			
5.			
6.			
7.			
8.			
9.			
10.			

Surface Area and Volume

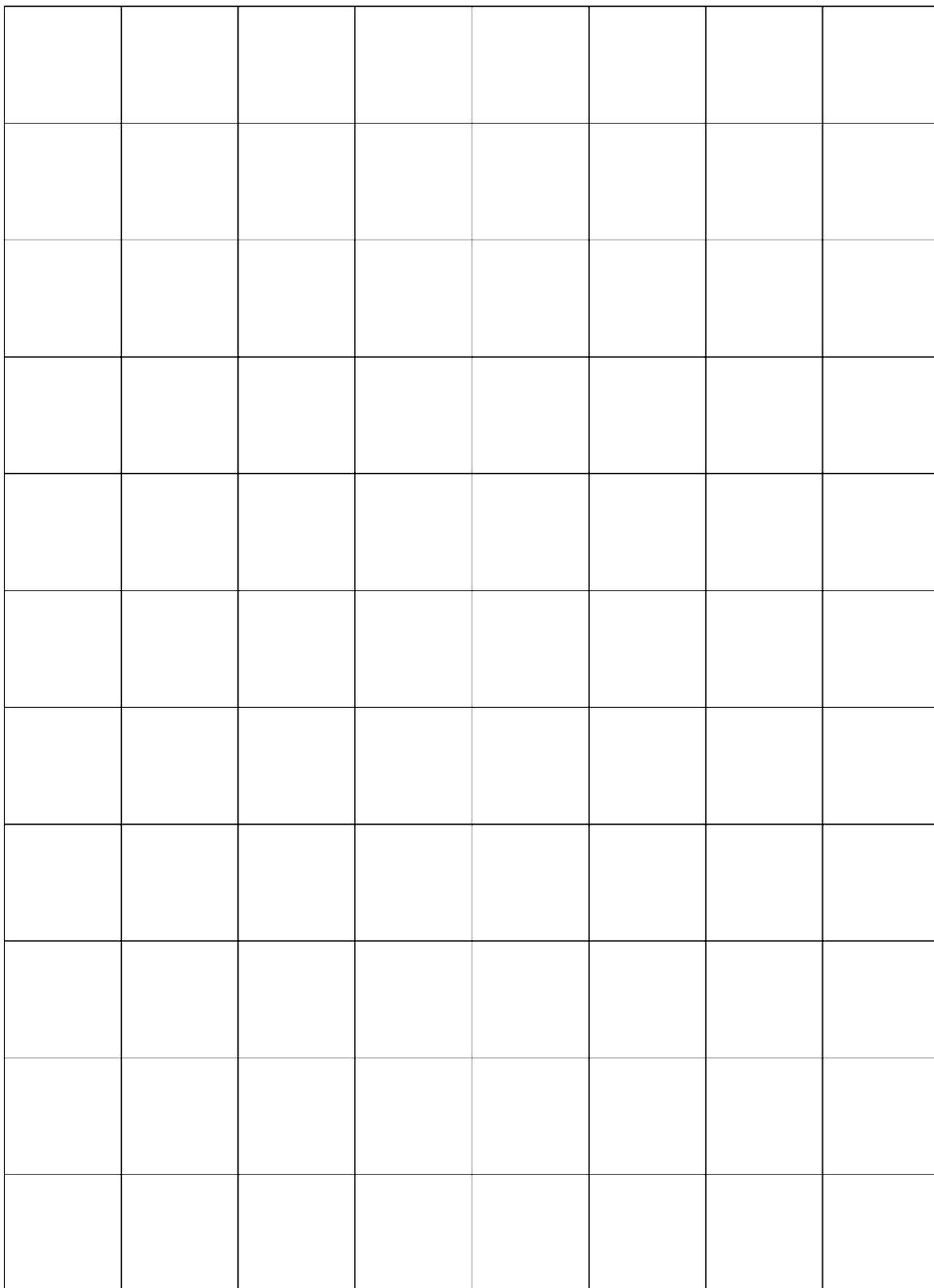
Rectangular prisms have six sides. Please label the dimensions of each of the sides: (Surface Area)

	Dimensions (ex: 3cm x 4cm)	Area (3cm x 4cm = 12cm squared)
Side 1		
Side 2		
Side 3		
Side 4		
Side 5		
Side 6		

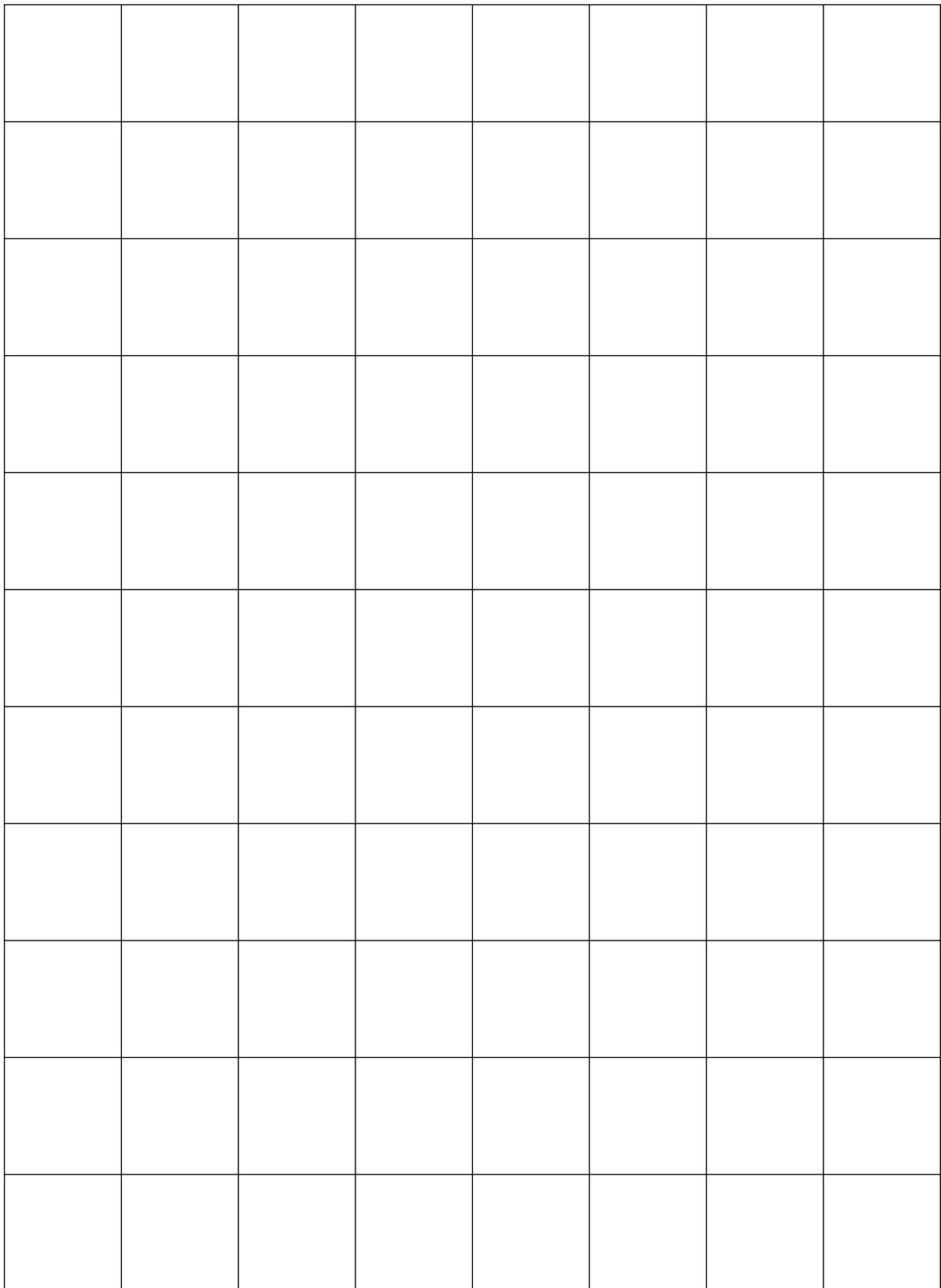
How many cubes fit in each box?

	Dimensions (ex: 3cm x 4cm x 2cm)	Volume (3cm x 4cm x 2cm squared)
Box 1		
Box 2		
Box 3		
Box 4		
Box 5		
Box 6		
Box 7		

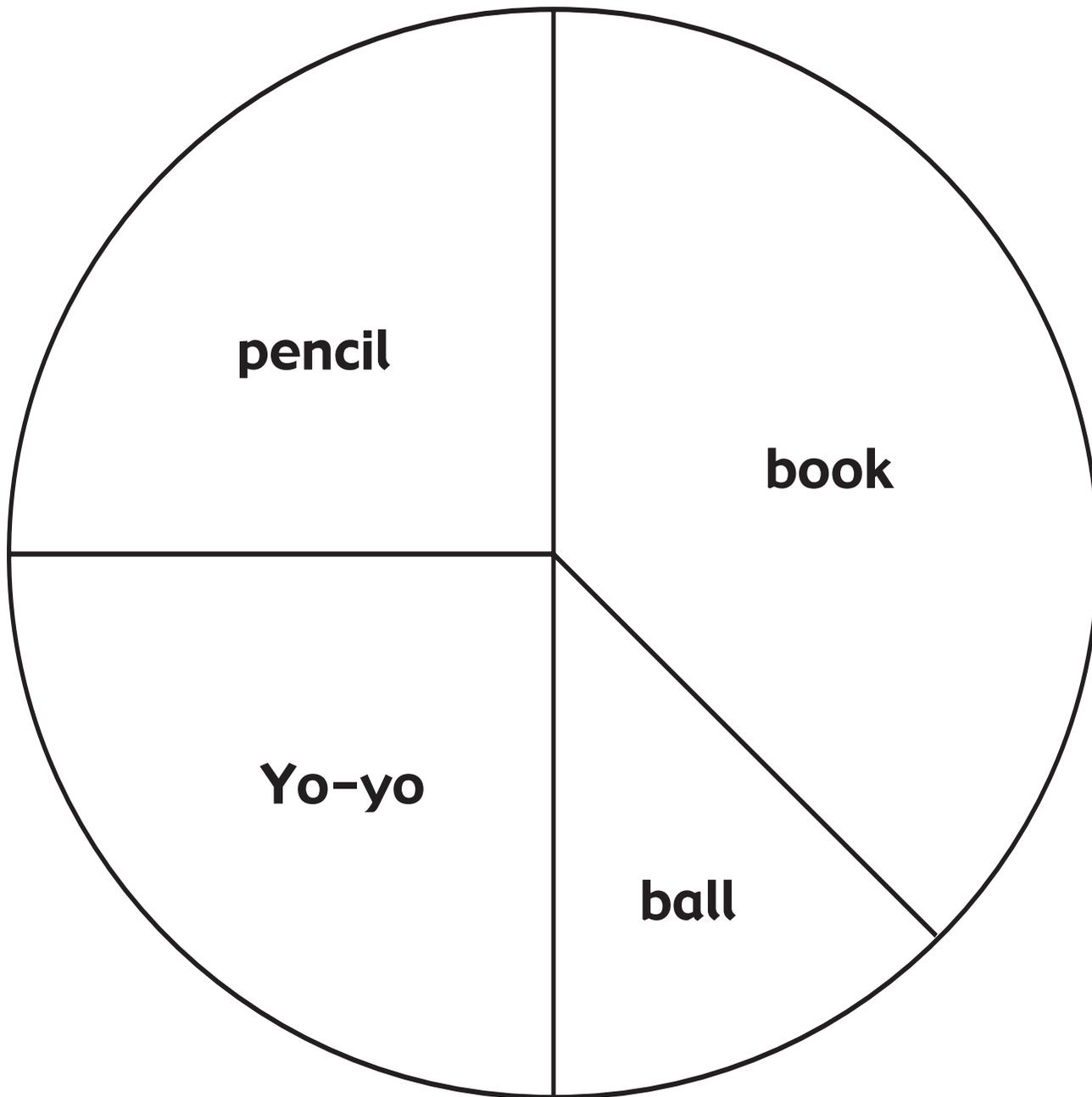
2 cm Grid Paper



2 cm Grid Paper



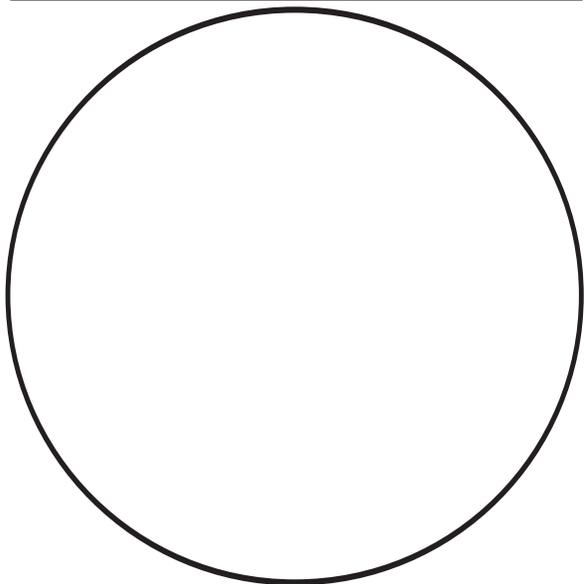
Spin Your Prize



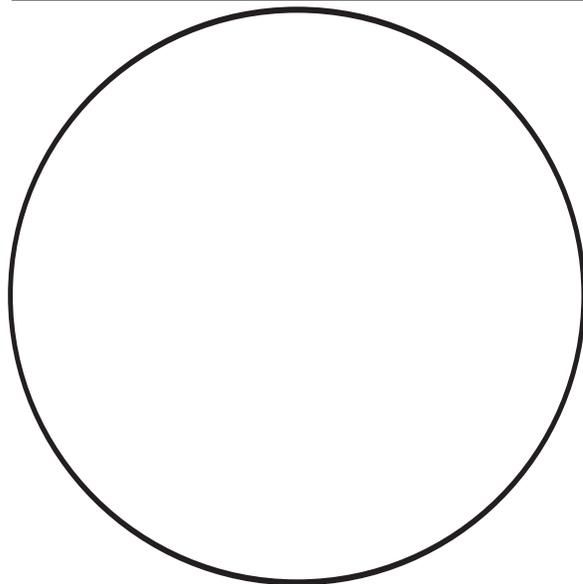
Name _____

Student Answer Sheet

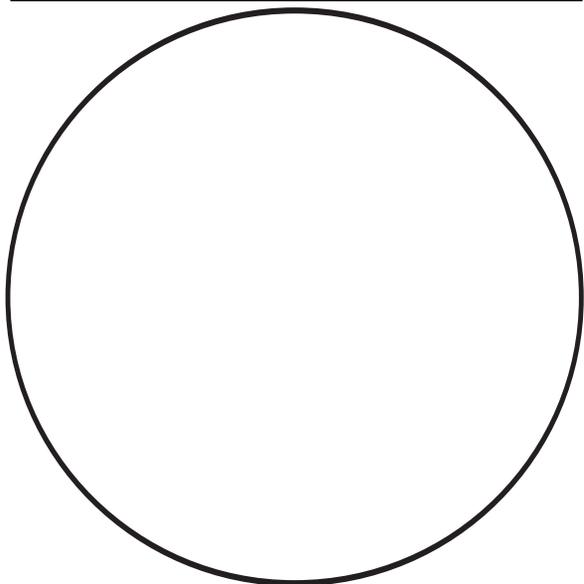
Clue Set #1



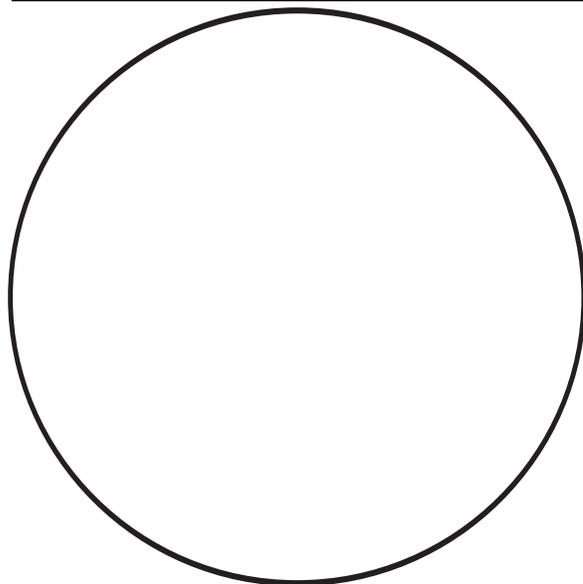
Clue Set #2



Clue Set #3



Clue Set #5



Clue Sets

Clue Set #1

At a department store, you can spin a spinner to win free merchandise. You have the same chances of getting a tie as a ring.

You will probably get a scarf $\frac{3}{8}$ of the time.

You get a free pair of socks about 1 out of 8 spins.

The chances of getting a ring is about 25%.

Clue Set #2

The shoe store has a prize spinner. You are likely to win a pair of socks about 1 out of 4 times.

You will win a pair of shoelaces about 50% of the time.

You will win shoe polish about one half as often as you win a pair of socks.

Your chances of winning a new purse is about $\frac{1}{8}$ of the time.

Clue Set #3

A TV station is giving away tickets to sports events. You are likely to win tickets to gymnastics about 1 out of every 4 spins.

In 100 spins, you are likely to win baseball tickets about 25 times.

You will probably get basketball tickets about $\frac{1}{6}$ of the time.

You are twice as likely to get volleyball tickets as you are to win basketball tickets.

Clue Set #4

At the music store you can get a free CD when you purchase \$25 worth of products. You get a jazz CD $\frac{1}{16}$ of the time.

In 200 spins you are likely to get a rap CD 100 times and a rock CD 50 times.

You have the same chances of getting a country-western CD as you do a jazz CD.

You are twice as likely to win a rock CD as you are to win a blues CD.

Tracking Paper

2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									

Probability Warm-up

1. Stacy has 14 coins in her pocket. The probability of pulling out a penny is $\frac{1}{2}$. How many pennies are in her pocket? _____ Express the probability in three ways:
 - a. Express as a fraction _____
 - b. Express as a decimal _____
 - c. Express as _____ out of _____
2. Steven has 2 nickels in his pocket. The probability of him pulling out a nickel is $\frac{1}{4}$. How many coins are in this pocket? _____ Express the probability of pulling out a nickel in three ways.
 - a. Express as a fraction _____
 - b. Express as a decimal _____
 - c. Express as _____ out of _____

Challenge:

3. Gene has less than 12 nickels, dimes, and quarters in his pocket. The probability of pulling out a nickel or a quarter is $\frac{3}{4}$. The probability of pulling out a dime is $\frac{1}{4}$. How many coins does Gene have in his pocket? _____ How many of each does he have?
Nickels _____ Dimes _____ Quarters _____

Tossing Coins

1. What are your odds of tossing heads when you flip a coin? Express your odds as a fraction. _____ Express it as _____ out of _____. Express it as a percentage: _____%

2. **Predict:** How many heads and how many tails will show up if you flip the coin 10 times? Why?

3. **Test:** Flip the coin ten times. Record you results on the chart below. Color all heads red and tails green.

1	2	3	4	5	6	7	8	9	10
---	---	---	---	---	---	---	---	---	----

Was your prediction accurate? _____ If not, explain what went wrong.

4. Make a bar graph representing the results of your first 10 flips. Do this bar graph on the worksheet provided. Make the number of heads red and the tails green.

5. Compare your results with the rest of the class. Were their results according to your prediction? _____ Next, add the results of the whole class. How do the results compare now?

6. **Experiment:** Now we will begin to see what happens when you flip the coin 200 times. Record all the flips for your group in the order that they happen. Color heads red and tails green. Remember to stop after every 50 and evaluate. Ask how many heads and how many tails? What is the largest number of either heads or tails in a row?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

How many times did heads come up 6 or more times in a row? _____

How many times did tails come up 6 or more times in a row? _____

How many total heads were there? _____ Total tails? _____

Was your prediction correct? _____

How could you tell if your friend really tossed the coin 200 times?

Name _____

Interpreting Tally Graphs

Directions: Write your conclusions from each frequency table. Be prepared to share your conclusions with your learning partner.

Learning Partner A:

<i>Students Absent</i>		
Grade	Boys	Girls
6th Grade	////	
7th Grade	//	//
8th Grade		///
9th Grade	//	///

Conclusions:

Learning Partner B:

<i>Volleyball Tournaments Won</i>		
	Won	Total
Team 1	///	3
Team 2	////	4
Team 3	/	1
Team 4	////	4

Conclusions:

Scale Makes a Difference

Suppose you have this data:

Homework Assignments Completed

Henry	4
John	6
Hillary	2
Pamela	10
David	3

Represent this data on this bar graph: Use the same colored crayon for each student.

<i>Homework Assignments Completed</i>					
13					
12					
11					
10					
9					
8					
7					
6					
5					
4					
3					
2					
1					
	<i>Henry</i>	<i>John</i>	<i>Hillary</i>	<i>Pamela</i>	<i>David</i>

Now represent the same data on this graph. Use the same colors for each student.

10					
8					
6					
4					
2					
	Henry	John	Hillary	Pamela	David

How does this look compared to the first graph?

Now put the same data on another graph. Use the same color for each student.

10									
5									
	Henry		John		Hillary		Pamela		David

How does this graph look compared to the first and second graph?

Which graph makes the data look like there is the greatest difference in the homework or the students? _____ Which is the least? _____

Generalization: If you want to emphasize the differences in data, what kind of scale would you use?

Academy Notes

Academy Notes

Academy Notes

**Professional Development Activity
Evaluation Form 2002-03
Utah State Office of Education**

Course Title	Elementary CORE Academy
Facilitator	_____
Dates	_____ to _____
Location	_____

	N/A	Strongly disagree	Disagree	Agree	Strongly Agree
1. Inservice aligned with the Utah Core Curriculum.	0	1	2	3	4
2. Useful assessment practices related to subject were presented.	0	1	2	3	4
3. Time allocated for this professional development was appropriate to meet my learning needs. If your answer was "strongly disagree" or "disagree", please check one of the following:	0	1	2	3	4
		___ More time needed			
		___ Less time needed			
4. Inservice was well organized.	0	1	2	3	4
5. Facilitator(s) and presenter(s) clearly stated objectives of professional development.	0	1	2	3	4
6. Presenter(s) had adequate knowledge of subject matter.	0	1	2	3	4
7. Professional development provided information relevant to my classroom.	0	1	2	3	4
8. Accommodations and facilities promoted learning.	0	1	2	3	4
9. I will recommend this professional development experience to other teachers.	0	1	2	3	4

10. Rate the use and effectiveness of each mode of instruction in this professional development.

	Not Used	Used Occasionally	Used Often	Not Used Effectively	Used Effectively
a) Lecture	0	1	2	0	1
b) Hands-on	0	1	2	0	1
c) Cooperative Groups	0	1	2	0	1
d) Discussion	0	1	2	0	1
e) Technology	0	1	2	0	1
f) Field Trips	0	1	2	0	1

11. How do you plan to implement the information from this professional development into your classroom instruction?

12. In what way will this professional development provide long-term benefit to the quality of your instruction?

13. What suggestions do you have for improving this professional development?

If you wish to send additional comments regarding this professional development experience, or to indicate other professional activities you would like to take in the future, please email or phone:

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1. Check the category that best describes your school position:

- Teacher School and/or District Administrator / Supervisor
 Grade Level
 Pre-service teacher candidate Other (Specify: _____)

2. Gender: Male Female

3. How many students, total, did you teach during the previous school year? _____
How many students do you expect to teach in the coming school year? _____

4. Are you teaching at a Title I School?

- Yes I am No I am not I don't know

School _____ District _____