ELEMENTARY CORE ACADEMY • ELEMENTARY CORE ACADEMY • ELEMENTARY CORE ACADEMY

5TH GRADE

ELEMENTARY CORE ACADEMY

TEACHERS OF UTAH, LOCAL SCHOOL DISTRICTS,
UTAH STATE OFFICE OF EDUCATION, & UTAH STATE UNIVERSITY

UtahState UNIVERSITY

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Acknowledgements

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Organizations:

Utah State Office of Education (USOE)
Utah State University (USU)
State Science Education Coordination Committee (SSECC)
State Mathematics Education Coordination Committee (SMECC)
Special Education Services Unit (USOE)
WestEd Eisenhower Regional Consortium

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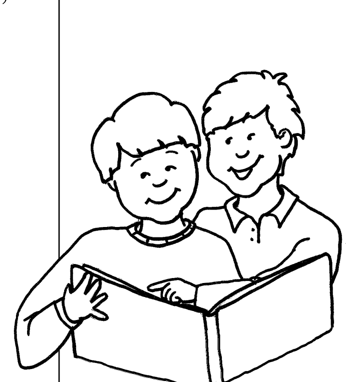

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Steven O. Laing, Ed.D. State Superintendent of Public Instruction
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Dear Core Academy Teachers:

Your involvement in the Core Academy represents a significant investment by you, your school, and district in educational excellence for the students of Utah. I commend you for your dedication and willingness to engage in meaningful professional growth. Efforts by teachers and administrators to develop, provide, and participate in high quality professional development programs must continue if we desire quality learning experiences for all children.

As the needs of students change, it is critical that educators adjust to meet those needs. Teachers should continue to gain expertise in the collection and use of accurate data and analysis of each student's level of achievement. This investment in accountability will empower teachers, parents, and others educators to be more effective.

Exemplary models of instruction, practical application, and collegial support must be an integral part of all professional development. Embedding sound instructional methods that specifically align to the state Core Curriculum will equip teachers with the skills and tools to meet the needs of Utah students.

It is my belief that educators care deeply about their students and work hard to create successful experiences in the classroom. Despite some challenges facing our schools, dedicated and professional educators make profound differences each day.

Sincerely,

Steven O. Laing

Steven O. Laing, Ed.D.
State Superintendent of Public Instruction

Funding Sources

Appreciation is expressed for the tremendous educational input and monetary commitment of several organizations for the successful delivery of the Elementary CORE Academy. This year's Elementary CORE Academy was developed and funded through a variety of sources. The Utah State Office of Education (USOE), in collaboration with Utah State University (USU) and local school districts of Utah, have supported kindergarten through sixth grade teachers with professional development experiences that will enhance the educational experience for Utah children.

Major funding for the Academy comes from the following sources:

State Funds:

Utah State Office of Education
Staff Development Funds
Special Education Services Unit

Federal Funds: ESEA Title II

WestED Eisenhower Regional Consortium

District Funds:

Various sources including Quality Teacher Block, Federal ESEA Title II, and District Professional Development Funds

School Funds:

Trust land, ESEA Title II, and other school funds
Utah State Office of Education Special Education Services

The state and district funds are allocations from the state legislature. ESEA is part of the "No Child Left Behind" funding that comes to Utah.

Additionally, numerous school districts, individual schools, and principals in Utah have sponsored teachers to attend the Academy. Other educational groups such as the Utah Division of Water Resources, National Energy Foundation, Utah Energy Office, and the Utah Mining Association have assisted in the development and delivery of resources in the Academy.

Most important is the thousands of teachers who take time from their summer to attend these professional development workshops. It is these teachers who make this program possible.

Goals of the Elementary CORE Academy

Overall

The purpose of the Elementary CORE Academy is to create high quality teacher instruction and improve student achievement through the delivery of professional development opportunities and experiences for teachers across Utah.

The Academy will provide elementary teachers in Utah with:

1. Models of exemplary and innovative instructional strategies, tools, and resources to meet newly adopted Core Curriculum standards, objectives, and indicators.
2. Practical models and diverse methods of meeting the learning needs of all children, with instruction implementation aligned to the Core Curriculum.
3. Meaningful opportunities for collaboration, self-reflection, and peer discussion specific to innovative and effective instructional techniques, materials, teaching strategies, and professional practices in order to improve classroom instruction.

Learning a limited set of facts will no longer prepare a student for real experiences encountered in today’s world. It is imperative that educators have continued opportunities to obtain instructional skills and strategies that provide methods of meeting the needs of all students. Participants of the Academy experience will be better equipped to meet the challenges faced in today’s classrooms.

Table of Contents

Chapter 3: Inquiry

Chapter 4: Assessment Strategies

Chapter 5: Misconceptions

Chapter 6: Journaling Ideas

Chapter 7: Standard I Activities

Chapter 8: Standard II Activities

Chapter 9: Standard III Activities

Chapter 10: Standard IV Activities

Chapter 11: Standard V Activities

Chapter 12: Appendix

Fifth Grade Mathematics Core Curriculum

Utah Elementary Mathematics Core Curriculum

Introduction

Most students enter school confident in their own abilities; they are curious and eager to learn more. They make sense of the world by reasoning and problem solving. Young students are active, resourceful individuals who construct, modify, and integrate ideas by interacting with the physical world as well as with peers and adults. They learn by doing, collaborating, and sharing their ideas. Students' abilities to communicate through language, pictures, sound, movement, and other symbolic means develop rapidly during these years.

Young students are building beliefs about what mathematics is, about what it means to know and do mathematics, and about themselves as mathematical learners. Mathematics instruction needs to include more than short-term learning of rote procedures. Students must use technology and other mathematical tools, such as manipulative materials, to develop conceptual understanding and solve problems as they do mathematics. Students, as mathematicians, learn best with hands-on, active experiences throughout the instruction of the mathematics curriculum.

Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, the term mathematical proficiency has been chosen to capture what it means to learn mathematics successfully. *Mathematical proficiency* has five strands: computing (carrying out mathematical procedures flexibly, accurately, efficiently, and appropriately), understanding (comprehending mathematical concepts, operations, and relations), applying (ability to formulate, represent, and solve mathematical problems), reasoning (using logic to explain and justify a solution to a problem), and engaging (seeing mathematics as sensible, useful, and doable, and being able to do the work).

The most important observation about the five strands of mathematical proficiency is that they are interwoven and interdependent. This observation has implications for how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve that goal. At any given moment during a mathematics lesson or unit, one or two strands might be emphasized. But all the strands must eventually be addressed so that the links among them are strengthened. The integrated and balanced development of all five strands of mathematical proficiency should guide the teaching and learning of school mathematics. Instruction should not be based on extreme positions that students learn solely by internalizing what a teacher or book says or solely by inventing mathematics on their own.

The Elementary Mathematics Core describes what students should know and be able to do at the end of each of the K-6 grade levels. It was developed, critiqued, and revised by a community of Utah mathematics teachers, university mathematics educators, State Office of Education specialists, mathematicians, and an advisory committee representing a wide variety of people from the community. The Core reflects the current philosophy of mathematics education that is expressed in national documents developed by the National Council of the Teachers of Mathematics, the American Association for the Advancement of Science, and the National Research Council. This Mathematics Core has the endorsement of the Utah Council of Teachers of Mathematics Association. The Core reflects high standards of achievement in mathematics for all students.

Organization of the Elementary Mathematics Core

The Core is designed to help teachers organize and deliver instruction.

- The INTENDED LEARNING OUTCOMES (ILOs) describe the goals for mathematical skills and attitudes. They are found at the beginning of each grade level, are an integral part of the Core, and should be included as part of instruction.
- A STANDARD is a broad statement of what students are expected to understand. Several Objectives are listed under each Standard.
- An OBJECTIVE is a more focused description of what students need to know and be able to do at the completion of instruction. If students have mastered the Objectives associated with a given Standard, they have mastered that Standard at that grade level. Several Indicators are described for each Objective.
- An INDICATOR is a measurable or observable student action that enables one to assess whether a student has mastered a particular Objective. Indicators are not meant to be classroom activities, but they can help guide classroom instruction.

Guidelines Used in Developing the Elementary Mathematics Core

The Core is:

Consistent With the Nature of Learning

The main intent of mathematics instruction is for students to value and use mathematics as a process to understand the world. The Core is designed to produce an integrated set of Intended Learning Outcomes for students.

Coherent

The Core has been designed so that, wherever possible, the ideas taught within a particular grade level have a logical and natural connection with each other and with those of earlier grades. Efforts have also been made to select topics and skills that integrate well with one another and with other subject areas appropriate to grade level. In addition, there is an upward articulation of mathematical concepts, skills, and content. This spiraling is intended to prepare students to understand and use more complex mathematical concepts and skills as they advance through the learning process.

Developmentally Appropriate

The Core takes into account the psychological and social readiness of students. It builds from concrete experiences to more abstract understandings. The Core focuses on providing experiences with concepts that students can explore and understand in depth to build the foundation for future mathematical learning experiences.

Reflective of Successful Teaching Practices

Learning through play, movement, and adventure is critical to the early development of the mind and body. The Core emphasizes student exploration. The Intended Learning Outcomes are central in each standard. The Core is designed to encourage instruction with students working in cooperative groups. Instruction should include recognition of the role of mathematics in the classroom, school, and community.

Comprehensive

The Elementary Mathematics Core does not cover all topics that have traditionally been in the elementary mathematics curriculum; however, it provides a comprehensive background in mathematics. By emphasizing depth rather than breadth, the Core seeks to empower students rather than intimidate them with a collection of isolated and eminently forgettable facts. Teachers are free to add related concepts

The Core is:

- **Consistent**
- **Coherent**
- **Developmentally Appropriate**
- **Reflective of Successful Teaching Practices**
- **Comprehensive**
- **Feasible**
- **Useful and Relevant**
- **Reliant Upon Effective Assessment Practices**
- **Engaging**

and skills, but they are expected to teach all the standards and objectives specified in the Core for their grade level.

Feasible

Teachers and others who are familiar with Utah students, classrooms, teachers, and schools have designed the Core. It can be taught with easily obtained resources and materials. A Teacher Handbook is also available for teachers and has sample lessons on each topic for each grade level. The Teacher Handbook is a document that will grow as teachers add exemplary lessons aligned with the new Core.

Useful and Relevant

This curriculum relates directly to student needs and interests. Relevance of mathematics to other endeavors enables students to transfer skills gained from mathematics instruction into their other school subjects and into their lives outside the classroom.

Reliant Upon Effective Assessment Practices

Student achievement of the standards and objectives in this Core is best assessed using a variety of assessment instruments. Performance tests are particularly appropriate to evaluate student mastery of mathematical processes and problem-solving skills. Teachers should use a variety of classroom assessment approaches in conjunction with standard assessment instruments to inform instruction. Sample test items, keyed to each Core Standard, may be located on the "Utah Mathematics Home Page" at http://www.usoe.k12.ut.us/curr/math. Observation of students engaged in instructional activities is highly recommended as a way to assess students' skills as well as attitudes toward learning. The nature of the questions posed by students provides important evidence of their understanding of mathematics.

Engaging

In the early grades, children are forming attitudes and habits for learning. It is important that instruction maximizes students' potential and gives them understanding of the intertwined nature of learning. Effective elementary mathematics instruction engages students actively in enjoyable learning experiences. Instruction should be as thrilling an experience for a child as seeing a rainbow, growing a flower, or describing a toad. In a world of rapidly expanding knowledge and technology, all students must gain the skills they will need to understand and function responsibly and successfully in the world. The Core provides skills in a context that enables students to experience the joy of learning.

Intended Learning Outcomes for Fifth Grade Mathematics

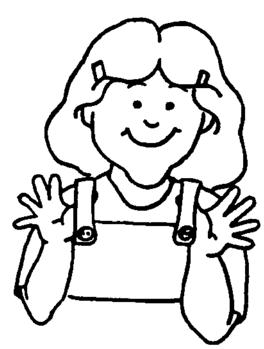
The main intent of mathematics instruction is for students to value and use mathematics and reasoning skills to investigate and understand the world.

The Intended Learning Outcomes (ILOs) describe the skills and attitudes students should learn as a result of mathematics instruction. They are an essential part of the Mathematics Core Curriculum and provide teachers with a standard for evaluation of student learning in mathematics. Significant mathematics understanding occurs when teachers incorporate ILOs in planning mathematics instruction.

By the end of fifth grade students will be able to:

1. **Demonstrate a positive learning attitude toward mathematics.**
   a. Display a sense of curiosity about numbers and patterns.
   b. Pose mathematical questions about objects, events, and processes.
   c. Demonstrate persistence in completing tasks.
   d. Apply prior knowledge and processes to construct new knowledge.
   e. Maintain an open and questioning mind toward new ideas and alternative points of view.
2. **Become mathematical problem solvers.**
   a. Determine the approach, materials, and strategies to be used in setting up a problem.
   b. Model problem situations in a variety of ways.
   c. Develop understanding of new mathematical concepts and vocabulary by answering questions such as: What made you think that? Did anyone think of this in a different way? Where have we seen a problem like this before?
   d. Construct and use concrete, pictorial, symbolic, and graphical models to represent problem situations.
   e. Know when to select and how to use grade-appropriate mathematical tools and methods as a natural and routine part of the problem-solving process.
   f. Build new mathematical knowledge through problem solving.
   g. Solve problems in both mathematical and everyday contexts.
   h. Recognize that there may be multiple ways to solve a problem.

- **Demonstrate a positive learning attitude toward mathematics**
- **Become mathematical problem solvers**
- **Reason mathematically**
- **Communicate mathematically**
- **Make mathematical connections**
- **Represent mathematical situations**



   i. Persevere in developing alternative problem-solving strategies if initially selected approaches do not work.

3. **Reason mathematically.**
   a. Draw logical conclusions and make generalizations.
   b. Determine the approach, materials, and strategies to be used in solving problems.
   c. Use models, known facts, and relationships to explain reasoning.
   d. Make precise calculations and check the validity of the results in the context of the problem.
   e. Make conjectures based on observation and information and test mathematical conjectures and arguments.
   f. Follow and construct logical arguments and judge their validity.
   g. Analyze mathematical situations by recognizing and using patterns and relationships.
   h. Justify answers and solution processes.
4. **Communicate mathematically.**
   a. Represent mathematical ideas with objects, pictures, and symbols.
   b. Express mathematical ideas to peers, teachers, and others through oral and written language.
   c. Engage in mathematical discussions through brainstorming, asking questions, and sharing strategies for solving problems.
   d. Explain mathematical work and justify reasoning and conclusions.
   e. Analyze, evaluate, and explain mathematical arguments and conclusions presented by others.
5. **Make mathematical connections.**
   a. Use one mathematical idea to extend understanding of another.
   b. Recognize the role of mathematics in the classroom, school, and community.
   c. Explore problems and describe and confirm results using various representations.
   d. Recognize the connections between mathematics and other content areas and apply mathematical thinking and problem solving in those areas.

6. Represent mathematical situations.

a. Create and use representations to organize and communicate mathematical ideas.

b. Represent mathematical concepts using concrete, pictorial, and symbolic models.

- **Nurturing Every Child**
- **Developmental Teaching**
- **Celebrating Each Child's Growth and Contributions**

Fifth Grade Math Standards

Standard I: Students will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.

Standard I: **Students will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.**

Objective 1: Represent whole numbers and decimals in a variety of ways.

a. Model, read, and write numerals from hundredths to one millions.

b. Write a *whole number* up to 999,999 in *expanded form* (e.g., 876,539 = 8 hundred-thousands, 7 ten-thousands, 6 thousands, 5 hundreds, 3 tens, 9 ones or 8x100,000 + 7x10,000 + 6x1,000 + 5x100 + 3x10 + 9).

c. Demonstrate multiple ways to represent whole numbers by using models and symbolic representations (e.g., 108=2x50+8; 108=102 + 8).

d. Classify whole numbers from 2 to 20 as prime or composite and 0 and 1 as neither prime nor composite, using models.

e. Represent repeated factors using exponents up to three (e.g., 8=2x2x2=23).

Objective 2: Identify relationships among whole numbers, fractions, decimals, and percents.

a. Order and compare *whole numbers*, fractions (including mixed numbers), and decimals using a variety of methods and symbols.

b. Rewrite mixed numbers and improper fractions from one form to the other.

c. Find the least common denominator for two fractions.

d. Represent commonly used fractions as decimals and percents in various ways (e.g., objects, pictures, calculators).

Objective 3: Model and illustrate meanings of operations and describe how they relate.

a. Identify the *dividend*, *divisor*, and *quotient* regardless of the division symbol used.

b. Determine whether a whole number is divisible by 2, 3, 5, 9, and/or 10, using the *rules of divisibility*.

c. Represent remainders as *whole numbers*, decimals, or fractions and describe the meaning of remainders as they apply to problems from the students' environment (e.g., If there are 53 people, how many vans are needed if each van holds 8 people?).

d. Model addition, subtraction, and multiplication of fractions and decimals in a variety of ways (e.g., using objects and a number line).

e. Model strategies for whole number multiplication (e.g., partial product, lattice) or division (e.g., partial quotient).

f. Select or write the number sentences that can be used to solve a two-step problem.

g. Describe the effect of place value when multiplying and dividing whole numbers and decimals by 10, 100, and 1,000.

Objective 4: Use fractions to communicate parts of the whole.

a. Divide regions, sets of objects, and line segments into equal parts using a variety of models and illustrations.

b. Name and write a fraction to represent a portion of a unit whole for halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths.

c. Represent the simplest form of a fraction in various ways (e.g., objects, pictorial representations, symbols).

d. Represent mixed numbers and improper fractions in various ways (e.g., rulers, objects, number lines, symbols).

e. Rename *whole numbers* as fractions with different denominators (e.g., 5=5/1, 3=6/2, 1=7/7).

f. Model and calculate equivalent forms of a fraction and describe the process used.

Objective 5: Solve problems using the four operations with whole numbers, decimals, and fractions.

a. Determine when it is appropriate to use estimation, mental math strategies, paper and pencil, or a calculator.

b. Use estimation strategies to determine whether results obtained using a calculator are reasonable.

c. Multiply up to a three-digit *whole number* by a one- or two-digit whole number.

d. Divide up to a three-digit *dividend* by a one-digit divisor.

e. Add and subtract decimals with digits to the hundredths place (e.g., 35.42+7.2; 75.2–13.45).

f. Add, subtract, and multiply fractions.

g. Simplify *expressions*, without *exponents*, using the *order of operations*.

Objective 6: Model and illustrate integers.

a. Identify, read, and locate *integers* on a number line.

b. Describe situations where integers are used in the students' environment.

Standard II: Students will use patterns and relations to represent and analyze mathematical situations using algebraic symbols.

Objective 1: Recognize, analyze, and use patterns and describe their attributes.

a. Analyze and make predictions about patterns involving *whole numbers*, decimals, and fractions using a variety of tools including organized lists, tables, objects, and variables.

b. Extend patterns and describe a rule for predicting the next element.

Objective 2: Represent, solve, and analyze mathematical situations using algebraic symbols.

a. Recognize a variety of symbols for multiplication and division including x, i, and * as symbols for multiplication and ∏, e, and a fraction bar (/ or -) as division symbols.

b. Recognize that a variable (Ø, n, x) represents an unknown quantity.

c. Solve one-step equations involving *whole numbers* and a single variable (e.g., n∏7=3).

d. Recognize that the answer to a multiplication problem involving a factor of zero is equal to zero (e.g., 0x45=0).

e. Use expressions or one-step equations to represent real-world situations.

f. Use the *associative*, *commutative*, and *distributive properties* to compute with whole numbers.

Standard II:
Students will use patterns and relations to represent and analyze mathematical situations using algebraic symbols.

Standard III: Students will use spatial reasoning to recognize, describe, and identify geometric shapes and principles.

Objective 1: Describe, identify, and analyze characteristics and properties of geometric shapes.

a. Identify and draw *perpendicular lines*.

b. Draw, label, and describe rays and describe an angle as two rays sharing a common endpoint.

c. Label an angle as *acute*, *obtuse*, *right*, or *straight*.

d. Identify and describe *equilateral*, *isosceles*, *scalene*, *right*, *acute*, and *obtuse* triangles.

e. Identify the *vertex* of an angle or the *vertices* of a polygon.

f. Compare *corresponding angles* of two triangles and determine whether the triangles are *similar*.

g. Identify and describe *pyramids* and *prisms*.

Objective 2: Specify locations and describe spatial relationships using coordinate geometry.

a. Locate points defined by ordered pairs in the first *quadrant*.

b. Write an ordered pair for a point in the first quadrant.

c. Specify possible paths between locations on a *coordinate grid* and compare distances of the various paths.

Objective 3: Visualize and identify geometric shapes after applying transformations.

a. Identify a *slide* (*translation*) or a *flip* (*reflection*) of a shape across a line.

b. Demonstrate the effect of a *turn* (*rotation*) on a figure using manipulatives.

c. Relate *pyramids* and *prisms* to the *two-dimensional* shapes (nets) from which they were created.

Standard III:
Students will use spatial reasoning to recognize, describe, and identify geometric shapes and principles.

Standard IV: Students will understand and apply measurement tools and techniques.

Objective 1: Identify and describe measurable attributes of objects and units of measurement.

a. Describe the relationship among *metric* units of length (i.e., millimeter, centimeter, meter, kilometer).
b. Describe the relationship among *customary* units of weight (i.e., ounce, pound).
c. Identify the correct units of measurement for *volume*, *area*, and *perimeter* in both metric and customary systems.
d. Estimate length, volume, weight, and area using metric and customary units.
e. Convert units of measurement within the metric system and convert units of measurement within the customary system.

Objective 2: Determine measurements using appropriate tools and formulas.

a. Measure length to the nearest 1/8 of an inch and to the nearest centimeter.
b. Measure *volume* and weight using *metric* and *customary* units.
c. Measure angles using a protractor.
d. Calculate *elapsed time* within a.m. or p.m. time periods.
e. Read and record the temperature to the nearest degree (above and below zero) when using a thermometer with a Celsius or Fahrenheit scale.
f. Calculate the *perimeter* of rectangles and triangles.
g. Calculate the *area* of squares and rectangles using a formula.

Standard IV:
Students will understand and apply measurement tools and techniques.

Standard V:
Students will collect, analyze, and draw conclusions from data and apply basic concepts of probability.

Standard V: Students will collect, analyze, and draw conclusions from data and apply basic concepts of probability.

Objective 1: Formulate and answer questions using statistical methods to compare data.

a. Formulate a question that can be answered by collecting data.

b. Collect, compare, and display data using an appropriate format (i.e., line plots, bar graphs, pictographs, circle graphs, line graphs).

c. Identify minimum and maximum values for a set of data.

d. Identify or calculate the mean, mode, and range.

e. Propose and justify inferences based on data.

Objective 2: Apply basic concepts of probability.

a. Describe the results of investigations involving random outcomes using a variety of notations (e.g., 4 out of 9, 4/9, 4:9).

b. Recognize that outcomes of experiments and samples are fractions between 0 and 1.

c. Predict the probability of an outcome in a simple experiment.

Starter Activities

Starter Activity—Bridging the Gap

Standard II
Students will use patterns and relations to represent and analyze mathematical situations using algebraic symbols.

Objective 2
Represent, solve, and analyze mathematical situations using algebraic symbols.

Intended Learning Outcomes
2. Become mathematical problem solvers.
4. Communicate mathematically.
6. Represent mathematical situations.

Standard II

Objective 2

Connections

Background Information

Students should have some skills in recording numerical relations as simple equations.

Invitation to Learn

Materials and instructions are distributed to the tables before participants arrive. Upon arriving, participants will immediately begin working on task.

Instructional Procedures

1. Typed out on instruction card (one for each participant):
2. How many of you have ever wanted to be part of a construction company that is responsible for building houses? Well today is your lucky day! You are going to be responsible for building houses on three islands. However, you are fortunate to have bridges that connect the islands to one another.
3. On your blackline master you will notice the architect has written on each bridge, that connects the two islands, the total number of houses on the two islands. The architect has also indicated the total number of houses to be built on the three islands. Your job is to figure out how many houses go on each island. You many use the cm cubes on your table to represent the houses.

Materials

- ❏ Instruction card (one for each participant)
- ❏ Blackline masters "Bridging the Gap" (one for each participant)
- ❏ cm cubes

Curriculum Integration

Math/Science—The skills developed in the activity will facilitate the modeling of patterns and functions and the drawing of conclusions about them.

Possible Extensions/Adaptations

Use four islands to increase the level of problem solving.

Assessment Suggestion

Observe the strategies the students use to solve the problems and the methods used to record the solutions. Assess the extent to which the students use symbols or variables to represent each problem.

Additional Resources

Navigating through Algebra from NCTM
Groundworks Algebra Puzzles and Problems, Creative Publications

Homework & Family Connections

As a homework assignment, send a problem home to be completed by the family.

Bridging the Gap

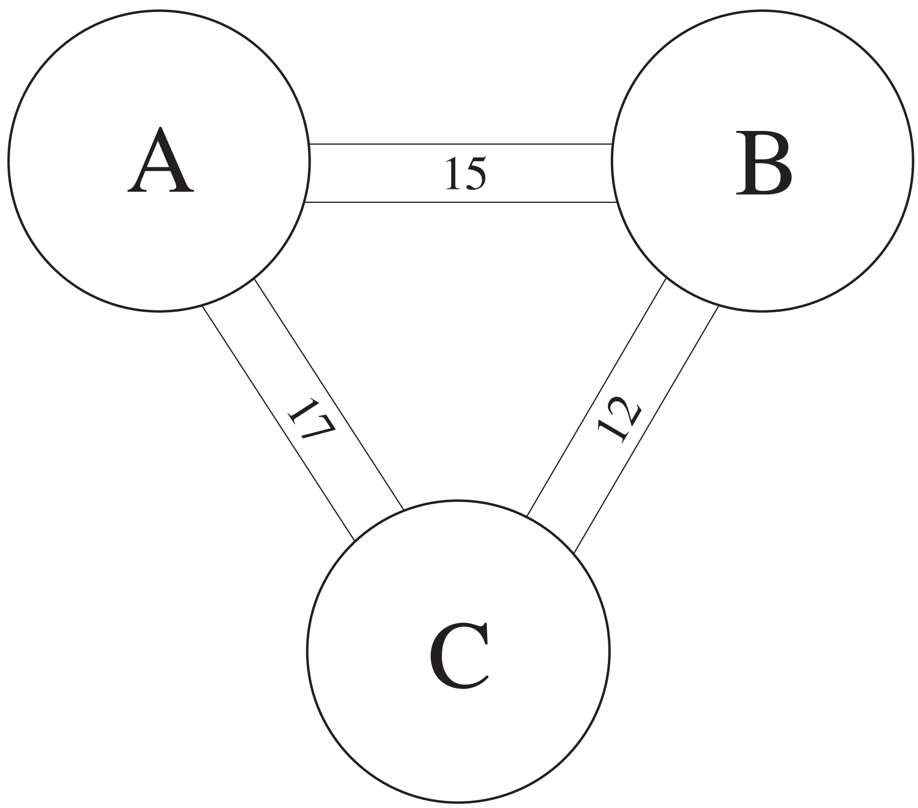
Instructions

How many of you have ever wanted to be part of a construction company that is responsible for building houses? Well today is your lucky day! You are going to be responsible for building houses on three islands. However, you are fortunate to have bridges that connect the islands to one another.

On your blackline master you will notice the architect has written on each bridge, that connects the two islands, the total number of houses on the two islands. The architect has also indicated the total number of houses to be built on the three islands. Your job is to figure out how many houses go on each island. You may use the cm cubes on your table to represent the houses.

Bridging the Gap

Exercise #1



Total Number of Houses = 22

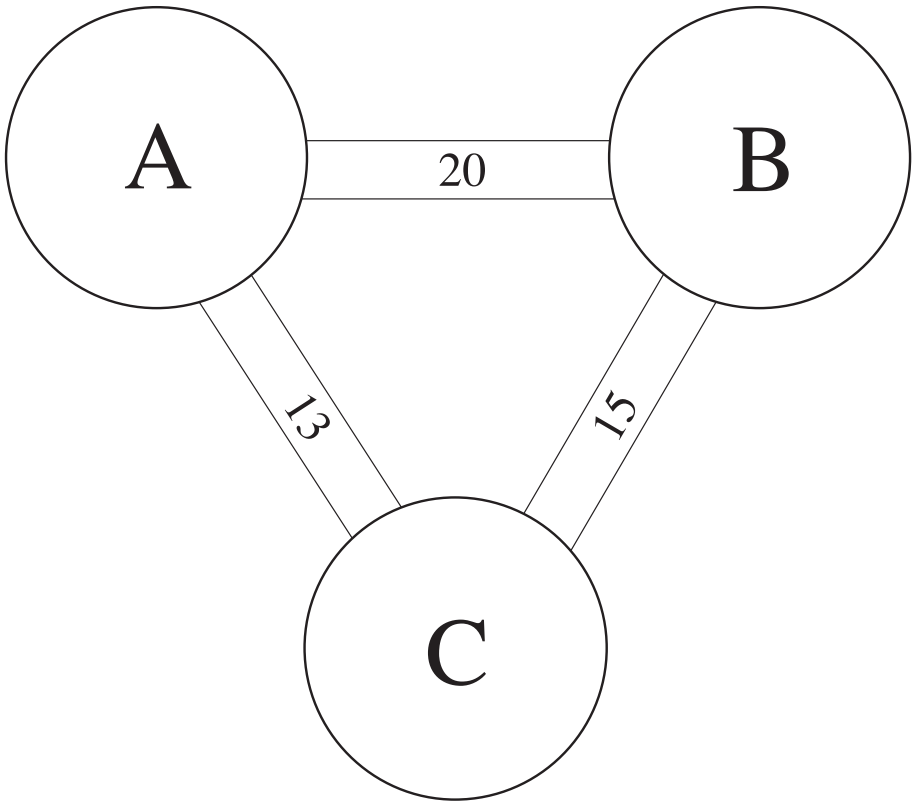
A = ___________

B = ___________

C = ___________

Bridging the Gap

Exercise
#2



Total Number of Houses = 24

A = ___________

B = ___________

C = ___________

Starter Activity—Eyewitness

Standard
II

Objective
2

Connections

Standard II
Students will use patterns and relations to represent and analyze mathematical situations using algebraic symbols.

Objective 2
Represent, solve, and analyze mathematical situations using algebraic symbols.

Intended Learning Outcome
2. Become mathematical problem solvers.

Background Information

Students should have some skills recording information with variables for the unknown.

Invitation to Learn

Materials and instructions are distributed to the tables before participants arrive. Upon arriving, participants will immediately begin working on task.

Instructional Procedures

Typed out on instruction card:

Have you ever been the eyewitness for a crime scene? When a crime is committed, what is the role of the eyewitness and the detective? Today you will be working with a partner. One of you will be the eyewitness and the other will be the detective.

1. The detective gives the eyewitness one minute to read the story problem. When time is up, the eyewitness turns the story problem over so he or she can't see it.
2. The detective then asks, "What's this about?"
3. The eyewitness then tells the basic story.
4. The detective then asks the eyewitness to give all the information he or she remembers. The detective asks questions to "learn the story and get the facts."
5. After relating all that can be remembered, the eyewitness is given one more opportunity to have an "instant replay" and read the problem one more time. Again, the detective times the eyewitness for one minute, then turns the problem over again.
6. The detective then asks for any missed details.

Materials

One for each participant

- ❏ Instruction card
- ❏ Two story problems

7. Following the eyewitness interrogation, the detective writes an equation to represent the case with the help of the witness.
8. If time permits, switch roles and use the second story problem.

Remember: No matter what type of story problem you are solving, you must be a "detective."

Curriculum Integration

Math/Science—Across the curriculum, students need to stop periodically when they are reading to summarize, clarify, predict and question.

Possible Extensions/Adaptations

When students are working on story problems, it is very useful for them to have a highlighter to "highlight" the important details needed to solve the problem. Repeated analysis of story problems enables students to differentiate relevant from irrelevant information.

Assessment Suggestion

Circulate around the room listening to how well the eyewitness can summarize the given information. Also, noting the ability of the detective to record the information as an algebraic or numerical equation.

As a quiz, have all students become the eyewitness and individually record their information as an equation to successfully solve the problems.

Additional Resources

Math Detective books by Terri Husted
Classroom math textbooks
Have students write story problems to be solved by classmates
www.edhelper.com

Homework & Family Connections

When students are reading at home, it's good for them to stop throughout the text and summarize the information that has been covered.

Eyewitness

Instructions

Have you ever been the eyewitness for a crime scene? When a crime is committed, what is the role of the eyewitness and the detective? Today you will be working with a partner. One of you will be the eyewitness and the other will be the detective.

1. The detective gives the eyewitness one minute to read the story problem. When time is up, the eyewitness turns the story problem over so he or she can't see it.
2. The detective then asks, "What's this about?"
3. The eyewitness then tells the basic story.
4. The detective then asks the eyewitness to give all the information he or she remembers. The detective asks questions to "learn the story and get the facts."
5. After relating all that can be remembered, the eyewitness is given one more opportunity to have an "instant replay" and read the problem one more time. Again, the detective times the eyewitness for one minute, then turns the problem over again.
6. The detective then asks for any missed details.
7. Following the eyewitness interrogation, the detective writes an equation to represent the case with the help of the witness.
8. If time permits, switch roles and use the second story problem.

Remember: No matter what type of story problem you are solving, you must be a "detective."

Story Problem #1 for Eyewitness:

Jazz Payroll

At approximately 1:00 a.m. on March 25, 2003, the Jazz payroll office was broken into and ransacked. After an investigation all payroll checks were located except the check for John Stockton. The clerk knows that Karl Malone earns 20% more money per game than John Stockton. The clerk computed that Malone earned $120,000 per game. How much money does John Stockton earn per game?

Story Problem #2 for Eyewitness:

Stick-to-it

Johnny bought a box of soccer stickers to use on his project report on soccer. There were two thousand, one hundred sixty stickers in the box, but he could not use some of them. One-ninth of them were stuck so tightly together that he could not detach them from each other. Eighty-three of them were blank. One-fifth of them had no glue. How many were left that he could actually use?

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ANSWER: **John Stockton earns $100,000/game**

Story Problem #2 for Eyewitness:

Stick-to-it

Johnny bought a box of soccer stickers to use on his project report on soccer. There were two thousand, one hundred sixty stickers in the box, but he could not use some of them. One-ninth of them were stuck so tightly together that he could not detach them from each other. Eighty-three of them were blank. One-fifth of them had no glue. How many were left that he could actually use?

ANSWER: **1,405 soccer stickers could actually be used**

Starter Activity—All in a Name

Standard V
Students will use concepts of probability and collect, analyze, and draw conclusions from data.

Objective 1
(5th) Formulate and answer questions using statistical methods to analyze data.
(6th) Design investigations to reach conclusions using statistical methods to analyze data.

Intended Learning Outcomes
3. Reason mathematically.
6. Represent mathematical situations.

Standard V

Objective 1

Connections

Background Information

Students should be familiar with the terms, "mean," "median," "range," and "mode." A quick review before beginning the activity may be needed.

Invitation to Learn

Materials and instructions are distributed to the tables before participants arrive. Upon arriving, participants will immediately begin working on task.

Instructional Procedures

Typed out on instruction card:

We all know how important our name is. It identifies who we are. It is so important that today we are going to collect data using our names.

1. Quickly walk around the room and record name data for ten participants including yourself. This will give you a chance to meet others in the room and introduce yourself. Have each participant record his or her first and last name in the appropriate columns. Returning to your seat, record the total number of letters for each name in the last column.
2. Using the data collected in your third column, make a quick graph (bar, line, etc.). Describe your data to another participant, in terms of clumps or bunches, gaps or holes, and bumps.
3. Using the same data collected in your third column, find the: mean, range, mode, and median. Compare these findings to your graph. Do you see any patterns?

Materials

One for each participant:

- ❏ Instruction card
- ❏ Blackline master "It's all in a Name"
- ❏ Blackline master of graph paper

4. Discuss with another participant:
   a. Why/When is it useful to find the mean, range, mode, and median of data?
   b. If more names are added to your data set, predict how your measures and graph will change. Explain why.

Curriculum Integration

Math/Science—Students can determine the mean, range, median, and mode of data collected from a science experiment. Analysis of the measures will then give the students valuable information to make informative conclusions.

Possible Extensions/Adaptations

A meaningful activity to use at the beginning of the school year to learn the names of your students. Also, after students have successfully analyzed the data from their group, then collect and compare the data using the whole class.

Assessment Suggestion

Observe students while they are working together to organize collected data. Quiz the class on given data. See if the student is able to organize the data to determine the mean, range, median, and mode.

Additional Resource

Connect to *NCTM Standards 2000* (Creative Publications)

Homework & Family Connections

Collect data from your family (names, age, height, etc.), and organize the date and determine the mean, range, median, and mode.

All in a Name

Instructions

We all know how important our name is. It identifies who we are. It is so important that today we are going to collect data using our names.

1. Quickly walk around the room and record name data for ten participants including yourself. This will give you a chance to meet others in the room and introduce yourself. Have each participant record his or her first and last name in the appropriate columns. Returning to your seat, record the total number of letters for each name in the last column.
2. Using the data collected in your third column, make a quick graph (bar, line, etc.). Describe your data to another participant, in terms of clumps or bunches, gaps or holes, and bumps.
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4. Discuss with another participant:
   a. Why/When is it useful to find the mean, range, mode, and median of data?
   b. If more names are added to your data set, predict how your measures and graph will change. Explain why.

Name______________________________

All in a Name

Recording Sheet

First Name	Last Name	Total # of Letters

MEAN:	
RANGE:	
MODE:	
MEDIAN:	

Mean: the sum of all the numbers divided by the number of items you're adding.

Range: the difference between the highest and lowest numbers.

Mode: the number that appears most often (there isn't always a mode, but sometimes there's more than one!).

Median: the middle number.

Name______________________________

All in a Name

Grid

Starter Activity—Roped Into Quadrilaterals

Standard III

Objective 1

Standard III
Students will use spatial reasoning to recognize and describe geometric shapes and principles.
Objective 1
Identify and analyze characteristics and properties of geometric shapes.
Intended Learning Outcomes
3. Reason mathematically.
4. Communicate mathematically.

Connections

Background Information

Students should be familiar with the common attributes of quadrilaterals such as parallelograms, trapezoids, rectangles, rhombi, and squares. If the students are not familiar with Venn diagrams, explain that the quadrilaterals that possess the characteristics for both circles should be placed in the intersection of the two rings. For example, if one circle is labeled “right angles” and the other “congruent sides,” a square would be placed in the intersection of the two rings.

Invitation to Learn

Materials and instructions are distributed to the tables before participants arrive. Upon arriving, participants will immediately begin working on task.

Instructional Procedures

Typed out on instruction card (one for each table):

1. Assign members of your group (table) to do the following:
   a. Cut out the quadrilateral pieces.
   b. Tie the ends of **each** piece of yarn to make three circles.
   c. Cut out the Task Activity Quadrilateral Labels.
2. The object of the activity is to place the quadrilateral pieces appropriately in your circles (yarn) according to the labels. You may need to overlap the circles to form intersections. The number of labels determines the number of circles used.
3. Participants progress through the labels, placing quadrilateral pieces according to common attributes.

Materials

One set for each table:

- ❏ 3 pieces of yarn (18 in.)
- ❏ Blackline master “Quadrilaterals”
- ❏ Blackline master “Quadrilateral Labels”
- ❏ scissors

Curriculum Integration

Math/Science—Give the students an opportunity to write and defend their reasoning.

Possible Extensions/Adaptations

Have students make up their own labels and then challenge a partner to use them to create quadrilateral circles. Also, students can make their own "Mystery Circles" for their classmates to label.

Assessment Suggestion

During the activity, circulate among the groups and ask the students to defend their placement of different pieces.

Give each student a "Mystery Circle" hand-out to label. In their writing journals, have them explain and defend their reasoning for their labels.

Additional Resource

Activity adapted from NCTM "Navigating through Geometry in Grades 3-5"

Homework & Family Connections

Identify, label, and group quadrilaterals found "hidden" throughout your house and in magazines. Make a collage of your findings.

ROPED INTO QUADRILATERALS

Instructions

1. Assign members of your group (table) to do the following:
   a. Cut out the quadrilateral pieces.
   b. Tie the ends of **each** piece of yarn to make three circles.
   c. Cut out the Task Activity Quadrilateral Labels.

2. The object of the activity is to place the quadrilateral pieces appropriately in your circles (yarn) according to the labels. You may need to overlap the circles to form intersections. The number of labels determines the number of circles used.

3. Progress through the labels, placing quadrilateral pieces according to common attributes.

Quadrilateral Labels

Task Activity #1:

At least one right angle	No right angles

Task Activity #2:

No congruent sides	Congruent sides

Task Activity #3:

At least one obtuse angle	At least one acute angle

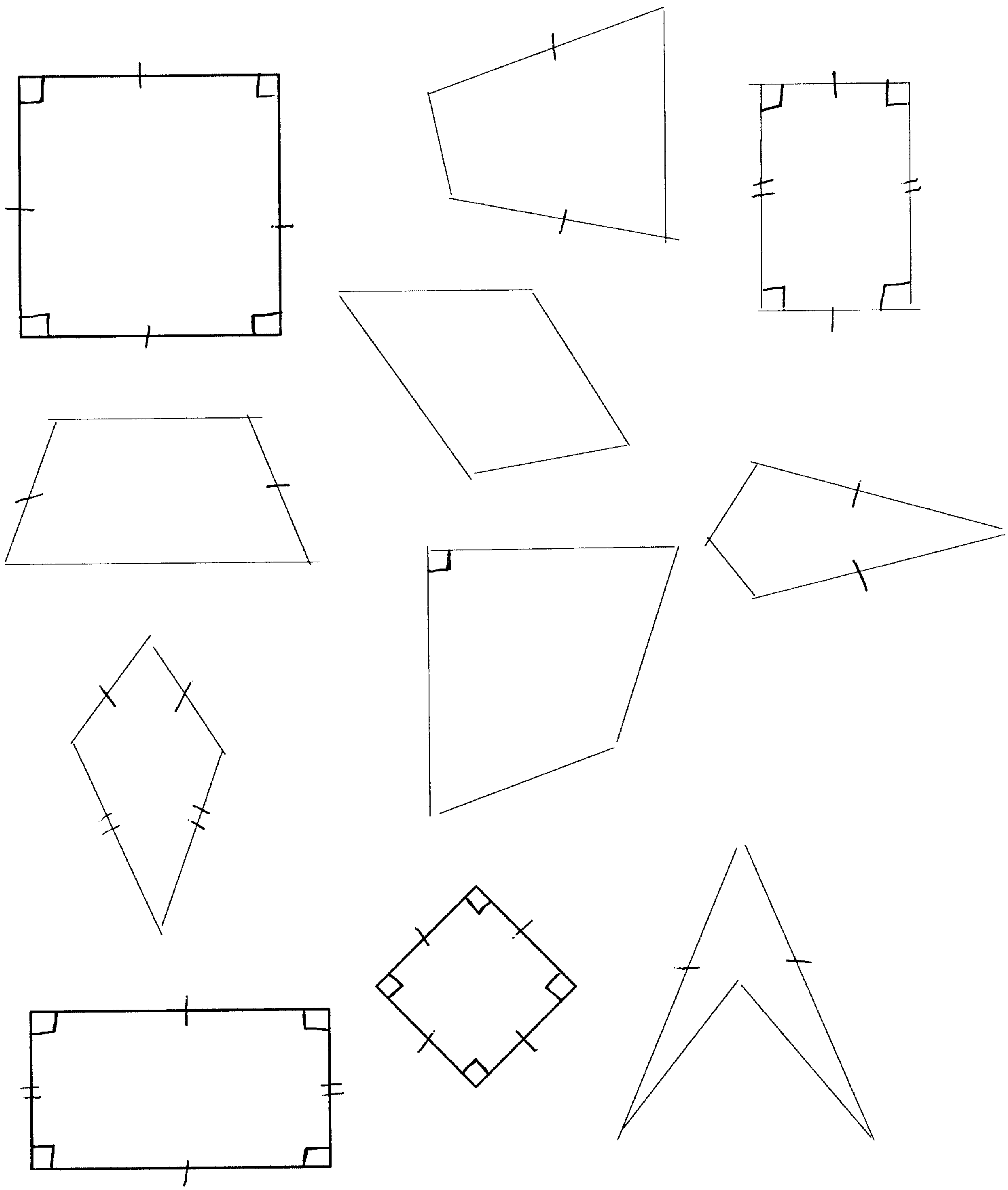
Task Activity #4:

Rectangles	Squares	Rhombi

Task Activity #5:

At least one acute angle	At least one pair of parallel sides

Quadrilaterals



Starter Activity—The "Right" Place

Standard I
Student will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.

Objective 5
Solve problems using the four operations with whole numbers, decimals, and fractions.

Intended Learning Outcomes
2. Become mathematical problem solvers.
3. Reason mathematically.

Standard I

Objective 5

Connections

Background Information

Reinforce there is not always just one right answer. Multiple answers are accepted in many problem solving activities. The important thing is for students to "stick with" it long enough to have success.

Invitation to Learn

Materials and instructions are distributed to the tables before participants arrive. Upon arriving, participants will immediately begin working on task.

Materials

One for each participant:

- ❏ Blackline master of The Right Place
- ❏ Scissors

Instructional Procedures

Follow directions on the Blackline master The Right Place

Curriculum Integration

Math/Science—Great way to reinforce the importance of problem solving throughout the curriculum. Multiple processes and ways to analyze, interpret and present information.

Possible Extensions/Adaptations/Integration

You may allow a number to be used more than once. The use of cards or number tiles may be used instead of writing the numbers.

Assessment Suggestion

Observations while students are working on problems. Observing their ability to reason through the process and communicate with one another about their thinking, and the student's ability to "stick with" the problem and come up with multiple solutions.

Additional Resources

Problem Solving with Numbers (Grades 3-6), Exclusive Educational Products

Arithme Twists, Creative Publications

Homework & Family Connections

Great way for parents to help their children with basic facts, instead of “drill and kill.”

The "Right" Place

Use the 1-6 to complete the equations. Each number can only be used once in an equation.

$$\square \times \square = \square + \square + \square$$

$$(\square - \square) \times \square = \square$$

$$\square \times \square \times \square (\square - \square) = \square\,\square$$

Cut apart:

1	2	3	4	5	6	7	8	9	0

Place the numbers 0-8 to complete the equation.

Hint: The three-digit number is >500.

$$\begin{array}{r} \square\;\square\;\square \\ \times \qquad\quad \square \\ \hline \square\;\square\;\square\;\square \end{array}$$

Randomly select 4 of your numbers. Place them in the equation below so that:

1. the product will be a 3-digit number
2. the product is between 750 and 2,500
3. it is an even product
4. the answer is close to 6,000

$$\begin{array}{r} \square\;\square \\ \times\;\square\;\square \\ \hline \end{array}$$

Starter Ideas

Inquiry

Helping students work together to make sense of mathematics:

"What do others think about what ________ said?"

"Can you explain it in your own words?"

"Do you agree? Disagree?"

"What is alike and what is different about your method of solution and hers?"

"Does anyone have the same answer but a different way to explain it?"

"Would you ask the rest of the class that question?"

"Do you understand what they are saying?"

"Can you convince the rest of us that that makes sense?"

Helping students to rely more on themselves to determine whether something is mathematically correct:

"Why do you think that?"

"Why is that true?"

"How did you reach that conclusion?"

"Does that make sense?"

"Can you make a model to show that?"

Helping students learn to reason mathematically:

"Does that always work?"

"Is that true for all cases?"

"Can you predict the next one in the pattern?"

"What about the tenth one in the pattern?

"Can you give me an example?"

"Can you think of a counterexample?"

"How did you reach that conclusion?

"How could you prove that?"

Helping students to connect mathematics, its ideas, and its applications:

"How does this relate to…?"

"What ideas that we have learned before were useful in solving this problem?"

"Have we ever solved a problem like this one before?"

"What uses of mathematics did you find in the newspaper last night?"

"Can you give me an example of…?

Inquiry Activity—Using Questioning Strategies

Standard
Multiple
Objective
Develop understanding of new mathematical concepts and vocabulary by answering questions.
Intended Learning Outcomes
2. Become mathematical problem solvers.

Standard

Objective

Connections

Background Information

One of the most important aspects of teaching is monitoring student understanding so that it is possible to provide feedback to students, allowing them to correct and refine their understanding of what is being taught. Mathematics teachers are beginning to use a wide variety of techniques to accomplish this, from multiple-choice tests assessing skills to rubrics assessing the quality of problem solving and mathematical reasoning. However, too often we forget one of the most useful and efficient ways to assess student learning: asking students probing questions that challenge and engage their thinking. A powerful learning experience can often fail to reach its desired goals because the students were not given the opportunity to explore and refine their understanding. When employed by a skilled teacher, questions will encourage students to work together to understand mathematics, to rely on themselves to determine if their thinking is correct, to conjecture, invent, and solve problems, and to see how mathematics connects to the real world and other branches of mathematics.

NCTM describes the shift towards better assessment practice in this way:

> A shift toward judging the progress of each student's attainment of mathematical power, and away from assessing students' knowledge of specific facts and isolated skills.
>
> A shift toward communicating with students about their performance in a continuous, comprehensive manner, and away from simply indicating whether or not answers are correct.
>
> A shift toward students learning to assess their own progress, and away from teachers and external agencies as the sole judges of progress. (NCTM Assessment)

To make this shift occur in classrooms teachers need to carefully plan questions to engage student's thinking, listen to students to determine if there are any misconceptions in their thinking, ask students to clarify and justify their ideas orally, and monitor when to clarify concepts.

Introduce the two dice tossing games: the sum game and the product game:

Sum Game	***Product Game***
Two Players:	**Two Players:**
1. Choose one player to be "even" and the other to be "odd."	1. Choose one player to be "even" and the other to be "odd."
2. Throw two dice.	2. Throw two dice.
3. Add the numbers on the two faces.	3. Multiply the numbers on the two faces.
4. If the sum is even, the even player gets 1 point.	4. If the sum is even, the even player gets 1 point.
5. If the sum is odd, the odd player gets 1 point.	5. If the sum is odd, the odd player gets 1 point.

Ask the participants if the games are fair? Lead a discussion in which good questioning practices are used. Ask the kinds of questions that will lead to deep understanding of what it means for a probability situation to be fair, and what happens when we add and multiply even and odd numbers.

Brainstorm reasons why teachers would ask such questions of their students. Write these on chart paper.

Instructional Procedures

1. Examine categories looking specifically for the following:
   - **Helping students work together to make sense of mathematics**
   - **Helping students to rely more on themselves to determine whether something is mathematically correct**
   - **Helping students learn to reason mathematically**
   - **Helping students to connect mathematics, its ideas, and its applications**
2. Participants work as partners to organize the questions that are provided into these four categories.
3. View the video "A Thinking Approach to Computation," Creative Publications, Part 2. This is an excellent example of the type of questioning practice described in the NCTM Professional Standards.

Materials

- ❏ *A Thinking Approach to Computation* Video by Creative Publications
- ❏ Questions and question categories already cut and placed in envelopes. One envelope for every 2 par
- ❏ A pair of dice for every 2 participants
- ❏ A transparency of the rules for the Sum/Product Game

Curriculum Integration

This same questioning strategy can be used with any content area.

Sum Game

Two Players:

1. Choose one player to be "even" and the other to be "odd."
2. Throw two dice.
3. Add the numbers on the two faces.
4. If the sum is even, the even player gets 1 point.
5. If the sum is odd, the odd player gets 1 point.

Product Game

Two Players:

1. Choose one player to be "even" and the other to be "odd."
2. Throw two dice.
3. Multiply the numbers on the two faces.
4. If the sum is even, the even player gets 1 point.
5. If the sum is odd, the odd player gets 1 point.

What are characteristics of effective homework in mathematics?

Research and Best Practice

Daily, children hurry home from school and arrive to face the obligatory question from parents, "What did you learn in school today?" They return to school the following day, and their teacher asks, "Do you have your homework assignment?" Perhaps a better question would be "What did you learn at home?" The home should be a place to extend mathematics learning.

- **The home should be a place to extend mathematics learning.**

Student learning in mathematics should always focus on understanding the set of skills and knowledge needed to investigate the world. Homework must emphasize developing students' mathematics skills to solve problems, which will help them understand the world. These mathematics skills are described as "process skills" in the *NCTM Principles and Standards for School Mathematics* and "habits of mind" in *Benchmarks for Science Literacy*. Mathematics educators and mathematicians agree that knowing mathematics is more than being able to recall facts. Research indicates that individuals with expertise in mathematics understand mathematics concepts, how to apply them to challenging, non-routine real-life problem-solving situations, and how to learn from their own problem-solving efforts.

Homework assignments provide the opportunity for students to do long term projects that require multiple levels of understanding. Students take ownership when they spend weeks following stock prices in the newspaper, paying close attention to favorites, predicting industry trends, interviewing traders, or perhaps even participating in an investment club. Watching TV and timing commercial breaks one night may be interesting, but when students keep data over a few weeks — timing commercials in different types of programs, making charts, and drawing graphs — their learning will go beyond the curriculum.

Homework time is an opportunity for students to reflect on learning and synthesize their mathematics understandings. Well-designed homework can bring parents and other adults into a student's community of mathematics learners. Assignments should include students discussing their learning with others. This can be done through student learning teams, parent involvement, or the teacher using e-mail to have discussion groups. Mathematics is in every aspect of life. Teachers should take advantage of the opportunity to provide students with authentic learning opportunities at home.

Classroom Implications

The value placed on various aspects of mathematics learning can be seen in the allocation of instructional time in class and by the nature of homework assigned. Teachers who value problem-solving skills will provide time in class to develop students' ability to solve problems and then will assign homework that uses these skills in new settings. What goes on in class should match the homework assigned.

Mathematics homework should not be schoolwork done at home. The home provides a unique opportunity for students to gain mathematics understanding by solving mathematics problems. Placing the major emphasis on basic skills and drill for skill development in mathematics is somewhat of a waste of student, parent, and teacher time and effort.

Teaching for understanding requires carefully designed tasks. Homework assignments should have clear criteria and/or written rubrics that describe expectations and establish student goals. The teacher must be certain that students have access to the materials and resources they will need to complete the assignment.

It is important for students to do their best, and for teachers to examine student work. Less is often more when it comes to homework. A product that has been refined by the student results in more effective learning than a large volume of work completed with little thought. The quality of student work is often determined by the standards a teacher sets on the assignment, time spent reviewing the expectations, and suggestions for improvements. A homework assignment should be a major event in student learning. Selling students on the importance of an assignment as a learning event is important: their ownership will determine the depth and breadth of their learning.

References

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National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics.

National Science Foundation. (1999). Inquiry thoughts, views, and strategies for the K–5 classroom.

Perkins, D. (1993). Learning for understanding.

To obtain copies of EDThoughts:

What We Know About Mathematics Teaching and Learning, contact McREL at 303-337-0990 (voice), 303-337-3005 (fax), or info@mcrel.org

Problem Solving

Problem solving is an important skill for students (Barba 1991). The abilities to think, reason, create, and find solutions to problems are evidence that a student is on the path of effective learning. Problem solving is the process of reaching a goal when a direct path to that goal is temporarily blocked. Students must recognize that the process is multi-faceted and involves questions about the ways we think as well as the ways we learn.

- **Problem solving is the process of reaching a goal when a direct path to that goal is temporarily blocked.**

Successful problem solvers share many of the same qualities. Students who are successful problem solvers tend to:

- Use prediction during their problem solving and find support for those predictions.
- Eliminate gaps in their knowledge.
- Rely on information learned from a variety of experiences.
- Make fewer mistakes reading or interpreting data or important information.
- Rely on note taking and journals rather than memory.
- Express less doubt and confusion and embody fewer misconceptions.

Students who become skilled at practicing problem solving techniques tend to:

- Grasp a problem quickly and can generalize information rapidly and easily.
- Switch easily from one solution method to another.
- Strive for an "elegant solution" where possible.
- Remember relationships in a problem and principles of solutions.
- Reflect on their own problem solving strategies and are capable of reversing their train of thought.

Students who are successful problem solvers also use facts, rules, skills, and strategies that are called heuristics. "Heuristic" means steps, or a sequences of steps, that are used to organize thinking and solve problems. Some effective heuristics for problem solving strategies are listed below.

- Study the solution process
- Make a figure, graph, drawing, table, chart, or equation
- Check the solution

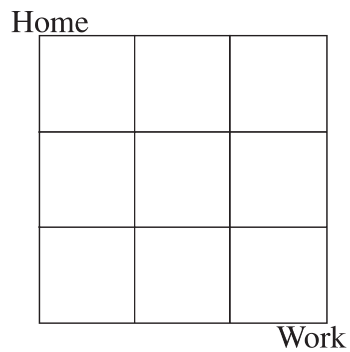
- Identify information
- Solve similar, but simpler probems
- Restate the problem
- Look for patterns
- Work backwards

Problem Solving Activities

1. *"No Repeats!"* (Standard 3, objective 2, indicator c – 5th Grade)

   I will be working every day this summer. Going the same way can get boring! I'd like to try a different 6-block route each day.

   How many days can I go to work without repeating a route?



(There are 20 possible paths.)

2. *"Multiplying and Dividing with Fractions"* (6th Grade – Standard I: objective 3: indicator b, Standard I: objective 5: indicator f)

   Draw a diagram for each word problem. Then determine which, if any, of the four number sentences matches your diagram. You may choose more than one number sentence for each. Explain the connections between the problem, your diagram, and the math.

   A. Maurine has 6 cups of sugar. Each batch of cookies requires 1/2 cup of sugar. How many batches of cookies can she make?

   B. Maurine has 6 batches of cookies. She plans to share them equally with her friend, Janelle. Han many batches will each of them get?

   C. Maurine has to travel 6 miles. When she walks, she covers 1/2 mile in an hour. Haw many hours will it take her?

   D. Maurine has to travel 6 miles. She plans to walk halfway and run halfway. How far will she run?

I. 6÷ 2	***II. 6*** ÷ 1/2	***III. 6 x 2***	***IV.*** *6 x* 1/2

Have participants share their drawings for each problem on the board and explain the connections between their drawings and the number sentences.

Even though this is a 6th grade task, it helps 5th grade teachers understand the meaning of the operations and the relationship between them!

3. *"Funny Stuff"* (5th Grade – Standard II: objective 1: indicator b, 6th Grade – Standard II: objective 1: indicator a)

   Mom went shopping at Funny Stuff. She tried to pay for a video game marked 2 ◈ with $2.00, but the clerk said $2.00 wasn't enough. She was stumped until the clerk handed her this chart.

   How much did the 2 ◈ games cost in dollars?

1 ◈ = $6
3 ◈ = $12
6 ◈ = $21
10 ◈ = $33
17 ◈ = $54

Principles and Standards for School Mathematics

Introduction

- ***build new mathematical knowledge***
- ***solve problems that arise***
- ***apply and adapt***
- ***monitor and reflect***

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- *build new mathematical knowledge* through problem solving
- *solve problems that arise* in mathematics and in other contexts
- *apply and adapt* a variety of appropriate strategies to solve problems
- *monitor and reflect* on the process of mathematical problem solving

Problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge. Through this process, they will often develop new mathematical understandings. Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort, and should then be encouraged to reflect on their thinking.

By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. In everyday life and in the workplace, being a good problem solver can lead to great advantages.

Problem solving is an integral part of all mathematics learning, and so it should not be an isolated part of the mathematics program. Problem solving in mathematics should involve all the five content areas described in these Standards. The contexts of the problems can vary from familiar experiences involving students' lives or the school day to applications involving the sciences or the world of work. Good problems will integrate multiple topics and will involve significant mathematics

Item 1: Build new mathematical knowledge through problem solving

How can problem solving help students learn mathematics? Good problems give students the chance to solidify and extend what they know and, when well chosen, can stimulate mathematics learning. With young children, most mathematical concepts can be introduced through problems that come from their worlds.

For example, suppose second graders wanted to find out whether there are more boys or girls in the four second-grade classes. To solve this problem, they would need to learn how to gather information, record data, and accurately add several numbers at a time.

In the middle grades, the concept of proportion might be introduced through an investigation in which students are given recipes for punch that call for different amounts of water and juice and are asked to determine which is "fruitier." Since no two recipes yield the same amount of juice, this problem is difficult for students who do not have an understanding of proportion. As various ideas are tried, with good questioning and guidance by a teacher, students eventually converge on using proportions. In high school, many areas of the curriculum can be introduced through problems from mathematical or applications contexts.

Problem solving can and should be used to help students develop fluency with specific skills. For example, consider the following problem, which is adapted from the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989, p. 24):

"I have pennies, dimes, and nickels in my pocket. If I take three coins out of my pocket, how much money could I have taken?"

Knowledge is needed to solve this problem—knowledge of the value of pennies, dimes, and nickels and also some understanding of addition. Working on this problem offers good practice in addition skills. But the important mathematical goal of this problem—helping students to think systematically about possibilities and to organize and record their thinking—need not wait until students can add fluently.

The teacher's role in choosing worthwhile problems and mathematical tasks is crucial. By analyzing and adapting a problem, anticipating the mathematical ideas that can be brought out by working on the problem, and anticipating students' questions, teachers can decide if particular problems will help to further their mathematical goals for the class. There are many, many problems that are interesting and fun but that may not lead to the development of the mathematical ideas that are important for a class at a particular time. Choosing problems wisely and using and adapting problems from instructional materials is a difficult part of teaching mathematics.

Item 2: Solve problems that arise in mathematics and in other contexts

People who see the world mathematically are said to have a "mathematical disposition." Good problem solvers naturally tend to analyze situations carefully in mathematical terms and pose problems based on situations they see. They first consider simple cases before trying something more complicated, yet they will readily consider a more sophisticated analysis.

For example, a task for middle-grades students presents data about two ambulance companies and asks which company is more reliable (Balanced Assessment for the Mathematics Curriculum 1999a). A quick answer found by looking at the average time customers had to wait for each company turns out to be misleading. A more careful mathematical analysis involving plotting response times versus time of day reveals a different solution. In this task, a disposition to analyze more deeply leads to a more complete understanding of the situation and a correct solution. Throughout the grades, teachers can help build this disposition by asking questions that help students find the mathematics in their worlds and experiences and by encouraging students to persist with interesting but challenging problems.

Posing problems comes naturally to young children: I wonder how long it would take to count to a million? How many soda cans would it take to fill the school building? Teachers and parents can foster this inclination by helping students make mathematical problems from their worlds. Teachers play an important role in the development of students' dispositions by creating and maintaining classroom environments, from pre-kindergarten on, in which students are encouraged to explore, take risks, share failures and successes, and question one another. In such supportive environments, students develop confidence in their abilities and a willingness to engage in and explore problems, and they will be more likely to pose problems and to persist with challenging problems.

Item 3: Apply and adapt a variety of appropriate strategies to solve problems

Of the many descriptions of strategies, some of the best known can be found in the work of Pólya (1957). Frequently cited strategies include:

- using diagrams
- looking for patterns
- listing all possibilities
- trying special values or cases

- working backward
- guessing and checking
- creating an equivalent problem
- creating a simpler problem

An obvious question is, How should these strategies be taught? Should they receive explicit attention, and how should they be integrated with the mathematics curriculum?

As with any other component of the mathematical tool kit, strategies must receive instructional attention if students are expected to learn them. In the lower grades, teachers can help children express, categorize, and compare their strategies. Opportunities to use strategies must be embedded naturally in the curriculum across the content areas.

By the time students reach the middle grades, they should be skilled at recognizing when various strategies are appropriate to use and should be capable of deciding when and how to use them. By high school, students should have access to a wide range of strategies, be able to decide which one to use, and be able to adapt and invent strategies.

Young children's earliest experiences with mathematics come through solving problems. Different strategies are necessary as students experience a wider variety of problems. Students must become aware of these strategies as the need for them arises, and as they are modeled during classroom activities, the teacher should encourage students to take note of them.

For example, after a student has shared a solution and how it was obtained, the teacher may identify the strategy by saying, "It sounds like you made an organized list to find the solution. Did anyone solve the problem a different way?" This verbalization helps develop common language and representations and helps other students understand what the first student was doing. Such discussion also suggests that no strategy is learned once and for all; strategies are learned over time, are applied in particular contexts, and become more refined, elaborate, and flexible as they are used in increasingly complex problem situations.

Item 4: Monitor and reflect on the process of mathematical problem solving

Effective problem solvers constantly monitor and adjust what they are doing. They make sure they understand the problem. If a problem is written down, they read it carefully; if it is told to them orally, they ask questions until they understand it. Effective problem solvers plan frequently. They periodically take stock of their progress to see whether they seem to be on the right track. If they decide they are not making progress, they stop to consider alternatives and do not hesitate to take a completely different approach. Research (Garofalo and Lester 1985; Schoenfeld 1987) indicates that students' failures are often due not to a lack of mathematical knowledge but to the ineffective use of what they do know.

Good problem solvers become aware of what they are doing and frequently monitor, or self-assess, their progress or adjust their strategies as they encounter and solve problems (Bransford et al. 1999). Such reflective skills (called metacognition) are much more likely to develop in a classroom environment that supports them. Teachers play an important role in helping to enable the development of these reflective habits of mind by asking questions such as:

- "Before we go on, are we sure we understand this?"
- "What are our options?"
- "Do we have a plan?"
- "Are we making progress or should we reconsider what we are doing?"
- "Why do we think this is true?"

Such questions help students get in the habit of checking their understanding as they go along. This habit should begin in the lowest grades. As teachers maintain an environment in which the development of understanding is consistently monitored through reflection, students are more likely to learn to take responsibility for reflecting on their work and make the adjustments necessary when solving problems.

Inquiry Notes

Inquiry Notes

Assessment Strategies

100 Yard Dash

Juan and Maria race for 100 yards. Maria wins by ten yards. Juan demands a second race, but this time Maria spots Juan a handicap – she begins ten yards behind the starting line. Assuming both run the same speed as before, who will win? Why?

Draw a diagram.

Explain your reasoning.

Evaluating and Designing a Rubric

Student Task

Make a graph to illustrate how many hours you spend during a typical school day doing different things. These things might be sleeping, eating, school, homework, playing sports, scouts, dance, playing with friends, playing games, watching TV, etc.

Your task is to collect, organize, and display your data. A chart or table may be helpful because you must in someway show evidence of data organization. You must also choose the best type of graph to represent this data (bar, circle, etc.). Please show any calculations that you make, and write an explanation for why you chose the graph you did and how you made your graph.

Student Rubric

	1 Point	2 Points	3 Points	4 Points
Organization of information	Information about time spent very disorganized	Some data organization evident, but not carried through	Data well-organized	In addition, the data is clearly presented
Graph	Graph chosen is inappropriate to the topic or very poorly executed	Graph chosen is adequate, but execution is poor	Appropriate form of graph and adequate information	In addition, the graph is very accurately and neatly presented
Calculations	Major errors in calculations	A number of errors in calculations	Very few errors in calculations	No calculation errors
Explanation	Explanation very muddled	Explanation difficult to follow	Explanation clear enough to follow	The explanation is clear and displays comprehensive understanding of the relative merits of different types of graphs

A few questions to consider as you evaluate the rubric:

- Is this a “good” rubric for the student task given? Why or why not?
- What changes should be made?
- Are the descriptions well stated and not ambiguous?
- Are there definite differences among the points?

Write any changes you would make on the rubric—use your State Core to see what expectations for student achievement are.

Pizza Party

Ms. Williams' 5th grade class is having a pizza party. They are trying to decide which pizzeria has the cheapest price. The local pizzerias and their prices are listed below.

Pizza Prices

Pizza to Go	$ 8.50	8 slices per pizza
Rosa's Pizza	$10.50	10 slices per pizza
Pizza with Pizazz	$ 6.25	6 slices per pizza

There are 30 students in Ms. Williams' class. Each person (including Ms. Williams) will eat two slices of pizza. All the pizza slices from each pizzeria are the same size. Where should you buy the pizza to get the best deal? Why?

Write a brief description of how you arrived at your decision. Provide your work (any calculations you made), a picture, or a diagram to support your thinking.

Activity—Process Categories

Standard

Objective

Connections

Standard II:
Students will develop a sense of self in relation to families and community.

Objective 1:
Describe factors that influence relationships with family and friends.

Indicator C:
Describe how children change over time.

Process Skills:
Symbolization, prediction, classification

Intended Learning Outcomes:
Demonstrate a positive learning attitude.

Background Information

Process categories refer to the mental abilities that are most likely required to answer a given question. Therefore, in addition to measuring content classified by standard, objective, and indicator, we can also measure the "process" used to answer a question as well. For the Utah CRT, we have employed the same "process" abilities as the NAEP assessments in Mathematics. These process standards are described below.

1. **Procedural:** Procedural questions are questions can be viewed as a student's *"knowing how."* Below are verbs that would generally associate with procedural questions.
   a. Compute
   b. Solve
   c. Evaluate
   d. Show
   e. Round
   f. Order
2. **Conceptual:** Conceptual questions are questions that can be viewed as a student's *"knowing that,"* or *"knowing about."* Below are verbs that would generally associate with conceptual questions.
   a. Identify
   b. Recognize
   c. Classify
   d. Compare
   e. Contrast
   f. Interpret

3. **Problem Solving:** Problem solving questions require both procedural and conceptual skills as noted above in 1 and 2. Problem solving questions typically require the following.
    a. Integration of discrete skill sets
    b. Extensions of existing knowledge
    c. Formulations of new strategy
    d. Investigations of new or unique approaches
    e. Reflections about methodologies used

These process standards are not wholly discrete. Often problems may fall in a gray area that surrounds these categories. Some problems will not fit only one process. In many instances, the background knowledge of the students may affect where a question could be classified. However, it is still important that questions be classified as accurately as possible within the aforementioned categories.

Background Information for Sample Test Score Grid

Question Analysis

It is important to remember that this analysis will only suggest potential problems or that questions are working just fine. Ultimately, the teacher as a professional must make the decision on whether or not a question is functioning as intended or requires edits or rejection.

- **Question 1:** 100% of the students answered it correctly. It may be working just fine. However, you might consider if it is too easy for what you are trying to measure.
- **Question 2:** This question seems to be working fine. Those students who missed it also did poorly on the test overall, suggesting that it is there lack of skill that caused the wrong answer. However, it has the potential of being too easy.
- **Question 3:** This question appears to be functioning perfectly. The best students answered correctly, the poorest students did not.
- **Question 4:** This question, like question #2 also appears to be functioning well.
- **Question 5:** This question does not appear to be working well at all. It seems to be a fifty/fifty chance of whether or not a student will answer correctly.
- **Question 6:** This question like #2 and #4 also seems to be working fine.

- **Question 7:** Unfortunately, this question like #5 seems to have a random level of performance surrounding it.
- **Question 8:** This question does not seem to be working at all and should probably be rejected. It is clear that the very best students are having extreme difficulty with it but that the students who are doing poorly overall are getting it correct. This question is probably functioning as some sort of “trick question” that is causing the best students to over-think and proceed with a set of false assumptions about what is being asked.
- **Question 9:** This question is probably fine. However the low percentage might suggest that it is written at a level that may be too difficult.
- **Question 10:** This question is quite likely too difficult as noted by the fact that not even the best students achieved a correct response.

REMEMBER: The above analysis with the accompanying spreadsheet is for illustration purposes only. Teachers should have as their objective to place the majority, if not all, of their students at or near the top of the performance ladder. As such, they will want their test analyses for almost all items to look like Questions #1 and #2.

Sample Math Test for Math Core Academy

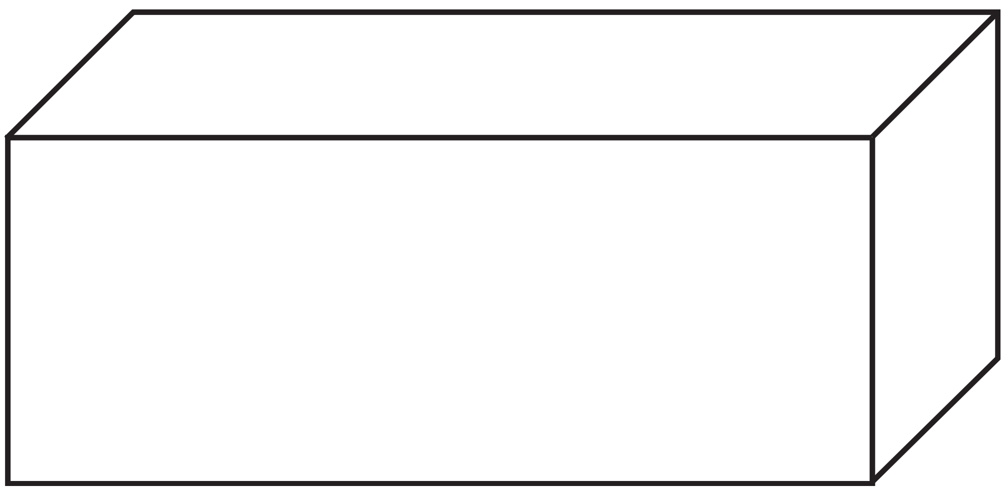
1. Compute: 3 1/2 x 4 3/4

2. Suppose that Billy ate 3/8 of a pizza. What do the 3 and the 8 represent?

3. Draw a model that shows the product of 9 and 8.

4. If ***b*** is 125% of ***a***, then how does ***b*** compare to ***a***?

5. Use the picture below to answer a, b, and c.



   a. How many cubic feet of water can be stored in a tank that measures 8 feet x 4 feet x 2 feet?
   b. How many gallons of water will the above tank hold if 1 cubic foot of water is equal to 7.5 gallons?
   c. How many hours will it take to empty the water tank above if it is being drained at a rate of 2 pints per minute?

SAMPLE TEST SCORE GRID FOR MATH CORE ACADEMY

QUESTION NUMBER	1	2	3	4	5	6	7	8	9	10	**TOTAL**
POINTS POSSIBLE	1	1	1	1	1	1	1	1	1	1	**%**
BILLY	1	1	1	1	1	1	**0**	**0**	1	**0**	70%
SALLY	1	1	1	1	**0**	1	1	**0**	1	**0**	70%
MARK	1	1	1	1	1	1	**0**	**0**	1	**0**	70%
BRETT	1	1	1	1	**0**	**0**	1	**0**	1	**0**	60%
LISA	1	1	1	1	1	1	**0**	**0**	**0**	**0**	60%
JOHN	1	1	1	1	**0**	1	1	**0**	**0**	**0**	60%
SUSAN	1	**0**	1	1	1	1	**0**	**0**	1	**0**	60%
VAN	1	1	1	1	**0**	1	1	**0**	**0**	**0**	60%
YVONNE	1	1	1	**0**	1	1	**0**	**0**	1	**0**	60%
HANS	1	1	1	1	**0**	1	1	**0**	**0**	**0**	60%
FRANCIS	1	1	**0**	1	1	**0**	**0**	1	1	**0**	60%
STEPHANIE	1	1	**0**	1	**0**	1	1	1	**0**	**0**	60%
MILHAUS	1	1	**0**	1	1	**0**	**0**	1	**0**	**0**	50%
WILLIAM	1	1	**0**	1	**0**	**0**	1	1	**0**	**0**	50%
BECKY	1	1	**0**	**0**	1	**0**	**0**	1	**0**	**0**	40%
RALPH	1	1	**0**	**0**	**0**	**0**	1	1	**0**	**0**	40%
RHETT	1	**0**	**0**	**0**	1	**0**	**0**	1	**0**	**0**	30%
SCARLETT	1	**0**	**0**	**0**	**0**	**0**	1	1	**0**	**0**	30%
ISHMAEL	1	**0**	**0**	**0**	1	**0**	**0**	1	**0**	**0**	30%
CHARLES	1	**0**	**0**	**0**	**0**	**0**	1	1	**0**	**0**	30%
QUESTION %	100%	75%	50%	65%	50%	50%	50%	50%	35%	0%	

QUESTION ANALYSIS

1.)

2.)

3.)

4.)

5.)

6.)

7.)

8.)

9.)

10.)

MATHEMATICS 5 ITEM DEVELOPMENT SCHEMATIC

STANDARD	OBJECTIVE	INDICATOR	PROCEDURAL	CONCEPTUAL	PROBLEM SOLVING
I	1	a			
		b			
		c			
		d			
		e			
	2	a			
		b			
		c			
		d			
	3	a			
		b			
		c			
		d			
		e			
		f			
		g			
	4	a			
		b			
		c			
		d			
		e			
		f			
	5	a			
		b			
		c			
		d			
		e			
		f			
		g			
	6	a			
		b			
II	1	a			
		b			
	2	a			
		b			
		c			
		d			
		e			
		f			
III	1	a			
		b			
		c			
		d			
		e			
		f			
		g			
	2	a			
		b			
		c			
	3	a			
		b			
		c			

IV	1	a			
		b			
		c			
		d			
		e			
	2	a			
		b			
		c			
		d			
		e			
		f			
		g			
V	1	a			
		b			
		c			
		d			
		e			
	2	a			
		b			
		c			

MATHEMATICS 6 ITEM DEVELOPMENT SCHEMATIC

STANDARD	OBJECTIVE	INDICATOR	PROCEDURAL	CONCEPTUAL	PROBLEM SOLVING
I	1	a			
		b			
		c			
		d			
		e			
		f			
	2	a			
		b			
		c			
		d			
	3	a			
		b			
		c			
		d			
	4	a			
		b			
		c			
		d			
		e			
	5	a			
		b			
		c			
		d			
		e			
		f			
		g			
		h			
	6	a			
		b			
		c			
II	1	a			
		b			
		c			
		d			
	2	a			
		b			
		c			
		d			
		e			
		f			
III	1	a			
		b			
		c			
		d			
	2	a			
		b			
	3	a			
		b			
		c			

IV	1	a			
		b			
		c			
		d			
	2	a			
		b			
		c			
		d			
		e			
		f			
V	1	a			
		b			
		c			
		d			
		e			
	2	a			
		b			
		c			

Assessment Strategies Notes

Assessment Strategies Notes

Misconceptions

Misconceptions

"Sometimes we learn false concepts, commonly called 'misconceptions.' These data can enter into the processing of new information and interfere with the correct interpretation of new information or experience …Students must confront the inconsistencies that lie between their mental notions and the new information before they can deal productively with the new ideas." (Tolman, *Discovering Elem. Science: Method, Content, and Problem-Solving Activities*, 3rd edition, Allyn &Bacon, 2002, p.26).

Note: If some of the bricks used in building a house were improperly cured, the inferior bricks become part of the house. Covering them over with stucco will not strengthen that part of the wall much. To correct the flaw, the weak part of the wall must be identified, the bad bricks removed and then replaced with good bricks. Since poorly cured bricks look the same as properly cured ones, identifying and removing the weak elements can be difficult. Similarly, it is necessary but difficult to identify and remove misconceptions. However, they must be removed, because like cancers, they continue to erode the formation of correct concepts.

- Every time we communicate, new concepts compete with the preconceived ideas of our listeners. All students hold these ideas, but they are unaware of their private theories.

"Every time we communicate, new concepts compete with the preconceived ideas of our listeners. All students hold these ideas, but they are unaware of their private theories."

From the video "A Private Universe: Misconceptions that Block Learning."

Produced at the Harvard-Smithsonian Center for Astrophysics.

Distributed by: The Astronomical Society of the Pacific
390 Ashton Ave.
San Francisco, CA 94112

Examples of common misconceptions:

Misconceptions Notes

Journaling
Ideas

Journaling

Background Information

Science and math concepts have been compared to a brick building where, course by course, each brick provides the cement foundation for the information that is to follow. The foundation of radioactivity research was the life work of the great French physicists, Pierre and Marie Curie. Madame Curie's carefully preserved journals minutely detail step-by-step the process of their work which has become the basis for innumerable advancements in science.

Purdue University research focused on using children's journals as a tool for teaching, learning, and assessing science. The findings showed that journals were valuable in teaching communication and literacy techniques during science lessons. Journals also provided documentation that students had a better understanding of scientific concepts and had begun an inquiry-based self-learning process.

It is vital to introduce student journals at the elementary school level to create the habit of journaling as a lifelong learning tool. Both formal and informal journaling models have been found to be equally effective. Journaling experiences are described in *Jordan School District Science TRB3* as:

1. Drawing pictures and labeling them
2. Writing short paragraphs about observed changes and interactions
3. Noting reactions to different stimuli
4. Making predictions
5. Recording observations
6. Analyzing reasoning
7. Keeping measurements
8. Making comparisons

Materials

- ❏ scissors
- ❏ paper
- ❏ glue sticks
- ❏ markers
- ❏ staplers and staples

Journaling Examples

from *Dinah Zike's Big Book of Books and Activities*

Layered Look Book p. 70-78

Pop-Up Book p. 112-115 *Remember the rule–Always cut on a fold, and Never glue on a fold.

Top Tab Book p. 80-83

Pyramid Book p. 38-41

Circle Book p. 64-65

Additional Resources

Jordan School District Science TRB3
Dinah Zike's Big Book of Projects by Dinah Zike (M. Ed.)
Big Book of Books and Activities by Dinah Zike (M. Ed.)

Journaling Notes

Journaling Notes

Standard I
Activities

Activity—Multiplication Strategy Review

Standard I
Students will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.

Objectives
3. Model and illustrate meanings of operations and describe how they relate.
5. Solve problems using the four operations with whole numbers, decimals, and fractions.

Intended Learning Outcomes
2. Become mathematical problem solvers.
6. Represent mathematical situations.

Standard
I

Objectives
3 & 5

Connections

Background Information

Students will already have been exposed to the following multiplication strategies: modeling multiplication with arrays, partial product multiplication, lattice multiplication, and traditional multiplication. These strategies are reviewed on the following pages.

Invitation to Learn

Pose the problem "35 x 27." Ask students to come up and share different ways to solve the problem. Inform the students that they are going to play a game in groups of four to practice all of the strategies for multiplication they have learned.

Instructional Procedures

1. Review partial product, lattice, and traditional multiplication strategies, if necessary.
2. Explain the rules for the game.
3. Deal one procedure card to each member of the group. That tells which strategy he or she will use to solve the first problem.
4. The problem cards are facedown in the center of the group.
5. The person with the "calculator" card flips over the first problem card. That person is in charge of checking the other players' answers when each of them finishes computing the problem.
6. When a person is finished solving the problem, he or she shows his or her answer to the "calculator person." That person will give thumbs up or thumbs down to show if the correct answer is displayed.

Materials

- ❏ Strategy cards (one of each strategy per group of four)
- ❏ Problem cards appropriate for the level of the students (two-digit by two-digit provided)
- ❏ 1 calculator for each group

7. If someone cannot answer the problem using the strategy on his or her card, the team works together to help that person arrive at the correct answer.
8. When everyone has shown the correct answer, the "strategy cards" are rotated to the right so each person has a new way to practice multiplication.
9. The person with the calculator card becomes the new leader and flips over the next problem card and the process repeats.
10. Arrange students by groups of four and let play begin.

Curriculum Integration

Math/Science—Read *One Hundred Hungry Ants* to show multiplication with arrays, and use TI-73 calculators.

Possible Extensions/Adaptations

Arrange your groups according to ability and give them appropriate problem cards for their levels. You may have one group working on simple two-digit by two-digit computations, while others may be doing three-digit by two-digit computations.

Use the *Math by Hand Application* on the TI-73 graphing calculators, which models partial product multiplication.

Assessment Suggestion

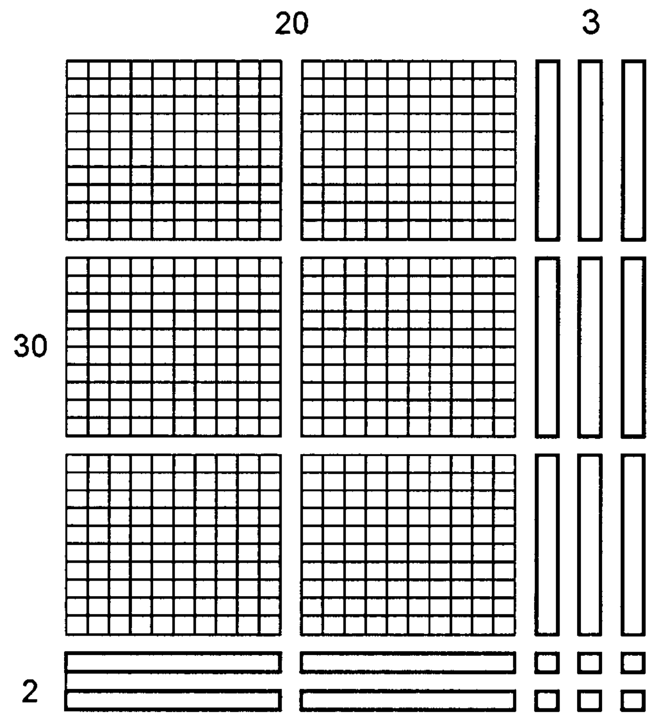
Use the Two Ways to Solve worksheet as your assessment piece. Students will have to demonstrate a knowledge of multiple strategies and journal which way they prefer and why.

Additional Resources

One Hundred Hungry Ants by Elinor Pinczes
Math by Hand Application Guidebook by Texas Instruments

Array Model and Partial Product Multiplication

Students will have had previous experience with using arrays to model multiplication in earlier grades. Using *Base Ten Blocks,* model two-digit by two-digit multiplication arrays. It is not necessary to model larger numbers; we just want to make the connection to prior knowledge and use of arrays.



20 x 30 = 600
20 x 2 = 40
30 x 3 = 90
2 x 3 = 6

$$\begin{array}{r} 600 \\ 40 \\ 90 \\ +\ 6 \\ \hline 736 \end{array}$$

Partial Product and Lattice Multiplication

Partial Product can also be set up in a grid format, which helps students be sure they have multiplied all of the combinations.

435 x 26

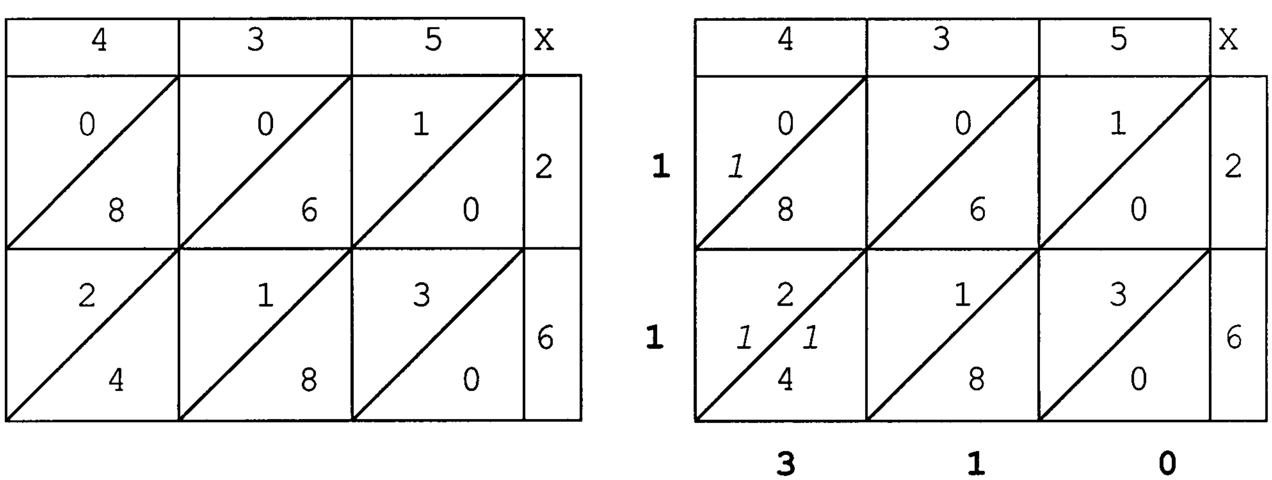
x	400	30	5
20	8,000	600	100
6	2,400	180	30

= 8,700

= 2,610

$$\begin{array}{r} 8{,}700 \\ +\ 2{,}610 \\ \hline 11{,}310 \end{array}$$

This is the same problem using the lattice method to solve the equation. The problem is broken down into one fact at a time, which can make it less daunting for some students. You fill in the lattice one square at a time placing your tens in the top half, ones in the bottom. Once the lattice is filled in, add the numbers **on the diagonal** starting with the lower right corner. If you have a number greater than 9 "carry" your ten(s) over into the next column.



435 x 26 = 11,310

Partial Product and Lattice Charts

x			

			x

Multiplication Strategy Cards

Partial Product	**Lattice**
Traditional	**Calculator**
Partial Product	**Lattice**
Traditional	**Calculator**

Multiplication Problem Cards

35 x 45	62 x 59	82 x 17
33 x 64	29 x 54	73 x 68
66 x 38	92 x 81	74 x 36
59 x 27	44 x 38	26 x 45
46 x 27	74 x 86	19 x 53
26 x 37	39 x 28	62 x 72

Name ___________________________

Multiplication Assessment

Show two ways to solve **74 x 36**.

Method #1	Method #2

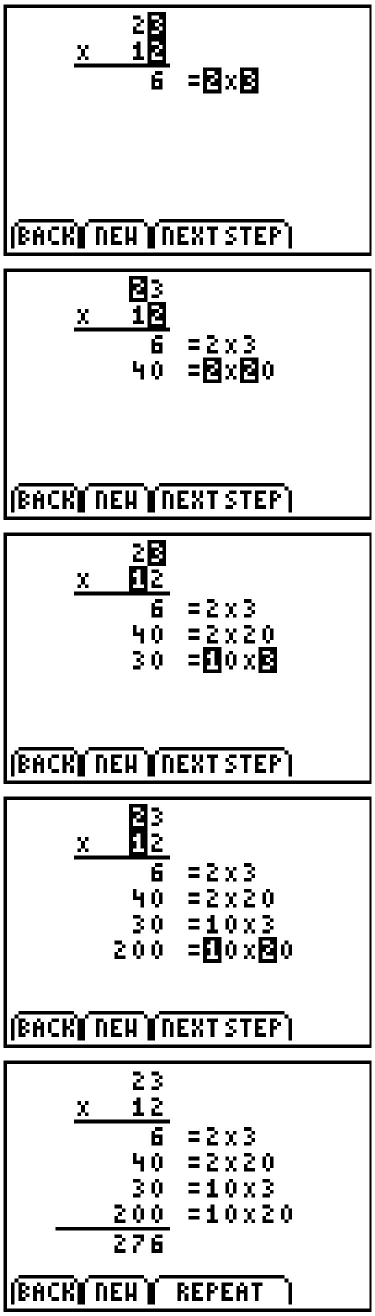
Explain which way you prefer and why.

Name ________________________

Multiplication Example for TI-73

Multiplication—Using the TI-73 APP *Math by Hand* in step-by-step mode to show partial products.

Example: 12 x 23 = ?



The application applies the diestributive property to multiplication problems and displays the partial products in a vertical format.

You could also apply the distributive property and display the partial products in a horizontal format, as shown below.

12 x 23 = (10 + 2) x (20 + 3)
= (10 x 20) + (10 x 3) + (2 x 20) + (2 x 3)
= 200 + 30 + 40 + 6
= 276

Math by Hand is a free APP (concept application program) for the TI-73 calculator http://education.ti.com/

Activity—Remainder of One

Standard
I

Objective
3

Connections

Standard I
Students will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.

Objective
3. Model and illustrate meanings of operations and describe how they relate.

Intended Learning Outcomes
2. Become mathematical problem solvers.

Background information

This is a beginning activity to introduce division with remainders and learn what a remainder represents.

Invitation to Learn

Have you ever felt left out of something that you really wanted to be a part of?

Instructional Procedures

Materials

- ❑ *Remainder of One* by Elinor J. Pinczes
- ❑ 25 one-centimeter cubes per student
- ❑ Overhead squares
- ❑ Graph paper
- ❑ 2 sheets of white paper per student
- ❑ 1 copy of bug graph paper per student
- ❑ Scissors
- ❑ Glue

1. Each student needs 25 one-centimeter cubes and a sheet of graph paper.
2. Begin reading the story *Remainder of One*.
3. When they create the two rows of bugs, have the students build what that would look like using centimeter cubes to represent bugs.
4. Share the array on the overhead.
5. Discuss what the single centimeter cube represents (Lone Joe, the left over bug).
6. Draw the array on the graph paper writing what it represents in either words or a number sentence.
7. Repeat this with each new line configuration in the story.
8. Next we are going to create books that show the arrays we created while reading the story. Look at the different ways we write division problems (Standard II, Objective 2a.), as well as the necessary vocabulary.
9. Create an interlocking book.

10. Using the bug graph paper, cut out the array that shows 25 ÷ 2 = 12 r1. Label it with the number sentence. Discuss the vocabulary term for each number in the number sentence. Record the terms on the last page in the book. Write, “dividend ÷ divisor = quotient and remainder” on the current page.
11. On the second page, cut out the array for:

    $$\begin{array}{r} 8 \text{ r } 1 \\ 3\overline{)\;25} \end{array} \qquad\qquad \begin{array}{r} \text{quotient remainder} \\ \text{divisor}\overline{)\;\text{dividend}} \end{array}$$

    Record the vocabulary terms for the number sentence.
12. On the third page, cut out the array for:

    $$\frac{25}{4} = \text{quotient remainder} \qquad \frac{\text{dividend}}{\text{divisor}} = \text{quotient remainder}$$

    Record the vocabulary terms for the number sentence.
13. On page four, cut out the array for “25÷ 5 = 5” and record the number sentence.
14. On the last page, make sure you have your vocabulary terms.

Curriculum Integration

Math/*Science*—Read *Remainder of One* by Elinor Pinczes.

Possible Extensions/Adaptations

Have students grab a small handful of centimeter cubes. Count the number of cubes. On a sheet of graph paper, have them draw the arrays and write the number sentences for the following problems:

C = number of cubes they pulled out
C÷ 2 =, C÷ 3 =, C÷ 4 =, C÷ 5 =, C÷ 6 =, C÷ 7 =,

Continue as desired. This could also be done as a homework connection. Put cubes in a snack size bag for students to take home and work with.

Assessment Suggestions

Pull students back to do individual interview assessments or have them do this in their math journals.

Question: Is this statement true or false 24 ÷ 5 = 4r3? Explain your reasoning.

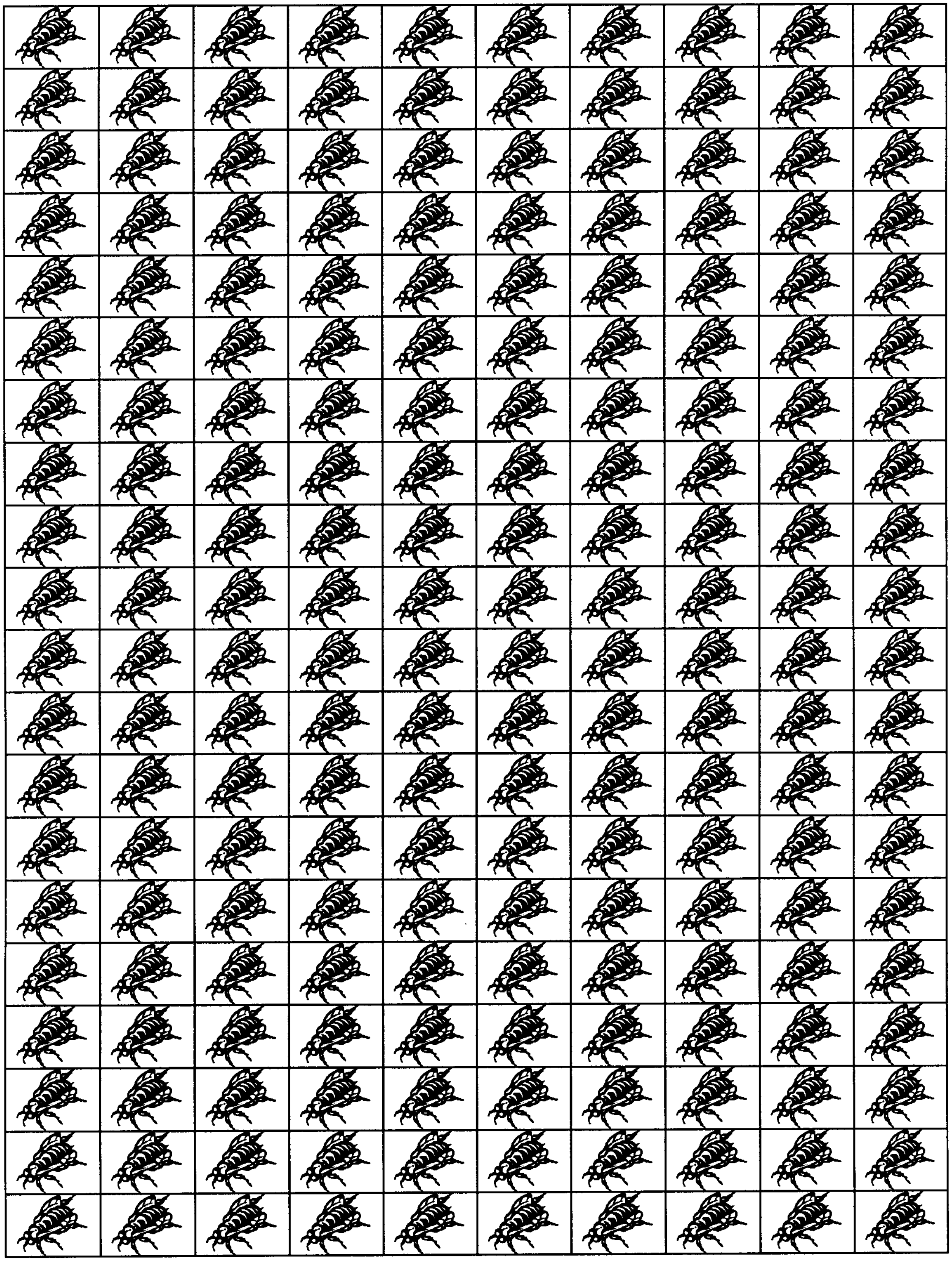
Additional Resources

The Kings Commissioners by Aileen Friedman

Homework & Family Connections

See Possible Extensions/Adaptations.

Bug Graph Paper



Vocabulary Graphic Organizer

Vocabulary Term	**Visual Representation**
Definition	**Personal Association**

Vocabulary Term	**Visual Representation**
divisor	$415 \div 3$ ☺☺☺ three people sharing
Definition	**Personal Association**
How many people you need to share with. How many groups you need to brean it in to.	If I had 415 pieces of gum and I was going to share them with my friends Brock and Wes, we would share the gum with 3 people. These are my two friends and me. ☺☺☺

Activity—Remainder Riddles

Standard I
Students will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.

Objective
3. Model and illustrate meanings of operations and describe how they relate.

Intended Learning Outcomes
1. Demonstrate a positive learning attitude toward mathematics.
2. Become mathematical problem solvers.

Standard I

Objective 3

Connections

Background Information

Students will have completed the Remainder of One exploration activity. You can use this as an opportunity for introducing the rules of divisibility or to practice using them.

Rules of divisibility:

2: Ends in an even number.

3: Add all of the digits together, if the sum is a multiple of three, it is divisible by three (54; 5+4 = 9, 9 is a multiple of 3, 54 ÷ 3 = 18).

5: Ends in zero or five.

9: Add all of the digits together, if the sum is a multiple of nine, it is divisible by nine (126; 1+2+6 = 9, 126÷9 = 14).

10: Ends in zero.

Invitation to Learn

Display the poster and pose the problem: "Today we are going to try to find a mystery number knowing only the remainder it leaves when divided by different numbers."

Instructional Procedures

1. Review the rules of divisibility.
2. Look at the poster and ask students where they think we should begin. Does the first clue help us at all? (No, any number divided by one will have no remainder because one is a factor of all numbers.) How about the second clue? (Yes, because when it is divided by two, it leaves no remainder so the answer must be even.)

Materials

- ❑ Remainder Riddle poster
- ❑ Remainder Riddles 1-25 handout
- ❑ Calculators (optional)

3. Continue working through the clues until you solve the riddle.
4. Discuss with the class how they can create their own riddles by doing one together as a class.
5. Working backwards makes this much easier. Begin by picking a number, then continue dividing it by the different numbers and recording the remainders. When finished, double check by working through all of the clues to make sure the number is the same.
6. Students are now going to create their own riddles. Hand out the Remainder Riddle 1-25 worksheet. You can have students work by themselves, with a partner, or in small groups. If you find a need, you may let your students use calculators or let them double-check their own calculations with a calculator.
7. After they create their riddles, have students share them with a neighbor and see if they can solve them.

Possible Extensions/Adaptations

Use calculators for students who do not have a knowledge of the facts. Create a class book of remainder riddles to checkout. Have a different student share his or her riddle each morning for ongoing practice of division and rules of divisibility.

Assessment Suggestions

Collect the papers that your students create. Assess the creator to make sure the problem works. Assess the solver to see if they were able to use the rules of divisibility to solve the riddle.

Homework & Family Connection

Have students create another riddle, picking a number between 1-50 using the Remainder Riddles 1-50 handout.

Remainder of One Riddle (1-25)

Written by: ___________________________________

Solved by: ____________________________________

1. When you divide my number by 1, R =
2. When you divide my number by 2, R =
3. When you divide my number by 3, R =
4. When you divide my number by 4, R =
5. When you divide my number by 5, R =
6. When you divide my number by 6, R =
7. When you divide my number by 7, R =

1 2 3 4 5 6 7 8 9 10

11 12 13 14 15 16 17 18 19 20

21 22 23 24 25

Remainder of One Riddle (1-50)

Written by: ____________________________________

Solved by: _____________________________________

1. When you divide my number by 1, R =
2. When you divide my number by 2, R =
3. When you divide my number by 3, R =
4. When you divide my number by 4, R =
5. When you divide my number by 5, R =
6. When you divide my number by 6, R =
7. When you divide my number by 7, R =

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Activity—Partial Quotient or Using What You Know

Standard I
Students will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.

Objectives
3. Model and illustrate meanings of operations and describe how they relate.
5. Solve problems using the four operations with whole numbers, decimals, and fractions.

Intended Learning Outcome
1. Demonstrate a positive learning attitude toward mathematics.
2. Become mathematical problem solvers.
6. Represent mathematical situations.

Standard
I

Objectives
3 & 5

Connections

Background Information

Teach this method of division to your students before the traditional algorithm. It shows what occurs in the division process rather than just prescribing steps to follow. Students should have worked with multiplication patterns like:

$$6 \times 2 = 12$$
$$6 \times 20 = 120$$
$$6 \times 200 = 1200$$

Students should have explored division with Base Ten Blocks as shown on the accompanying page.

Invitation to Learn

Pose a problem such as, "My library book has 382 pages. I have three weeks before I need to return it. How many pages do I need to read each week to finish it in time?" Let the students work on this for 10-15 minutes and share some of the strategies they used to figure it out. This can act as a pretest to see how much your students know about division.

Instructional Procedures

1. Discuss some of the strategies used on the original problem.
2. Write the problem 382 ÷ 3 on the board. Create a chart that says:

Week 1	Week 2	Week 3

Along with the division problem written like this:

$$3\overline{)382}$$

3. Start by showing your students that you could start tallying that you read one page each week and keep repeating this until you have 382 tally marks. Show how this would look using repeated subtraction on the division problem.

Week 1	Week 2	Week 3
///	///	///

382	
– 3	1 page each week, subtract 3 pages altogether
379	*(pages left to read)*
– 3	1 page each week, subtract 3 pages altogether
376	*(pages left to read)*
– 3	1 page each week, subtract 3 pages altogether
373	*(pages left to read)*

Point out that we could go on like this until we finished dividing all the pages but that would take us a long time. Erase what you just entered into your chart and on your division problem.

4. Ask students if we could divide a greater number of pages instead of one by one. Students will see that we could read 100 each week and then see how many we have left to read.

Week 1	Week 2	Week 3
100	100	100

$3\overline{)382}$	
– 300	100 pages each week, 300 pages altogether
82	*(pages left to read)*

5. What could we use next? One by one would still take a long time. Students will give different responses like:

   10 pages each week and subtract 30 pages
   20 pages each week and subtract 60 pages.

Responses will vary depending on the students' ability to use multiplication patterns. The great thing about this strategy is that you can use either number and just keep subtracting and see what you still have left to divide. That is why we call it "Using What You Know." Every student could use a different number and still arrive at the same answer.

6. Continue filling in the chart and division problem.

Week 1	Week 2	Week 3
100 10 10 7	100 10 10 7	100 10 10 7

$3\overline{)382}$	
– 300	100 pages each week , 300 pages altogether
82	*(pages left to read)*
– 30	10 pages each week , 30 pages altogether
52	*(pages left to read)*
– 30	10 page each week , 30 pages altogether
22	*(pages left to read)*
– 21	7 pages each week, 21 pages altogether
1	*(page left to read)* remainder

7. Add up our pages down the side and see if they match with one week of our chart. Record the answer to the problem where it belongs.
8. Discuss how to interpret the remainder. We have to read 117 pages each week but on the third week you have to read 118.
9. Practice several more of these problems *always* creating a word problem that goes with it. This helps students to understand what the remainder actually means. Students enjoy coming up with the stories to go with the problems.

Curriculum Integration

Math/Science—Use the Math by Hand Application for the TI-73 calculator to demonstrate the same process.

Possible Extensions/Adaptations

Use this activity to begin discussing different ways of interpreting remainders. Because students are coming up with different problem situations, their remainders may be interpreted differently. Did you need to ignore it, round up, or report it as a fraction or decimal?

Assessment Suggestions

Give the students a number sentence such as 413 ÷ 8. Have them write a word problem for the number sentence, and identify which number is the dividend and which is the divisor. Tell students to solve the problem and explain the steps they used. Have them name the quotient and remainder, and explain what the remainder means in the solution.

Additional Resources

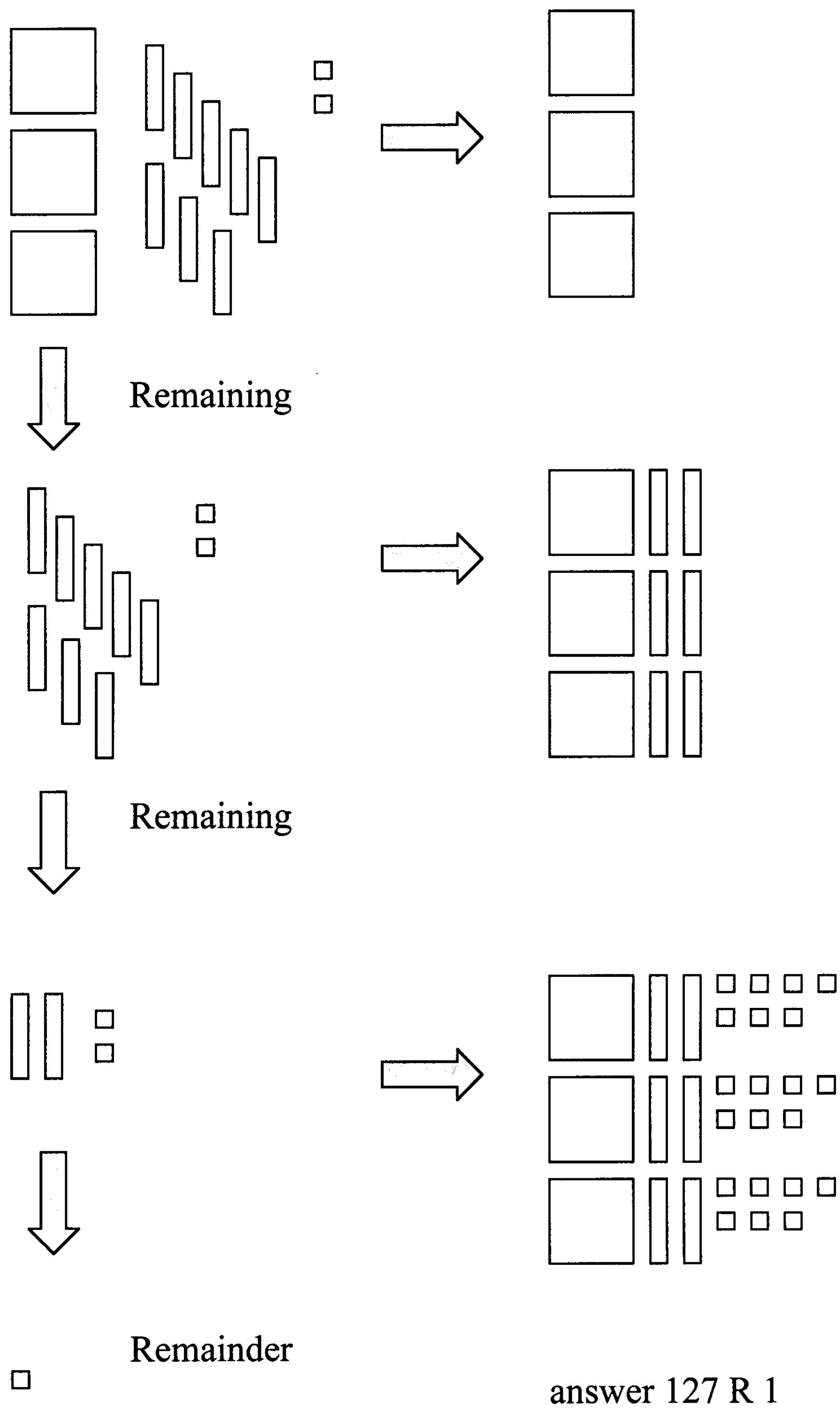
Divide and Ride by Stuart Murphy deals with interpreting remainders.

Homework & Family Connections

Many parents have not seen this strategy for division. Have the students explain it to their parents. Let the students know that they are to practice this strategy alone and not look at the traditional algorithm yet.

Division—Using Base Ten Blocks

Example: 382 ÷ 3



Name ________________________

Division Assessment

Using the problem **571 ÷ 4** coomplete the following:

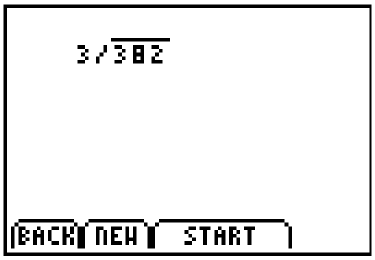
1. Create a story problem using those numbers.
2. Identify the dividend and explain what it means.
3. Identify the divisor and explain what it means.
4. Solve the problem and explain the steps you used to find your answer.
5. Identify the quotient and explain what the remainder represents in your problem.

Name ________________________

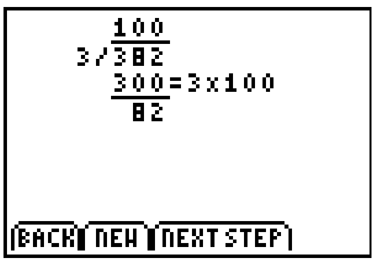
Division Example for TI-73

Division—Using the TI-73 APP *Math by Hand* in step-by-step mode to show partial products.

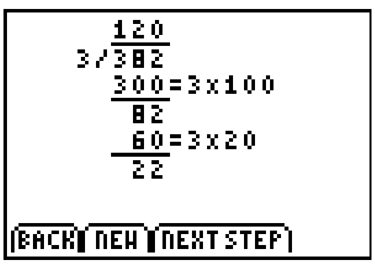
Example: 382 ÷ 3 = ?



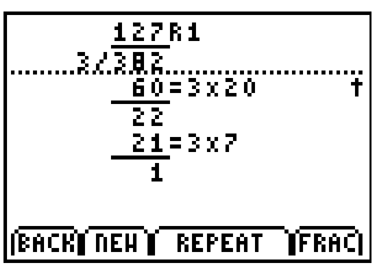
The application shows that there are 100 groups of 3 in 382, not 1 group of 3 in 3. This helps students who are learning the algorighm for the first time, or who have trouble lining up their writing.



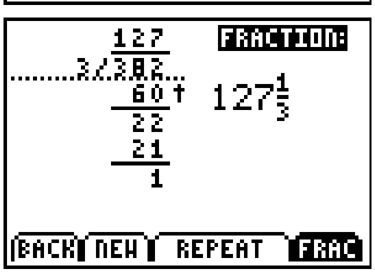
The next step shows that there are 20 groups of 3 in 82.



This step shows that there are 7 groups of 3 in 22. Since the remainder is less than the divisor, the problem is solved.



Finally, to see the relationship between the remainder and the fraction form select **FRAC.**



Math by Hand is a free APP (concept application program) for the TI-73 calculator http://education.ti.com/

Activity—Mixed Number/Improper Fraction Bingo

Standard I

Objectives 2 & 4

Connections

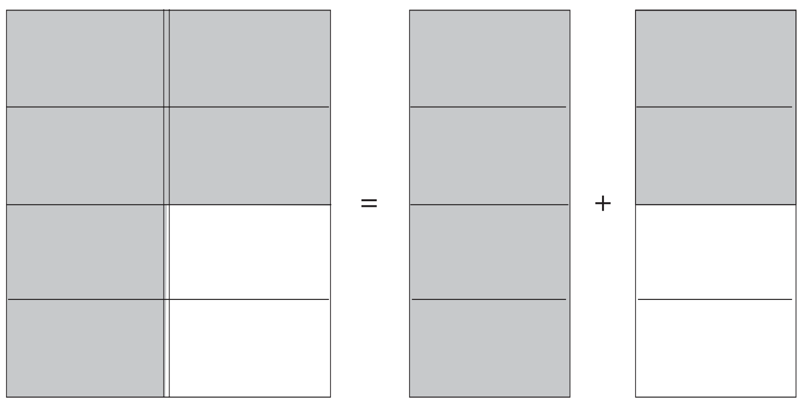
Standard I
Students will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.

Objectives
2. Identify relationships among whole numbers, fractions, decimals, and percents.
4. Use fractions to communicate parts of the whole.

Intended Learning Outcomes
3. Reason mathematically.

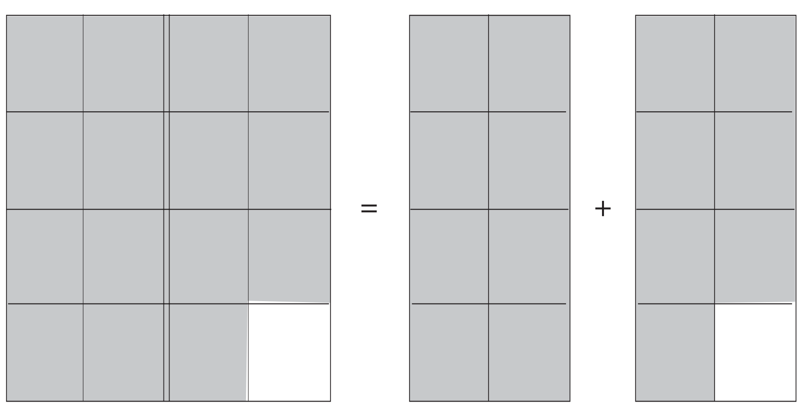
Background Information

Some work with renaming from one form to the other should have taken place before playing the bingo game. It is useful to look at models of renaming between the forms, instead of going straight to the abstract computation of it. This helps students develop a better understanding of the process.



6/4 = 4/4 (1) + 2/4

6/4 = 1 2/4 or 1 1/2



1 7/8 = 8/8 + 7/8

1 7/8 = 15/8

This sample shows modeling with rectangles. This is a great use for Fraction Circles or Fraction Factory pieces as well as any handmade equivalent fraction sets. Don't leave out the modeling! When making the connection to the symbolic make sure to discuss the fraction bar as another division symbol.

Invitation to Learn

BINGO WITH PRIZES!

Instructional Procedures

See game

Possible Extensions/Adaptations

Have two versions of the game, one changing improper to mixed and vice versa. Also, use the numbers to create a mixed number/improper fraction concentration game.

Additional Resources

The Hershey's Milk Chocolate Bar Fractions Book by Jerry Pallotta

Homework & Family Connections

Mixed Number/Improper Fraction Concentration Game. Have each student find a recipe that calls for quantities such as 1 cup of sugar and have them change those amounts to improper fractions.

Materials

- ❏ Some manipulative such as Fraction Circles for modeling mixed numbers and improper fractions
- ❏ Bingo game board for each student
- ❏ Set of improper fraction cards
- ❏ Game board markers
- ❏ Prizes for winners if

Mixed Number and Improper Fraction Bingo

Objective

Be the first player to place five markers in a row horizontally, vertically, or diagonally.

Players

Whole class

Rules

Materials

- ❏ One game board for each student
- ❏ Improper Fraction cards
- ❏ Markers

1. Hold up an improper fraction card for the students to change to a mixed number.
2. If that mixed number is on his or her game board, he or she may cover it.
3. The first player with five in a row calls out "BINGO."
4. That player reads out his or her covered numbers for you to check. If correct, the game is over. Start a new round.

Here's an Example

The card 9/7 is held up. I convert it to the mixed number 1 2/7. I mark 1 2/7 on my game board.

Helpful Hints

If you don't want to make game boards for your entire class, have your students create a set the first time they play and then collect them and laminate for future use. Just make sure they copied the numbers correctly onto their boards! Make two versions of the game, one changing improper to mixed, the other vice versa. Make the fraction cards on 4x6 or 5x7 index cards and write big so everyone can read them. Have the answers somewhere on your cards otherwise when it's time to check you are going to have to do some fast calculating! You have 30 fractions and only 25 squares on your boards. Make sure you mix up the numbers and get those last numbers on some of the boards, too.

Numbers for Improper ⇔ Mixed Bingo

Game Board	Card
1 1/3	4/3
1 2/7	9/7
1 3/10	13/10
1 5/9	14/9
1 9/10	19/10
2 2/7	16/7
2 2/5	12/5
2 3/4	11/4
2 5/6	17/6
2 11/12	35/12
3 2/7	23/7
3 2/5	17/5
3 1/2	7/2
3 4/7	25/7
3 5/8	29/8
3 7/10	37/10
4 1/3	13/3
4 3/4	19/4
4 5/8	37/8
5 4/5	29/5
6 1/2	13/2
6 5/6	41/6
7 1/9	64/9
7 2/3	23/3
7 5/7	54/7
7 3/4	31/4
8 1/3	25/3
8 5/9	77/9
9 1/2	19/2
9 5/6	59/6

Mixed Number ⇔ Improper Bingo

Activity—Fruity O Fractions

Standard I
Students will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.

Objective
4. Use fractions to communicate parts of the whole.

Intended Learning Outcomes
1. Demonstrate a positive learning attitude toward mathematics.
2. Become mathematical problem solvers.

Standard I

Objective 4

Connections

Background Information

This would make a good and fun assessment on students' knowledge of creating equivalent fractions using a common denominator. The first page, Mmmm…Fruity Os is using problem solving strategies and the given clues to determine the quantity of each color of Fruity Os. Students should be familiar with strategies, like Guess and Check, Drawing a Picture, Act it Out, Use a Model, and Working Backwards. These two activities do not need to be done together or at the same time. One is for problem solving; the other is for fractions.

Invitation to Learn

Do you think one color of Fruity O's is more common than the other colors? Everyone grab a handful of Fruity Os and write down how many of each color you have. Can you express those numbers as fractions?

Instructional Procedures

1. Begin with the invitation to learn.
2. Have students report what color they had the most of.
3. Record fractions for how many students had the most red, orange, yellow, green, blue, and purple. (14/28 of us had orange as our most common color, 3/28 had green . . .)
4. See if any of those fractions can be simplified.
5. If you are doing both activities, introduce the first sheet as a problem solving activity that they can use their Fruity Os to help them solve. You could work through the first problem together as a class if your students don't know where to start.

Materials

- ❏ One or two bags of Fruity O Cereal (depending on class size) divided into smaller bowls and set on each table
- ❏ Mmmm…Fruity Os and Fruity O Fraction worksheets
- ❏ Fruity O Fractions part 2 (optional)

6. The second sheet, Fruity O Fractions could be used as an assessment of finding equivalent fractions using a common denominator or work through that one together and use Part 2 as your assessment.
7. If you are not using it as an assessment you could work through the problems together and have the students do number 3 on their own or in groups.
8. Problem number three has the students writing their own riddle. This could be difficult for some of the students. You many want to do one together as a class first.
9. Have students continue working on Part 2, assign it as homework, or use it the next day as an assessment piece.

   (Answers to Mmmm…Fruity Os: ***#1*** 12 red, 3 yellow, 6 blue ***#2*** 5 red, 12 yellow, 6 blue ***#3*** 8 red, 3 yellow, 8 blue ***#4*** 5 red, 15 yellow, 5 blue)

   (Answers to Fruity O Fractions: ***#1*** 4 blue, 6 yellow, 10 red ***#2*** 4 blue, 12 yellow, 8 red ***#3*** answers will vary)

Possible Extensions/Adaptations

Use Fruity O Fractions Part 2 (Answers: ***#1*** 8 yellow, 8 orange, 2 green, 6 red ***#2*** 3/10 red, 5 blue, 6 yellow, 10 green, 9 red ***#3*** 16 blue, 10 red, 6 orange, 2 green, 2 yellow, 4 purple). Use this activity to show multiplication of whole numbers by a fraction (1/5 x 20 = 4).

Assessment Suggestions

Mentioned earlier in Instructional Procedures.

Additional Resources

Jump Kangaroo Jump by Stuart Murphy

Homework & Family Connections

Give students a baggie of 24 Fruity O's and have them record the fraction of each color, simplifying if possible. Write it as a riddle and share in class the next day.

Mmmm... Fruity Os

Solve these Fruity O riddles.

1. There are six blue. There are twice as many red as blue. There are half as many yellow as blue. How many of each color are there?

   red__________ yellow__________ blue__________

2. There are 5 red. There is one less red than blue. The number of yellow is double the number of blue. How many of each color are there?

   red__________ yellow__________ blue__________

3. There are 19 pieces in all. There are the same number of red and blue and only three yellow. How many of each color are there?

   red__________ yellow__________ blue__________

4. There are three times as many yellow as blue. There are 5 red. There are 25 pieces in all. How many of each color are there?

   red__________ yellow__________ blue__________

Fruity O Fractions

Use common denominators to help you make these Fruity O combinations.

1. Make a yellow, blue, and red combination. The combination should contain 20 Fruity Os:
   1/5 blue, 3/10 yellow, and 1/2 red
   How many of each color is in your mix?

   blue__________ yellow__________ red__________

2. Make a combination using 24 Fruity Os. It should contain:
   1/6 blue, 1/2yellow, and 1/3 red
   How many of each color is in your combination?

   blue__________ yellow__________ red__________

3. Make up your own riddle with 30 Fruity Os in it. Record fractional part of each color.
   __________blue __________yellow __________red
   Exchange problems with a friend and have them fill out the next part. Make sure you know the answer first so you can tell them if they are correct.

   Solved by: ____

   blue__________ yellow__________ red__________

Fruity O Fractions Part 2

Use common denominators to help you make these Fruity O combinations.

1. Make a yellow, orange, green, and red combination. The combination should contain 24 Fruity Os:

   1/3 yellow 1/3 orange 1/12 green 1/4 red

   How many of each color is in your mix?

   yellow_______ orange_______ green_______ red_______

2. Make a combination using 30 Fruity Os. It should contain:

   1/6 blue, 1/5yellow, 1/3 green, ___?___red

   How many of each color is in your combination?

   blue_______ yellow_______ green_______ red_______

3. Make a combination with 40 Fruity Os in it. It should contain:

   2/5 blue 1/4 red 3/20 orange

   1/20 green 1/20 yellow 1/10purple

   How many of each color is in your combination?

   blue_______ red_______ orange_______

   green_______ yellow_______ purple_______

Activity—Comparing Fractions War Card Game

Standard I

Objective 2

Connections

Standard I
Students will acquire number sense and perform operations with whole numbers, simple fractions, and decimals.

Objective 2
Identify relationships among whole numbers, fractions, decimals, and percents.

Intended Learning Outcomes
1. Demonstrate a positive learning attitude toward mathematics.
3. Reason mathematically.

Background Information

Students should have had experience with finding LCD of fractions in order to compare the two fractions. This is not an introductory activity.

Instructional Procedures

See game instructions.

Possible Extensions/ Adaptations

If you have groups that you think may struggle, pull out the cards from the deck that do not have denominators that can be changed to a 12 (for example, pull out 2/5, 4/7…). This way, a common denominator is easy to find.

Use these cards for other activities. For example, draw two cards and add them together.

Use the cards to order fractions. Draw three cards and place them in order least to greatest.

Additional Resource

Gator Pie by Louise Mathews

Homework & Family Connections

Send the deck of cards with a copy of the directions home for families to play.

Comparing Fractions War

Materials

- ❑ One deck of Comparing Fractions War game cards

Objective

Collect the most card pairs by the end of the game.

Players

Two

Rules

1. Shuffle the deck of fraction cards and deal all of the cards equally between the two players. Players may NOT look at their cards. Keep them face down in a pile.
2. Players each flip over a card and compare them to see who has the greater fraction.
3. The person with the greater fraction collects that card pair.
4. If the players flip over equivalent fractions, they play another round and the winner of that hand takes all four cards.

Here's an example:

Player A turns over 3/4, Player B turns over 5/6.

Find a common denominator and compare the fractions.

Player A 3/4 = 9/12

Player B 5/6 = 10/12

Player B takes both cards because 5/6 is greater than 3/4.

Comparing Fractions War Game Cards

6/8	7/9	4/9	1/6
2/4	3/3	1/8	5/7
1/4	7/10	2/8	8/10
3/7	8/12	4/10	2/6

1/3	3/8	2/3	4/5
5/6	7/12	3/5	4/7
5/12	1/2	4/8	3/9
3/4	9/12	2/5	4/6

Standard II
Activities

Activity—Building Patterns

Standard II
Students will use patterns and relations to represent and analyze mathematical situations using algebraic symbols.

Objectives
1. Recognize, analyze, and use patterns to describe their attributes.
2. Represent, solve, and analyze mathematical situations using algebraic symbols.

Intended Learning Outcomes
2. Become mathematical problem solvers.
3. Reason mathematically.

Standard II

Objectives 1 & 2

Connections

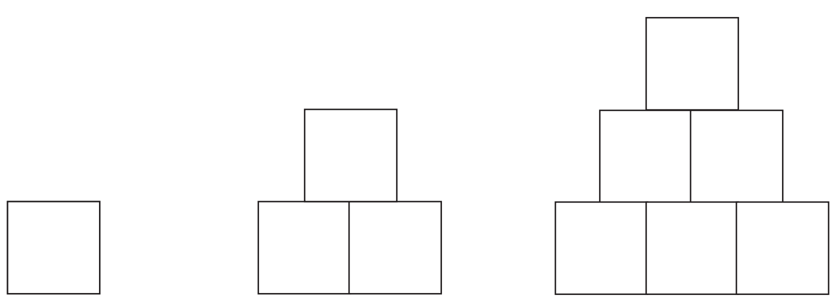
Background Information

In a pattern, repetition occurs in predictable ways. Patterns are fundamental to understanding place value, multiplication, and many other mathematical topics. They are especially important in understanding that a function is an important aspect of developing algebraic thinking. You may want to have calculators available as students discover relationships in growing patterns. That way, students can easily calculate large scale number relationships.

Activity A

Invitation to Learn

1. Read the book *The Book That Jack Wrote* by Jon Scieszka and help students recognize that it is a growing pattern. Discuss how growing patterns are different than growing patterns and that today we will work with growing patterns and math.
2. Use the square tiles on the overhead to build the following designs (or just draw them) and have students predict the next design in the sequence.

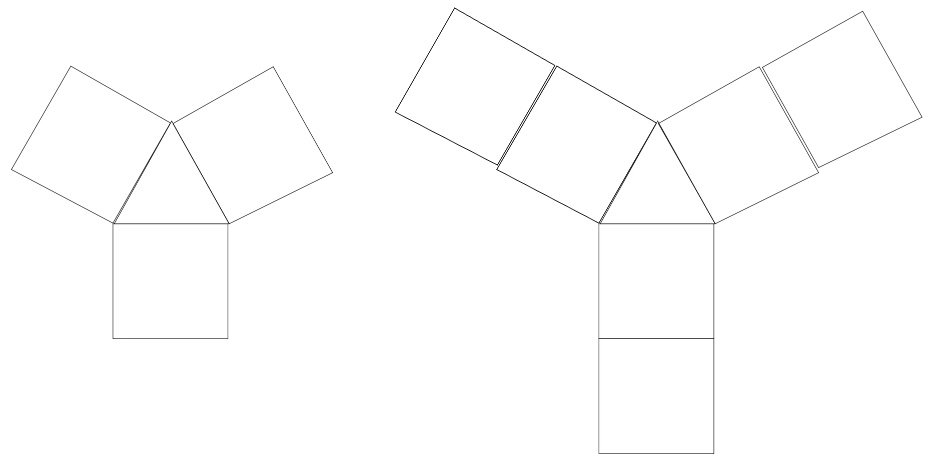


Materials

- ❑ *The Book That Jack Wrote* by Jon Scieszka (or a copy of the story *The House that Jack Built*)
- ❑ One-inch square tiles or one-inch paper squares (for each student or pair of students)
- ❑ Paper and pencil (for each student or pair of students)

Instructional Procedures

1. Give each team a plastic bag with 50 square tiles.
2. Have students build "windmills" with their tiles, building them in order of size with the smallest one first. Show them the following shapes already drawn on a transparency. The students must decide how to build each new windmill for themselves.

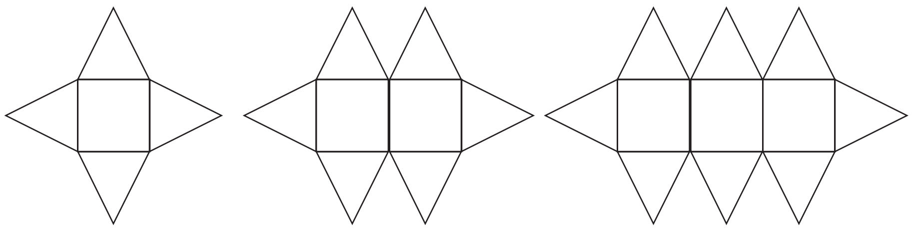


3. Have students record their work in a two-column table to record their work with the headings "Shape" and "Total tiles in shape." The left column for "Shape" should show 1,2,3…and the right column should show the totals 3,6,9…
4. Ask several teams to describe how the first and second shapes are alike and how they are different. Do the same for the second and third windmill.
5. Ask them to build the fourth windmill (it should have three blades with four tiles per blade) to the right of the first three.
6. Have students describe in their own words what each of the windmills looks like. Encourage the following type of details: The first blade has one tile per blade for three blades, the second has two tiles per blade for three blades, etc.
7. Now ask the students to predict how the tenth windmill might look based on their descriptions given in step six. (Example: Number ten has ten tiles per blade for three blades.) Have them complete the table through the tenth shape. Some students may need to build a few more. Discuss how the numbers in the "Total" column change (increase by three each time).
8. Ask them to try to describe how to find the total tiles needed for making the Nth windmill. The algebraic expression should follow the language pattern used in step six: "The Nth shape will use N tiles per blade for three blades or three times N tiles total."

Activity B

Instructional Procedures

1. Begin by using overhead pattern blocks to build the following designs on the overhead.



2. Students should draw these three designs and color them with colored pencils or crayons. Then have them draw and color the next three designs, using 4 squares with 10 triangles around them, 5 squares with 12 triangles, and 6 squares with 14 triangles.
3. Have students make a table recording their results.

   Example:

How many squares?	1	2	3	4	5	6
How many triangles?	4	6	8	10	12	14

4. Ask students what they notice about the numbers in the table. How do the number of triangles relate to the number of squares used in each design? Several answers are acceptable if they seem reasonable and apply to all designs drawn or all pairs of numbers in the table. One relationship might be as follows: For N squares in a row, there are N triangles above and N triangles below the squares with and extra triangle on the left and right ends of the row of squares. Another way to say this is, double the number of squares used and add two more to find the number of triangles needed each time. An algebraic expression would be "If N is the number of squares, then the triangles would be (2 x N) + 2 or 2N+2.
5. For independent practice, have students explore and create new growing patterns using two different shapes from their pattern blocks. Then they should create the tables and find the relationship rule. They could share their patterns with the class by drawing the first three designs on the overhead and allowing the class to try to describe the pattern that occurs between the two shapes used.

Materials

- ❏ Pattern block sets for each student or pair of students
- ❏ Colored pencils or crayons
- ❏ Pencil and paper
- ❏ An overhead set of pattern blocks for the teacher

Curriculum Integration with Science

Fibonacci numbers can be found in many different living things. The sequence occurs in the seeds of a sunflower or the spirals that go in opposite directions on a pineapple. The two numbers will be adjacent numbers in the Fibonacci sequence, usually 8 and 13 for a pineapple and 55 and 89 for sunflowers.

Possible Extensions/Adaptations

Advanced students could go on to explore the patterns that emerge in square numbers, triangular numbers, Pascal's triangle, or Fibonacci numbers. Tessellations could be researched, explored, and created as an added connection to art.

Assessment Suggestion

Watch as students build their patterns and create their T-charts. Observe whether they have created patterns that are consistent in following a rule, and can enter the correct numbers on their T-charts. Have students come to the overhead or board to draw their pattern designs and lead discussions about how their pattern grows. This will give good insight into their understanding. Make sure to have students move from building patterns to writing down an equation describing it mathematically.

Additional Resources

Books

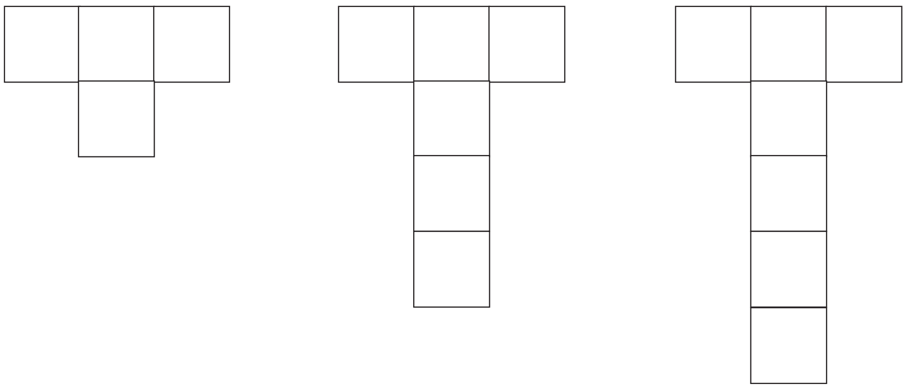
Hands On Math by Frances M. Thompson
Challenge Math by Edward Zaccaro
Elementary School Mathematics by John A. Van De Walle

Web sites

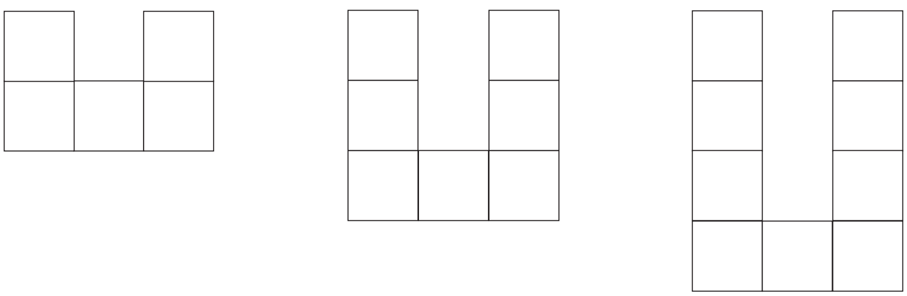
www.uen.themepark/patterns
http://illuminations.nctm.org

Homework & Family Connections

Students can take home graph paper and draw growing patterns using the letters in the alphabet. For example a “t” can grow like this:



Or a “u” can grow like this:



Then have students determine the pattern by placing it on a T-chart.

Activity—Table Settings

Standard II

Objective 1

Connections

Standard II
Students will use patterns and relations to represent and analyze mathematical situations using algebraic symbols.

Objective 1
Recognize, analyze, and use patterns that describe their attributes.

Intended Learning Outcomes
1. Demonstrate a positive learning attitude toward mathematics.

Background Information

Many of the values of attribute activities and pattern analysis are the same: development of logical reasoning, perseverance in solving problems, a willingness to solve problems, and the ability to test ideas.

Younger children love to build and extend patterns. When using patterning with older children, it is also a matter of testing ideas, extending patterns to test relationships, and developing general formulas. All of these are important aspects of mathematical thinking and problem solving. Numeric sequences are good early examples of the concept of functional relationships. Each term in the sequence or pattern has a unique value, depending on its position.

Younger children need to use repeating patterns. Older children should develop patterns that grow. Be careful not to move too quickly because children of all ages need plenty of opportunities to construct ideas about patterning.

Invitation to Learn

Ask, "How many people can sit around your kitchen table at home?" Call on several students to respond. Draw a square on the overhead or board. Ask students, "How many people could sit around this table if one chair fits on each side?" Draw lines to represent chairs.

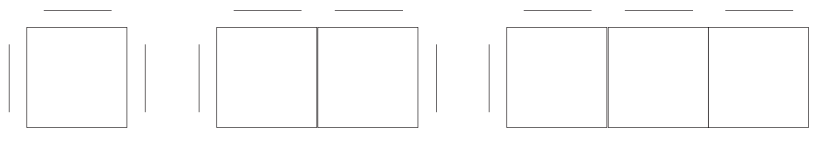
Instructional Procedures

Materials

- ❏ *Spaghetti and Meatballs for All* by Marilyn Burns
- ❏ Paper and pencil

1. Begin reading the book *Spaghetti and Meatballs for All*. As the story progresses, continue to draw the new formations. Instruct students to draw the table formations in their math journals and label how many people are able to sit there. Use different questioning strategies to help students make predictions about how the tables will be set up and what will happen to the seating.

2. When finished with the story, tell the students that they are going to host a banquet and that this time all the tables will need to be set up in one row. Have the students draw what their banquet arrangements will look like using one to six tables including lines for the chairs.



3. Tell the students that they are going to transfer the information to an organized list or table and look for patterns. Make a T-chart on the board and have students copy it in their math journals. Ask the students what the headings should be. Then fill in the numbers in the left column one to six.

# of tables	# of chairs
1	
2	
3	
4	
5	
6	

4. Students then fill in the number of chairs for each of the table arrangements. Have students make predictions about the patterns that they see.

# of tables	# of chairs
1	4
2	6
3	8
4	10
5	12
6	14

5. Students will probably look at the patterns going vertically which is good, but the focus should be on what is the relationship between the sets of numbers. Have students test their theories (e.g., if someone says you add three each time, go down the list and test it to see if it applies to each set). Encourage the students to communicate what they believe the pattern to be.
6. Point out the pattern: the number of chairs is equal to the number of tables, times two, plus two more chairs for the ends. Explain that this would be easier to see and explain in an equation.
7. Write the equations for each of the numbers:

$$(1 \times 2) + 2 = 4$$
$$(2 \times 2) + 2 = 6$$
$$(3 \times 2) + 2 = 8$$

8. Have the students continue writing the equations through number six (or beyond) on their tables.
9. End by challenging students to use the equation to find out how many chairs would fit around 10 tables, 15 tables, or 25 tables. Have students understand the connection between making organized lists and finding patterns to problem solving.

Possible Extensions/Adaptations

Challenge students to do the same thing (make an organized list and find the pattern) with triangular tables, hexagonal tables, trapezoid tables, etc.

Attribute activities also help students see patterns. Use attribute blocks to have students make logical connections between the blocks using the words and, or, or not.

Some students may need to use pattern blocks to visualize the chairs around the square tables. Have them available for those students (everyone will use pattern blocks in the Let's Build It activity). Some students may find it easier to see the clues of pattern relationships in drawings rather than in charts, so allow this as necessary.

Assessment Suggestion

Monitor students' drawings and lists as they fill in the numbers. Pay close attention to their pattern theories. Have everyone continue their lists through the number ten and assess if the corresponding number of chairs is correct.

Additional Resources

Lessons for Algebraic Thinking by Maryann Wickett, Katharine Kharas, and Marilyn Burns
Family Math: The Middle Grades by Virginia Thompson and Karen Mayfield-Ingram
Elementary School Mathematics by John A. Van De Walle
http://illuminations.nctm.org
www.uen.themepark/patterns

Homework & Family Connections

Challenge students to use their own kitchen tables as a base to create patterns. What if they used two of their tables? How many people could sit around it? What if they had a triangular table? What patterns would emerge then?

Activity—What's My Function?

Standard II

Objectives 1 & 2

Connections

Standard II
Students will use patterns and relations to represent and analyze mathematical situations using algebraic symbols.

Objective 1
Recognize, analyze, and use patterns that describe their attributes.

Objective 2
Represent, solve, and analyze mathematical situations using algebraic symbols.

Intended Learning Outcomes
3. Reason mathematically.
4. Communicate mathematically.

Background Information

The numeric relationship between the frame number and the number of objects in the frame is a powerful pattern. If a rule can be discovered that can give the value of any frame in terms of the frame number, the number of objects in any frame can be determined without building or calculating all of the intermediate frames in the pattern.

These vocabulary terms should be taught and emphasized during the lesson:

Variable: a letter, symbol, or other placeholder in a mathematical expression that has an unknown value

Function: a relationship between two variables in which the value of one variable (often called the output) depends on the value of the other variable (often called the input)

Equation: a mathematical sentence that contains an equal sign

Invitation to Learn

It is a good idea to give the students plenty of experiences with input/output tables before having them look for patterns and function relationships. One way they could do this is to have a race.

Have students come to the board and draw an input/output T-chart with five rows. Then have each student write down the five numbers you call out on the input (left) column such as 5, 12, 18, 25, and 40. Remind students that the input is the number to which an operation rule is applied, and that the output number is the result of that operation. Next announce a rule such as "add 15" and have each player write the rule at the top of their chart. On a signal, the students fill in the output column using the rule, then turn and face the class. When both are finished have the class help you check their work. They could score one point for each correct answer and five points for whoever finishes first. Continue playing with different input numbers and different rules. This will help students feel comfortable with how these types of tables work. In addition, you could have students who are still in their seats do this activity in their math notebook at the same time as those who are doing it on the board.

Instructional Procedures

Materials

- ❏ *Two of Everything* by Lily Toy Hong
- ❏ Overhead calculator for teacher
- ❏ Paper and pencil

1. Begin by reading *Two of Everything*. Make a T-chart on the board to show what went into the pot (input value) and what came out (output value).

In	Out

In	Out
3	6

2. Ask what would happen if you put 3 coins in the pot and write "3" midway down the *In* column. Students should state that 6 will come out. Continue by asking what would happen if you put in four coins, then five, ..., ten?
3. Ask students to describe to you what patterns they notice in the columns.
4. Ask if anyone sees a different pattern. Continue to record responses and discuss.
5. Lead students to these descriptions: "The *Out* number is equal to the *In* number times two," or "The *Out* number is equal to the *In* number added to itself," or "The *In* plus *In* equals *Out*."
6. Explain that an easier way to describe these patterns is to use symbols (variables) in place of the words. Draw a square above the In column and a triangle above the *Out* column.

$\square$ In	$\triangle$ Out
5	2
6	4
7	6
8	8
9	10
10	12
11	14
12	16
13	18
14	20

7. Use one of the sentence descriptions, such as "The *Out* number is equal to the *In* number times two." Ask which symbol represents the *Out* number (the triangle) and which symbol represents the *In* number (the square) and write the symbolic/algebraic equation:

$$\triangle = \square \times 2$$

8. Show the other algebraic equations:

$$\triangle = \square + \square$$

OR

$$\square + \square = \triangle$$

9. Now refer back to the pot and T-chart and ask the students what would happen if the pot did something different or followed a new rule. Again make a T-chart, enter in numbers, and conduct a discussion until students discover the new rule (function). An example could be:

$\square$ In	$\triangle$ Out
1	5
2	10
3	15
4	20

10. Have students guess the next number in the pattern sequence and practice writing the algebraic equations.
11. After the rule/function has been determined, see if students can skip several numbers and follow the rule for a larger number (e.g., What would the output be if the *In* was 30? 150, etc).
12. Conclusion: Using organized lists, or tables and variables, helps solve problems and understand patterns. This is using the power of patterns to predict beyond the information currently available.

Possible Extensions/Adaptations

Calculator game

Tell students that they have a “mission possible” and that they have to figure out the mathematical mystery. But instead of you giving them the rule and having them figure out the number answers, you are going to give them the number answers and have them figure out the rule.

Have students prepare to write down a series of numbers and try to figure out the pattern. Using the overhead calculator and without letting the students see what you are entering, enter 2 + 5 =; let the students see the 7 then continue to press the = button several times (they will see 12, 17, 22, 27, etc.).

When several students believe they have figured out the pattern rule allow them to share. You can also have them observe the number in the ones place. How long is it before the pattern begins to repeat? Do this activity several times with different beginning arithmetic problems.

Assessment Suggestion

Allow the students to work in pairs to choose a rule, write it as an equation, and generate a T-chart. Brainstorming a list of possible rules might be helpful so that students who have difficulty deciding on a rule of their own or who want to make their rules too complicated have some guidelines to follow.

Additional Resources

Books

Lessons for Algebraic Thinking by Maryann Wickett, Katharine Kharas and Marilyn Burns
Challenge Math by Edward Zaccaro
Elementary School Mathematics by John A. Van De Walle
Beginning Algebra Thinking for Grades 3-4 and 5-6 by Shirley Hoogeboom and Judy Goodnow

Magazine

The Mailbox Intermediate April/May 2001

Web sites

www.learner.org/teacherslab/math/patterns/mystery
This site is an interactive activity where students enter in two numbers and the function is a "mystery." It is like a function machine and students can continue to try it with different numbers until they think they can solve it and then the computer tests them.
www.standards.nctm.org
www.mathsolutions.com

Video

Disney's Donald in Mathmagic Land (includes information about Fibonacci number sequences).

Homework and Family Connections

You could use the Find the Patterns worksheet for homework. Have students make the tables, find the patterns, write the equations, and answer the questions for each.

Students could also do the extension activity at home and create In/Out charts with their own rules and fill in several examples on the chart.

Name ________________________

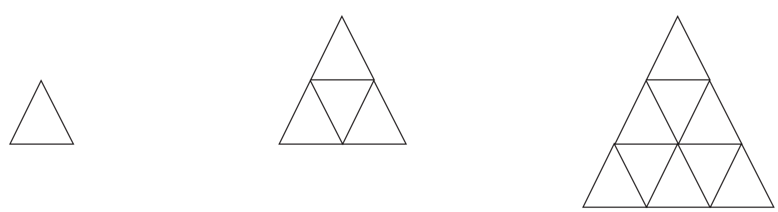
Find the Patterns

* For each of the following problems, make a table, decide the pattern, write the equation, and answer the question.

1. Sam went hunting for shells along the beach. On the first day he didn't find any. On the second day he found one. On the third day he found two. And on the fourth day he found 3. If he continues to find shells at this rate, how many will he find on days 5, 6, 7, and then on day 18?

2. Drew and Sue are brother and sister. When Sue was 5, Drew was 7. When Sue was 6, Drew was 8. When Sue was 7, Drew was 9. Now that Sue is 40, how old is Drew?

3. Look at the following pattern. Make a table to show what is happening. Draw the next three designs with the growing pattern and then try to decide what the pattern is and write the equation.



Activity—Understanding Variables

Standard
II

Objective
2

Connections

Standard II
Students will use patterns and relations to represent and analyze mathematical situations using algebraic symbols.

Objective 2
Represent, solve, and analyze mathematical situations using algebraic symbols.

Intended Learning Outcomes
2. Become mathematical problem solvers.

Background Information

The definition of a variable is a letter, symbol, or other placeholder in a mathematical expression that has an unknown value. Students should be familiar with finding missing values though they may not associate them with the term variable. Several different symbols and letters should be used so students can feel comfortable solving equations with any variable used.

Invitation to Learn

1. Tell the students that you are going to play a true/false game. You will write an equation on the board and they will put their thumbs up if it is true and down if it is false. Proceed to write several true and several false equations (i.e. *true:* 3 + 6 = 9, 10 – 8 = 2, 7 x 2 = 14; *false:* 5 x 10 = 45, 24 – 20 = 14, etc).
2. Explain that today we will be working with equations that are neither true nor false, but are "open" and have a missing number for them to solve.

Activity A

Instructional Procedures

Materials

- ❏ Large symbol cards for board
- ❏ Symbol card sets for individuals or small groups

1. Begin by practicing using algebraic language. Have students write the following mathematical expressions write in their math journals and make them true:

   a. Six more than 17
   b. Five less than 25
   c. Three times as large as 11
   d. A number increased by 5
   e. Four times the sum of two numbers
   f. The product of eight and another number

(a. 17 + 6 = 23, b. 25 – 5 = 20, c. 11 x 3 = 33, d, e, and f are open ended)

Note: A picture book that gives students the opportunity to practice this is *Ready, Set, Hop!* by Stuart J. Murphy.

2. Check for understanding and then proceed. Explain that you did not give them the answer so the problem should look like this:

$$17 + 6 = \square$$

After they solve it the equation should look like this:

$$17 + 6 = 23$$

3. Tell students that many times not all of the number values are given and there are missing addends. For example:

Kelly had 5 more marbles than Mike and together they had 12 marbles. The problem would look like this:

$$\square + 5 = 12$$

4. Explain that when a symbol or letter is used in place of an unknown number it is called a *variable*. Ask students what number would go in the place of the variable to make this equation true.
5. Do several more simple examples with different variables and have students write them in their math journals and solve:

a. $10 - \square = 6$

b. $5 \text{ x } n = 15$

c. $30 - \triangle = 10$

6. Have each student create five equations using one variable, then switch with a partner to solve. Switch back to correct.

Activity B

1. Explain that not only is it important to be able to figure out what a variable's value is equal to, it is also important to know how to set up the expression.
2. Pass out the envelopes of *variable symbol* and *operation cards*. Have students remove the cards and use them to set up the following expressions:

a. Jeff's ticket for a movie cost $\square$. His dad's cost $\triangle$. What was the total of the two tickets?

$$(\square + \triangle)$$

b. Cindy has $\square$ to buy a book that costs $\triangle$. How much change will she receive?

$$(\square - \triangle)$$

c. One can of pop costs $\triangle$. How much would a $\square$-pack cost?

$$(\triangle \text{ x } \square)$$

3. Next have students write in their math journals how they would write these expressions:

a. I started with $\square$ beans, then I increased them by 10.

$$(\square + 10)$$

b. There were $\triangle$ beans in the bag. Now there are 7 less.

$$(\triangle - 7)$$

c. I started with *n* beans in a bag and decreased it by 4.

$$(n - 4)$$

d. There were *y* beans in the bag. Now there are 2 more.

$$(y + 2)$$

4. To conclude, write $x + 5$ on the board and have students think of a word problem to match the expression. Have them share their ideas with their partners and vice versa. Repeat the activity with $n - 2$.

Possible Extensions/Adaptations

1. Students who are ready could use the variable cards to represent two-step problems:

a. Katie purchased a box of popcorn for $\bigcirc$ and a liter of pop for $\triangle$. What was her total cost if she used a coupon for $\square$ off?

$$(\bigcirc + \triangle) - \square = ⬡$$

b. Blake had $\square$ dollars saved. He bought a radio for $\triangle$.

The next week he got $\bigcirc$ dollars for his allowance.

How much money does he have now?

$$(\square - \triangle) + \bigcirc = ⬡$$

2. Student can work in pairs to play “mystery numbers.” Have the partners draw a circle and square at the top of their paper. Then have them draw two cards from a number card set of 0-9 (or just assign them two different numbers). The pair chooses which number will be represented by the circle and which by the square and writes that on their paper. Then they write down five number sentences using the circle and the square with the answer. Have pairs use addition, subtraction, multiplication, or division. They can come to the board and write their first sentence, then call on volunteers to see if they can determine what the circle and square represent. If they can’t, then the pair writes their next sentence and calls on volunteers as before. Continue if necessary until all five number sentences have been used or until the numbers have been discovered. This could also be done in small groups. See example below:

$\bigcirc \times \square = 24 + \bigcirc = 11 \square$

$\square + \square = 16 + \bigcirc \times \bigcirc = \square$

$\square - \bigcirc = 4$

Answer: $\square = 8 \quad \bigcirc = 3$

Assessment Suggestion

Carefully observe as students write their equations and use variables to make sure they understand. Have them use white boards or chalkboards to show you their answers quickly or just have them write in their math journals as you walk around and assess their learning.

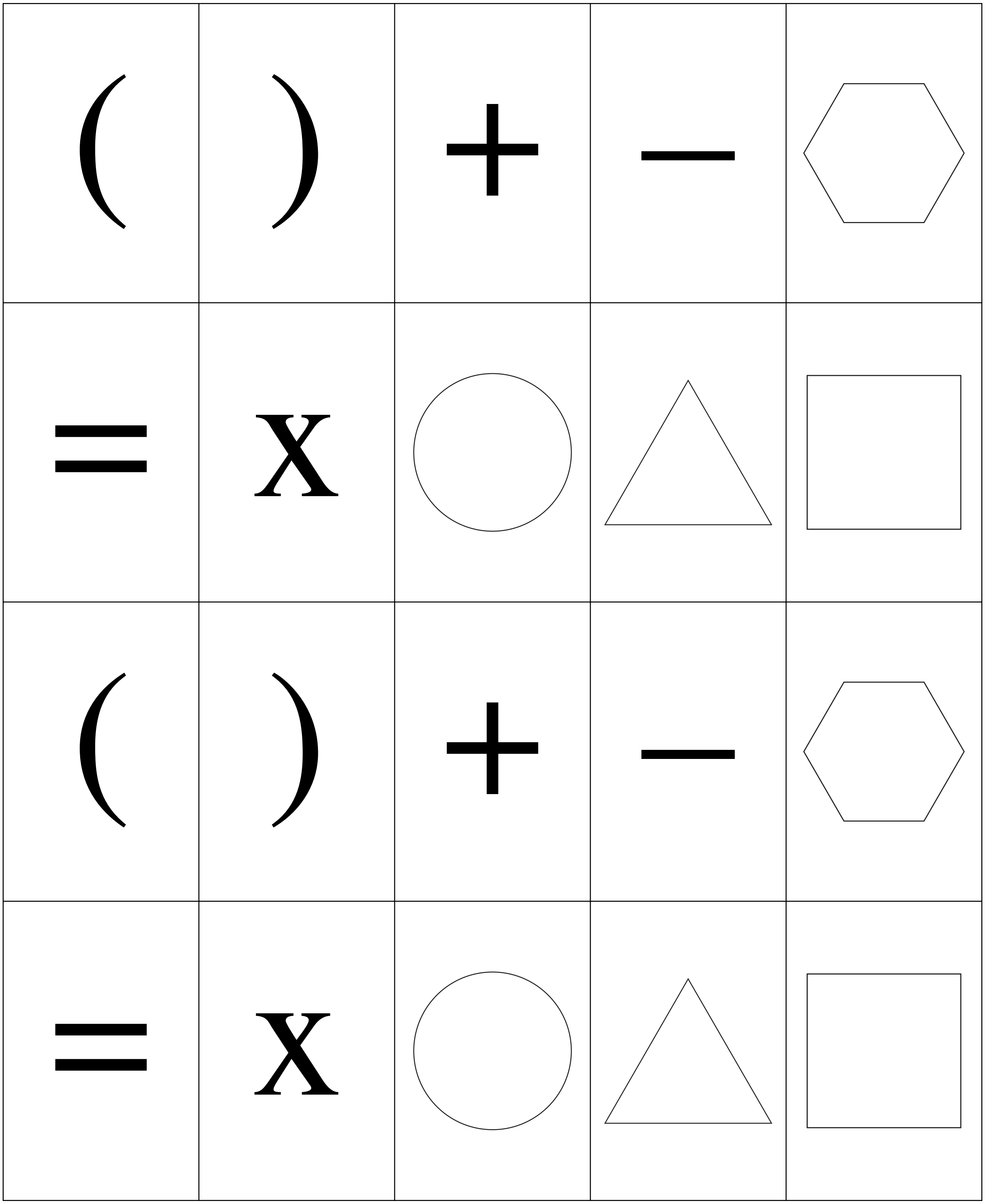
Additional Resources

The Best of Mailbox Magazine Math: Grades 4-6 Teacher resource book
Cooperative Learning and Mathematics K-8 by Beth Andrini
Lessons for Algebraic Thinking by Maryann Wickett, Katherine Kharas, and Marilyn Burns
Family Math: The Middle School Years by Virginia Thompson and Karen Mayfield-Ingram
Ready, Set, Hop! by Stuart J. Murphy (MathStart Series)
The Mailbox: Intermediate April/May 2001

Homework & Family Connections

Have students play the "mystery number" game at home (see extensions). Students choose two numbers to represent the circle and square and see how many number sentences (clues) it takes for their parents to decide what the variables represent.

Symbol Cards



Activity—Algebraic Equations

Standard II

Objective 2

Connections

Standard II
Students will use patterns and relations to represent and analyze mathematical situations using algebraic symbols.

Objective 2
Represent, solve, and analyze mathematical situations using algebraic symbols.

Intended learning outcomes
2. Become mathematical problem solvers.

Background Information

To get a feel for what students know about algebra, have students write their response to this question in their math journals: "What do you think it means to think algebraically?" Have students share their responses either orally or by turning in their responses for you to review. This will inform you what students know about algebra at this point in their schooling. Students may not realize what they know or that they have actually been thinking algebraically for several years. This lesson can help your students see the connection between arithmetic and algebra.

Invitation to Learn

Show the students the balance scale. Show them how both sides need to have the same amount of weight on them to balance. Demonstrate how adding or removing objects from one or both pans affects the balance. Explain that mathematical equations must be equal on both sides to balance.

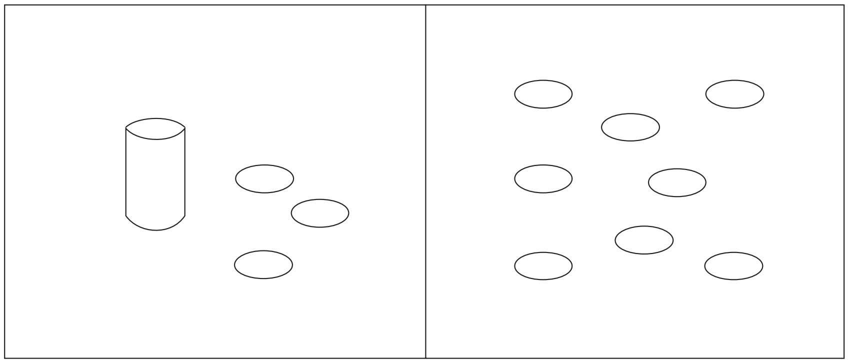
Activity A

Instructional Procedures

Materials

- ❏ A two-pan balance scale with weights
- ❏ 3-ounce cups
- ❏ Plain piece of paper
- ❏ Beans or counters (25 for each pair of students)

1. Give each pair of students one piece of paper, one cup, and 25 counters. Have the students fold their paper in half, open it up and draw a line down the middle. Explain that this paper will act as a balance scale and each side of the paper has to remain the same to be balanced.
2. Have the students hide four counters under their cup on the left half of their paper. Ask how many counters would need to be on the right side to make their paper scale balance? *(4)*



3. Now have them empty the cup and put it on the left side of the paper with 3 counters next to it. Put 8 counters on the right side of the paper. Tell the students that we will pretend that the correct number of counters are under the cup to make the paper scale balance. To be able to find out how many counters are under the cup, you could remove the same amount of counters from each side (thereby keeping it balanced) until all the beans you can see on the left are gone (what remains under the cup will equal what remains on the right side).
4. Students should remove three counters from the left and in turn remove three from the right. What remains shows what is "hiding" under the cup *(5)*. To verify this, have students put 8 counters on each side. Cover up 5 of them with the cup and then do the same procedure as before. When they lift the cup, they will see 5 counters on each side.
5. Write the equation for this problem: x + 3 = 8.
6. Write the equation for what happened:

$$\begin{array}{r} x + 3 = 8 \\ \underline{\;-3 \;\; -3} \\ x = 5 \end{array}$$

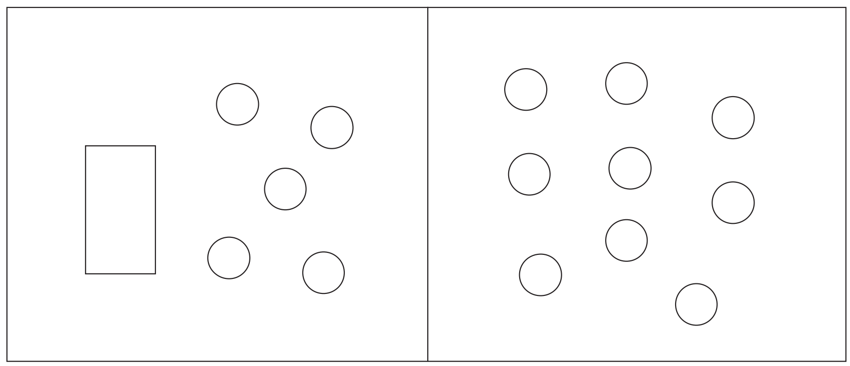
7. Now give the students other equations to solve (you may have to model a few more first). Here are some sample problems:

   a. x + 5 = 12
   b. x + 8 = 20
   c. 24 = 14 + x
   d. 15 = x + 3

8. Have students share their results and write number sentences on the board to record the steps they used.

Activity B

1. To move from the concrete to pictorial practice, you will now have students draw the counters, balance scale, and cup to solve equations.
2. In their math journals, have students draw a similar type diagram:



3. Remind students that the “cup/unknown/variable” has to be all alone on one side to reveal what is beneath (it’s value). Ask them to get rid of or cross out the circles on the left and a corresponding amount on the right. Then have them write the equation they just completed, including the value of x:

$$\begin{array}{r} x = 5 = \ \ 9 \\ \underline{\quad - 5 = - 5} \\ x = \ \ 4 \end{array}$$

4. You may do several more problems like this before moving on.
5. Now give students basic one-step equations to solve and have them do it without counters, cups, or drawing. Emphasize that you are doing “inverse operations” so that when problems look like this: $x – 3 = 10$, the students will know to add. It isn’t recommended that you try to do this with the counters or the drawing though, just with the equations. This can lead into solving equations with larger numbers too such as: $112 + x = 245$, or $x – 3,000 = 5,000$.

Possible Extensions/Adaptations

Some students may need to continue to use the counters and cups throughout the entire activity. Make them available for that purpose.

Also, a literature book that gives students practice with equations and finding unknowns is *Safari Park* by Stuart J. Murphy.

Assessment Suggestion

Give students several problems to solve equations with variables. Many textbooks have lessons and practice problems like this or you could create your own.

Additional Resources

Hands-On Math by Frances M. Thompson
Safari Park by Stuart J. Murphy (MathStart Series)

Homework & Family Connections

Send home a cup with beans and the paper used in class with each student. Have them show their parents the activity they did in class and explain how to solve for missing addends.

Directions for playing the "Equation Game" (or "Mathematical" Scrabble™)

Objective of the game

Set the highest score (each card's value is located in the lower right-hand corner of the card). This game allows students to practice forming equations and using different symbols to represent addition, subtraction, multiplication, and division.

Procedure to play

Materials

(for groups of 4)

- ❑ Number and operation symbol squares (prepare first by copying onto cardstock, cutting apart, and placing in a plastic bag. Hint: copy each set onto a different color of cardstock so that if pieces are dropped, they don't become mixed up. Copy the = and parenthesis cards onto white cardstock so they can easily be set to the side)
- ❑ Four copies of the one-inch grid paper to serve as a game board
- ❑ Cardstock "tri-fold" tents (to conceal the number cards from opponents)
- ❑ Paper and pencil

1. Place the four game boards together in the center of the playing area to form one large game board. Turn all of the number cards upside down except for the = cards and parenthesis cards which should be set to the side and used by players as needed.
2. Each player chooses 10 number cards and places them in front of his or her cardstock tent. Choose who will play first. That player must form an equation by laying down number cards in the center of the board. Equations must be made horizontally (read left to right) or vertically (read top to bottom). That player totals his or her score using the lower right hand corner scoring values and then replaces the number of cards used. Play continues with each player and equations can cross over other equations.
3. Calculators can be used only if a player challenges another players' equation. If a player cannot form or add onto an existing equation, that player must choose one number card to discard and replace it with a new one. Then his or her turn is over. Blank cards can become any number or operation (up to the number 12) and then must retain that representation throughout the game.
4. When equations can no longer be formed by anyone, or when the cards are all gone from the selection pile, the game is over. Any cards left in front of each player must be deducted from that player's total. The highest total wins.

Game Board for Equation Game

Game Pieces for Equation Game

0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4

10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	$-_1$	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	$-_1$	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	$-_1$	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	$-_1$	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	$-_1$	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	X_2	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	X_2	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	X_2	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	X_2	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	X_2	X_2	$-_1$

13 3	15 3	17 4	19 4	21 2	23 3	25 3	27 4	29 4	2
13 3	15 3	17 4	19 4	21 2	23 3	25 3	27 4	29 4	2
13 3	15 3	17 4	19 4	21 2	23 3	25 3	27 4	29 4	2
13 3	15 3	17 4	19 4	21 2	23 3	25 3	27 4	29 4	2
13 3	15 3	17 4	19 4	21 2	23 3	25 3	27 4	29 4	2
14 2	16 2	18 3	20 2	22 2	24 3	26 3	28 3	30 2	2
14 2	16 2	18 3	20 2	22 2	24 3	26 3	28 3	30 2	2
14 2	16 2	18 3	20 2	22 2	24 3	26 3	28 3	30 2	2
14 2	16 2	18 3	20 2	22 2	24 3	26 3	28 3	30 2	2
14 2	16 2	18 3	20 2	22 2	24 3	26 3	28 3	30 2	2

$=_0$	$=_0$	$=_0$	$=_0$	$=_0$	$=_0$	$(_0$	$)_0$	$(_0$	$)_0$
$=_0$	$=_0$	$=_0$	$=_0$	$=_0$	$=_0$	$(_0$	$)_0$	$(_0$	$)_0$
$=_0$	$=_0$	$=_0$	$=_0$	$=_0$	$=_0$	$(_0$	$)_0$	$(_0$	$)_0$
$=_0$	$=_0$	$=_0$	$=_0$	$(_0$	$)_0$	$(_0$	$)_0$	$(_0$	$)_0$
$=_0$	$=_0$	$=_0$	$=_0$	$(_0$	$)_0$	$(_0$	$)_0$	$(_0$	$)_0$
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$=_0$	$=_0$	$=_0$	$=_0$	$(_0$	$)_0$	$(_0$	$)_0$	$(_0$	$)_0$
$=_0$	$=_0$	$=_0$	$=_0$	$(_0$	$)_0$	$(_0$	$)_0$	$(_0$	$)_0$
$=_0$	$=_0$	$=_0$	$=_0$	$(_0$	$)_0$	$(_0$	$)_0$	$(_0$	$)_0$
$=_0$	$=_0$	$=_0$	$=_0$	$(_0$	$)_0$	$(_0$	$)_0$	$(_0$	$)_0$

Activity—Math Stations for Pattern Review

Standard II

Objectives 1 & 2

Connections

Standard II
Students will use patterns and relations to represent and analyze mathematical situations using algebraic symbols.

Objective 1
Recognize, analyze, and use patterns to describe their attributes.

Objective 2
Represent, solve, and analyze mathematical situations using algebraic symbols.

Intended Learning Outcomes
2. Become mathematical problem solvers.
6. Represent mathematical situations.

Background Information

Math stations serve many purposes. Not only do they provide opportunities to differentiate curriculum according to student needs, but they also give the teacher a chance to work with smaller groups and better assess progress and understanding. This not only benefits gifted and talented students, but students who are struggling as well, because planning is well thought out to help each group achieve.

Before beginning, the teacher should put students into four different groups according to their needs and level of understanding (e.g., one advanced group, two medium groups, and one group with students who need more instruction and practice). I put students names on business card magnets (so I can use and rearrange them all year) and put them on the board under the station that they will be starting at. The following stations can be used as four rotations in one day or two one day and two the next.

Also, stations should be generally be self-correcting so that the teacher is freed up to work with the *teacher team* station. Answer sheets could be made available so that when students are finished, they can check their own work. This requires a high-level of trust and an independent-working classroom atmosphere that must be established prior to doing stations.

Invitation to Learn

Review with students the procedures of how to rotate among the stations, how to record information in their logs, and what to do if they finish one station early.

Instructional Procedures

1. *Teacher team:* At this station, students work with the teacher and do several word problems that involve one-step equations and variables. The teacher serves as a guide and can watch individuals as they go through their thinking process to solve the problems. The level and type of problems can vary according to each group. Students who are struggling can receive more instruction and guided practice with simple problems as the teacher corrects any mistakes or misunderstandings; students who have mastered word and real-life problems can be introduced to two-step problems or problems using larger numbers. Have your lowest group start here so you can re-teach and give them more help before they move to the other stations. Use "Find the Missing Addend" worksheet for this station.
2. *Practice:* At this station, students work on solving a variety of simple one-step equations with single variables. These are not word problems but lists of equations for the students to solve. Use the "Equation Mysteries" worksheet for this station. Problems 11-15 have simultaneous equations with two-variables. This will be a challenge but some students could be ready for it.
3. *Project:* Here students may complete a project that could possibly extend over a few days. This station usually involves building or making something that directly relates with the subject matter previously taught. At this station, students will be able to freely explore with pattern blocks to create designs, patterns, and tessellations. If the pattern blocks are cut from paper, the students could glue down their design for display. If they are working with pattern block sets, then the students could continue to create designs as time allows.
4. *Proof:* This station usually involves doing a few problems with emphasis on having students explain how they came to a solution. Here students will use "in/out" cards, decide what the rule is for each card, and write the rule in an algebraic sentence. Students can start on different cards, but make them aware that the rules do become more complex as the card number increases. Manipulatives like beans or counters should be available for the students to use if wanted.

Materials

- ❏ Paper and pencil
- ❏ Station Log
- ❏ Pattern blocks
- ❏ In/Out function cards (prepare ahead of time)
- ❏ Word problems with variables
- ❏ Equation practice problem sheets

Curriculum Integration

A fifth station could be added that focuses on Science. Students work with their group to look for patterns in the Science or Social Studies topic that you are discussing at that time. For example, during the "Earth's Features" unit, students could look for patterns of earthquakes or volcanic eruptions. Students could describe patterns in landforms and how future patterns could be predicted according to erosion and weathering. Students could look at patterns created by magnets and iron shavings during the "Magnet" unit.

Many patterns emerge in history and students could look for patterns and make connections in their study of America. This could lead to written reports or graphic organizers showing the repeating patterns.

Possible Extensions/Adaptations

As mentioned, adaptations can be made with each group of students. If one group needs addition practice with equations, you could have them do that for two station rotations. Or, if one group doesn't need you as the teacher to guide them at the teacher team station, you could let them work independently while you assisted other groups. Stations allow for differentiation of curriculum and of teaching processes.

Assessment Suggestion

Students use a "station log" to record what activities they worked on, whether the activity was completed or not, what they learned while working, and how they scored (if applicable). This log should be kept in a journal/folder that could also hold any worksheets they worked on and lined paper to show any work done.

If stations are used frequently, students could then turn in their logs to you every week or every other week for you to look over and evaluate. The best assessment often comes from the actual "working out" of the problems and the notes taken while completing the various station requirements.

Additional Resources

Family Math: the Middle School Years by Virginia Thompson and Karen Mayfield-Ingram

Cooperative Learning and Mathematics by Beth Andrini

Challenge Math for the Elementary and Middle School Student by Edward Zaccaro

Lessons for Algebraic Thinking by Maryann Wickett, Katharine Kharas, and Marilyn Burns

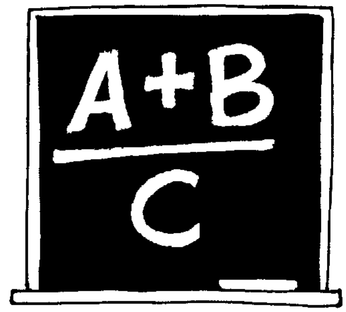
Teaching Gifted Kids in the Regular Classroom by Susan Winebrenner

The Differentiated Classroom: Responding to the Needs of All Learners by Carol Ann Tomlinson

www.learner.org/teacherslab/math/patterns/mystery

* This online activity could also be used by a few students or an individual as an extra math station or early finisher.

Find the Missing Addend



* First write the equation using one or more variables, then solve the problem.

1. Rose had some marbles. Betty had 22 marbles. Together they had 50. How many marbles did Rose have?

2. Tommy made $12 mowing lawns which he added to his savings. He now has $38. How much did Tommy have in his savings before?

3. There were 54 children at a party. Some more children came to the party and then there were 99 children. How many more children came?

4. Dana bought a cd on sale for $8.50. She saved $4.75. How much was the cd at regular price?

5. Alan bought a new helmet for $7.00 and also bought a pair of knee-pads. He had $20.00 to begin with and came home with $3.25. How much were the knee-pads?

6. Jennifer invited some friends to a party. Twice as many people showed up than what she invited. Not including her, there was a total of 24 people at her party. How many friends did she invite?

7. For dinner, Matt bought some pizzas and cut each one into 8 slices. He ate three slices while waiting for everyone else to come to the table and then there were 21 slices left. How many pizzas did he buy?

8. My secret number is 24 more than Mary's. Her number is 31. What is my number?

9. Blake's secret number is half of Michael's. If Michael's secret number is 128, what is Blake's?

10. Caitlin's secret number is 9. Haley's secret number is 7 more than 4 times Caitlin's. What is Haley's number?

11. Rebecca has ten less than the sum of Tessa's number and Emily's number. If Emily's number is 42 and Tessa's number is 28, what is Rebecca's number?

12. If Ken's secret number is twice John's plus four more; and John's secret number is 78, what is Ken's number?

Equation Mysteries

* Solve each equation to find the missing addend/variable. Make sure to use the variable in your answer (e.g., “n = 4”).

1. $\square + 12 = 20$
2. $36 - \bigcirc = 16$
3. $29 + \triangle = 42$
4. $8 * n = 40$
5. $(20 - n) + 2 = 17$
6. $3(2) + n = 10$
7. $24/n = 8$
8. $(x \bullet 12) = 60$
9. $(3 \bullet 7) + y = 32$
10. $\dfrac{2 + 8}{x} = 2$

* The following problems involved simultaneous equations and use two variables. Have fun and good luck!

11. $\bigcirc + \square = 8$
    $\bigcirc - \square = 4$ ______________________________

12. $\bigcirc + \square = 10$
    $\bigcirc - \square = 4$ ______________________________

13. $\bigcirc + \square = 12$
    $\bigcirc - \square = 6$ ______________________________

14. $\bigcirc + \square = 11$
    $\bigcirc - \square = 5$ ______________________________

15. $\bigcirc + \square = 18$
    $\bigcirc - \square = 0$ ______________________________

Math Station Log

Date	Station Name	Description of Activity	Finished	Score

In/Out Functions Cards

In/Out Functions Card #1

In	Out
1	100
2	200
3	300
4	400
5	500
6	600

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #2

In	Out
1	4
2	8
3	12
4	16
5	20
6	24

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #3

In	Out
1	1
2	4
3	9
4	16
5	25
6	36

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #5

In	Out
1	5
2	6
3	7
4	8
5	9
6	10

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #4

In	Out
1	0
2	1
3	2
4	3
5	4
6	5

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #6

In	Out
5	15
8	25
10	30
15	45
20	60
25	75

1. **What's the rule?**
2. **What are the three more examples?**
3. **What is the equation?**

In/Out Functions Card #7

In	Out
1	3
2	5
3	7
4	9
5	11
6	13

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #8

In	Out
18	9
16	8
14	7
12	6
10	5
8	4

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #10

In	Out
20	4
25	5
30	6
35	7
40	8
45	9

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #9

In	Out
1	1.5
2	2.5
3	3.5
4	4.5
5	5.5
6	6.5

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #11

In	Out
1	7
2	16
3	25
4	34
5	43
6	52

1. What's the rule?

2. What are the next three numbers?

3. What is the equation?

In/Out Functions Card #12

In	Out
1	4
2	9
3	14
4	19
5	24
6	29

1. What's the rule?

2. What are the next three numbers?

3. What is the equation?

In/Out Functions Card #13

In	Out
1	4
2	7
3	10
4	13
5	16
6	19

1. What's the rule?

2. What are the next three numbers?

3. What is the equation?

In/Out Functions Card #14

In	Out
0	2
1	7
2	12
3	17
4	22
5	27
6	32

1. What's the rule?

2. What are the next three numbers?

3. What is the equation?

In/Out Functions Card #15

In	Out
8	7
12	9
20	13
24	15
30	18
100	53

1. What's the rule?

2. What are the three more examples?

3. What is the equation?

In/Out Functions Card #16

In	Out
1.5	3.0
2.4	4.8
3.0	6.0
4.75	9.50
5.1	10.2
7.3	14.6

1. **What's the rule?**
2. **What are the three more examples?**
3. **What is the equation?**

In/Out Functions Card #17

In	Out
1	1/2
2	1
3	1 1/2
4	2
5	2 1/2
6	3

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #18

In	Out
1	6
2	9
3	12
4	15
5	18
6	21

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #20

In	Out
3	33
0	-3
10	117
4	45
2	21
5	57

1. **What's the rule?**
2. **What are the three more examples?**
3. **What is the equation?**

In/Out Functions Card #19

In	Out
1	6
2	8
3	10
4	12
5	14
6	16

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

Activity—Patterns with Decimals

Standard II
Students will use patterns and relations to represent and analyze mathematical situations using algebraic symbols.

Objective 1
Recognize, analyze, and use patterns to describe their attributes.

Intended Learning Outcomes
2. Become mathematical problem solvers.

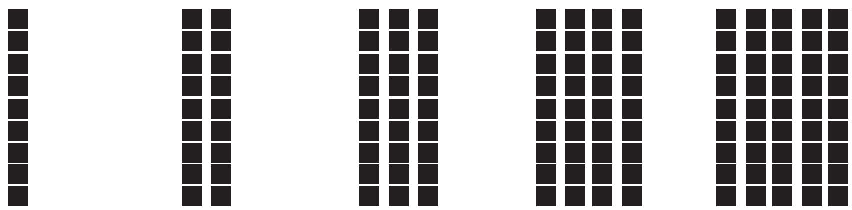
Standard II

Objective 1

Connections

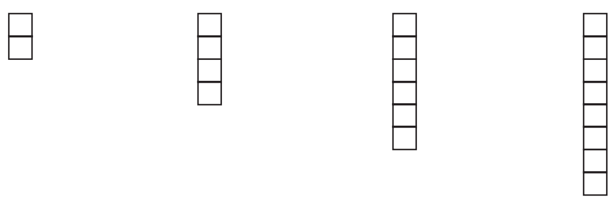
Invitation to Learn

Use the overhead base ten blocks and build a few patterns such as:



0.1 0.2 0.3 0.4 0.5

AND



0.02 0.04 0.06 0.08

Have students draw and write down the numeric names under each one.

Instructional Procedures

1. Pass out the envelopes with the sequence cards to each group along with several blank hundreds square worksheets.
2. There are three sets of patterns with eight cards in each sequence. The objective is for the students to work together to determine which sequence belongs with each of the cards.
3. Have them distribute the cards between the group members. The team must work together to decide which cards go in which sequence. The numbers will relate through an obvious

Materials

- ❏ Overhead base ten blocks (cubes, longs, and flats)
- ❏ Hundreds squares worksheet
- ❏ Decimal sequencing cards (one set per group)
- ❏ Paper and pencil

pattern and though some numbers may seem to go with two sets, there should only be 8 cards in each sequence and every card must be used.

4. Once the students have finished and have laid their sequences out on their desks, they will use the hundredths square worksheet and color in the decimal representations in the order of the pattern. It should become clear if their patterns are correct by looking at the pictures.
5. When teams have finished, have some share what they found. If someone has found a sequence that differs from the original one, discuss to make sure it is truly a consistent pattern.

Possible Extensions/Adaptations

Students could create their own decimal pattern cards to share with other groups (similar to this activity).

Some students may benefit from using the base ten blocks or by coloring in the decimal amount before trying to find which sequence the card belongs to.

Assessment Suggestion

Watch and listen as students discuss what pattern rules they find and how they sequence their cards.

You could have them individually color in the squares matching their cards as an assessment of knowing how to represent different decimal amounts.

Additional Resources

Hands-On Math by Frances M. Thompson
www.uen.themepark/patterns

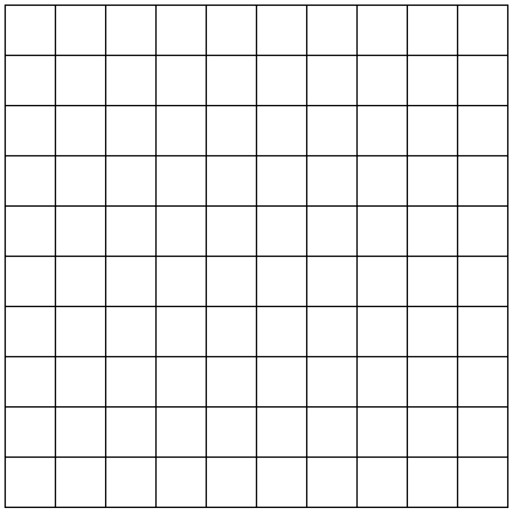
Homework & Family Connections

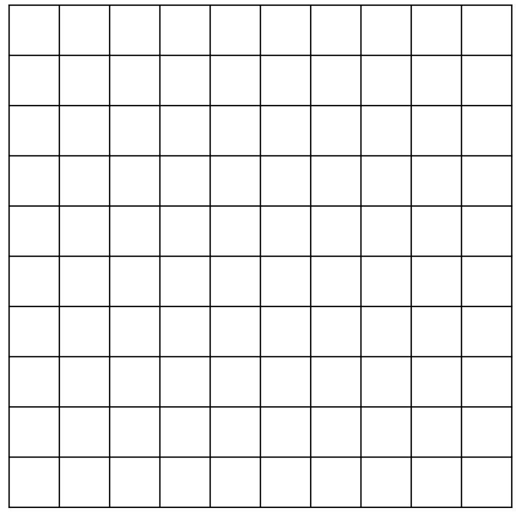
Students could create their own decimal pattern cards (see Possible Extensions/Adaptations) at home and have a family member see if they can put the correct numbers of a pattern sequence in the correct group.

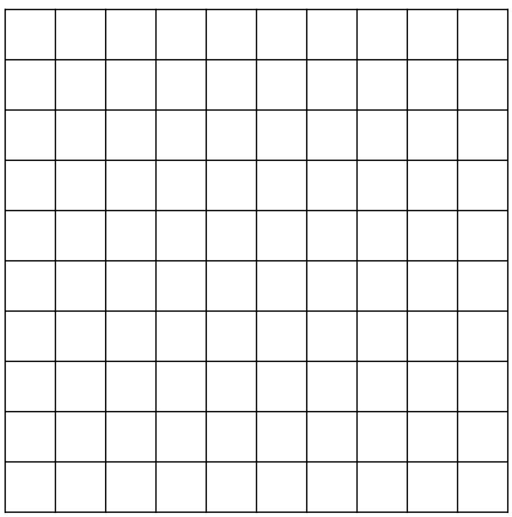
Decimal Sequencing Cards

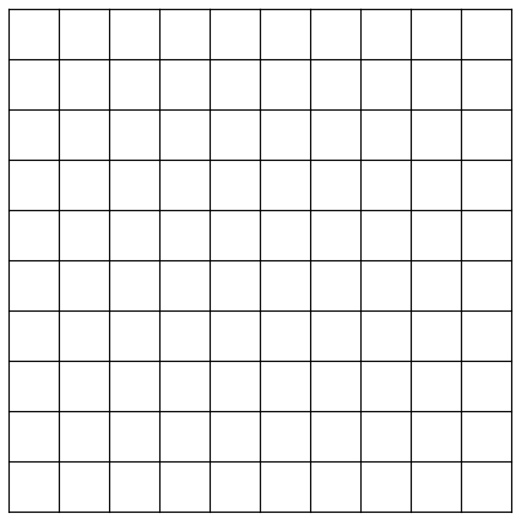
0.01	0.03	0.2
0.02	0.07	0.4
0.04	0.11	0.6
0.08	0.15	0.8
0.16	0.19	1.0
0.32	0.23	1.2
0.64	0.27	1.4
1.28	0.31	1.6

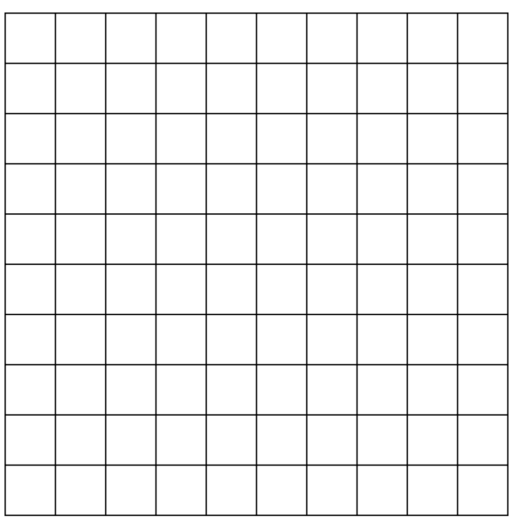
Hundreds Squares Worksheet

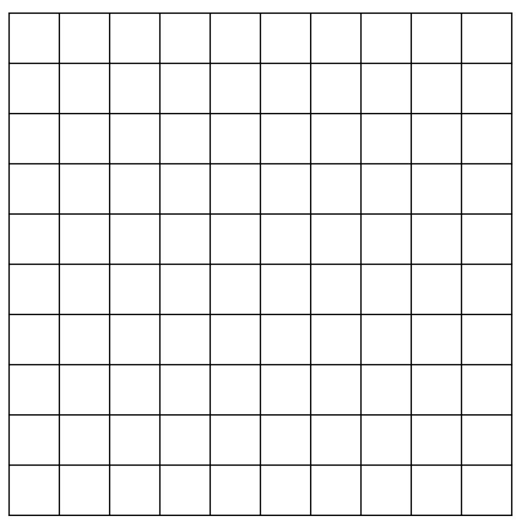












Answer Key for "In/Out Functions Cards"

1. n x 100 = y
2. n x 4 = y
3. n x n = y
4. n – 1 = y
5. n + 4 = y
6. n x 3 = y
7. (n x 2) + 1 = y
8. n ÷ 2 = y
9. n + 0.5 = y
10. n ÷ 5 = y
11. (n x 9) – 2 = y
12. (n x 5) – 1 = y
13. (n x 3) + 1 = y
14. (n x 5) + 2 = y
15. (n ÷ 2) + 3 = y
16. n x 2 = y
17. n ÷ 2 = y
18. (n x 3) + 3 = y
19. (n + 2) x 2 = y
20. (n x 12) – 3 = y

Answer Key to Equation Mysteries Worksheet

1. 8
2. 20
3. 13
4. 5
5. 5
6. 4
7. 3
8. 5
9. 11
10. 5
11. Circle = 6, Square = 2
12. Circle = 7, Square = 3
13. Circle = 9, Square = 3
14. Circle = 8, Square = 3
15. Circle = 9, Square = 9

Answer Key to Find the Missing Addend Worksheet

1. 28 marbles
2. $26
3. 45 children
4. $13.25
5. $9.75
6. 12 friends
7. 3 pizzas
8. 55
9. 64
10. 43
11. 60
12. 160

Standard III Activities

Activity—Pass It On

Standard III
Students will use spatial reasoning to recognize and describe geometric shapes and principles.

Objective 1
Describe, identify, and analyze characteristics and properties of geometric shapes.

Intended Learning Outcome
4. Communicate mathematically.

Standard III

Objective 1

Connections

Background Information

Using accurate terminology is a significant part of understanding geometry. Mathematics has its own language. In order for students to comprehend directions and results, they must be familiar with that language.

Ray: A part of a line that has one endpoint and goes on forever in one direction.

Angle: Two rays sharing a common endpoint.

Equilateral: A triangle with all sides and angles equal.

Isosceles: A triangle that has exactly two congruent sides.

Scalene: A triangle that has no congruent sides.

Right angle: An angle that measures exactly 90º.

Acute angle: An angle that measures less than 90º.

Obtuse angle: An angle that measures greater than 90º but less than 180º.

Pyramid: A polyhedron whose base is a polygon, and whose other faces are triangles that share a common vertex.

Prism: A three-dimensional figure with two parallel and congruent faces that are polygons. The rest of the faces are parallelograms.

Vertex: The point at which two line segments, lines, or rays meet to form an angle.

Edge: The line segment where two faces of a solid figure meet.

Face: The plane figure that serves as one side of a solid figure. The faces of a cube are square.

Triangle: A polygon with three sides.

Polygon: A closed plane figure made by line segments.

Invitation to Learn

The teacher should say a few foreign words or phrases. Ask the students if it was a compliment, statement, question, or direction. State that if we do not know the language, it is hard to understand what to do. This lesson will allow the students an opportunity to practice using the geometric language.

Instructional Procedures

Materials

- ❑ Pass It On Cards:

  Cards are made by listing a geometric term on one side of the card and a definition of a different term on the back side of the card.

1. Divide the class into groups of approximately 12 students (adjust the number of cards used to the size of the groups). Each student receives one card.
2. Randomly select one student to read the definition on their card. The student who holds the geometric term that matches that definition will stand and say the term. Once it is agreed that the response is correct, the standing student then reads the definition on the back of their card.
3. Again, a new student will identify the definition as a match to their card and will stand and say their term, and then read their definition.
4. The practice continues until the chain is complete and all students have read their definition.
5. If time permits, redistribute the cards and repeat the activity.

Possible Extensions/Adaptations

This game could be expanded or adapted by adding additional geometric or other mathematical terms.

An adaptation of this activity could be played with one player describing, defining, and drawing the term until his partner can name it. See how many terms each team can name in 60 seconds.

Assessment Suggestions

Provide each student with a list of 15-20 geometric terms and a piece of blank paper. Have them cut their geometric terms up and then sort them into groups. As a teacher you should NOT give them a predetermined number of groups or suggestions for sorting. When they have their words sorted, have them glue them on to the blank sheet of paper into groups and then add a label to describe each group.

Additional Resources

A Cloak for the Dreamer by Aileen Friedman
Sir Cumference and The Great Knight of Angleland by Cindy Neuschwander
Marvelous Math: A Book of Poems by Lee Bennett Hopkins

Home & Family Connections

Send home a list of the terms and their definitions. The family could cut apart the cards, turn them all over and then play a game of "memory" trying to match definitions with terms.

Pass It On Cards

Angle	Vertex	Acute Angle
Pyramid	Obtuse Angle	Edge
Face	Polygon	Equilateral
Ray	Triangle	Isosceles
Right Angle	Scalene	Prism

The plane figure that serves as one side of a solid figure. The faces of a cube are square.	A triangle that has exactly two congruent sides.	A three-dimensional figure with two parallel and congruent faces that are polygons. The rest of the faces are parallelograms.
An angle that measures exactly 90°.	A polygon with three sides.	Two rays sharing a common endpoint.
The point at which two line segments, lines or rays meet to form an angle.	An angle that measures less than 90°.	A triangle that has no congruent sides.
A polyhedron whose base is a polygon, and whose other faces are triangles that share a common vertex.	A closed plane figure made by line segments.	An angle that measures greater than 90° but less than 180°.
A part of a line that has one endpoint and goes on forever in one direction.	The line segment where two faces of a solid figure meet.	A triangle with all sides and angles equal.

Activity—Match My Masterpiece Using Ordered Pairs

Standard III

Objective 2

Connections

Standard III
Students will use spatial reasoning to recognize, describe and identify geometric shapes and principles.

Objective 2
Specify locations and describe spatial relationships using coordinate geometry.

Intended Learning Outcome
4. Communicate mathematically.
6. Represent mathematical situations.

Background Information

The standard coordinate grid is sectioned into four quadrants. In the elementary math curriculum, the students are introduced to the first quadrant, which is the top, right quadrant and uses only positive numbers for coordinates (also called ordered pairs). The first number given in an ordered pair moves horizontally on the grid along what is called the "X" axis. The second number in the ordered pair moves vertically on the grid along what is called the "Y" axis.

Coordinate geometry is tied closely to algebra and the two are used together to solve many real world problems. Points, lines, line segments, and rays can all be graphed on the coordinate system. It is useful in building design, landscaping, space exploration, and computer animation to name just a few.

Invitation to Learn

On a large, plain white piece of paper, put a small X. Ask the students to describe to you the precise location of the X. Is it in the exact middle? Towards the top? Closer to the right or left sides? If you gave them each a piece of paper that was exactly the same size, could they put an X on their paper in the exact same spot? After discussing the difficulty of accomplishing that task, put an X on a large piece of graph paper (or use a transparency on overhead projector). Now, could they replicate your X on their own paper. Demonstrate how the axis of a grid make locating a spot much easier.

Demonstrate the plotting points that would create the letter "H" and then how each point has an ordered pair which corresponds to that point. Connect the lines to show the letter. Follow up with the letter "I" and then write out the ordered pairs used to create that letter.

Instructional Procedures

1. Give each student (3) 6x6 grids and have them use plot points to create a three letter word on their paper.
2. Once the three letters are plotted, list the ordered pairs table used for each individual letter.
3. Display the original grids in the classroom. Have students switch their sheets of ordered pairs.
4. Have students plot out the points which correspond with the ordered pairs they are given and connect them to form words.
5. Them have students match up their work with the original.

Materials

For each student:

- ❏ Graph paper sectioned into (3) 6x6 grids (can also use blackline master included with this plan)
- ❏ Colored pencil or marker

Curriculum Integration

Math/Science—Language: writing, spelling. Art: graphic design.

Social Studies—map skills.

Possible Extensions/Adaptation

Instead of creating three-letter words, the students could create simple graphic designs, such as a star, geometric shape, or seasonal design.

Students could plot out a spelling word.

Assessment Suggestions

A teacher-created list of ordered pairs could be given to each student, who would then turn the numbers into point on the grid.

Additional Resources

A Fly on the Wall by Julie Glass

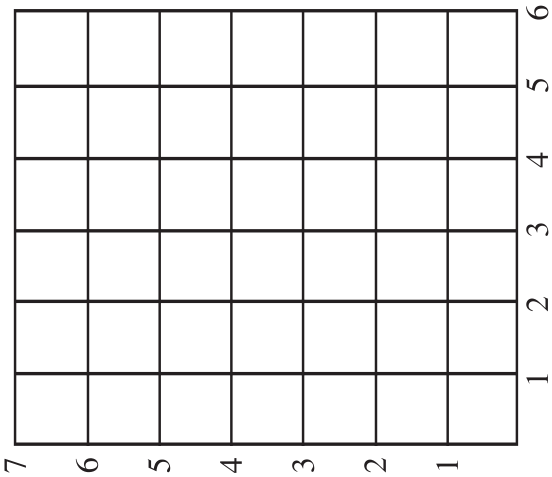
Community resources

Computer graphics programmers, computer graphic artists, lighting engineers, landscape architects.

Home & Family Connections

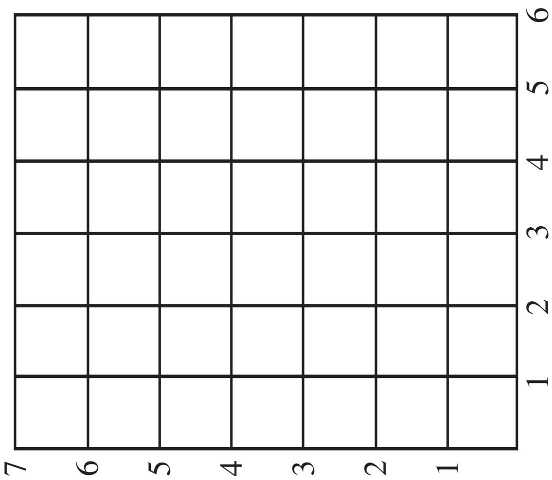
Parents could write code messages using ordered pairs.

Match My Masterpiece



Coordinates for letter #1

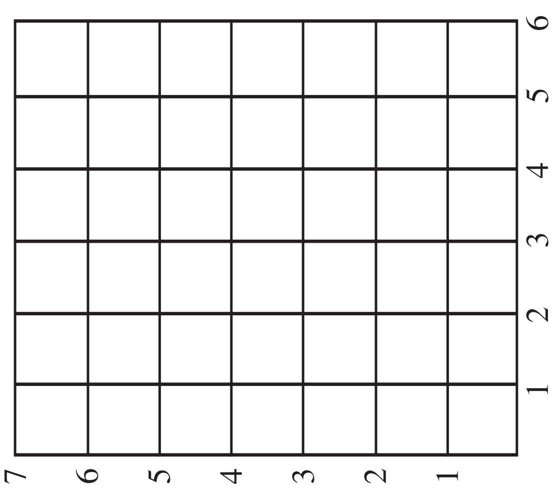
Letter formed: ______



(Cut along this line)

Coordinates for letter #1

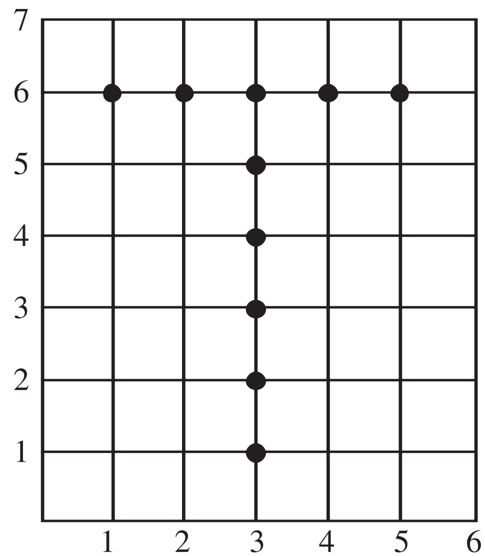
Letter formed: ______

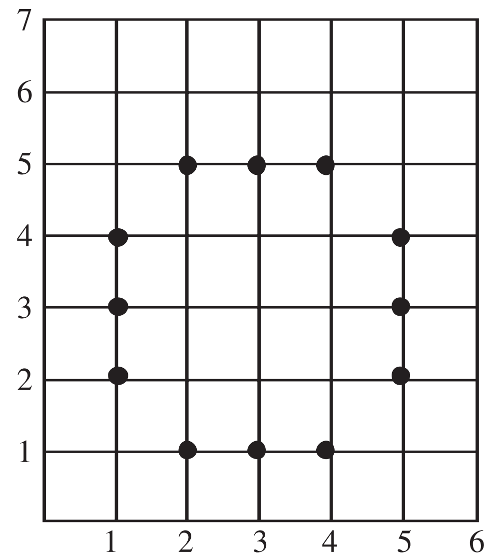


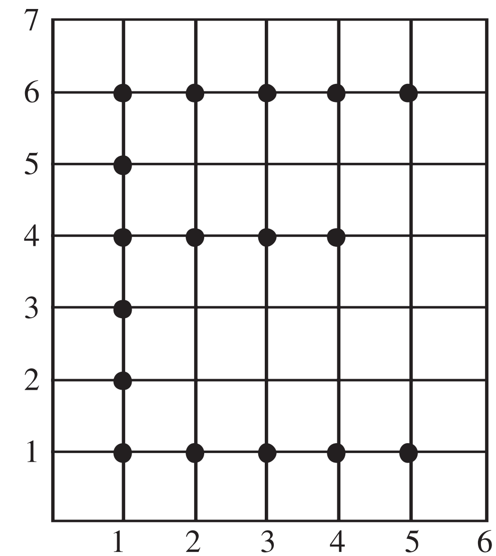
Coordinates for letter #1

Letter formed: ______

Sample Match My Masterpiece







(Cut along this line)

Coordinates for letter #1

(1,6)
(2,6)
(3,1)
(3,2)
(3,3)
(3,4)
(3,5)
(3,6)

Letter formed: ____________

Coordinates for letter #1

(2,1)
(3,1)
(4,1)
(1,2)
(5,2)
(1,3)
(5,3)
(1,4)
(5,4)
(2,5)
(3,5)
(4,5)

Letter formed: ____________

Coordinates for letter #1

(1,1) (2,6)
(1,2) (3,6)
(1,3) (4,6)
(1,4) (5,6)
(1,5) (2,4)
(1,6) (3,4)
(2,1) (4,4)
(3,1)
(4,1)
(5,1)

Letter formed: ____________

Activity—Mountain Rescue Mission

Standard III

Objective 2

Connections

Standard III
Students will use spatial reasoning to recognize and describe geometric shapes and principles.

Objective 2
Specify locations and describe spatial relationships using coordinate geometry.

Intended Learning Outcomes
2. Become mathematical problem solvers
3. Reason mathematically.

Background Information

Coordinate geometry uses numeric methods to represent a location. The most commonly used coordinate system today is that of longitude and latitude with its angles represented in degrees, minutes, and seconds. The movement from one point to an adjacent point, either horizontally or vertically, is considered one unit. As an example, to move three sections to the right and four sections up in a grid would be considered seven units (3 + 4).

Invitation to Learn

Ask the students to close their eyes and imagine that they are flying an airplane, but they are in a thick cloud. How would they get safely to their destination? How could an air traffic controller give them directions to move and avoid mountains and other aircraft? If they were in the middle of the ocean, how would they describe their location so that someone could find their boat if necessary?

Show a map and globe and the lines of latitude and longitude. These are used in the same method that we use a quadrant grid.

Instructional Procedures

Materials

- ❏ Grid paper (10x10) one for each team, numbered using the coordinate system
- ❏ Colored pencils or markers – a different color for each member
- ❏ 2 dice

1. Divide the class into groups of two.
2. In this game, the rescue teams are at the base of the hill (ordered pair to be determined by roll of dice) and the injured victim is at the top of the hill (ordered pair (10,10)). The objective is to get to him as quickly as possible.
3. Player #1 rolls two dice and uses the numbers rolled as the ordered pair for his base camp. Player #2 starts in the same manner.

4. On each successive roll, the players use 1 die. Once the number is rolled, the player must decide how to move the rescue team up the hill. For example, if a 5 is rolled, he must decide if it should be used as a vertical move of 5 units straight up, or to combine horizontal and vertical movement such as up 3 and over 2. At no time can the rescue team go off the grid.
5. Once he decides how he is going to use his roll, he must write the new ordered pair of his location and the new point is recorded on the coordinate plane.
6. The first player to land exactly on (10,10) is the winner.

   Alternate: The player who reaches the victim with the shortest pathway is the winner.

Curriculum Integration

Social Studies—Mapping skills, longitude and latitude.

Possible Extensions/Adaptations

Battleship game

Assessment Suggestions

Anecdotal notations and student self-evaluations.

Additional Resources

A Fly On the Wall by Julie Glass
GPS unit, pilot, search and rescue team member

Home/Family Connections

Tic-Tac-Toe

The game is played in the traditional way except that the grid is larger and the X's and O's are placed on the intersection rather than in the spaces.

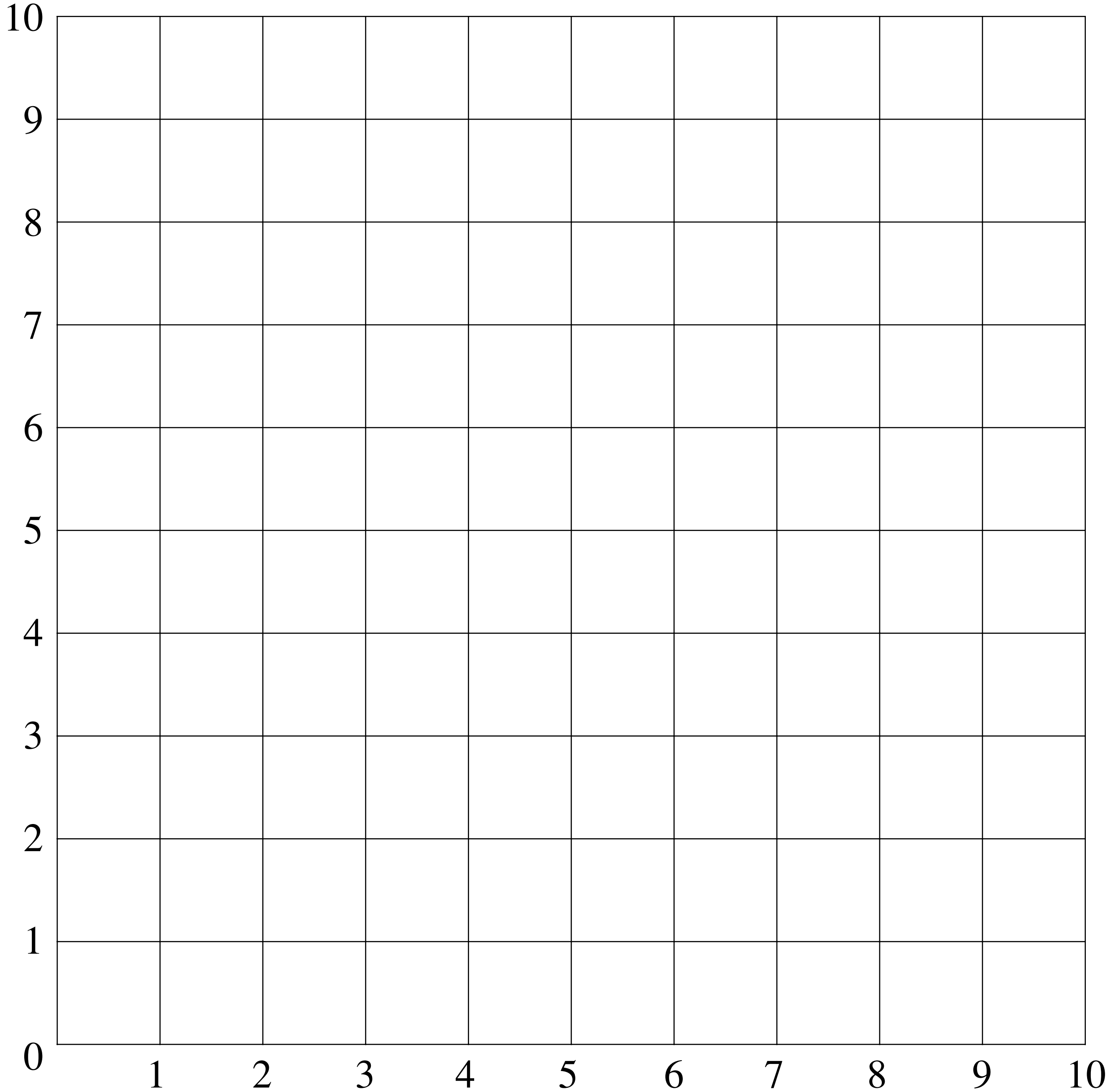
1. The goal is to get 4 X's or 4 O's in a row.
2. The spot where the X's or O's are placed must be given using their ordered pair.
3. An adult could act as the game host and will mark both the X's and O's on the grid, using the coordinates (ordered pairs) given. The other family members will compete against each other, one representing the X's and the other O's.

4. Players take turns naming ordered pairs. The points MUST be named with ordered pairs.
5. The goal is to get 4 X's or 4 O's in a row.

Adaptation

Try playing with more than two players at a time. Instead of using X's and O's, each player could use their own color to mark points.

10 x 10 Grid Paper



Activity—Try An Angle

Standard III

Objective 1

Connections

Standard III
Students will use spatial reasoning to recognize and describe geometric shapes and principles.

Objective 1
Describe, identify and analyze characteristics and properties of geometric shapes.

Intended Learning Outcomes
3. Reason mathematically.
4. Communicate mathematically.

Background Information

Triangles are defined as three-sided polygons which have straight, rather than curved, sides. The sum of the three angles of a triangle equals 180º. Triangles are classified by similarities (or differences) in lengths of the sides and measurements of the angles. An ***equilateral triangle*** has all three sides of the same length. An ***isosceles triangle*** has two sides that are the same length. A ***scalene triangle*** has no sides that are the same length. A ***right triangle*** has one right angle, an ***acute triangle*** has all acute angles (less than 90º), and an ***obtuse triangle*** has one angle that is greater than 90º.

Invitation to Learn

Give each student a set of die-cut tangrams. See how many different ways they can arrange 2,3,4…7 pieces to create triangles. Classify the different triangles by sides and angles.

Once a triangle is constructed, recreate the pattern on paper.

Instructional Procedures

Materials

Each student or team needs:

- ❏ 1 Copy of triangle patterns
- ❏ Tape
- ❏ Protractor

1. Using the above classifications, the students should classify and label the six triangles on their pattern sheet.
2. Tear or cut the corners of one of the triangles.
3. Arrange the angles so they touch, then tape them together.
4. Use a protractor to measure each angle. What is the sum for all three angles?
5. Do the same for the other 5 triangles.
6. List your results in a table.
7. Is there a pattern?

Triangle Type	Right Scalene	Acute Isosceles	Obtuse Isosceles	Obtuse Scalene	Right Isosceles	Equilateral Acute
Sum of Angles						

Curriculum Integration

Math—Standard IV Objective 2c: Measuring with a protractor.

Possible Extensions/Adaptations

Cut several strips of centimeter graph paper into strips that are 1 cm X 18 cm. Try folding them into triangles with the following side lengths:

A. 6 cm, 6 cm, 6 cm B. 6 cm, 4 cm, 7 cm C. 7 cm, 4 cm, 2 cm

D. 4 cm, 4 cm, 8 cm E. 5 cm, 5 cm, 8 cm F. 7 cm, 4 cm, 7 cm.

Measure, fold, and tape each triangle. Be sure to label the size. What did you learn from this experiment?

Assessment Suggestions

Have students complete a Frayer Model using a term such as isosceles. Adapt the model to include a drawing in one of the quadrants.

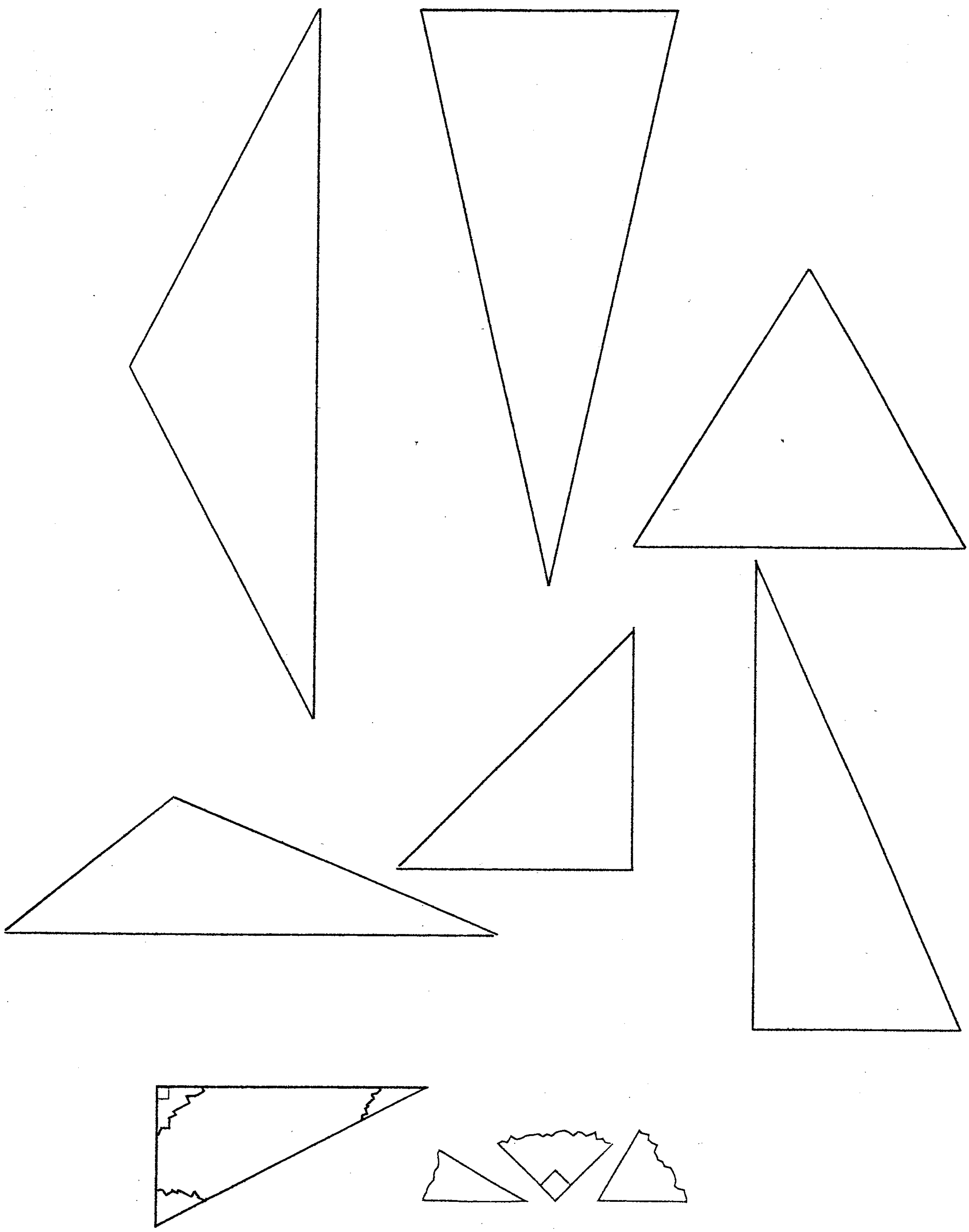
Additional Resources

The Greedy Triangle by Marilyn Burns
Architects and building contractors

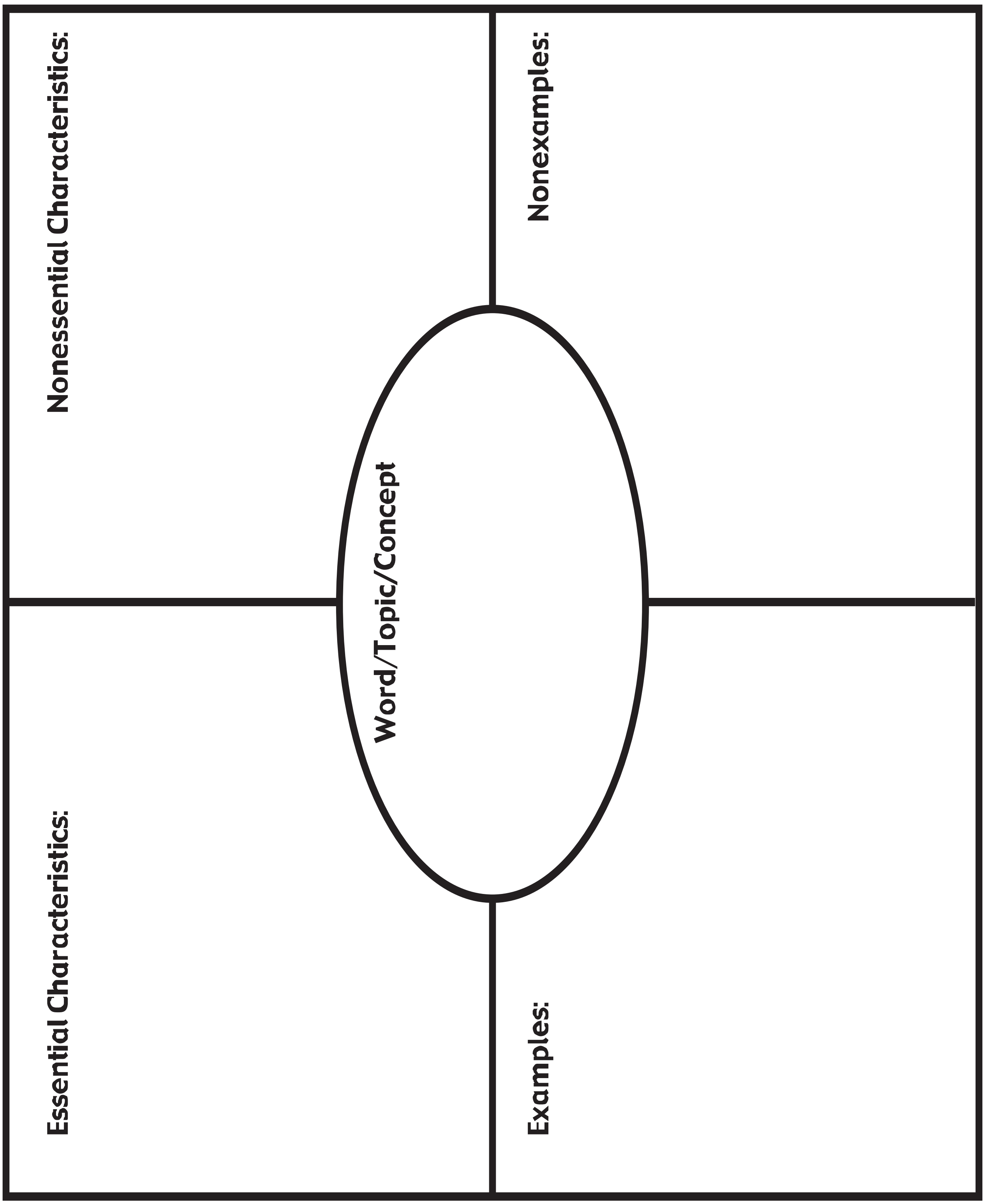
Home & Family Connections

Have students take their set of die cut tangrams home and, with their family members, try to recreate a variety of different triangles, then trace the pieces used in the triangular pattern on a piece of paper. Once the triangles are drawn, label each triangle as equilateral, isosceles, scalene, right, or obtuse.

Triangle Patterns



Frayer Model



Activity—Prisms, Pyramids, and Nets

Standard III

Objectives 1 & 2

Connections

Standard III
Students will use spatial reasoning to recognize, describe and identify geometric shapes and principles.

Objective 1
Describe, identify and analyze characteristics and properties of geometric shapes.

Objective 2
Visualize and identify geometric shapes after applying transformations and identify lines of symmetry.

Intended Learning Outcomes
2. Become mathematical problem solvers.

Background Information

Each geometric solid has a flat side called a ***face.*** They have ***edges*** which connect the faces. The corners are known as ***vertices*** (plural), or ***vertex*** (singular).

A jacket for a geometric solid that can be folded to cover the surface of the solid is called a ***net***. A net is a way of representing the polyhedron in two dimensions.

Invitation to Learn

Divide the class into four groups. Give groups A and B a sheet of the same colored paper. Give groups C and D a sheet of the contrasting paper. Have groups A and B measure and cut a rectangle that is 2” by 3.” Have group C cut 2 rectangles that measure 2” by 4.” Have group D cut 2 rectangles that measure 3” by 4.” Collect all six sides and tape them together to form a rectangular prism. Point out the faces, vertices and edges of the figure. Ask the students to identify the shape of each of the faces. Are all of the faces alike? In table form, have them record the numbers of faces, edges, and vertices.

Instructional Procedures

1. Give each group a supply of marshmallows and toothpicks. Have them design a jacket (net) that would go around the outside of the prism to protect it.
2. Have them try their jacket on the solid figure and adjust as necessary.
3. When they have a jacket that encloses the prism, have them unhook a few of the vertices and lay the net out flat.

4. Have the students record the number of faces, vertices, and edges in a table.
5. Using the graph paper, draw the design of the marshmallow/toothpick structure.
6. Repeat the process for a pyramid.

Possible Extensions/Adaptations

Have students construct or design nets for other polyhedra. Add their edge/vertex/face information to the chart and look for a pattern to predict additional information. Let the students discover the equation “face + vertex - 2 = edges” using leading questions such as:

1. Find the sum of the number of faces and vertices.
2. Compare this with the number of edges. What is the difference?
3. Add the sum of faces/vertices for other shapes. What is that difference?
4. Can you write an equation that would fit this problem?

Assessment Suggestions

Rubric

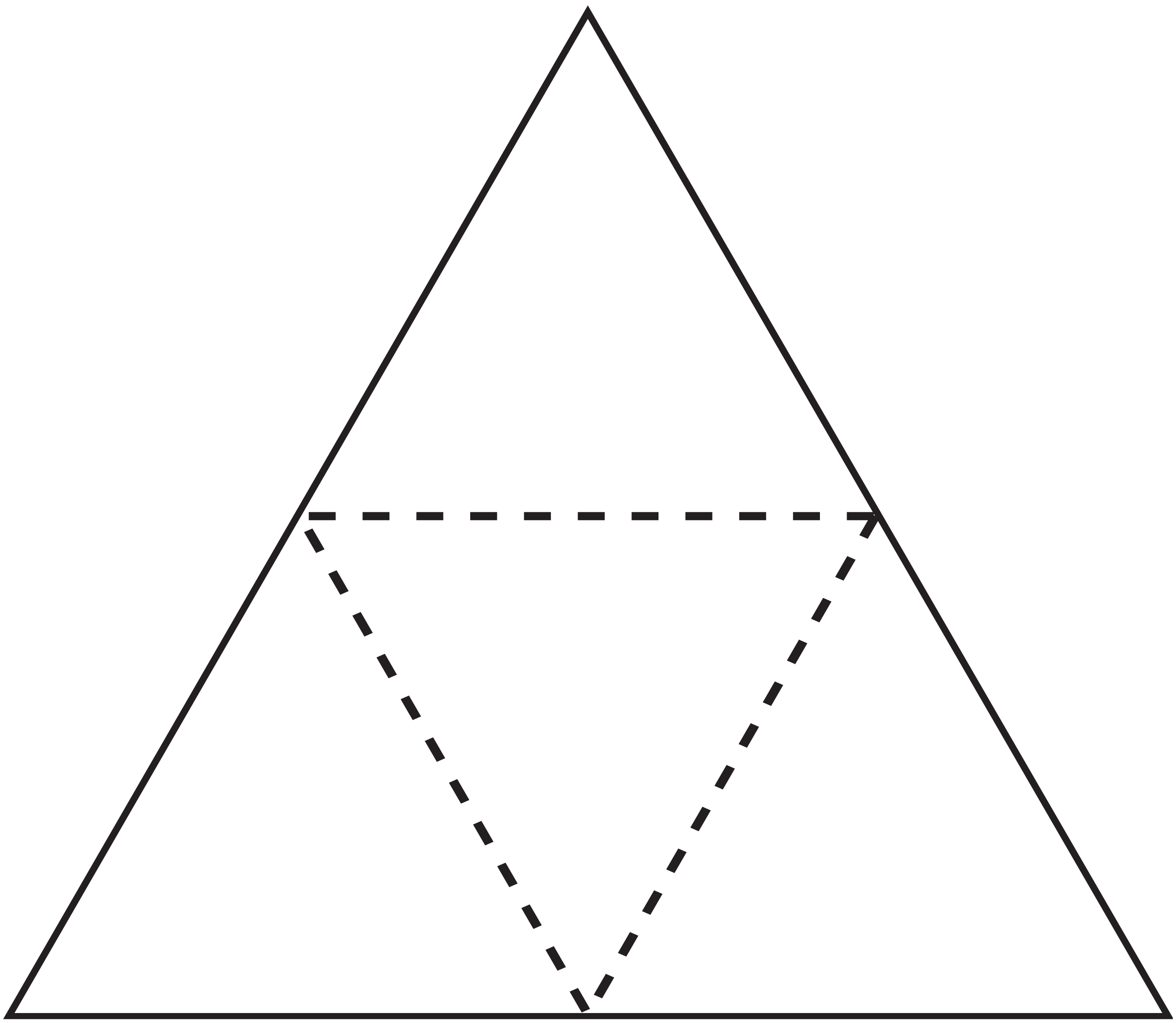
Home/Family Connections

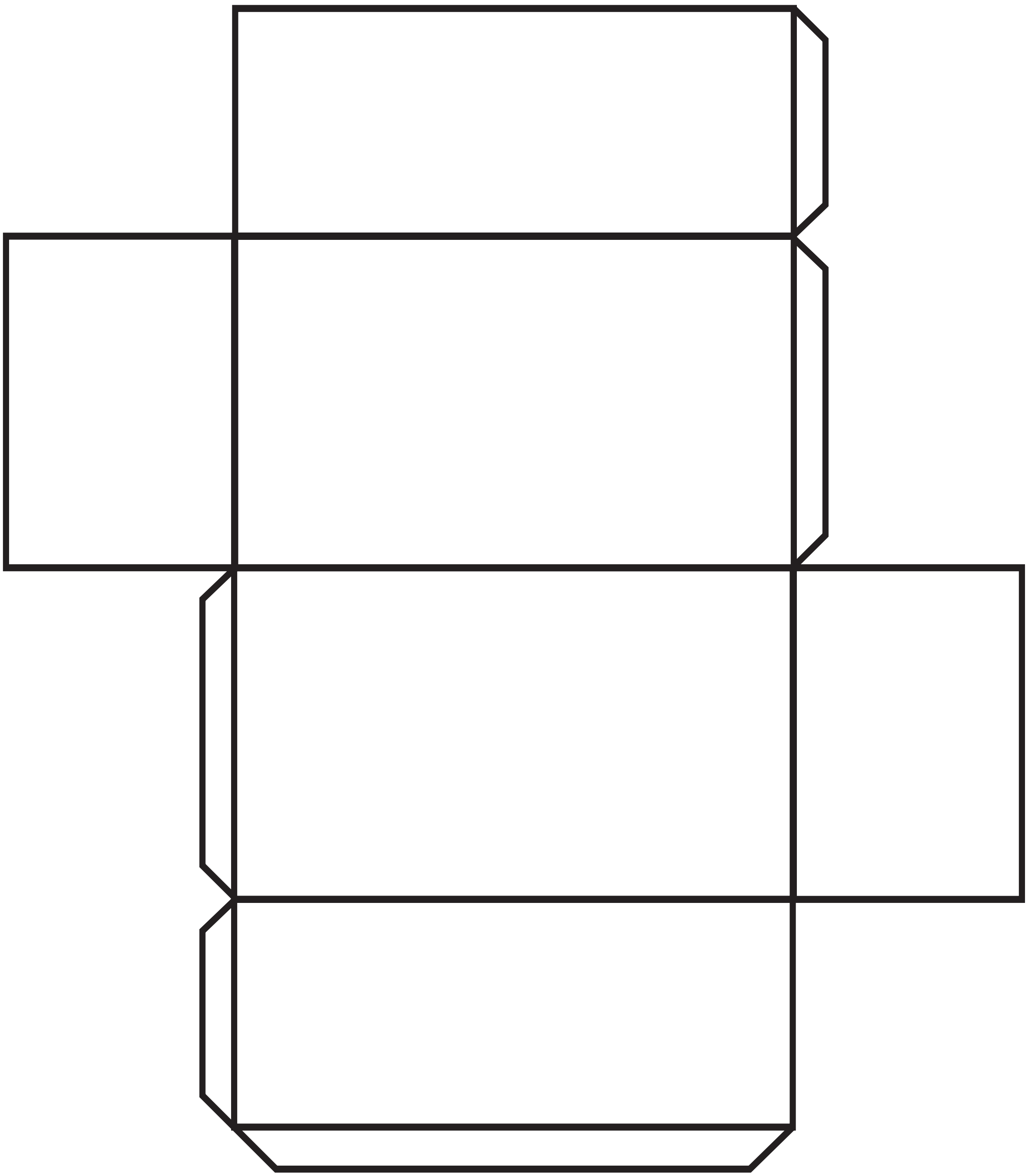
Students and their families can construct a variety of polyhedron nets at home.

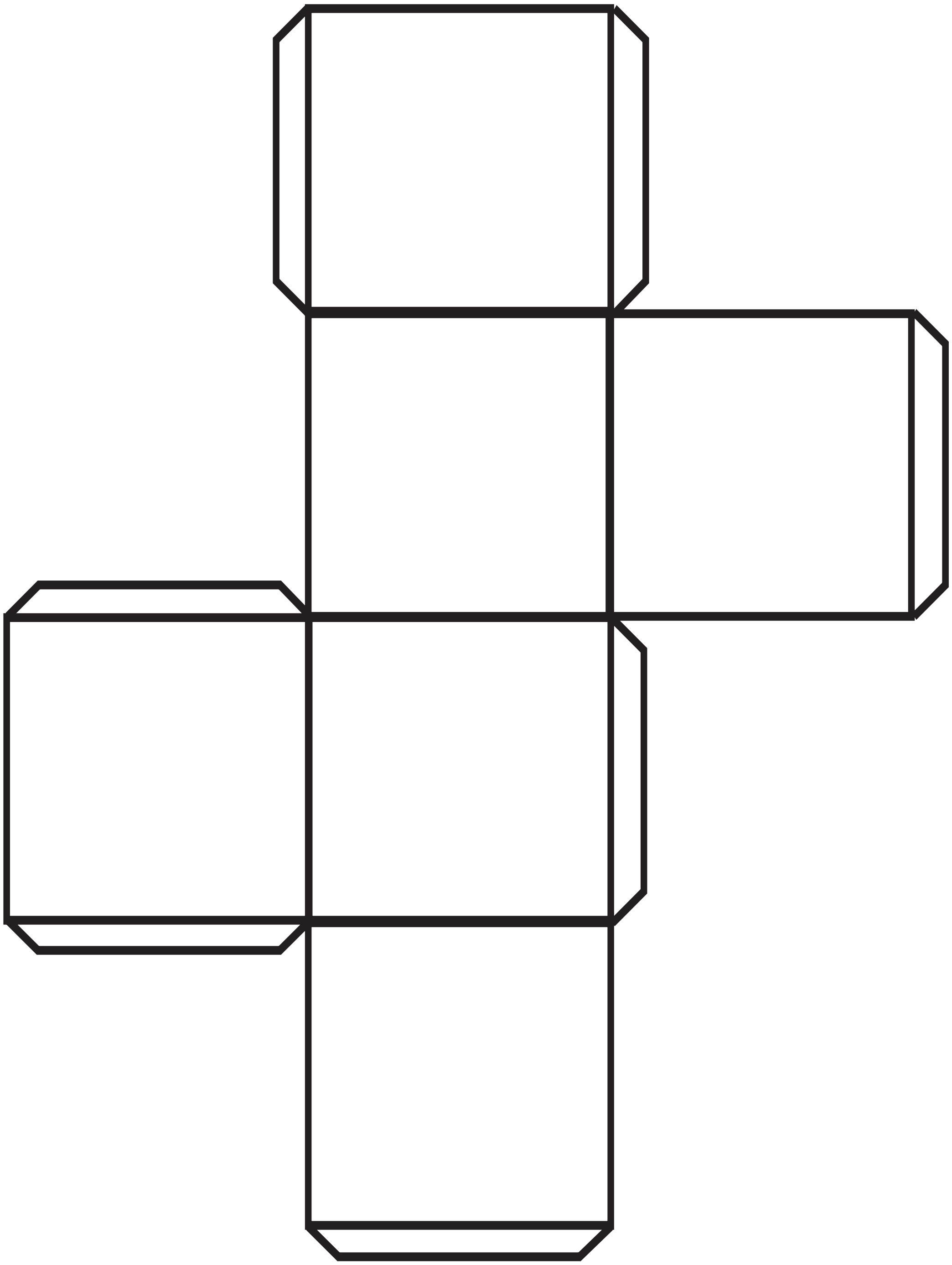
Materials

- ❏ Solid figures of prisms and pyramids
- ❏ Nets which cover the prisms and pyramids
- ❏ Graph paper
- ❏ Scissors
- ❏ Tape
- ❏ Solid colored paper, two contrasting colors
- ❏ Ruler
- ❏ Marshmallows or gumdrops (not too fresh)
- ❏ Toothpicks

Figures for Prisms and Pyramids







Activity—Tessellation Quilt Squares

Standard III
Students will use spatial reasoning to recognize and describe geometric shapes and principles.

Objective 3
Visualize and identify geometric shapes after applying transformations.

Intended Learning Outcomes
5. Make mathematical connections.
6. Represent mathematical situations.

Standard III

Objective 3

Connections

Background Information

Shapes are used in patterns and designs all around us. Symmetries help us classify and organize these patterns. A change in position is called a ***transformation***. When the object moves up, down, or over it is called a ***slide*** or ***translation***. Every translation has a direction and distance. When a mirror image of the object occurs, it is called a ***flip*** or ***reflection***. Every reflection has a mirror line. A turn or rotation rotates the object around a set point. Every rotation has a center and an angle.

Tessellations occur when a surface is completely covered with one or more shape. The pattern has no gaps or holes or any overlapping sections. Typically, tessellations use standard geometric shapes, but as long as the original area is maintained, alterations can provide interesting and creative alternatives in pattern design.

Invitation to Learn

Write the letters ATOYOTA and ask the students if they can see a line of symmetry. (If you draw a line through the middle of the "Y" and then reflect the entire phrase across the line, the left side becomes the right and vice versa.

Have the students brainstorm other words or letter patterns that can reflect across a line ("NAN" "ITI").

Instructional Procedures

Materials

- ❑ 1"x 1" square of heavy cardstock paper
- ❑ 6"x 6" square of white art paper
- ❑ Lead pencil
- ❑ Scissors
- ❑ Colored pencils
- ❑ Tape
- ❑ Quilts, quilt books and/or art by Escher

1. Make a symmetrical pattern by taking an asymmetrical letter and repeating a single translation over and over. Decide on a direction and a distance, for example 3 cm to the right. Then translate your letter 3 cm to the right, then again 3 more cm to the right. Keep repeating the translation until you run off the paper. Also translate the letter 3 cm to the left, and repeat. Did you get any other types of symmetries such as reflections or rotations in the process?
2. Show examples of quilts, quilt books, and art by Escher and point out the symmetry and transformations.
3. Explain tessellations using irregular patterns (see background information above).
4. Distribute a 1" square and 6" square to each student. Have them lightly shade one side of their 1" square (so they can tell front from back).
5. Draw and then cut an irregular design on one side of the small square. The cut should go from corner to corner on the same side of the square.
6. Slide the cut across the square to the opposite side and then, with a partner, tape it to the cube. Make sure that there are no spaces and that the cube or the cut did not get turned over to the colored (back) side. In order to tessellate, the new irregular shape must maintain the same area as the original square. There can be no extra cuts or spaces and they need to make sure that the edges are flush before they tape them. (There is no overlapping.) This small cardstock piece is now "the pattern."
7. Starting in one corner of the 6" piece of art paper, lightly trace the pattern with a pencil. Slide the pattern and trace, repeating until the entire paper is covered (tessellated). Make sure that there is no space between the original pattern and the new pattern to be traced.
8. Using red, white, and blue colored pencils, artistically color in the "quilt blocks" to create a pleasing pattern.
9. Mount all student quilt blocks on a large piece of colored paper to create a class quilt.

Curriculum Integration

Art, Social Studies—Quilts in American History, Underground Railroad Quilts

Possible Extensions/Adaptations

As the students become experts in tessellations using translations, they could create a second quilt square using rotation. Cut one side of the cardstock square and rotate it around a set point and then tape it down. As before, they trace the square on the art paper, but this time, the student rotates it before tracing subsequent patterns. Again, when the paper is completely tessellated, they color it using the predetermined color scheme (red, white, and blue) and then mount their squares with their classmates to create a large class quilt.

Explore additional shapes which will tessellate a plane and find the common features (Hint: measure inside angles).

Assessment Suggestions

Have the student attach a paragraph to their quilt square describing the transformation used and how they created the pattern for their square. Have them identify the initial pattern and then, using the terms translation, reflection or rotation, describe the location and position of at least two other parts of the pattern.

Additional Resources

Eight Hands Round: A Patchwork Alphabet by Ann Whitford Paul
Sam Johnson and the Blue Ribbon Quilt by Lisa Cambell Ernst
Quilting Now and Then by Karen B Willing
The Quilt Block History of Pioneer Days with Projects Kids Can Make by Mary Cobb
Sweet Clara and the Freedom Quilt by Deborah Hopkinson
The Keeping Quilt by Patricia Polacco
The Magic of M.C. Escher by M.C. Escher
Quilts and Quilters, Artists, Craftsmen, Choreographers, Musicians

Home/Family Connections

Identify patterns in their home (wallpaper, upholstery fabrics, etc.) that have some type of translations. Identify which type of pattern movement is used and the overall pattern that was created.

Standard IV Activities

Activity—The Amazing Inch and Measuring Up!

Standard IV
Students will understand and apply measurement tools and techniques.

Objective 1
Identify and describe measurable attributes of objects and units of measurement.

Objective 2
Determine measurements using appropriate tools and formulas.

Intended Learning Outcomes
1. Become mathematical problem solvers.
2. Make mathematical connections.

Standard IV

Objectives 1 & 2

Connections

Background Information

In this activity, students will gain a knowledge and understanding of both the metric and customary systems of measuring length with rulers, meter/yardsticks, and tape measures. Use accurate terminology to demonstrate and explain Metric and Customary Units of Length Measurement. Encourage students to also use correct terminology.

Vocabulary

Customary: A system of measurement used in the United States. The system includes units for measuring length, capacity, and weight.

Units of Length Measurement in Customary System:

one foot (ft or ')	=	12 inches (in or ")
one yard (yd)	=	36 inches
	=	3 feet
1 mile (mi)	=	5,280 feet
	=	1,760 yards

Invitation to Learn

Have students work in partners/cooperative groups to list as many occupations as possible where tools are used for measuring. Students should also list what tools are used with each job. Discuss why it is important to learn how to use these tools in real life situations.

Instructional Procedures

Materials

For each student:

- ❑ (2) 3" x 12" pieces of oak tag
- ❑ ruler with standard and metric measurements
- ❑ "Measuring in Feet and Inches" chart
- ❑ Enlarged Inch labeled
- ❑ Enlarged Inch

For teacher:

- ❑ plastic overhead ruler (to nearest 1/8 inch)

1. Each student receives one of the 3" x 12" pieces of oak tag.
2. Students are then instructed that this paper represents one magic "amazing inch." As the teacher models, students follow the teacher's example:
   a. Teacher will fold the paper in half once. Then draw a line on the fold, and label it 1/2. The left side edge of the paper will be labeled 0/2, while the right side edge will be labeled 2/2.
   b. While students continue to follow the teacher's example, the teacher will fold the paper in half again. Students will be asked how many equal parts there are. A line a littler shorter will be drawn on the additional folds and labeled 1/4, 2/4, and 3/4. The edges will be appropriately labeled 0/4 and 4/4.
   c. One more fold and with shorter lines drawn on the folds which will be labeled 1/8, 2/8, 3/8, 4/8, 5/8, 6/8, and 7/8. The edges will be labeled 0/8 and 8/8.
3. During the folding process, the teacher will review how each time the "magic inch" is folded each of the new sections are equal in size.
4. Teacher places a transparent plastic ruler on the overhead and points to the different lines on the ruler asking individual students (or popcorn style where students quietly call out answers), what part of the inch each line represents.
5. Students will apply the concept of the magic inch to an enlarged inch ruler on a sheet of paper by labeling it as was done with the "magic inch" and with the overhead.
6. Students will practice estimating and measuring by using *paper* rulers which are ruled to 1 inch, 1/2 inch, 1/4 inch, and 1/8 inch. (It would be a good idea to have students practice first with the 1 inch ruler. Then, after some practice use the 1/2 inch ruler. Continue to practice, then progress to the 1/4 inch ruler and finally after more practice the 1/8 inch ruler). Answers are recorded on a copy of the "Measuring in Feet and Inches" Worksheet.
7. Students will then partner and, using a ruler, review with each other what the different lines on the ruler represent.
8. Working in partners or groups, the class will then estimate the measurement of several items in the classroom and record their answers on the "Measuring in Feet and Inches" worksheet. After estimating, students will measure and record the exact measurement of each item.

9. Students label and glue a copy of an enlarged inch in their journals.
10. Students will explain in their journals how 2/4 and 4/8 are equal to 1/2, and 2/8 is equal to 1/4, etc. (tie in with fractions).

Curriculum Integration

Journal writing: Discuss how measurement is used in occupations. Use rulers as an additional way to teach fractions. Students, working with a partner, measure themselves and each other, then make a half-size self-portrait on butcher paper. They may draw clothes, hair, etc., to make the half-size me look like themselves. Construction paper and yarn may also be used for clothes and hair. (This activity may be done by customary, to the nearest inch, or metric measuring, to the nearest centimeter.)

Possible Extensions/Adaptations/Integration

"Inches" or "Metric Measurement" Games

Social Studies—Students interview parents or other relative to discover how measurement is used in their jobs. Graph the class results.

Math—The measuring activities may also be done in the metric system.

Assessment Suggestion

Students will estimate the measurement of several items, then measure them and record answers on a copy of the "Measuring in Feet and Inches" worksheet.

Students write the correct measurements on an enlarged inch (blackline included).

Additional Resources

Literature:

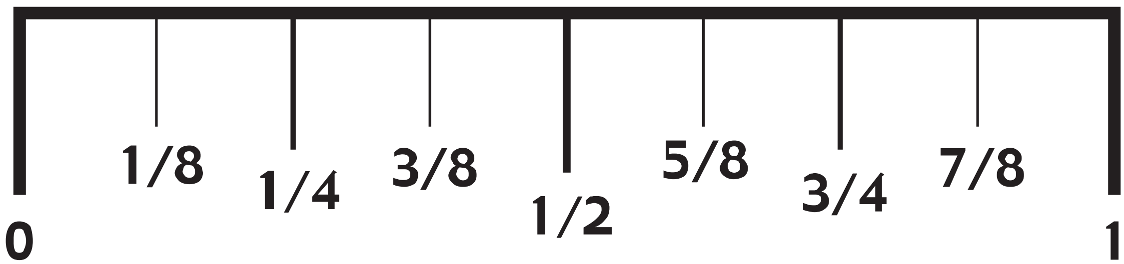
How Tall How Short How Faraway by David A. Adler
Measuring by Sheila Cato

Homework & Family Connections

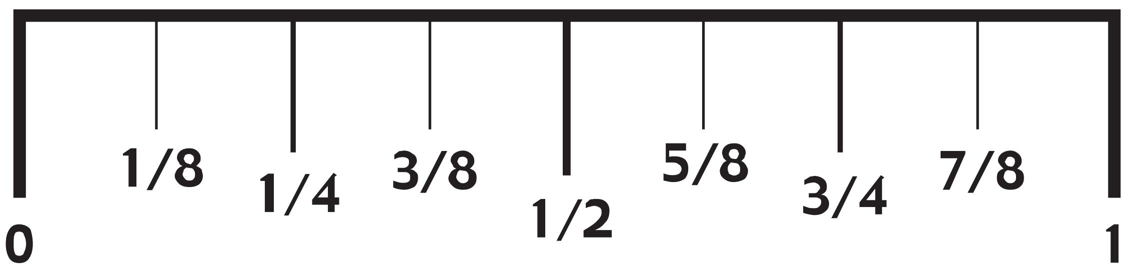
Students take a 3" x 12" piece of tag board home and teach parents or siblings how to fold and label their own "magic inch."

Students estimate and then measure items in their homes and record answers on "Measuring in Feet and Inches" worksheet.

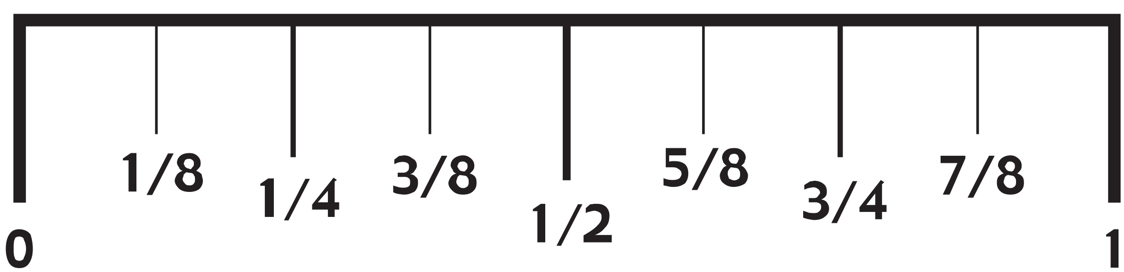
The "magic" Amazing Inch!



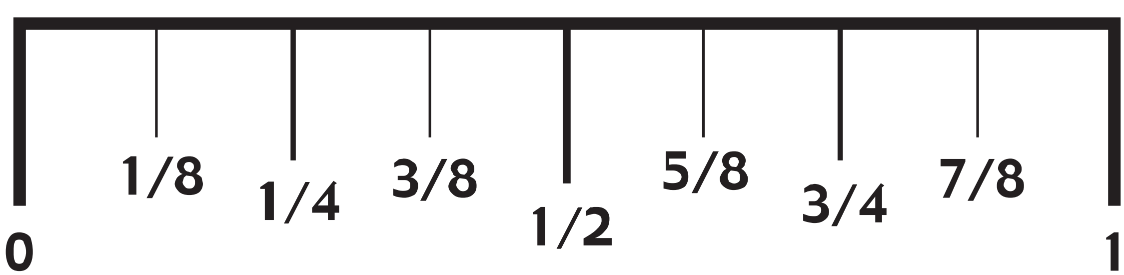
The "magic" Amazing Inch!



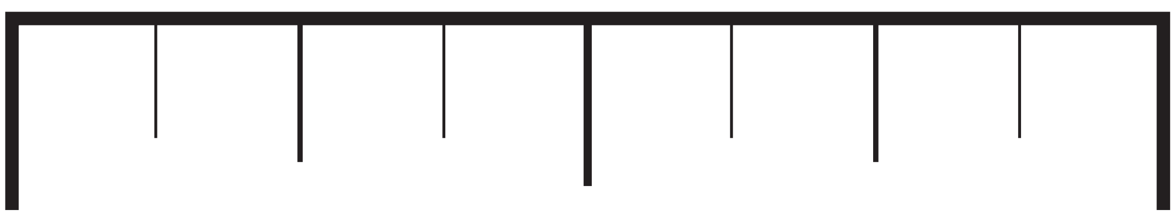
The "magic" Amazing Inch!

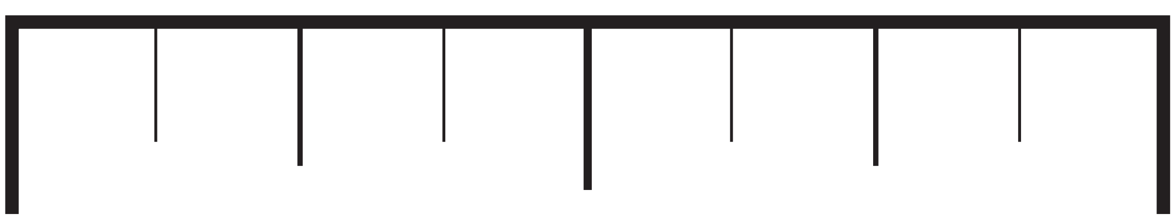


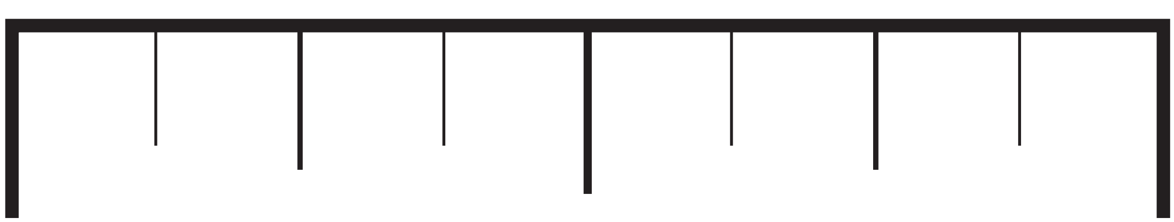
The "magic" Amazing Inch!



Blank Rulers







Name______________________________

Measuring in Feet and Inches

Measure each item to the nearest 1/8 inch.

#	Item Name	Estimate		Exact Measurement	
		feet ' and inches "	inches " only	feet ' and inches "	inches " only
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Activity—A Thin Inch

Standard IV
Students will understand and apply measurement tools and techniques.

Objective 1
Identify and describe measurable attributes of objects and units of measurement.

Objective 2
Determine measurements using appropriate tools and formulas..

Intended Learning Outcomes
1. Demonstrate a positive learning attitude towards mathematics.
2. Become mathematical problem solvers.
4. Communicate mathematically.

Standard IV

Objectives 1 & 2

Connections

Background Information

Vocabulary:

Area: The measure, in square units, of the inside of a closed plane figure.

Customary: A system of measurement used in the United States. The system includes units for measuring length, capacity, and weight.

Perimeter: The distance around a closed figure.

Formulas for Perimeter:

Rectangle:

$P = 2 \text{ x } (l + w)$
$P = (2 \text{ x } l) + (2 \text{ x } w)$
$P = (l + \mathrm{l}) + (w + w)$

Square:

$P = 4 \text{ x } s$
$P = s + s + s + s$

Area: Measure of the surface of a region in square units.

Formulas for Area:

Rectangle:

$A = \mathrm{l} \text{ x } w$

Square: Area equals sides squared

$A = s^2$
$A = s \text{ x } s$

Customary Units of Measure:

Length

one foot (ft or ')	=	12 inches (in or ")
one yard (yd)	=	36 inches
	=	3 feet
1 mile (mi)	=	5,280 feet
	=	1,760 yards

Area

1 square foot (ft^2)	=	144 square inches (in^2)
1 square yard (yd2)	=	9 square feet
1 acre	=	43,547 square feet
	=	4,840 square yards
1 square mile	=	640 acres

Invitation to Learn

Instruct students to estimate about how many Wheat Thins it would take to measure around the edge of a math book and to record their estimates on the "Wheat Thin Measuring" worksheet. Next students will estimate about how many Wheat Thins it would take to fit side-by-side on top of the math book and record their answers. Students will then measure the edges of their math books, using wheat thins, and record their answers. They will then measure how many it would take to cover the top of their math books and record those answers. When done, instruct students to experiment by estimating, measuring, and recording results from other objects (for example: other books, piece of paper, CD case).

Instructional Procedures

1. Begin reading the book, *Measuring Penny*. Point out the different items that were used for measuring in this story. After reading, discuss the various items that were used to measure Penny and the other dogs. Ask what other items could have been used for measuring.
2. Summarize *How Big is a Foot?* by Rolf Myller. Review with students how the king had seen each do their jobs correctly. What had happened so his bed didn't fit him? Help the students to recognize that each had different size feet.
3. Using the clip "How Close to Measure" from the video *It Figures*, fast forward to show the part of the video where the kids are making a clubhouse and using a pencil to measure how big the piece of wood needs to be. Watch as they build their clubhouse. Discuss why using a pencil might have made their job a little more difficult.

4. Discuss why it would be helpful to have a standardized type of measuring system. Help the students understand the importance of everyone using the same measurements.
5. Using color tiles on the overhead (compare them with Wheat Thins), demonstrate how to find the perimeter and area.
   a. Make a rectangle with color tiles (e.g., 2x4).
   b. Using another color tile, count out loud while measuring around the perimeter of the shape (12).
   c. Count how many color tiles are in the whole shape (8).
   d. Try a few more rectangles while the class counts with you.
   e. Students will then make rectangles with color tiles on one-inch grid paper.
   f. Students should draw each rectangle on a smaller piece of graph paper to record and compare their discoveries as they look for patterns.
   g. Model for and instruct students to write how many tiles are on each edge of the rectangle, and how many total tiles there are in each rectangle.
6. Demonstrate how to use the ruler to measure the edges of the overhead or some other object to find out how many inches the perimeter would be.

   Instruct students to work in partners/teams to estimate and then measure the perimeter of other items in the classroom with a rule. Extend this to estimating and then figure the area from the measurements. Have students record their results for each item.

   Review with students the definitions of length, width, perimeter, and area.

   a. After students have experimented with several items (books, a videotape case, etc.) and recorded their results, instruct them to look for patterns.
   b. Discuss what patterns they found when using wheat thins or rulers for measuring.
   c. Ask how they might apply the patterns they have found so they would work with any rectangle they might measure.
   d. Ask if there might be an easy way to remember or to figure without measuring (explore and discover possible formulas)
   e. Help students see the connection that a square has the largest area (other than a circle–7th grade curriculum).

Materials

For the teacher:

- ❑ *Measuring Penny* by Loreen Leedy
- ❑ *How Big is a Foot?* by Rolf Myller
- ❑ TV and VCR with remote
- ❑ *IT FIGURES* video #1, “How Close to Measure” USOE consortium
- ❑ *MATH WORKS* video #1, “Measurement” #9 USOE consortium

For each student/partners/team:

- ❑ Wheat Thins
- ❑ Wheat Thin chart for recording the measuring
- ❑ paper rulers (ruled to 1", 1/2", 1/4", and 1/8")
- ❑ inch grid paper
- ❑ graph paper

7. Show video, *Math Works*, Video #1: Measurement. Fast forward to the part where the boy and girl have some fencing and are figuring the area for the garden. Pause and discuss as the boy and girl talk about their thoughts and plans.
8. Review and write the formulas for perimeter and area on the overhead. Refer to the formulas in the Background Information throughout the instruction as applicable.

Curriculum Integration

Math/Science: Real Life—When planning for and planting a garden, students would want to have the most possible available area.

Possible Extensions/Adaptations/Integration

Math—A Wheat Thin activity could be also be done by using items from the metric system (for example: centimeter blocks to find the area and perimeter of a book). Students could then outline the book on centimeter grid paper. More items could be measured while the students seek for patterns.

Assessment Suggestion

Have students measure items with a ruler to the nearest inch, then find the perimeter and area of each item and record results.

Additional Resources

Videos:

Math Works

It Figures

Homework & Family Connections

Measure to the nearest inch the length and width of five or six items at home, then find the perimeter and area and record the results.

Wheat Thins Measure Up!

Estimate how many wheat thins it would take to put around the edge of your math book, then see how many it takes. Do the same with other objects in the class. Record your answers and watch for possible patterns.

Object	Estimate	Actual
1. Math Book		
2.		
3.		
4.		
5.		

Write patterns you observe: ______________________________

Estimate how many wheat thins it would take to cover your math book when placed side-by-side, then see how many it takes. Do the same with other objects in the class. Record your answers and watch for possible patterns.

Object	Estimate	Actual
1. Math Book		
2.		
3.		
4.		
5.		

Write patterns you observe: ______________________________

Estimate the perimeter of your math book in inches, then find the actual measurement. Estimate the area of your math book to the nearest inch, then find the actual measurement. Do the same with other objects in the class. Record your answers and watch for possible patterns.

Object	Perimeter Estimate	Perimeter	Area Estimate	Area
1. Math Book				
2.				
3.				
4.				
5.				

Write patterns you observe: ______________________________

Activity—King Henry Did What?

Standard IV

Objectives 1 & 2

Connections

Standard IV
Students will understand and apply measurement tools and techniques.

Objective 1
Identify and describe measurable attributes of objects and units of measurement.

Objective 2
Determine measurements using appropriate tools and formulas.

Intended Learning Outcomes
2. Become mathematical problem solvers.
3. Reason mathematically.
5. Make mathematical connections.

Background Information

The metric system might seems new to some students. Rreminding them that pop comes in liters may make a connection to students. The metric system of measurement increases in increments of ten. Students should have had experience multiplying and dividing decimals.

Vocabulary

Area: The measure, in square units, of the inside of a closed plane figure.

Capacity: The maximum amount that can be contained by an object. Often refers to measurement of a liquid.

Metric System: A system of measurement based on tens. The basic unit of length is the meter. The basic unit of mass is the gram. The basic unit of capacity is the liter.

Perimeter: The distance around a figure.

Plane Figure: A figure that lies on a flat surface.

Metric Units of Measure:

Length

1 centimeter (cm)	=	10 millimeters (mm)
1 decimeter (dm)	=	100 millimeters
	=	10 centimeters
1 meter (m)	=	1,000 millimeters
	=	100 centimeters
	=	10 decimeters

1 dekameter (dam)	=	10,000 millimeters
	=	1,000 centimeters
	=	100 decimeters
	=	10 meters
1 hectometer (hm)	=	100,000 millimeters
	=	10,000 centimeters
	=	1,000 decimeters
	=	100 meters
	=	10 dekameters
1 kilometer (km)	=	1,000,000 millimeters
	=	100,000 centimeters
	=	10,000 decimeters
	=	1,000 meters
	=	100 dekameters
	=	10 hectometers

Area

1 square centimeter (cm^2)	=	100 square millimeters (mm^2)
1 square decimeter (dm^2)	=	100 square centimeter (cm^2)
1 square meter (m^2)	=	10,000 square centimeters
	=	100 square decimeters

Volume

1 cubic centimeter (cm^3)	=	1,000 cubic millimeters (mm^3)
1 cubic decimeter (dm^3)	=	1,000 cubic decimeters

Capacity

1 liter (L)	=	1,000 milliliters (mL)
	=	1 cubic decimeter (dm^3)

Mass

1 gram (g)	=	1,000 milligrams
1 kilogram (kg)	=	1,000 grams
1 metric ton (t)	=	1,000 kilograms
1 cubic centimeter (cm^3)	=	holds 1 milliliter of water that has a mass of 1 gram

Materials

For the teacher:

- ❏ *How Tall, How Short, How Faraway* by David A. Adler
- ❏ Meterstick
- ❏ TV and VCR with remote
- ❏ *IT FIGURES* video series, Video #1: How Close to Measure (USOE Consortium)
- ❏ King Henry Story
- ❏ Overheads of King Henry Worksheets
- ❏ "When I Say, You Say"
- ❏ "Metric Rap"

For each student:

- ❏ King Henry Worksheets #1 & #2
- ❏ Dry erase board with marker (or black construction paper with chalk)
- ❏ Paper metric rulers

For each set of partners or teams:

- ❏ Metric/standard rulers and meter/yardsticks
- ❏ Little baggy of candy/ various items to measure
- ❏ Candy Measuring worksheet

Invitation to Learn

Working in partners, students will measure various candies or items in a baggy using the width of their index finger. Each student will record the results of the measurement to the nearest index finger width on "Candy Measuring" worksheet.

Instructional Procedures

1. Show video clip from "How Close to Measure" in the *It Figures* series, Tape 1. Fast forward to the part of the story about the mouse and the elephant getting measured. While watching, use the remote to pause, ask questions, and discuss as necessary for student understanding.
2. Question students about why the elephant and mouse could not be measured in the same units.
3. Tell the King Henry Story.
4. Instruct students that when converting from one unit to another in the metric system, one must always multiply or divide in increments of 10. Demonstrate how this is done by using the King Henry Worksheets. Remind the students that this is the same process they used when multiplying and dividing with decimals.
5. Students write notes in their journals as the teacher explains and writes on the overhead. Students will also make a King Henry conversion chart in their journals for reference. Students will explain in their own words how to convert from one unit to another (times by 10 or divide by 10).
6. Teacher will model a few examples on the overhead/chalkboard. Students use dry erase boards and markers (or black construction paper and chalk) to show their work as they practice a few problems the teacher writes on the overhead. Teacher is able to assess and view the students understanding as they practice.

   Example:

   30 m = ____ cm   answers: 30 m = <u>3,000</u> cm

   ____ m = 250 dm   25 m = 250 dm
7. Students will practice the conversion process using the King Henry Worksheets.
8. Teach class *When I Say, You Say*. Example: Say to class: "When I say meter, you say, 'width of a door.'" Practice the others.
9. Teach class the "Metric Rap." When practiced once a day for a few weeks, students are able to easily pass it off.

Concentration game

To make your own Concentration board:

1. Place 24 library card pockets on a large sheet of posterboard.
2. Organize them into four rows with six in each row.
3. Using 3x5 cards, make sets of matching cards.

   Examples for metric:

a. 3 m	match could be	300 cm
b. 2.4 hm	match could be	240 m

4. Cards could also be made for Customary units of measurement.
5. Individual game sets of cards could be made for cooperative teams or partners.

Curriculum Integration

Math/Science: Science—The Science Fair requires all measurements to be done in the metric system.

Science—Working with matter, it would be helpful to use the metric system.

Possible Extensions/Adaptations/Integration

Instruct students in the step process for converting metric units.

Students could also make a half-size me using the metric or customary units of measurement (recording sheet and “Make a Half-Sized Me” is attached).

Students could make a 2x the size of a doll or paper doll in either the metric or customary units of measurement.

Students measure candy/items in a baggy to the nearest centimeter and 1/8 inch and record their answers.

Assessment Suggestion

Students create a half-size me using a metric measuring tape.

1. Each student will estimate own metric measurements and fill in the estimate part of the "Make a Half-Size Me" chart.
2. Students work in partners measuring each other and filling in the measurement parts of the chart.
3. Each student divides his measurement by 2 and fills in the divide by 2 section of the chart.
4. Using the 1/2 size measurements, each student will draw himself on a piece of butcher paper about half the size of the student's height.
    a. Student will draw the half-size me to look like himself.
    b. Students may draw, color, use yarn, fabric, etc., to make it look more like himself.

Additional Resources

Math Curse by Joh Scieszka and Lane Smith

Fast & Fun Mental Math by Chuck Lotta

Homework & Family Connections

Students can estimate and then measure several items in their home.

When I Say You Say

When I say:	**You say:**
meter	width of a door
kilometer	10-minute walk
decimeter	brand new crayon
centimeter	tip of a finger
millimeter	width of a dime
gram	paper clip
kilogram	cantaloupe
liter	bottle of pop
kiloliter	tub of water
milliliter	3 little drops

Metric Rap

A milli's one-thousandth
    of the unit you've got.
Centi's one-hundredth
    right on the dot.
Deci's one-tenth,
    coming 'round' the bend,
Then you're back to just
    the unit again.
What about deka?
    Means you've got 10!
And like hecto?
    That's 100 my friend!
You're up to a kilo
    before too long.
It's exactly 10,000.
    That's the end of my song.

Fairy Tale of King Henry

King Henry was a jolly old English king most of the time. Even though metrics had been used in England for a long time, King Henry had difficulty converting between the different metric units. He had really become frustrated and confused. He felt, as a king, that he should be able to do about anything. He asked for help from all of the smart people in his kingdom. They tried hard, but without a lot of luck, until one of the great mathematicians ________________ (substitute student name) in his kingdom came forward.

_________________ explained to King Henry that everything in metrics was done by either multiplying by 10 or dividing by 10 which is easy to do by just moving the decimal point. He explained how to convert between the units so well, that King Henry was so excited he made up a chart to help others. They understood just as well, so he got even more excited. He was so excited that:

King Henry Danced Merrily Down Center Main.

Name______________________________

← ÷ King Henry Metrics x →

King	Henry	Danced	Merrily	Down	Center	Main
kilo	hecto	deka	liter meter gram	deci	centi	milli
Ex: 0.1	**1**	**10**	**100**	**1,000**	**10,000**	**100,000**
1.						**0.1**
2.				**728**		
3.		**79**				
4.					**300**	
5. **91**						
6.		**0.92**				
7.	**51**					
8.						**21**
9.				**0.839**		
10.			**0.467**			
11.	**0.3**					
12. **1.806**						
13.				**0.9**		
14.			**21**			
15.						**0.76**
16.					**0.4630**	
17. **25**						
18.		**1.324**				
19.						**28**
20.				**750**		

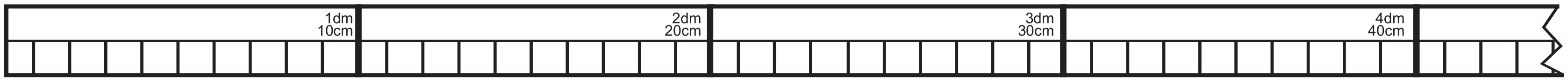
Name______________________________

← ÷ King Henry Metrics #2 x →

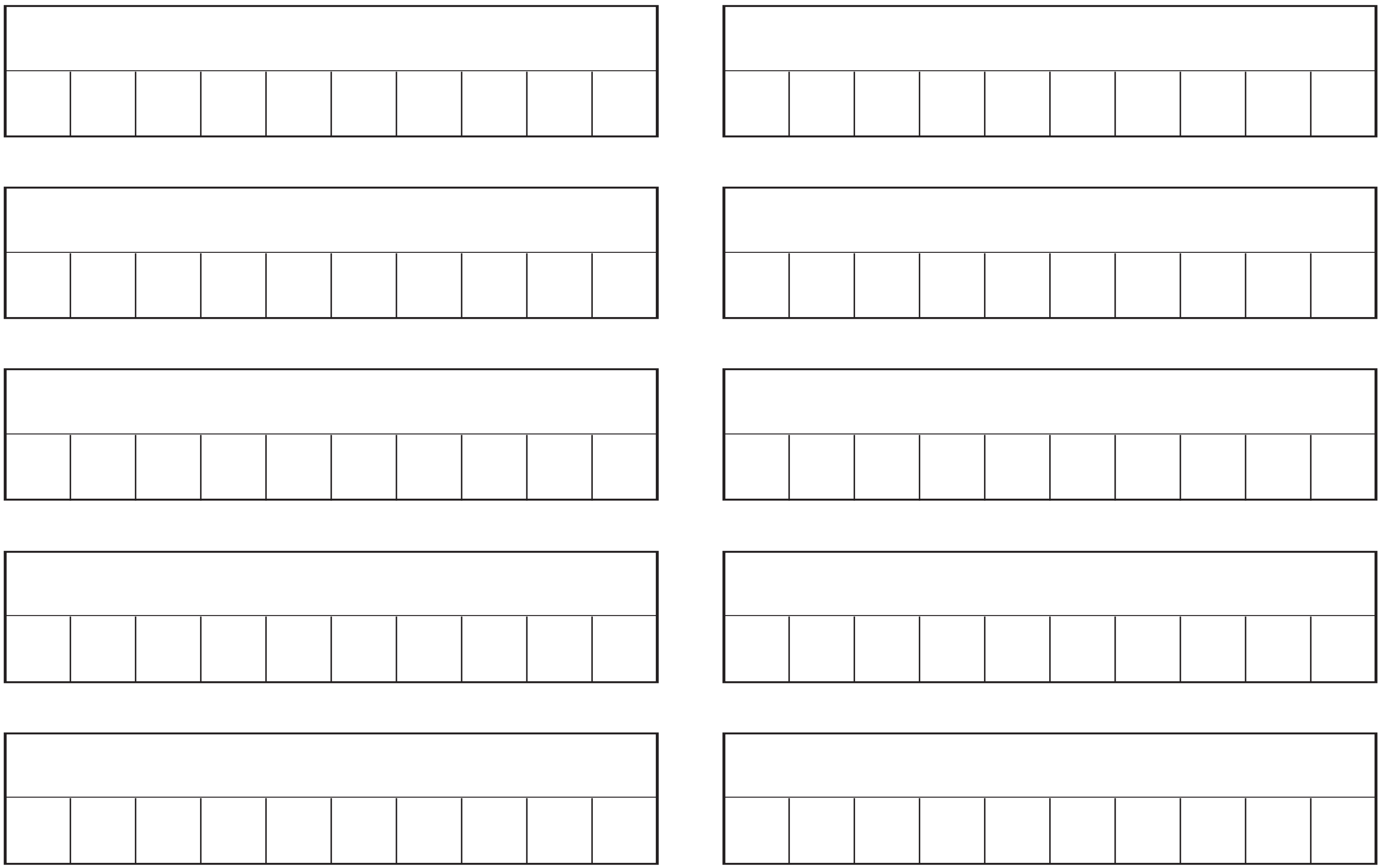
King	Henry	Danced	Merrily	Down	Center	Main
kilo	hecto	deka	liter meter gram	deci	centi	milli
Ex: 0.1	1	10	100	1,000	10,000	100,000
1.		0.04				
2.	1.397					
3.			3.56			
4. 9.0628						
5.				0.668		
6.						95,723.4
7.					5.6722	
8.			98.98			
9. 0.8875						
10.						6
11.				5.008		
12.		85				
13.	0.8035					
14.			4.434			
15.					8.721	
16.		6.852				
17. 0.3492						
18.						0.00065
19.				0.00807		
20.	85.33					

Metric Tape Measure 1

Cut each rectangle down the center of the bold lines. Tape rectangles together to create a meter tape. Label the meter tape as shown in the example.

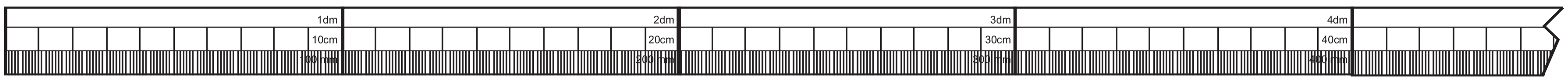


Adapted from Gayle Cloke

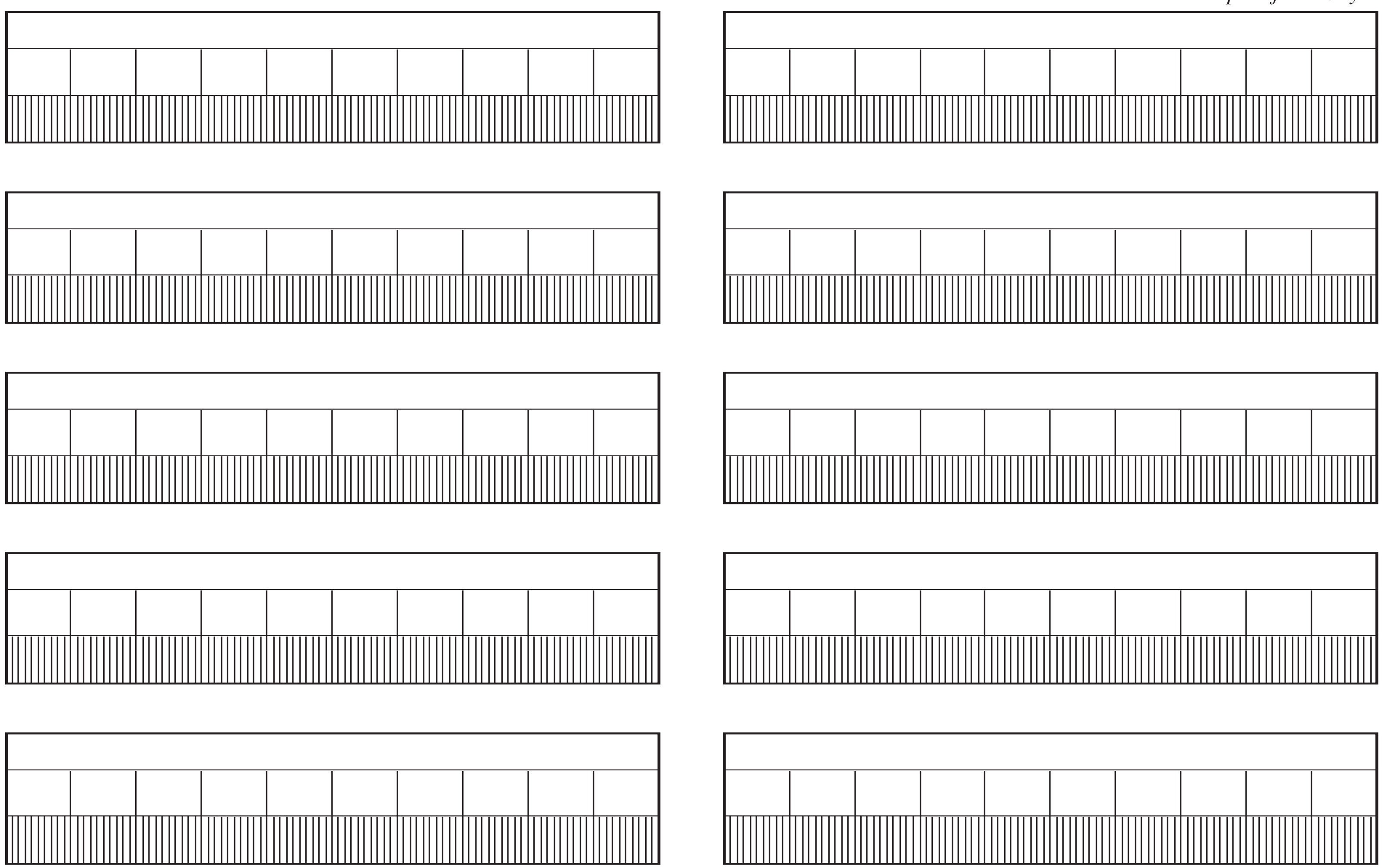


Metric Tape Measure 2

Cut each rectangle down the center of the bold lines. Tape rectangles together to create a meter tape. Label the meter tape as shown in the example.



Adapted from Gayle Cloke



Metric Measures

kilometer km	hectometer hm	dekameter dam	Meter m	decimeter dm	centimeter cm	millimeter mm
kilogram kg	hectogram hg	dekagram dag	Gram g	decigram dg	centigram cg	milligram mg
kiloliter kL	hectoliter hL	dekaliter daL	Liter L	deciliter dL	centiliter cL	milliliter mL

Metric Measures

kilometer km	hectometer hm	dekameter dam	Meter m	decimeter dm	centimeter cm	millimeter mm
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Metric Measures

kilometer km	hectometer hm	dekameter dam	Meter m	decimeter dm	centimeter cm	millimeter mm
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kiloliter kL	hectoliter hL	dekaliter daL	Liter L	deciliter dL	centiliter cL	milliliter mL

Name_______________________________

Metric Measuring

Measure each item to the nearest centimeter.

#	Item Name	Estimate	Exact Measurement
		nearest centimeter	centimeters

Name______________________________

Candy Measuring

Measure each object and record the correct length on the appropriate chart.

Nonstandard

Measure with your index finger.

Objects	Peanut	Cereal	Pencil	Cracker	Foot	Pretzel	Wrist	Pixie Stix	Jolly Rancher

Customary

Measure to the nearest 1/8 inch.

Objects	Peanut	Cereal	Pencil	Cracker	Foot	Pretzel	Wrist	Pixie Stix	Jolly Rancher

Metric

Measure to the nearest centimeter.

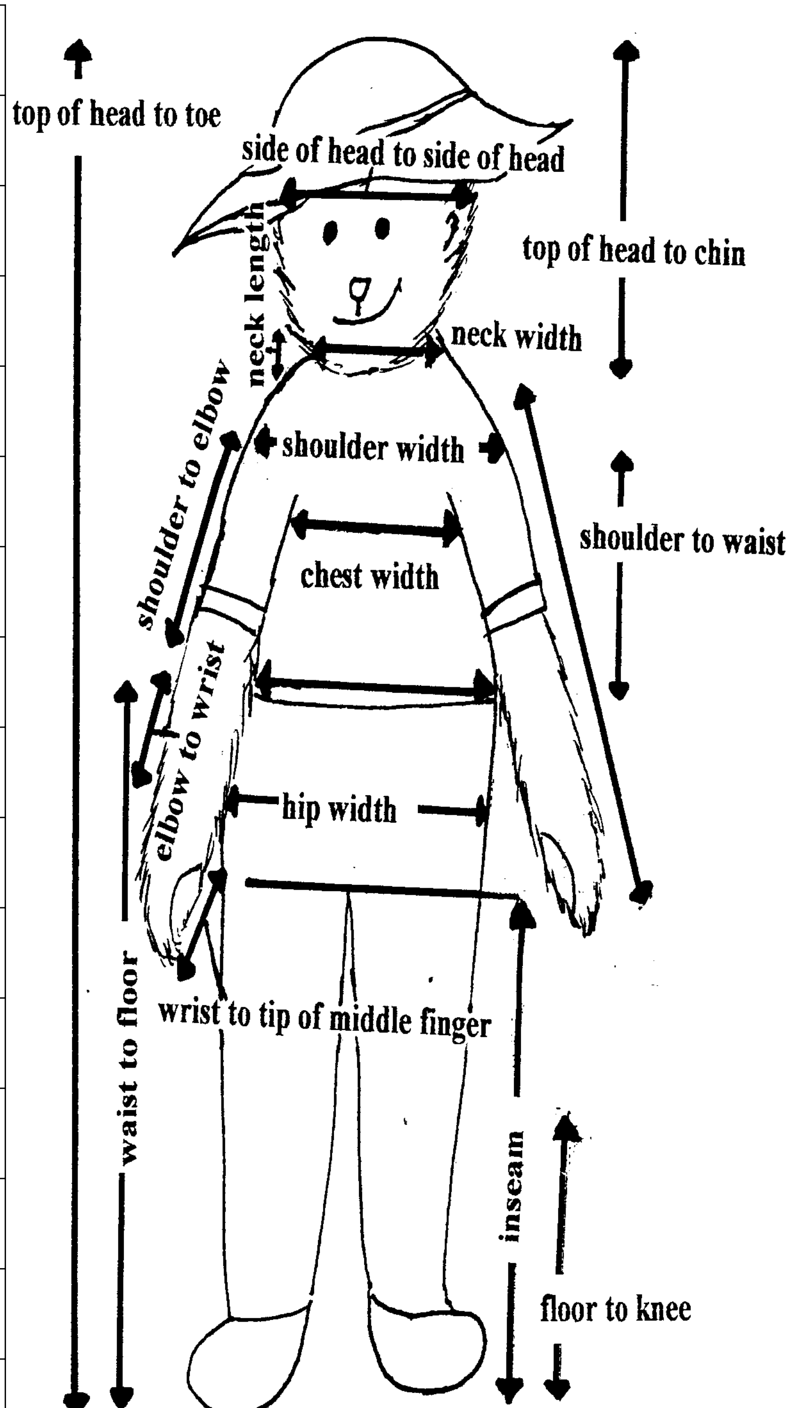
Objects	Peanut	Cereal	Pencil	Cracker	Foot	Pretzel	Wrist	Pixie Stix	Jolly Rancher

Name______________________________

Make a "Half-Size Me"

Guess your height in centimeters. use a metric measuring tape, ruler or meter stick, find each measurement. Divide each measurement by 2 for your "half-size me." Fill in the chart as you complete each part. Make your "half-size me" to look like yourself.

	Estimate	Measurement	÷ 2 =
top of head to toe			
top of head to chin			
side of head to side of head			
neck length			
neck width			
shoulder width			
shoulder to waist			
shoulder to elbow			
elbow to wrist			
wrist to tip of middle finger			
waist to floor			
inseam			
chest width			
hip width			
waist to knee			



Concentration Cards

3 m	**4.25 cm**	**8,000 mm**
300 cm	**.425 dm**	**800 cm**

4.43 km	**.25 hm**	**78 dam**
4,430 m	**25 m**	**780 m**

9.9 km	**.52 m**	**52 cm**
99 hm	**5.2 dm**	**520 mm**

7.8 hm	**46 hm**	**8.8 dm**
7,800 dm	**460 dam**	**88 cm**

Activity—Twice the Rice, or More?

Standard IV
Students will understand and apply measurement tools and techniques.

Objective 1
Identify and describe measurable attributes of objects and units of measurement.

Objective 2
Determine measurements using appropriate tools and formulas.

Intended Learning Outcomes
3. Reason mathematically.
5. Make mathematical connections.
6. Represent mathematical situations.

Standard IV

Objectives 1 & 2

Connections

Background Information

Review what uses people have for measuring tools, like cooking, mechanics, painting, etc.

Vocabulary

Capacity: The maximum amount that can be contained by an object. Often refers to a measurement of liquid.

Volume: The number of cubic units it takes to fill a figure.

Customary units of measurement:

Capacity

1 tablespoon (tbsp)	=	3 teaspoons (tsp)
1 fluid ounce (fl oz)	=	2 tablespoons
1 cup (c)	=	8 fluid ounces
1 pint (pt)	=	2 cups
1 quart	=	2 pints
1/2 gallon	=	2 quarts
1 gallon	=	4 quarts

Volume

1 cubic foot (ft^3)	=	1,728 cubic inches

$v = l \times w \times h$

Metric Units of measure:

Capacity

1 liter (L)	=	1,000 milliliters (mL)
	=	1 cubic decimeter (dm^3)

Volume

1 cubic centimeter (cm^3)	=	1,000 cubic millimeters (mm^3)
1 cubic decimeter (dm^3)	=	1,000 cubic decimeters

Formula for Volume:

$v = l \times w \times h$

Invitation to Learn

Cooperative groups list as many ways as possible to use measuring cups, etc. in jobs and at home.

Instructional Procedures

1. Through exploration with rice, allow students to experiment and discover how different customary or metric measuring tools are used to measure. Discuss how the measuring units convert from one unit to another.
2. Have students record their responses on the "Capacity" worksheet.
3. Students copy the interior G (gallon) into their journals and explain in their own words what they have learned about customary measurements.
4. Read and discuss with the class *Hershey's Weights and Measures* by Jerry Pallotta.
5. Review Capacity chart with class.
6. Introduce student-size "Gallon-t Robo" to the class. Instruct them to cut out and glue in their planners.
7. Give partners/teams an enlarged copy of "Gallon-t Robo." Have them cut out the shapes and explore how many quarts will fit in the gallon, how many pints will fit in the quart, half-gallon, or gallon, etc.
8. Teams should record exploration results on "Gallon-t Robo" exploration worksheet.
9. Orally quiz the class on what possible units or combination of units equal one gallon, one quart, one pint, etc.
10. Teams will create their own "Gallon-t Robo" using all of the pieces from the original copy. They will color and glue it on a large piece of construction paper.

Materials

- ❏ *Hershey's Weights and Measures* by Jerry Pallotta
- ❏ Rubbermaid type container(s) with rice/water
- ❏ Various measuring supplies, both metric and customary
- ❏ Customary and metric measuring cups
- ❏ Customary and metric measuring spoons

For each student:

- ❏ "Gallon-t Robo"
- ❏ "Capacity Measurement" chart
- ❏ "Capacity" worksheet

For partners/teams:

- ❏ Enlarged "Gallon-t Robo"
- ❏ "Gallon-t Robo Exploration" worksheet
- ❏ "Capacity Measurement" chart
- ❏ "Capacity" worksheet
- ❏ "The Inner G" handout

Curriculum Integration

Math/Science—Students can use measuring tools with the matter unit.

Possible Extensions/Adaptations/Integration

Real Life—Working in cooperative groups, students make cookies or brownies using their measuring skills.

Art—Have individual students cut out "Gallon-t Robo" and assemble the pieces to make their own art creation.

Assessment Suggestion

Have students measure rice or water and record the results. Give them a quiz on different measurements.

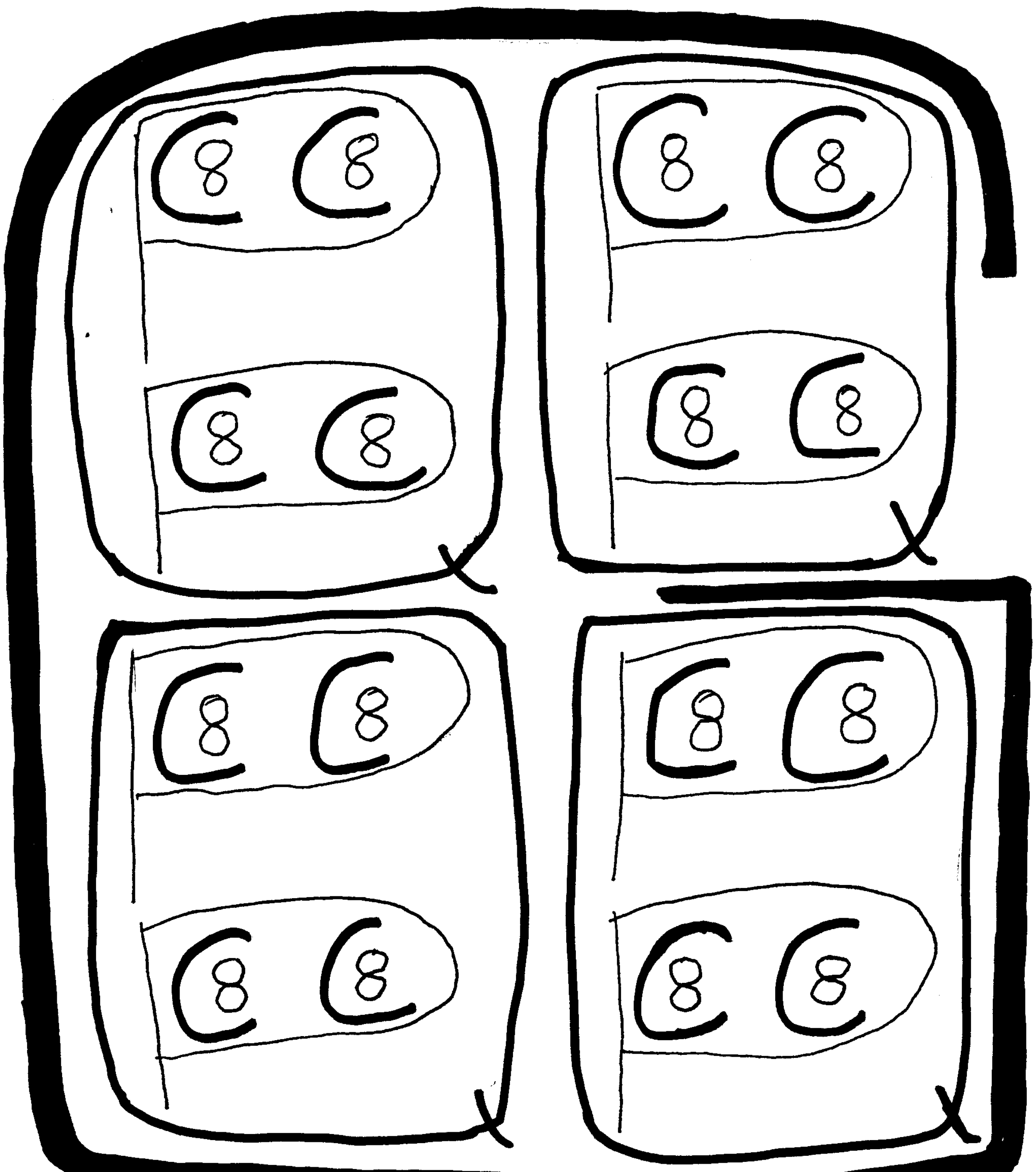
Additional Resources:

50 Fabulous Measurement Activities (Scholastic)
Measuring by Sheila Cato

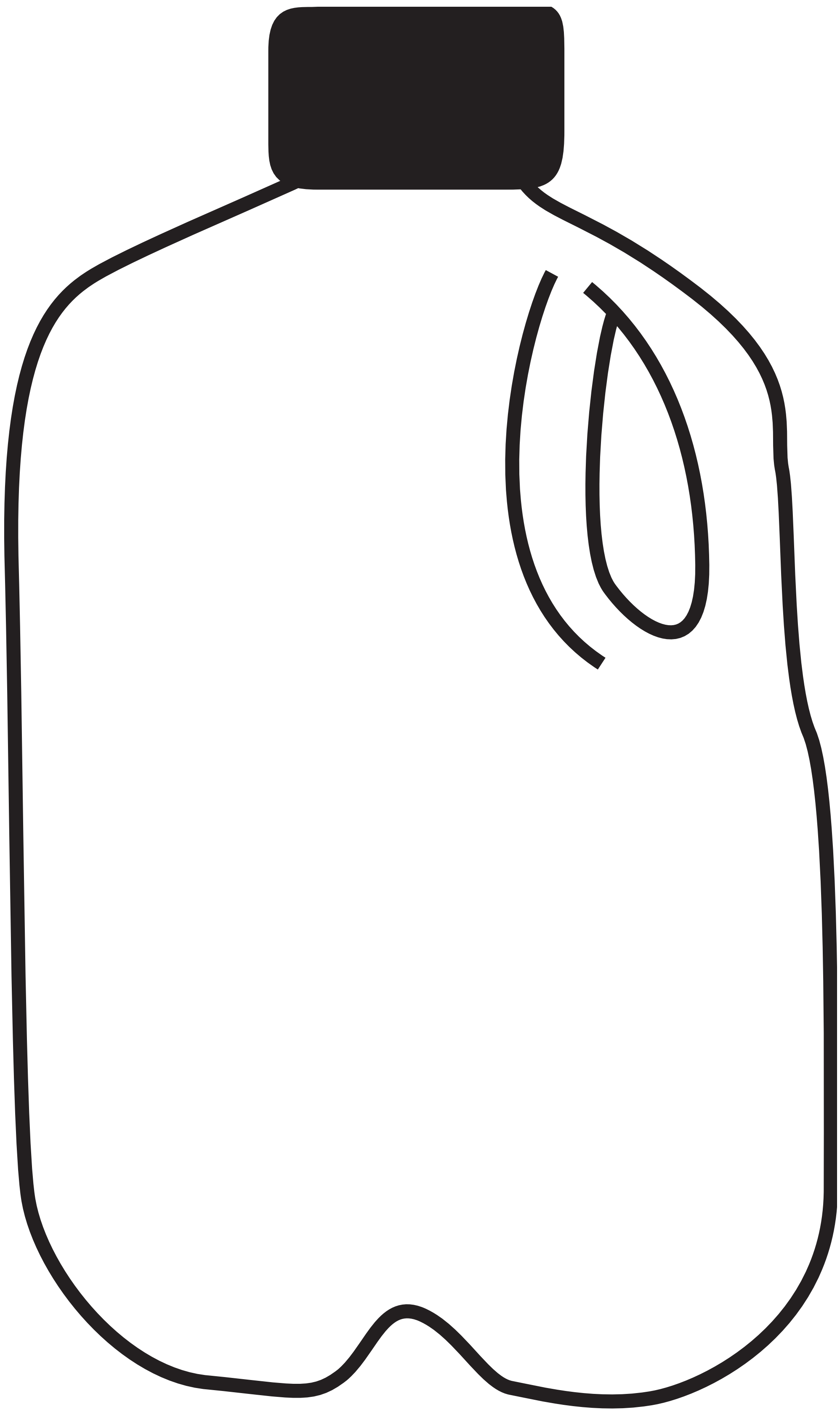
Homework & Family Connections

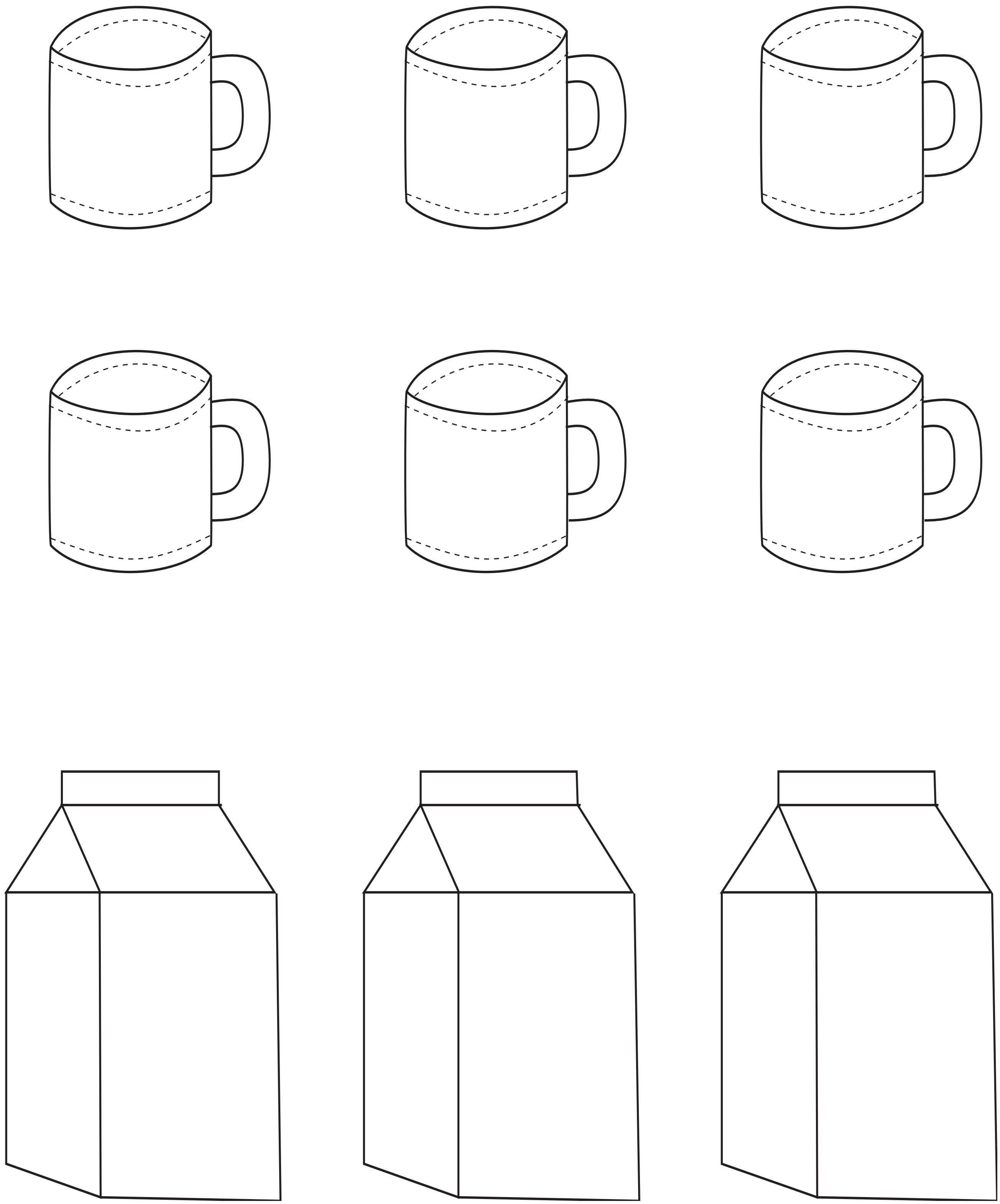
Students make cookies, jello, or something else at home for their family that requires measuring.

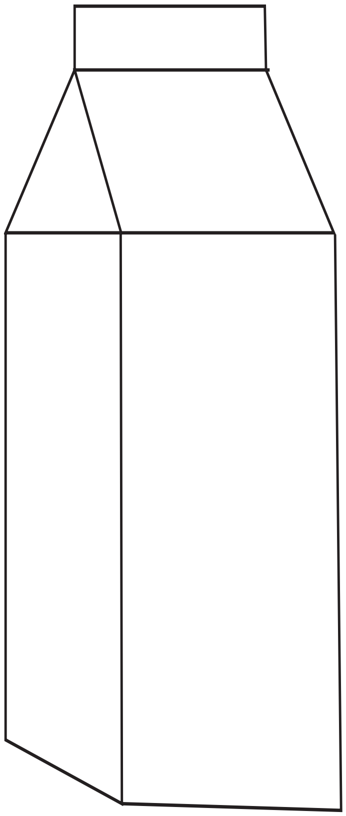
The Inner G

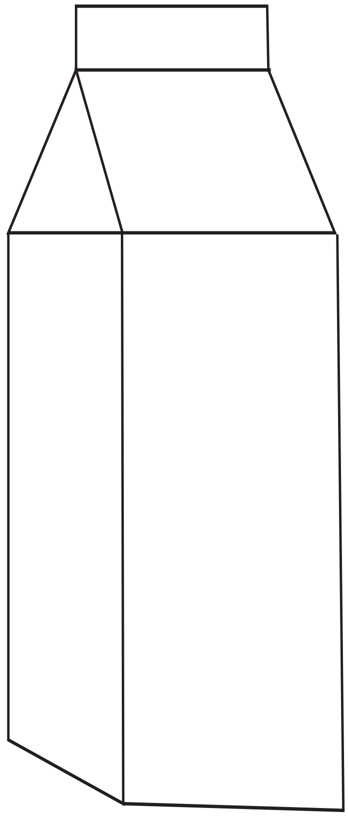


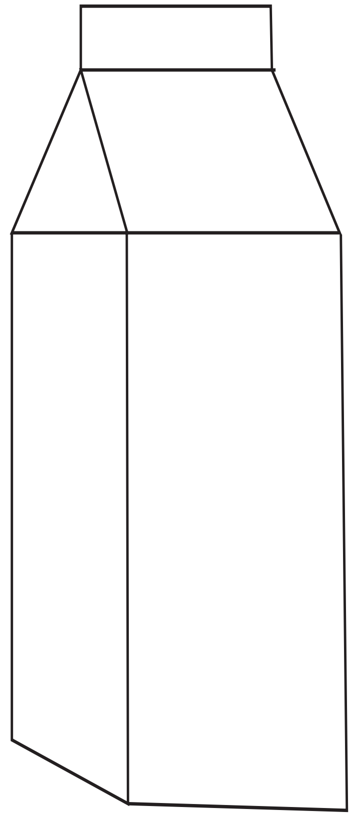
CAPICITY

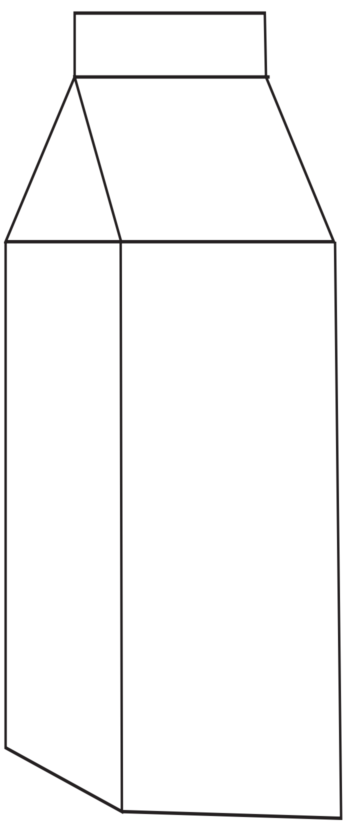


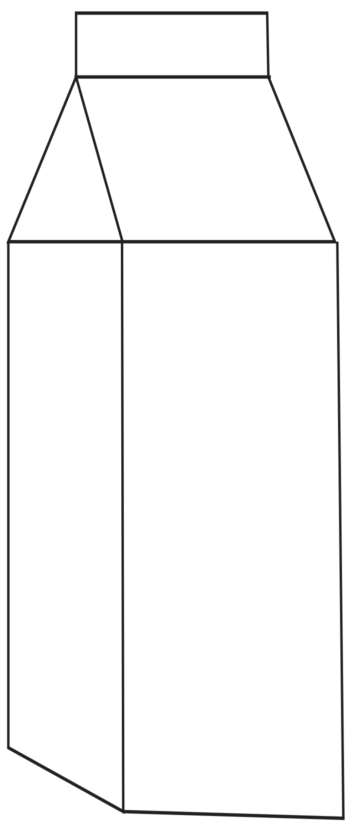


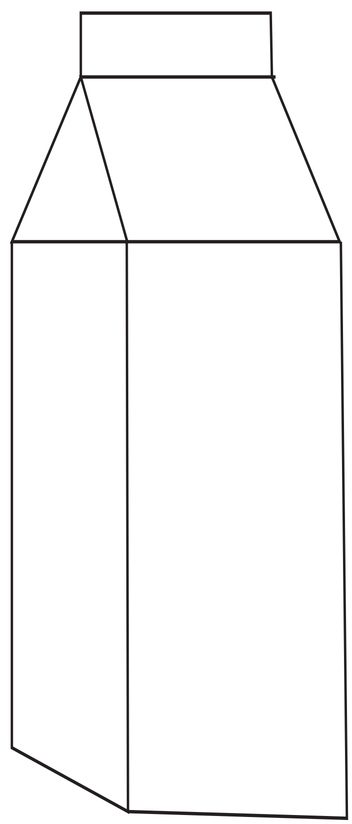












1 Quart = 4 Cups

1 Gallon = 4 Quarts

1 Quart = 2 Pints

1 Pint = 2 Cups

CAPACITY

1 Quart = 4 Cups

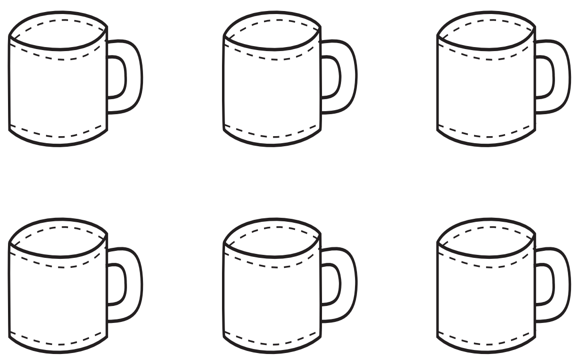
1 Gallon = 4 Quarts

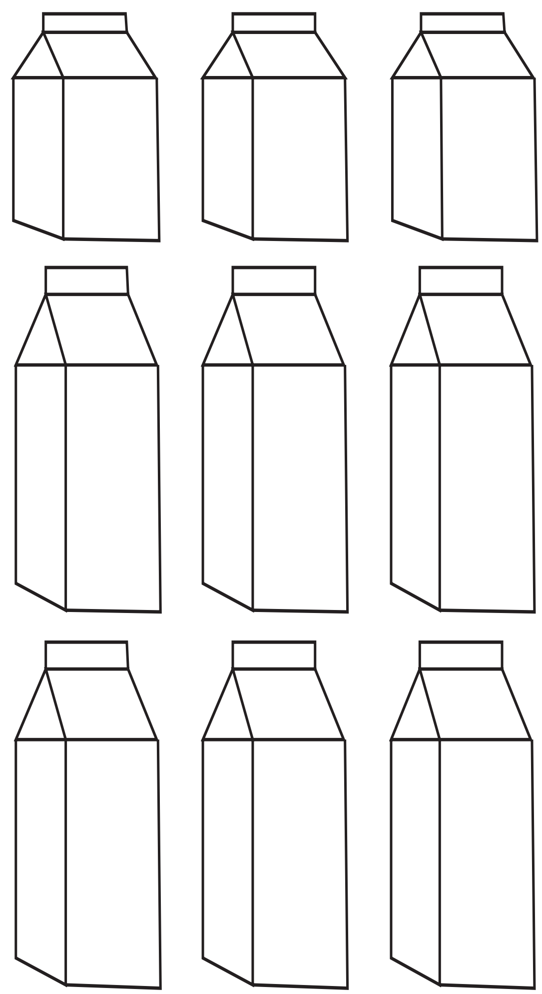
1 Quart = 2 Pints

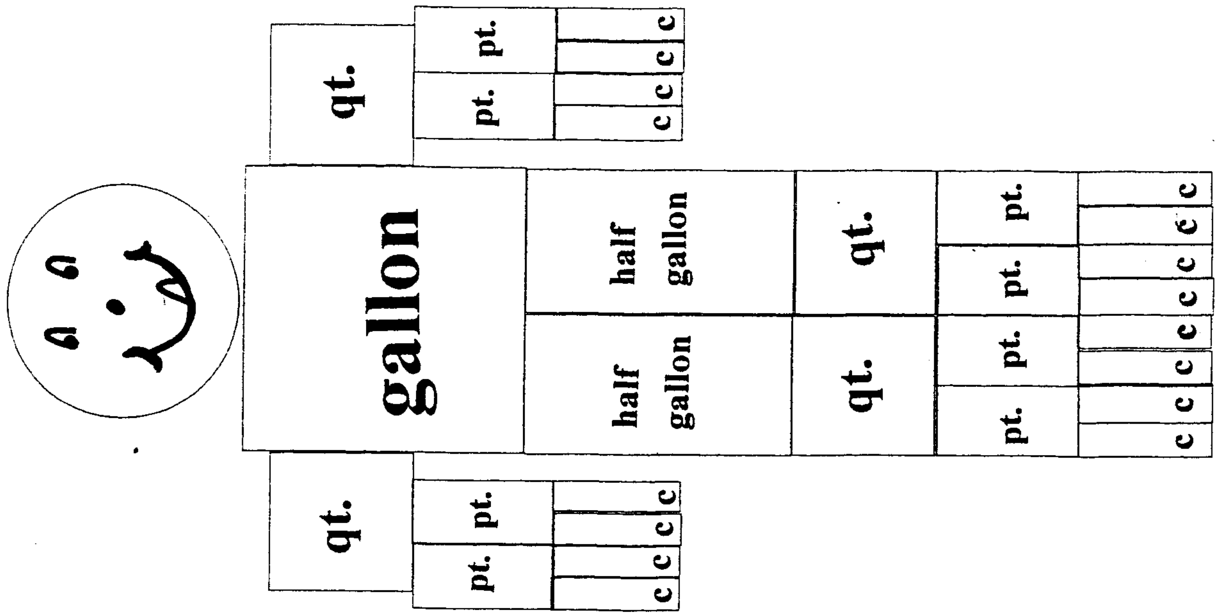
1 Pint = 2 Cups

= =

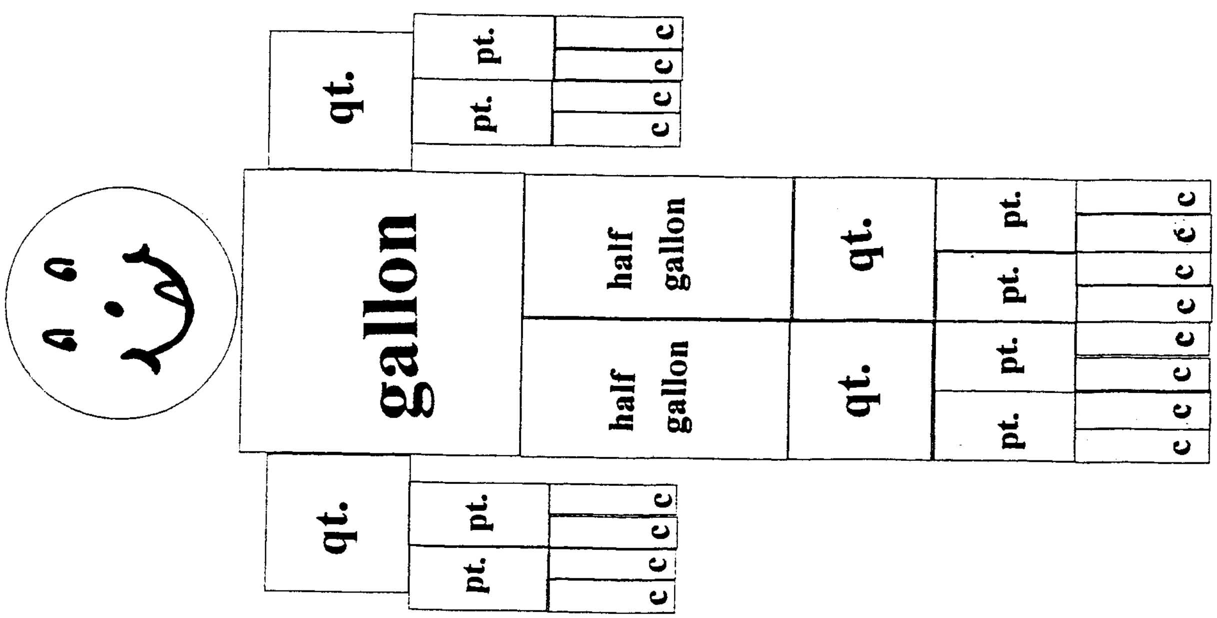
= =







Gallon-t Robo



Gallon-t Robo

Name______________________________

Gallon-t Robo Combinations

Sketch your different Gallon-t Robo combinations in the squares below:

Name______________________________

Customary Measuring

1. ______ 1/4 cups = 1 cup
2. ______ 1/2 cups = 1 cup
3. ______ 1 cups = 1 pint
4. ______ 1 cups = 1 quart
5. ______ 1/4 cups = 1/2 cup
6. ______ 1/2 cups = 3 cups
7. ______ quarts = 1 half-gallon
8. ______ quarts = 1 gallon
9. ______ cups = 1 gallon
10. ______ 1/2 cups = 2 cups
11. ______ 1/3 cups = 1 cup
12. ______ pints = 1 quart
13. ______ 1/3 cups = 2 cups
14. ______ 1/4 cups = 2 cups
15. ______ pints = 2 quarts
16. ______ pints = 1/2 gallon
17. ______ pints = 1 gallon
18. ______ 1/2 cups = 1 gallon

Measuring Spoons

19. ______ teaspoons = 1 Tablespoon
20. ______ 1/2 teaspoons = 1 teaspoon
21. ______ 1/4 teaspoons = 1 teaspoon
22. ______ 1/8 teaspoons = 1 teaspoon
23. ______ 1/3 teaspoons = 2 teaspoons
24. ______ 1/3 teaspooons = 1 teaspoon
25. ______ 1/4 teaspoons = 2 teaspoons
26. ______ 1/8 teaspoons = 2 teaspoons

Equal, Greater Than, or Less Than

27. 1 gallon ◯ 4 quarts
28. 3 cups ◯ 1 pint
29. 2 quarts ◯ 8 pints
30. 4 pints ◯ 1 cup
31. 2 cups ◯ 1 quart
32. 3 pints ◯ 1 quart
33. 8 cups ◯ 1 gallon
34. 2 quarts ◯ 10 cups

Name______________________________

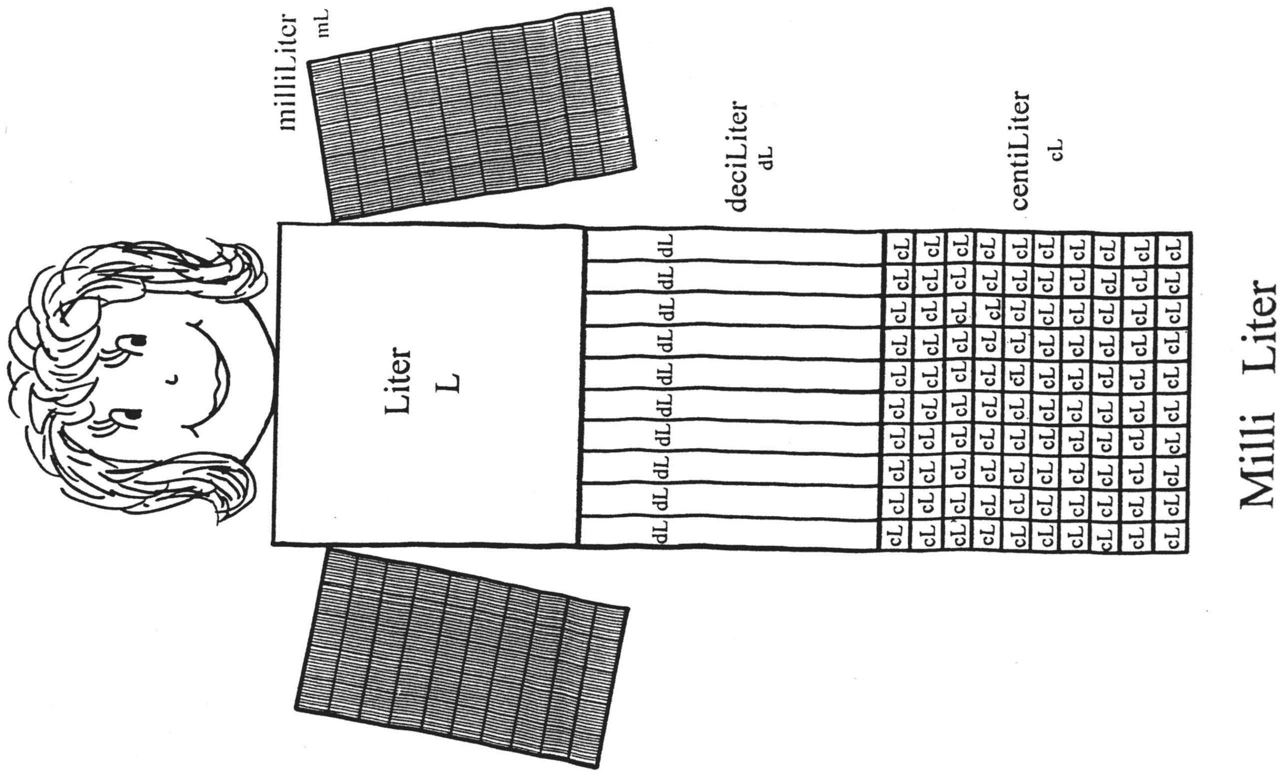
Metric Measuring

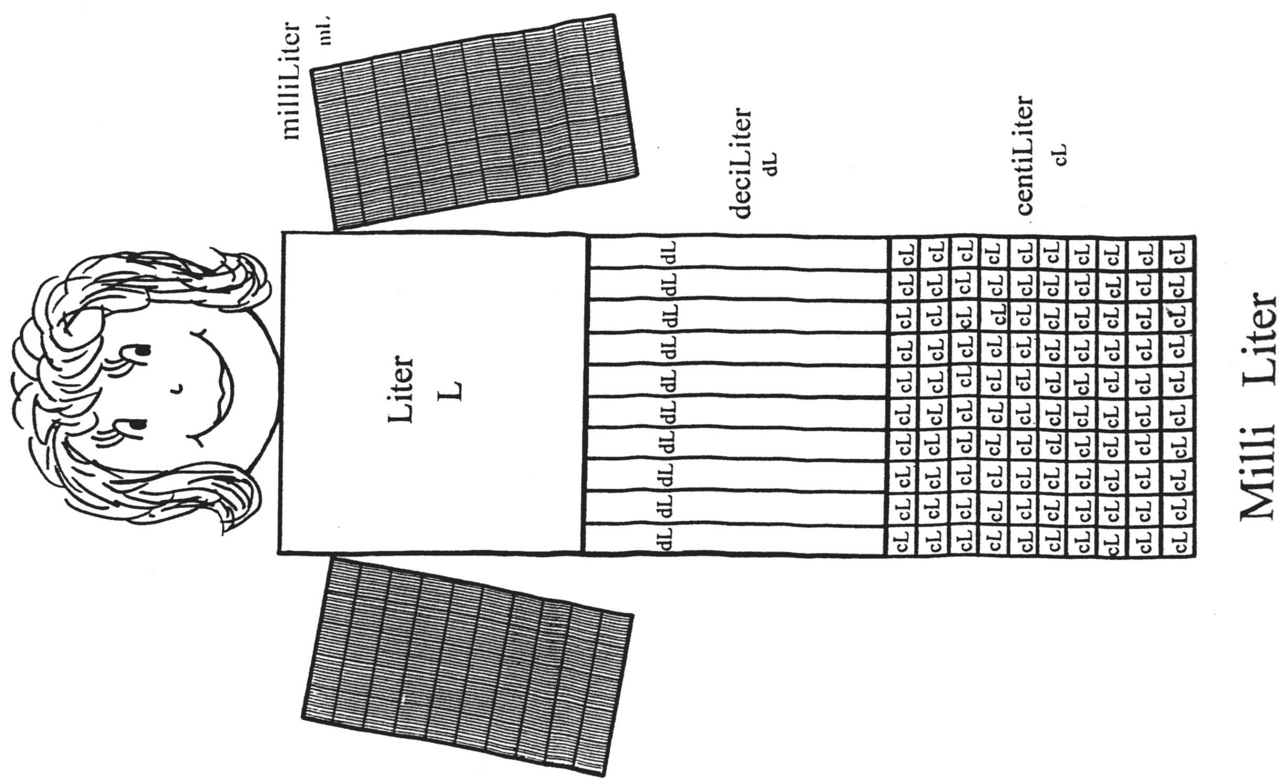
Measure the following:

1. ______ milliLiters = 1 Liter
2. ______ milliLiters = 2 Liters
3. ______ milliLiters = 4 Liters
4. ______ deciLiters = 1 Liter
5. ______ deciLiters = 2 Liters
6. ______ deciLiters = 3 Liters
7. ______ centiLiters = 1 Liter
8. ______ centiLiters = 2 Liters
9. ______ centiLiters = 3 Liters
10. ______ Liters = 20 deciLiters
11. ______ deciLiters = 100 milliLiters
12. ______ Liters = 2,000 milliLiters

Extend your thinking:

1. ______ Liters = 1 dekaLilter
2. ______ Liters = 1 hectoLilters
3. ______ Liters = 1 kiloLilters
4. ______ deciLiters = 1 hectoLiter
5. ______ deciLiters = 1 kiloLiter
6. ______ hectoLiters = 5 Liters
7. ______ Liters = 2 dekaLiters
8. ______ Liters = 2 hectoLiters
9. ______ Liters = 2 kiloLiters
10. ______ deciLiters = 1 kiloLiters
11. ______ milliLiters = 2 centiLiters
12. ______ deciLiters = 2 hectoLiters


milliLiter
mL
deciLiter
dL
centiLiter
cL
Liter
L
Milli Liter


milliLiter
mL
deciLiter
dL
centiLiter
cL
Liter
L
Milli Liter

Activity—Time Marches On!

Standard IV

Objective 2

Connections

Standard IV
Students will understand and apply measurement tools and techniques.

Objective 2
Determine measurements using appropriate tools and formulas.

Intended Learning Outcomes
2. Become mathematical problem solvers.
5. Make mathematical connections.

Background Information

Students need to have a knowledge of how to tell time and how a clock works, as well as the ability to understand, figure, and demonstrate a knowledge of the amount of time that passes between two times.

Vocabulary:

Elapsed time: The amount of time that passes between two times (e.g., 8:00 to 9:00).

Invitation to Learn

Put the following sample questions on the overhead for the class to ponder and discover:

1. You have a 10:00 a.m. appointment. It will take you 1/2 hour to get there from your home. What time will you need to leave?
2. School gets out at 3:30 p.m. It takes you about 15 minutes to get home. You have a dinner appointment at 6:00 p.m. It will take about 45 minutes to get to your dinner appointment on time. What time will you need to leave?

Discuss why it is important to be able to tell time, know how to use a clock, and understand adding and subtracting time.

Instructional Procedures

Make paper plate clocks (see blackline):

1. Each student will receive one paper plate, two clock hands, and a brad.
2. Teacher will model on the overhead or the chalkboard how to make a clock.
   - Draw a big circle on the overhead. Inside the big circle, draw a smaller circle representing the rim of a paper plate, (this circle frame will be the one where the clock numbers are written).

- In the circle frame, draw a big (feather-light) circle at the top and one at the bottom (12 and 6), then draw one right side and the left side (3 and 9).
- Write 12 in the top circle and 6 in the bottom one. Instruct the students to do the same with theirs.
- Write a 3 in the circle on the right and a 9 in the circle on the left. Instruct students to do the same.
- Review what the numbers are increased by: Count by 3's, 3, 6, 9, 12. Also discuss multiples of 3's (1 x 3 = 3, 2 x 3 = 6, 3 x 3 = 9, and 4 x 3 = 12).
- Ask students what number(s) need to be placed between the 12 and 3. (The answer would be 1 and 2). Model for them how to place them.
- Follow the same pattern with the other numbers. (3 and 6, 6 and 9, 9 and 12).
- Review what the numbers are now increased by: Count by 1's (1, 2, 3, 4, 5, 6 ...).

3. Ask how many minutes there are in one hour, how many minutes there are between the 12 and 1, between the 1 and 2, and so on. (5 minutes) Remind the class that it takes 5 minutes to move from one number on the clock to the next number.
    - Count by 5's as a class while pointing to 1 x 5 (to show the 5 minutes), then 2 x 5 would be 10, 3 x 5 would be 15, etc., exploring the 5 times tables.
    - Remind the class that they cannot go higher than 5 x 12 (60) and that there are no more than 60 minutes in one hour. Explore the possibilities of a pattern.
4. Draw a half-circle next to the right side of the clock. Tell class that this part of the clock is known as after or past.
5. Draw a half-circle next to the left side of the clock. Tell class that this part of the clock is known as before or to.
6. Draw a long minute hand on the overhead. Inside it, write "minute" (for the minute hand) and explain that the long word minute = long hand.
7. Draw a short hand on the overhead. Inside it, write hour (for the hour hand) and explain that the short word hour = short hand.
8. Instruct students to place their clocks on their desks as you discuss and demonstrate a few different times. Using the clock you made or a Judy clock, put a few different times on your clock and instruct the class to discover the times each (examples: 5:00,

Materials

For each student:

- ❏ Paper plates
- ❏ Brad
- ❏ Clock hands
- ❏ Colored pencils, markers, or crayons

For teacher:

- ❏ Overhead clock with hands or Judy clock
- ❏ *Tuesday* by David Wiesner

8:30, 6:15, 4:10, etc.). Practice until you feel comfortable with the class' understanding.

9. Now ask the class to show you the time you say (examples: 7:00, 2:15, 3:45, 11:55). Practice until you feel comfortable with the class' understanding to the nearest 5 minutes.
10. Ask what it means to say (2:15 which is shown on your clock) two-fifteen, fifteen after two, or fifteen past two.
    - Try 1:00. You might hear one or one o' clock.
    - Try 7:30. You might hear seven-thirty, half past seven, or 30 minutes after seven.
    - Try 3:45. You might hear three forty-five, a quarter to four, forty-five minutes past three, forty-five minutes after three, or fifteen to four.
    - For 11:55, you might hear eleven fifty-five, five to twelve, or five to 12:00.

    Remind students that all of these are the same. Try a few others.
11. Remind the class to keep their clocks on their desks as you model for them a couple of problems. Show 1:00 on your clock. Tell them you are now going to add 2 1/2 hours to your clock. Ask them to predict what time they think it will be? With the class watching, move your clock forward 2 1/2 hours and show the time. Ask them to check and see if their predictions were right. Model a few examples for the class.
12. Have students practice using their clocks to the nearest five minutes as a class:
    - Say: "It is 4:00 a.m. now. Show that time on your clock. What time will it be 4 hours from now? Show that time on your clock." Class holds clocks up as they get the correct time.
    - Suppose that it is 3:00. What time will it be in 5 minutes? Wait for students to figure the answer using their clocks and then raise their hands to share answers.
    - Suppose recess starts at 10:00 a.m. and lasts 15 minutes. What time will you need to come in from recess? Show it on your clocks.
    - School starts at 5 minutes to 9:00 or 8:55. PE begins 30 minutes later. What time will it be?

    Students write in their journals listing times of morning activities before school, or a full day's activities.

Curriculum Integration

Math/Science—Extend the teaching and questions to the nearest minute.

Have students estimate how many times they can blink, hop, say school, write name, etc., in one minute. Let them actually try one or more while being timed for the minute.

Use clocks to show fractions: 1/2 of the clock would be how many minutes? 1/3 of the clock would be how many minutes? 1/4? etc.

Use clocks to show acute, obtuse, and right angles.

Compare how many minutes it takes different liquids to boil (water, vinegar, salt water, milk, etc.)

Possible Extensions/Adaptations

Discuss the importance of being to school, meetings, and jobs on time. Also discuss how catching airplanes, buses, etc. depends on knowledge of time and clocks.

On a paper with 24 clocks (12 morning clocks 12 a.m. -11 a.m. and 12 afternoon-evening clocks 12:00 p.m.-11:00 p.m. clocks), students write under the clocks what they would be doing at each time.

Compare and do clock activities with digital clocks and circle clocks.

Decorate the face of the clock. Do not put art in the rim where the numbers are located.

Assessment Suggestion

Teacher observes students as he or she says a time and shows time on his or her clock.

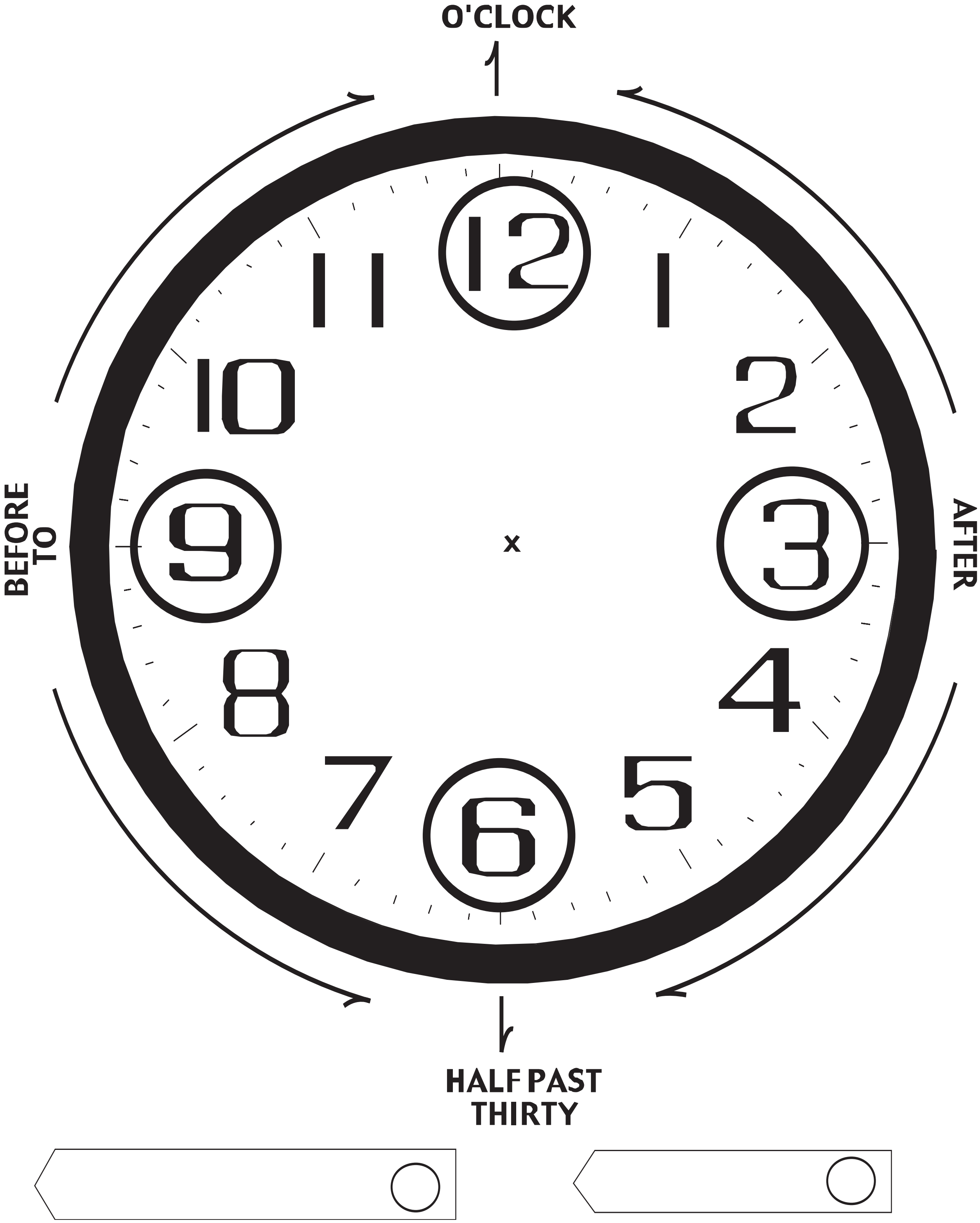
Teacher gives real life story problems to class and observes as they show the answers on their clocks.

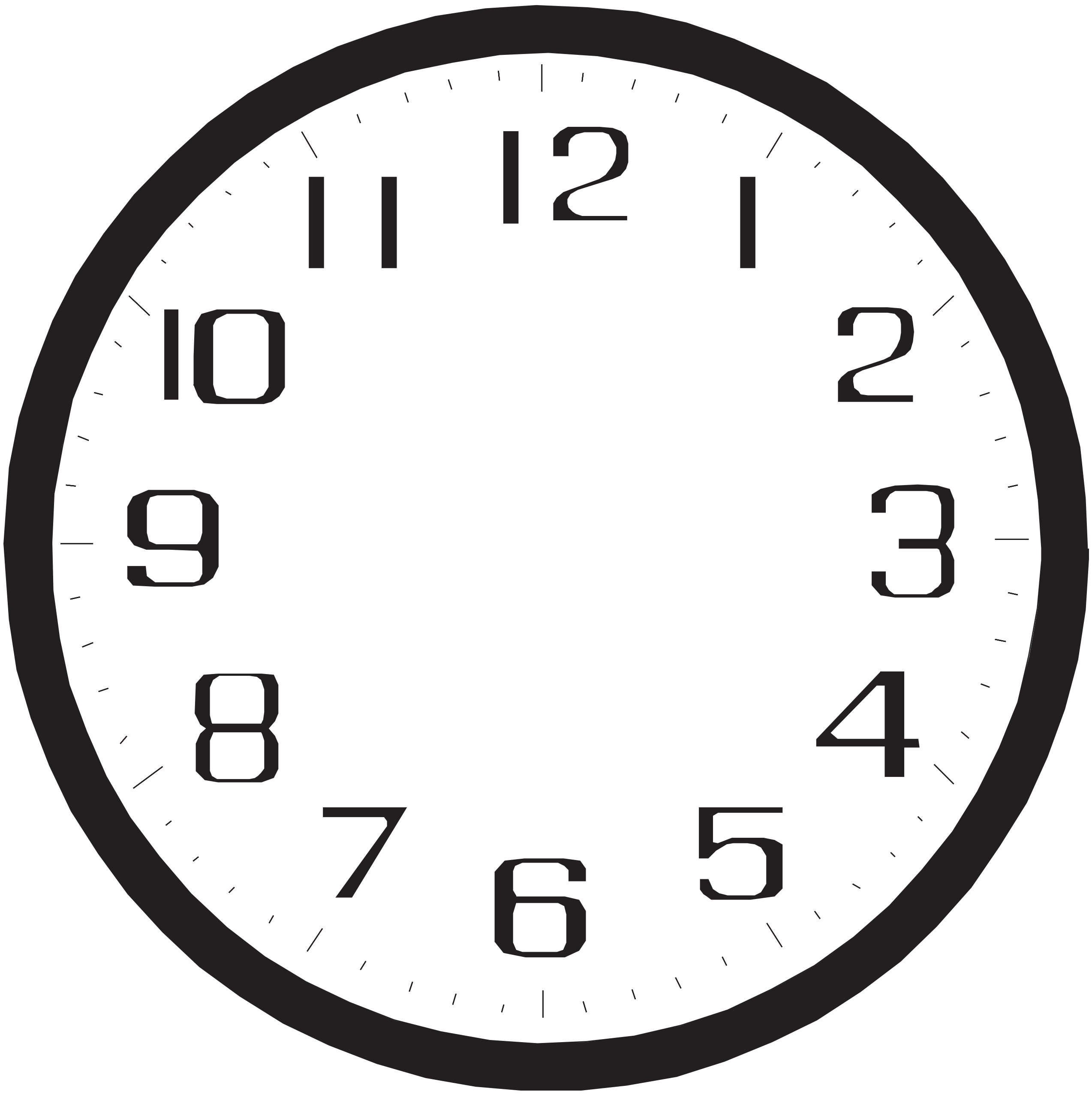
Students have clocks on a sheet of paper with the time illustrated on them. Have students write the time underneath their clocks (or 10 minutes after each time, or 15 minutes before or after the given times, etc.).

Homework & Family Connections

Let students take their clocks home, and teach and practice with parents or siblings.

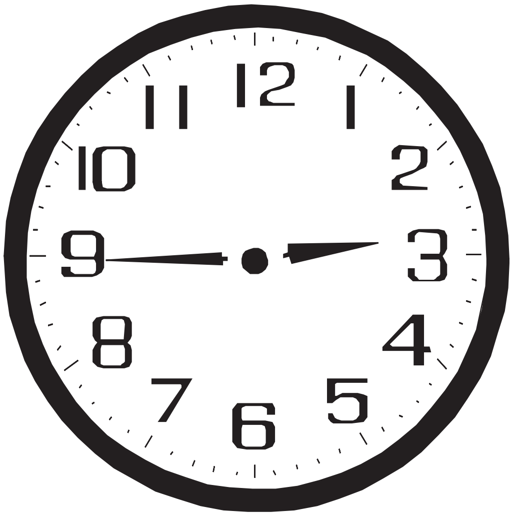
Clock Pattern




12
1
2
3
4
5
6
7
8
9
10
11

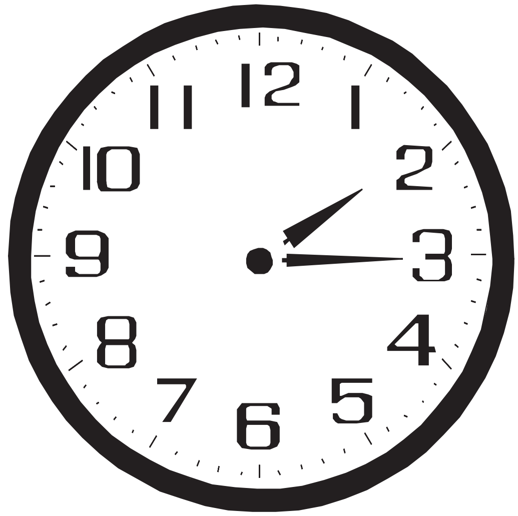
"I Have" Clock Activity Cards

I have



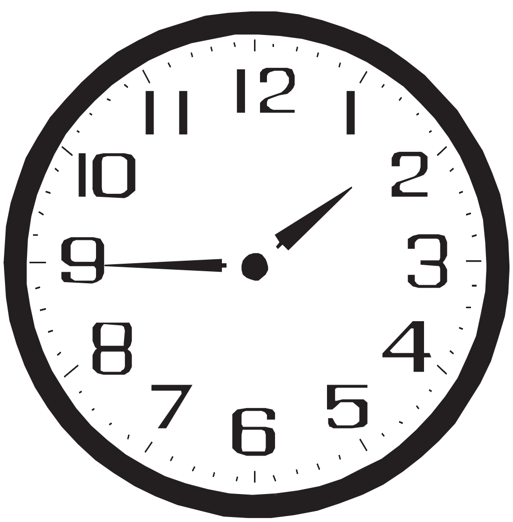
Who has 1 hour
less than me?

I have



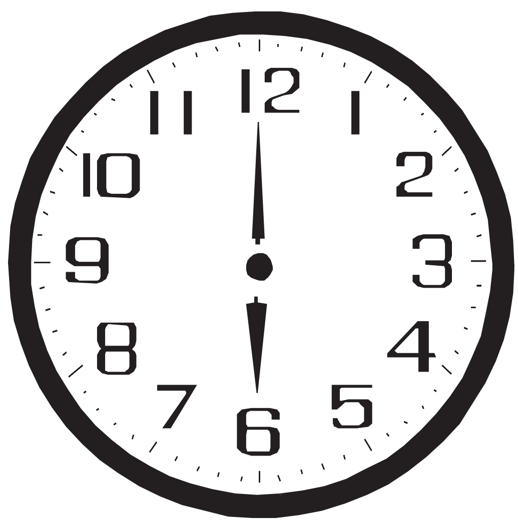
Who has 3 hours
45 minutes more
than me?

I have



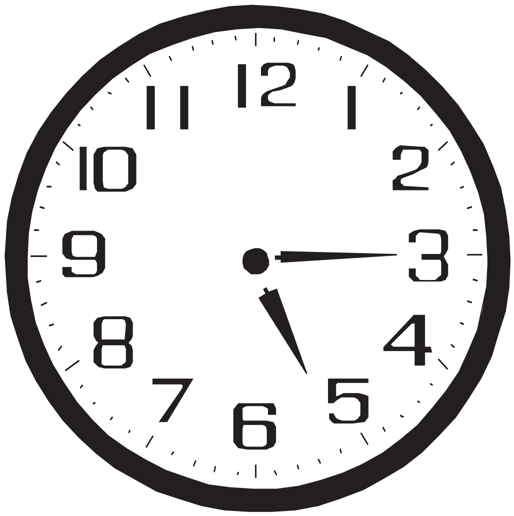
Who has 30 minutes
more then me?

I have



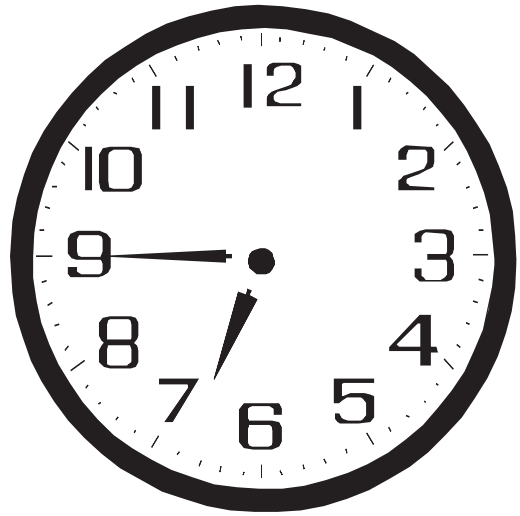
Who has a quarter
past 5?

I have



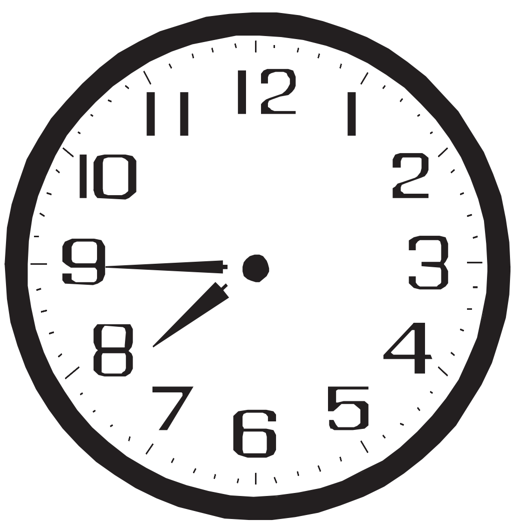
Who has 2 1/2
hours more than me?

I have



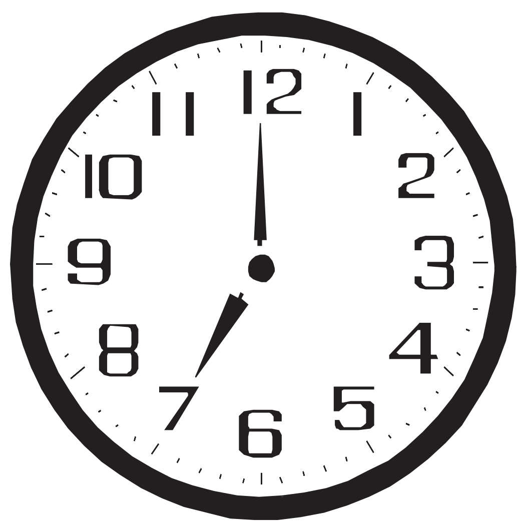
Who has 15 minutes
more than me?

I have



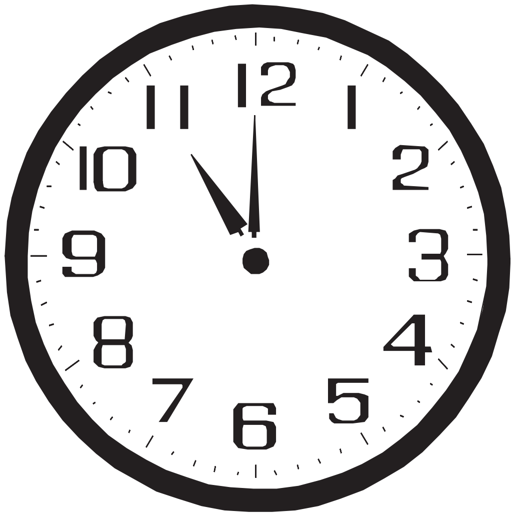
Who has 1 hout less
than me?

I have



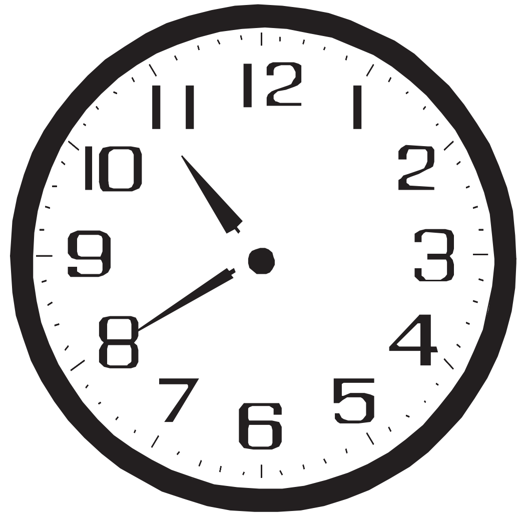
Who has 4 hours
more than me?

I have



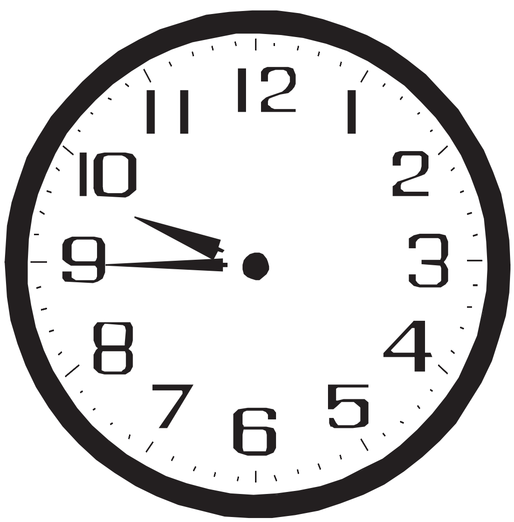
Who has 1 hour 15
minutes less than me?

I have



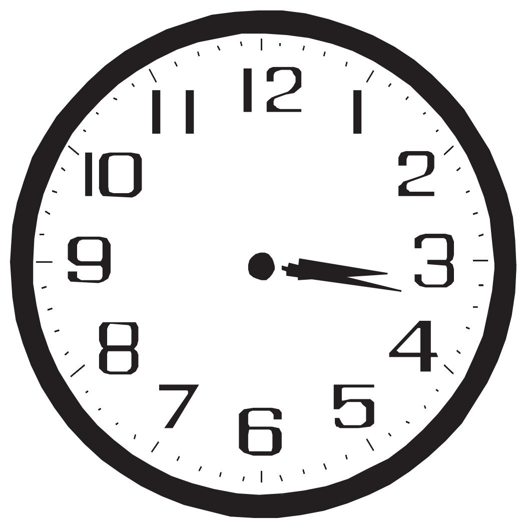
Who has 3:17?

I have



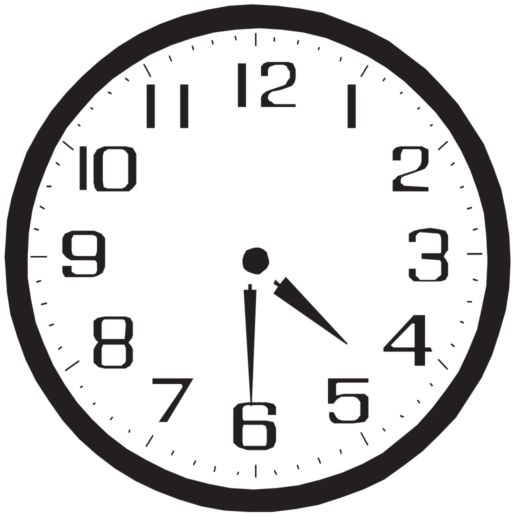
Who has 55 minutes
more than me?

I have



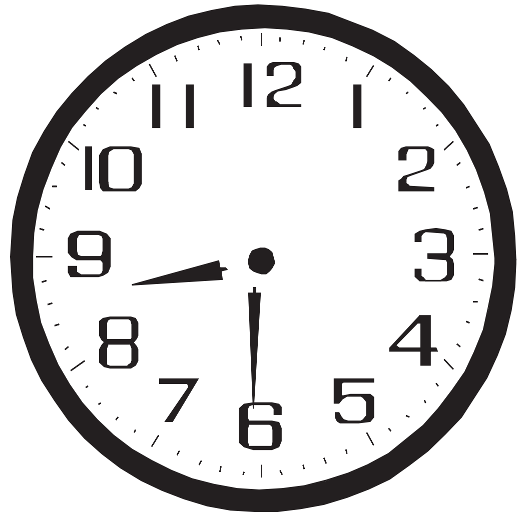
Who has 1 hour 13
minutes more than me?

I have



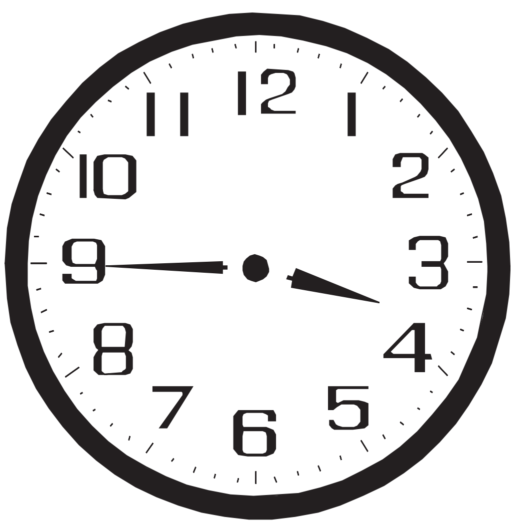
Who has 45 minutes
less than me?

I have



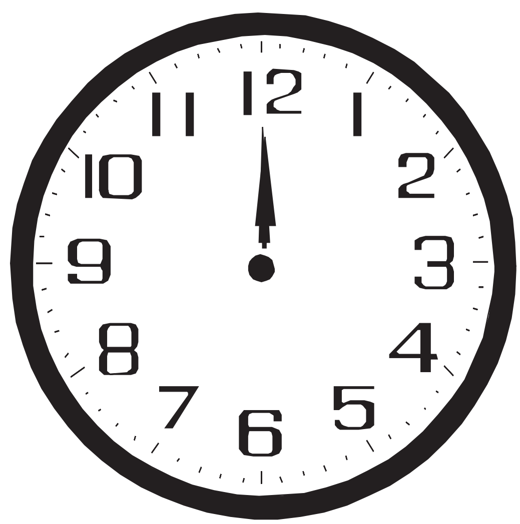
Who has 3 hours
30 minutes more
than me?

I have



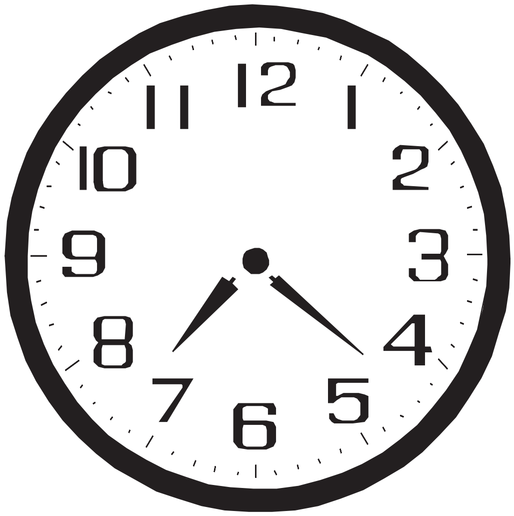
Who has half past 8?

I have



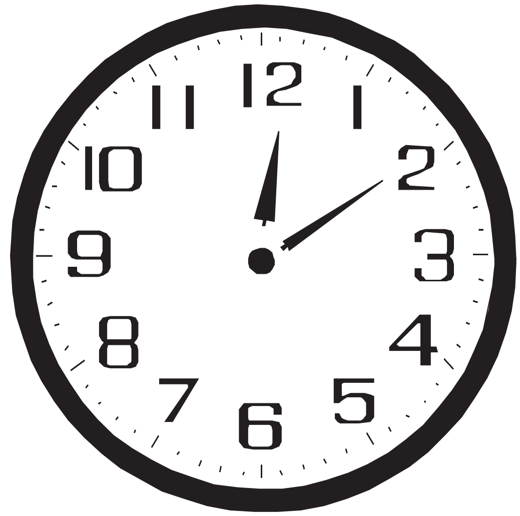
Who has 7:22?

I have



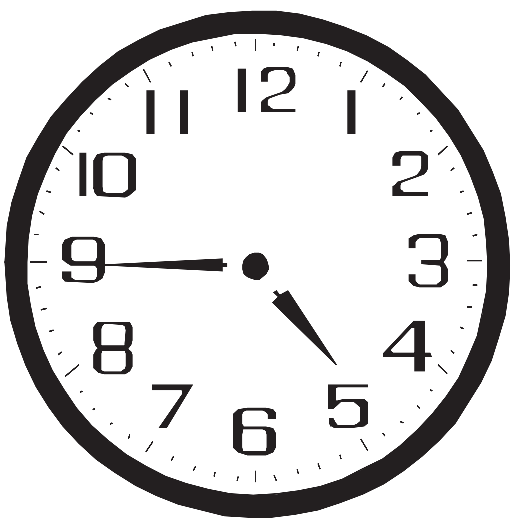
Who has a quarter
to 5?

I have



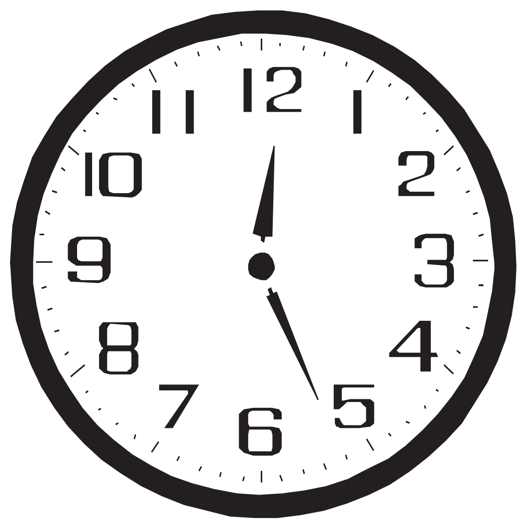
Who has 15 minutes
more than me?

I have



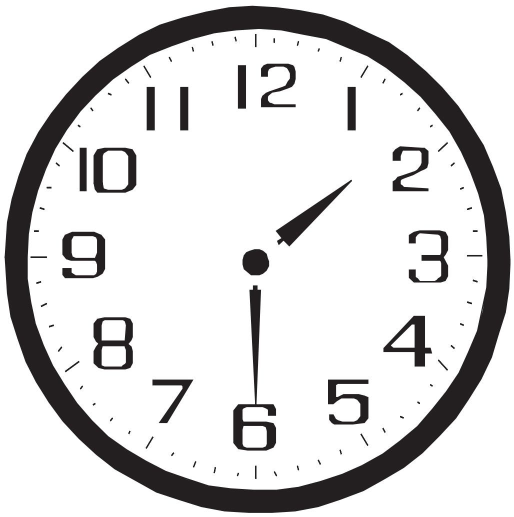
Who has 12:11?

I have



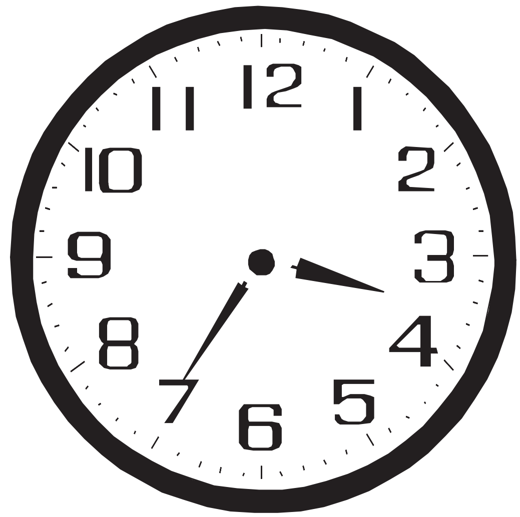
Who has 1 hour and 4
minutes more than me?

I have



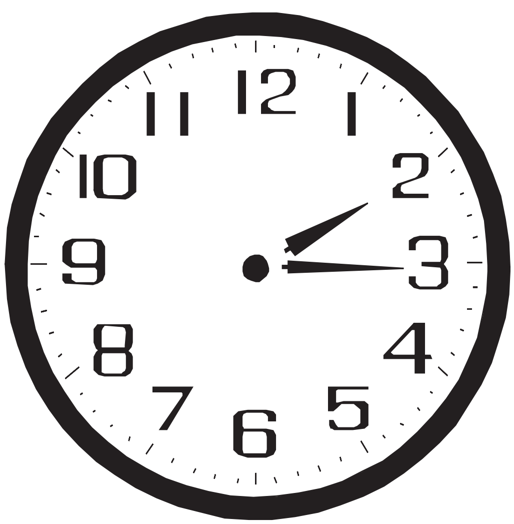
Who has 45 minutes
later than me?

I have



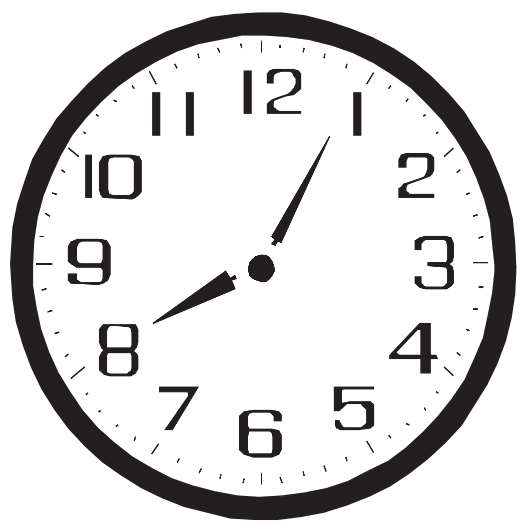
Who has 4 1/2 hours
more than me?

I have



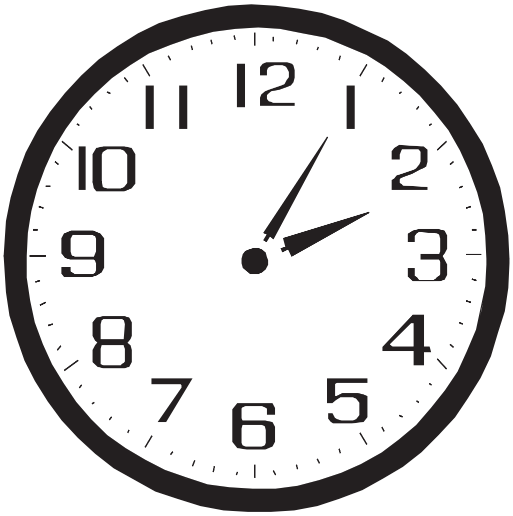
Who has 1 hour 20
minutes more than me?

I have



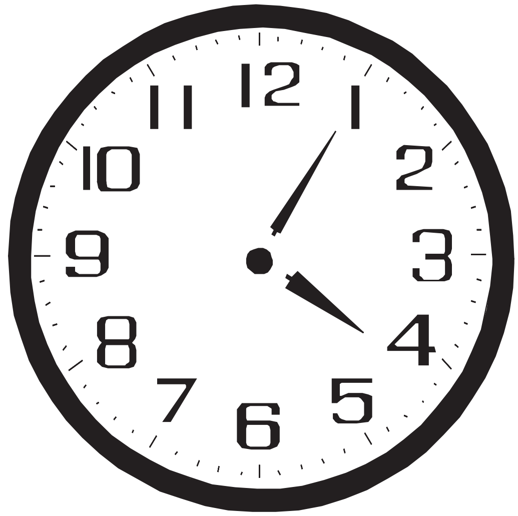
Who has who has 6 hours
less than me?

I have



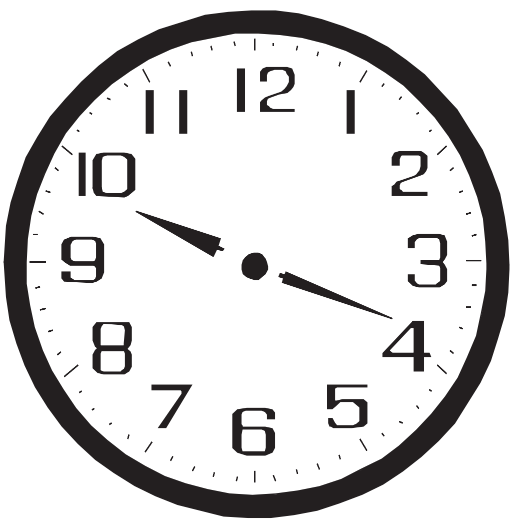
Who has 50
minutes after 3:00?

I have



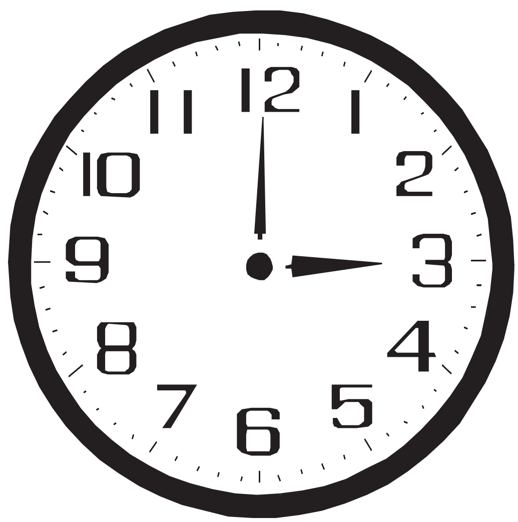
Who has 1 hour 5 minutes
less than me?

I have



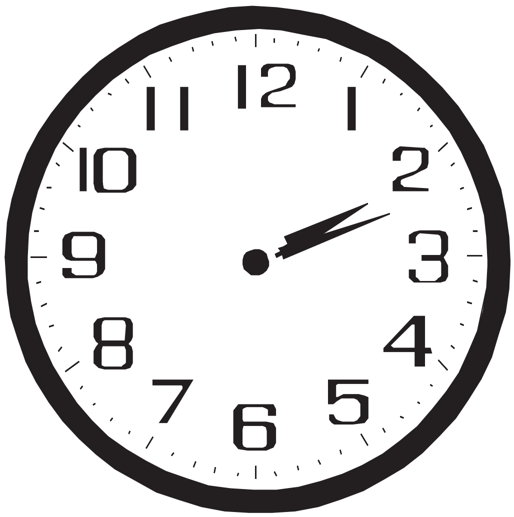
Who has 15 minutes
more than me?

I have



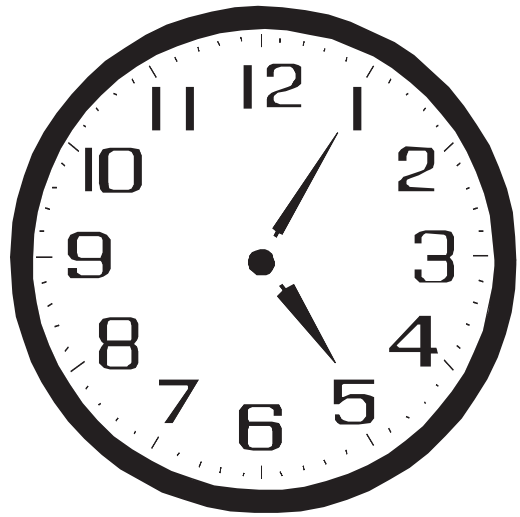
Who has 48 minutes
less than me?

I have



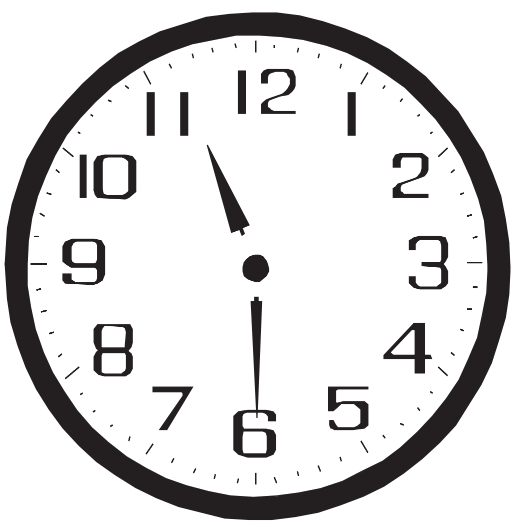
Who has 30 minutes
past 11?

I have



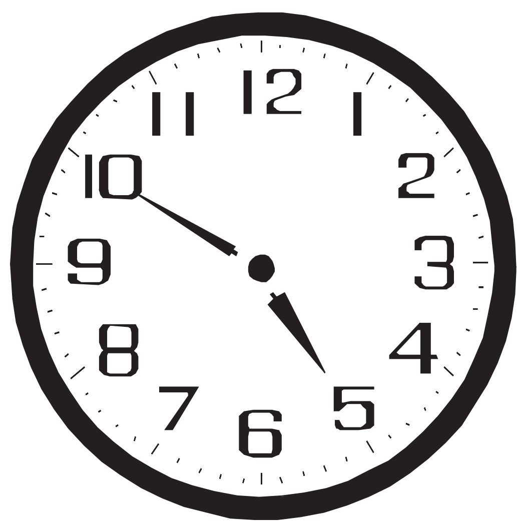
Who has 15 minutes
less then me?

I have



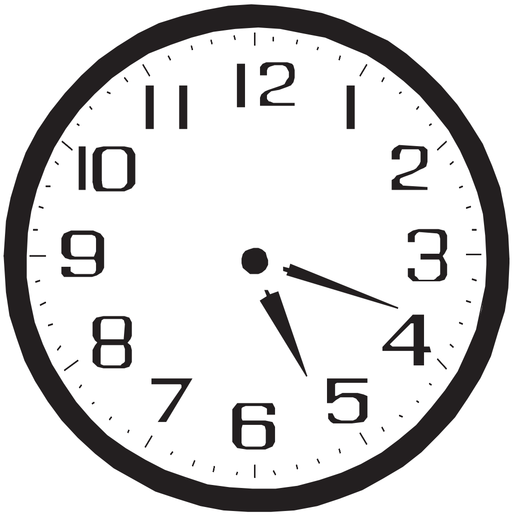
Who has 5 hours 35
minutes more than me?

I have



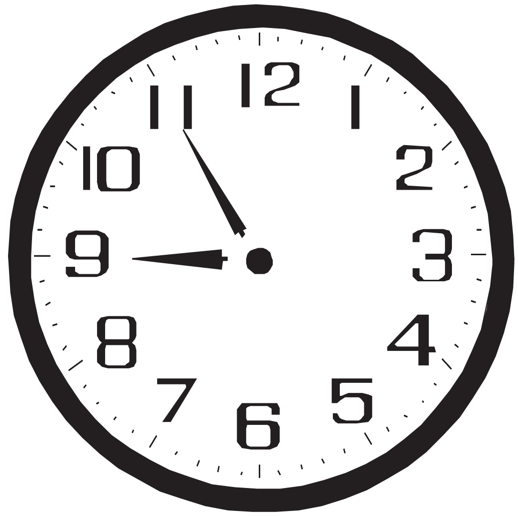
Who has 28 minutes
more than me?

I have



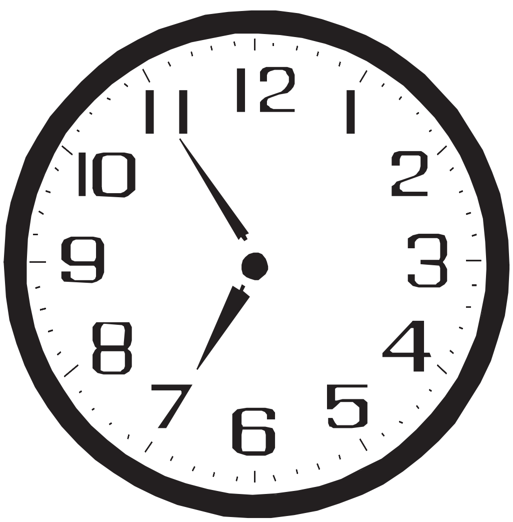
Who has 5 minutes
before 7:00?

I have



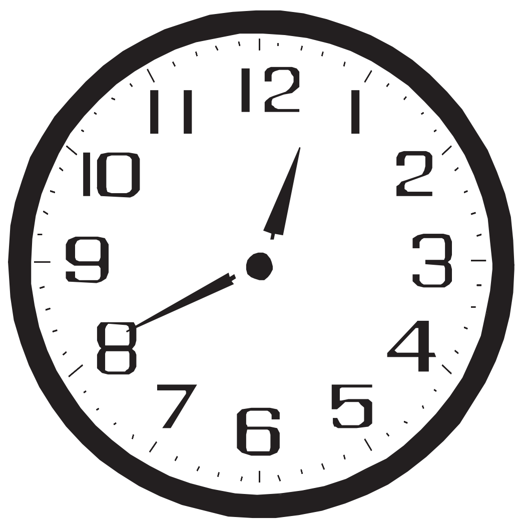
Who has 20 to 1:00?

I have

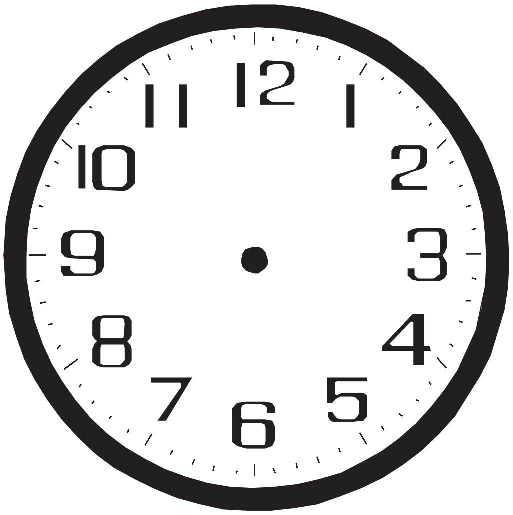


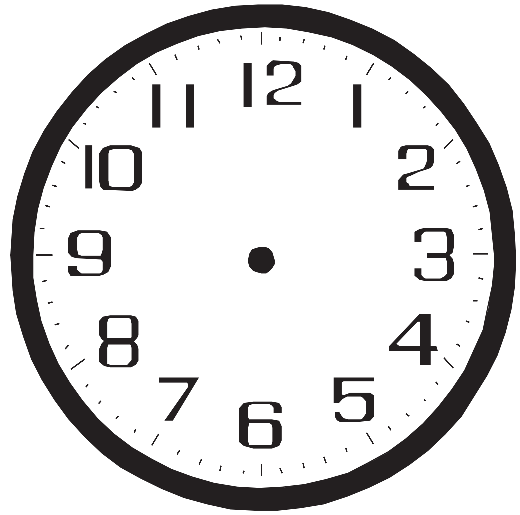
Who has 2 hours
more than me?

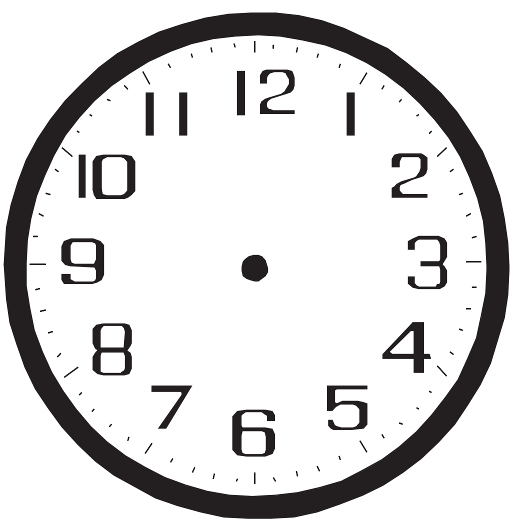
I have

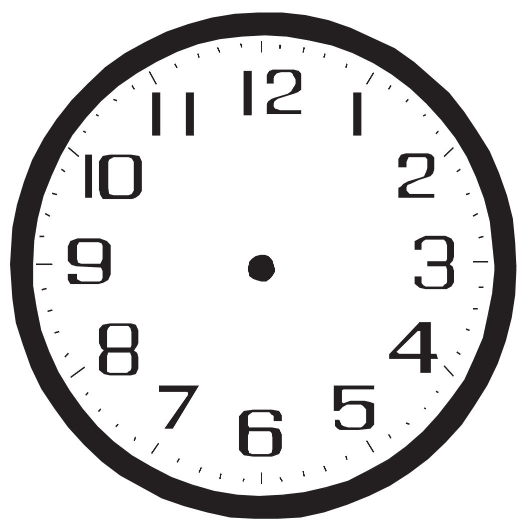


We're all done!









ANSWERS FOR THE "I HAVE" CLOCK ACTIVITY
(36 CLOCK CARDS)

1) 2:45
2) 1:45
3) 2:15
4) 6:00
5) 5:15
6) 7:45
7) 6:45
8) 7:00
9) 11:00
10) 9:45
11) 10:40
12) 3:17
13) 4:30
14) 3:45
15) 8:30
16) 12:00
17) 7:22
18) 4:45
19) 12:11
20) 12:26
21) 1:30
22) 2:15
23) 3:35
24) 8:05
25) 2:05
26) 3:50
27) 4:05
28) 3:00
29) 2:12
30) 11:30
31) 5:05
32) 4:50
33) 5:18
34) 6:55
35) 8:55
36) 12:40

Standard V Activities

Activity—Dice Sums

Standard V
Students will use concepts of probability and collect, analyze, and draw conclusions from data.

Objective
1. Formulate and answer questions using statistical methods to analyze data.

Intended Learning Outcomes
3. Reason mathematically.
6. Represent mathematical situations.

Standard V

Objective 1

Connections

Background Information

Providing experience with probability for your students is an opportunity to enrich their mathematics learning. Concepts in probability are basic to a well-rounded education in mathematics and contribute to developing students' critical thinking skills. Probability activities push children to devise ways to deal with uncertainty and provide opportunities for them to formulate and test ideas. In addition, experiments in probability provide contexts in which students apply the arithmetic they are learning.

Children's initial experiences with probability need to come from problem situations in which their intuition is developed. Once they sense what "should happen" in a situation, children can be challenged to test the validity of their ideas.

This lesson uses dice to model an experience with probability that is appropriate for children in the elementary grades. It builds on children's informal experience with dice. Students are taught to play a game with two dice and then are asked to devise a winning strategy for it. The activities in this lesson focus the children on investigating the probabilities of the sums that come up when two dice are tossed.

Invitation to Learn

Hold up two dice. Ask students what they know about them. Say, "If I roll two dice and figured the sum of the dots that come up, what is the smallest sum I could get?" Write 2 on the board. Ask, "What is the largest sum I could get?" Leave a space and write 12 on the board. Then ask, "What about 3? Is a sum of 3 possible? How could you get it? Continue in this fashion for 4, 5, 6, and the rest of the numbers up to 12. Say, "Let's see how many possible sums there are." Together count the numbers from 2 to 12 that you have written on the chalkboard to see that there are eleven sums possible from rolling two dice.

Instructional Procedures

Tell students you are going to teach them a game to play with two dice called Dice Sums.

1. Students play with a partner.
2. Partners will need 11 counters and a 2-to-12 number line.
3. Have students arrange their counters on their number line. They can do this in any way they wish: one counter per number, all counters stacked on one number, or counters grouped in any way they would like.
4. Once all teams have placed their counters on the number line, the teacher rolls the dice and calls out the sum of the dice.
5. If they have a counter on that number, they remove it. For example: If you roll an 8 and they have a counter on 8, they remove the counter from the number line. If they have more than one counter on 8, they only remove one counter per roll.
6. The idea is to be the first team to remove all their counters. Encourage students to talk with their partner about how they would like to arrange their counters to be the first team to have them all removed.
7. Students are not allowed to re-arrange their counters once the game begins.
8. As the game is played, ask students what numbers they are waiting to be rolled. Discussion should take place during the game about numbers that were "good" ones and numbers that did not come up as quickly.
9. Play the game again and this time have students record where they placed their counters so you can discuss which sums seem to be coming up more often than others. When you get a winner, ask winners to report their winning arrangement.
10. Hand out additional number lines and two dice to each pair of students so they can play the game with each other. Encourage them to discuss their arrangements to see what they can learn.

Materials

- ❏ Two six-sided dice (per pair of students)
- ❏ Counters or markers, 11 per student (pennies, beans, etc.)
- ❏ 2-12 number line
- ❏ Dice Sums recording sheet

Curriculum Integration

Math/Science—Dice Shakers: Students can create their own dice shaker to use during this unit. Collect empty paper towel or toilet paper tubes. Cut the paper towel tube to the appropriate size. Have students cut circles from construction paper large enough to cover the bottom of the tube. Tape construction paper circles to the bottom of the tube before covering the sides of the tube. Have students use construction paper to cover the outside of the tube, and then decorate the tube with drawings, constructions paper cutouts, etc. Students now have their own personalized shaker to use when playing dice games

Read *Jumanji* by Chris Van Allsburg. The story begins when Judy and Peter find a board game in the park. They take it home, hoping to alleviate their boredom. One live lion, an erupting volcano, and a dozen destructive monkeys later, the children are no longer bored. Their jungle adventure game has come to life! Readers will tremble along with Judy and Peter, urging them to roll the dice that will plunge them from one perilous predicament into another.

Possible Extension/Adaptation

After students have played for a while, have them try a different, but related activity. Give each student the Dice Sums worksheet. Have them roll the dice and record the sum with an X in the proper column. Model this for students. Explain to students that they will continue to roll the dice and record an X in the proper column until one sum reaches the bottom of the column. Then record which sum reached the finish line first on the class chart. Students may complete more than one Dice Sums sheet while waiting for others to finish.

When most students have had time to extend the game with the tally sheet, call class together to discuss results.

Possible discussion questions:

- Which number comes up most often? Why do you think this is so?

Lead students to discuss how many ways there are to arrive at each sum. For example: There is only one way to get 12: a six and a six. There are several ways to get 7: a three and a four, a five and a two, a six and a one, etc.

Ask students to think about how they would now arrange the counters to try to win the game. Leave materials out for as long as there is interest in the game. This will give students a chance to solidify their strategies for winning the game; hopefully understanding that some numbers come up more often than others for very good reasons.

Assessment Suggestion

Students write a description of the arrangement they would use explaining why they made their choice.

Have students write what they have learned from playing the game.

Additional Resources

A Collection of Math Lessons From Grades 3-6 by Marilyn Burns

Homework & Family Connections

Teach the game to someone at home. Have that person write a description of what they have learned and their thoughts about the game. Share their writing with the class the next day.

2-12 Number Line

2 3 4 5 6 7 8 9 10 11 12

Name______________________________

Dice Sums

Roll 2 dice. Mark an X under the sum. Do this until one sum reaches the Finish Line.

2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

Finish Line

Recored the sum that finished first on the class chart.

Activity—Alphabet Study

Standard V Students will use concepts of probability and collect, analyze, and draw conclusions from data.	
Objective 1 Formulate and answer questions using statistical methods to analyze data.	
Objective 2 Apply basic concepts of probability.	
Intended Learning Outcomes 3. Reason mathematically. 4. Communicate mathematically. 5. Make mathematical connections.	

Standard V

Objectives 1 & 2

Connections

Background Information

The order of usage of alphabet letters is useful, even essential, information for people. Manufacturers of bulletin board letters, marquee letters, and artists' press-type letters certainly need to know. It is also valuable to people who break codes, important at times of national security, and recreational at times for those who solve puzzles. In this lesson, students are engaged in a statistical study in which they investigate the order of usage of alphabet letters. Students will collect and organize data. They will compare their individual data with data collected in their groups, and then with the data collected in the entire class. This will give them experience as they compare results from different sample sizes.

Invitation to Learn

Read *Martha Blah Blah* by Susan Meddaugh. This is the story of a talking dog who loses her ability to talk when the owner of the alphabet soup company decides to cut back and leave some letters out of the alphabet soup she eats.

Announce to your class that for the next few days they will be studying the alphabet. Tell them they will be involved in a math exploration with the alphabet in a way that helps them investigate ideas about probability and statistics. Without discussing their thoughts with others, tell students to predict what they think are the five most commonly used letters in the English language (the letters that occur most often when people write books, newspaper articles, magazine articles, etc.). Have students list their choice of five letters and put a star next to the one letter they think is used the most.

Materials

- ❑ Chart paper
- ❑ Markers
- ❑ Adding machine tape
- ❑ *Martha Blah Blah* by Susan Meddaugh

Instructional Procedures

1. After students have made their individual predictions explain what they will do in their groups. Students share their predictions in their groups and come to a group decision about what they think are the five most commonly used letters and which one occurs most. Write each group's prediction on the board. Ask students to explain why they predicted what they did.
2. Real world connections: Next ask students who in the world they think would care about the order of usage of alphabet letters. Ask if they can think of anyone who needs to know or who would benefit from knowing. If students cannot come up with any, some include: Wheel of Fortune participants, Scrabble players (some letters are worth more than others), and letter manufacturers. Show a sheet of press-type letters purchased from an art store, the kind of type that you transfer onto paper by rubbing. Explain that they are commonly used by artists and designers when they need lettering.
3. Explain to students that you already know which letter is used the most often and that you have prepared a list of all twenty-six letters of the alphabet in order of their usage in the English language. Write the letters on an addition machine tape, roll the tape up and paper clip it.
4. Tack the adding machine tape to the wall or board in the rolled up position. Explain to the class that before unrolling it that you are planning to have them conduct a mathematical investigation. Then you will check the class' prediction with what is on the rolled-up tape.
5. Explain that it would be very difficult to take all the writings ever done in the English language and count up how many A's were used and how many B's, C's and so on, but it would be possible to look at smaller samples. Explain to students you would like each of them to do the following:
   - Pick a sentence with at least five words in it from any book.
   - Copy the sentence onto a piece of paper.
   - Find out and record how many times each letter of the alphabet appears in your sentence.
6. Give time for several students to explain how they went about counting which letters were used more often than others.

7. Have students then compile their records to find group (4-5 students) totals for how often each letter appeared. Explain that once their group had arrived at their totals, you will have a large class chart on the board in which one student from each group should record their group totals. The chart should resemble this:

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Group 1																										
Group 2																										
Group 3																										
Group 4																										
Group 5																										
TOTALS																										

8. When all group data is entered on the chart, assign each group to find the total for 4 or 5 letters. Students should be allowed to use calculators for a task such as this. Suggest that several students find the total for the same letter so that the accuracy of the totals is valid. Record totals on the chart.
9. When all totals are entered on the chart, have the class gather around the chart. Have students tell you which letter is used most often while you write this on the board next to the rolled up tape. Students can help you determine the order of usage of letters for the class investigation while you record them.
10. Before unrolling the tape, ask students what their thoughts were about their totals and the totals on the tape. Would more than half or less than half of the letters match? How much of the order would have to match for them to be satisfied with their statistical sample? What would disappoint them?

Possible Extensions/Adaptations

Explain to students the following history of the typewriter keyboard. When Christopher Sholes invented the typewriter in 1867, he purposely scrambled the letters so that typists couldn't type too quickly and jam the letters. That was before the invention of electric typewriters, which eliminated the problem of jamming when someone typed too quickly. On a typewriter keyboard, more than half of the strokes are done with the left hand, the weaker hand for most people. The two most agile fingers on the right hand rest on J and K, which are two of the least used letters. The left pinky, the weakest finger for most, rests on the A. In 1930, August Dvorak designed a keyboard on which it is possible to type more quickly. Although used on some Apple computers, it has not received widespread use.

Students can choose one from the following extension assignments:

1. Design a typewriter keyboard that makes use of the results of our mathematical investigation. When they finish, they will be able to compare it with the Dvorak keyboard.
2. Investigate the game Scrabble. List the letters in the game in two ways—in order of their values and in order of how many there are of each. See how it relates to the findings of the class. Describe your thoughts as to why you do or do not think the Scrabble scoring or the numbers of each letter ought to be changed.
3. Do a statistical sample for another language and compare the results with our findings for English. If others in the class choose this assignment, combine your findings for a larger sample.
4. Cryptograms are communications written in code, in which each letter stands for another letter in the alphabet. Whichever letter is used as a substitute for A, for example, is used for A throughout the puzzle. Figure out this message:

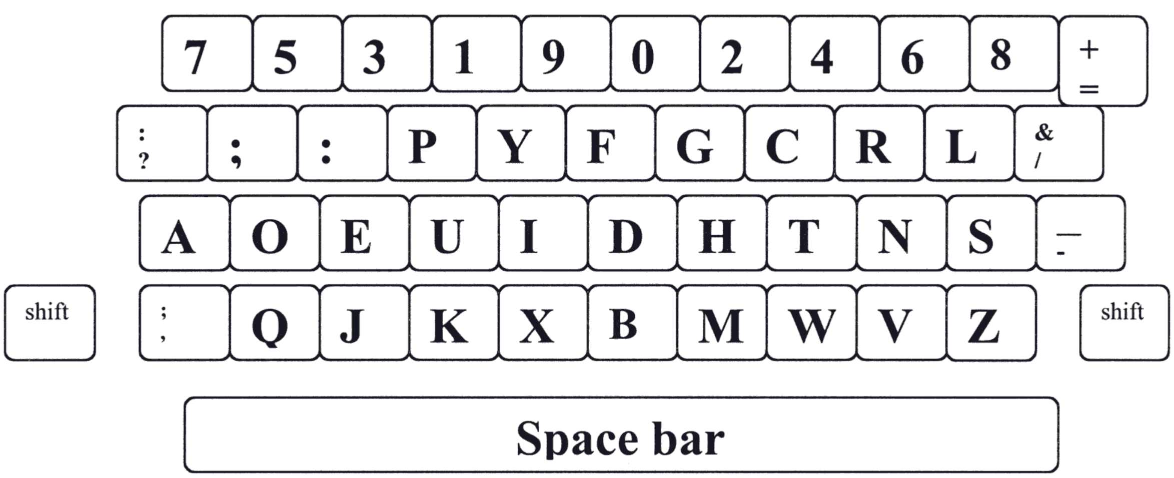
   AB CD EDDF AG FXD HIJY

   A CAKK LMN NZM IG AWD WJDIE WZGD.

   Here's a clue: The most frequently used words in the English language are the, an, a, and and.

   The order of the usage is as follows:

1.E	2.T	3.A	4.O	5.NI	6.S	7.R	8.H	9.L
10.D	11.C	12.U	13.P	14.F	15. M	16.W	17.Y	18.BG
19.V	20.K	21.QX	22.JZ.					



Assessment Suggestion

Possible questions to stimulate discussion:

1. What would happen if we had done a larger sample?
2. How might the results differ if we had used first grade readers instead of the assortment of books we did use?
3. What if we used sentences from a book written in Spanish or French or some other language?
4. If we were to try the sampling experiment again, would we be likely to produce results that would match exactly the order on the adding machine tape?
5. Why do you think mathematicians say that larger samples will more closely resemble what really happens?
6. Do you think a larger sampling of letters would produce results closer to the actual order?

Additional Resources

A Collection of Math Lessons, From Grades 3-6 by Marilyn Burns

Homework & Family Connections

Ask students to have family members help them find games in newspapers, magazines, or books at home that use letters to solve the puzzle. An example would be the Cryptoquotes or Word Jumble printed in most daily newspapers. Crossword puzzles are also an excellent extension to this lesson. Students can cut these from the newspapers and bring them to class to share. Discuss how letter usage can help solve some puzzles.

Activity—Dodecahedron Race

Standard V

Objective 2

Connections

Standard V
Students will use concepts of probability and collect, analyze, and draw conclusions from data.

Objective 2
Apply basic concepts of probability.

Intended Learning Outcomes
3. Reason mathematically.
4. Communicate mathematically.

Background Information

Students may not be familiar with polyhedra dice. Be sure to allow students plenty of time to explore the polyhedrons. Most students seem fascinated by the different polyhedrons and enjoy taking a closer look at them. Focusing their attention on the games later will be easier if they are initially given time to explore the materials on their own. If students play the games at their desktops or tables, a good suggestion to keep the polyhedrons from flying all over the room is to set a rule that if the polyhedrons fall onto the floor, the students loses a turn. Have students roll the dice into a shoebox or shoebox lid to contain them. Most games work best when students play as a two-person team. Interacting with a partner encourages students to discuss and reflect on their strategies and ideas – a wonderful way for students to communicate mathematically.

Invitation to Learn

Show students a dodecahedron die (numbered 1-12). Ask if anyone has ever seen a die with this many sides. Discuss the name of the die, dodecahedron. Ask for students to help define dodecahedron. Dodeca = twelve, hedron = a figure having ____number of sides. . . so. . . dodecahedron is a twelve sided object. Tell students you are going to teach them a game called Dodecahedron Race in which they will be using a dodecahedron. In this game the first number whose row is completely filled wins.

Instructional Procedures

1. Students will take turns rolling the dodecahedron and recording the results on the chart. Place an X in the box of the number that is rolled. Demonstrate to students how to record their numbers.

2. Have students predict which number will win the race. They should write their prediction on the top of their score sheet. Discuss with their partner why you think your number will win the race.
3. One person rolls the dodecahedron. The other person marks an X in that number's row on the race chart.
4. Continue rolling the dodecahedron until one row is filled with Xs.

Play the game at least three times.

Materials

- ❑ 1 dodecahedron, numbered 1-12
- ❑ 3-4 race charts
- ❑ Probability Questions worksheet

Possible Extensions/Adaptations

"Probability Questions" worksheet

Answers to Probability Questions:

1. 1 out of 12 because there are twelve possibilities and only one number 2.
2. 1 out of 12 because there are twelve possibilities and only one number 6.
3. 1/6, 1 out of 6, and 1:6.
4. 1/12, 1 out of 12, and 1:12.
5. 5 out of 12
6. 9 out of 12

BONUS: 1/12, 1 out of 12, and 1:12.

Assessment Suggestion

Use the following questions as a beginning to a discussion on probability. It is helpful to record the different ways to write each outcome.

1. What are the chances that 1 will win the next race you play? Why?
2. What are the chances that 10 will win? Why?

   Continue explaining the chance of each number winning. Discuss it this way:

   - The probability of rolling a 3: There is one way to roll a 3, and there are 12 possible outcomes. The probability of rolling a 3 equals 1 out of 12, 1:12 or 1/12.
   - The probability of rolling a 4: There is one way to roll a 4, and there are 12 possible outcomes. The probability of rolling a 4 equals 1 out of 12, 1:12 or 1/12.

- The probability of rolling an odd number: There are six ways to roll an odd number (1,3,5,7,9,11) and there are 12 possible outcomes. The probability of rolling an odd number equals 6 out of 12, or 6:12 or 6/12.
- The probability of rolling a number less than 12: There are 11 ways to roll a number less than 12. The probability of rolling a number less than 12 is 11 out of 12 or 11:12 or 11/12.

Additional Resources

Exploring Math with Polyhedra Dice by Nancy Segal Janes

Dodecahedron Race

Player #1__________________________

I predict ______ will be rolled more than any other number.

Player #2__________________________

I predict ______ will be rolled more than any other number.

1										
2										
3										
4										
5										
6										
7										
8										
9										
10										
11										
12										

Probability Questions

1. What are the chances that 2 will win the next race you play? Why? ____________________

2. What are the chances that 6 will win the next race you play? Why? ____________________

3. Write at least two different ways of expressing the probability of rolling a 6. (Hint: as a fraction and as a ratio) ____________________

4. Write at least two different ways of expressing the probability of rolling a 12. ____________

5. What is the probability of rolling a number less than 6? Explain ____________________

6. What is the probability of rolling a number more than 3? Explain. ____________________

BONUS:

Write three different ways to express the chances of rolling an 11. ____________________

Activity—Likely or Unlikely

Standard V
Students will use concepts of probability and collect, analyze, and draw conclusions from data.

Objective 2
Apply basic concepts of probability.

Intended Learning Outcomes
1. Demonstrate a positive learning attitude toward mathematics.
3. Reason mathematically.

Standard V

Objective 2

Connections

Background Information

This activity introduces children to sampling with replacement as a way to predict how many of each color are in a bag of color tiles. This activity engages children in taking samples and analyzing data, and provides them with opportunities to think proportionally. Students discuss and define words such as certain, impossible, likely, unlikely, 50/50 chance, etc.

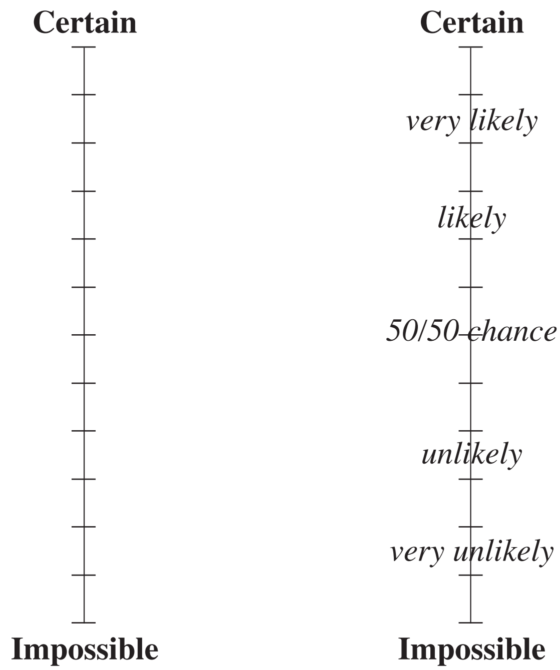
Invitation to Learn

Show students a paper bag and tell them you have placed color tiles in the bag. (Put 12 tiles of the same color in the bag, for example: 12 yellow tiles). Walk around the room letting children look into the bag. After everyone has had a chance to see inside the bag, tell students you are going to shake the bag up, and without looking inside you will put your hand into the bag and take out one tile. Ask students to predict what color of tile they think you will pull from the bag. You should hear a chorus of YELLOW. Hold a discussion about why it would be yellow. Ask students how likely it would be that you would draw a yellow tile from the bag. Students might respond by saying it is very likely, extremely likely, highly likely, etc. If students do not say the word CERTAIN, add that to the discussion. Tell students there is a numerical way to write the chance of pulling a yellow tile from the bag; write 12 out of 12. Then write 12/12 and say "12 out of 12." Then write 12:12 and say "12 to 12."

Instructional Procedures

1. Write the words "Certain" and "Impossible" on the board, each on one end of a vertical line. Do not write very likely, likely, 50/50 chance, etc. Discuss these words and let students tell you where to place them on the line.

Materials
- ❑ Paper bags
- ❑ Color tiles



- Ask a student to help you place an X on the line between “Certain” and “Impossible” that would display the chance of pulling a yellow tile from the bag.
- Now take one yellow tile out and put in one red tile. Again walk around the room letting students look into the bag. Again ask students to predict what color of tile they think you will pull from the bag if you were to shake it up and randomly draw one tile.
- Ask students if they can tell you the numerical way to write the chance of pulling a red tile from the bag (1out of 12; 1/12; 1out of 12, etc.) Write these on the board as students dictate them.
- Fill in the chart by adding the words likely and unlikely, 50/50 chance and then very likely and very unlikely, discussing with students the placement of each phrase.

2. Continue changing the make-up of tiles in the bag. Start by adding one or two of another color and taking out one or two of the original color. For now, only use two colors. Keep the discussion on the number of tiles and the number of each color. Each time you take a tile, write the numerical display in several ways on the board, saying it out loud.
3. Now put 12 tiles in a bag (example: 8 red and 4 yellow). Tell students you have 12 tiles of two different colors. Explain that you are going to pull one tile at a time from the bag without

looking inside. Write a T-chart on the board with RED and YELLOW for the headings. Put a tally mark under the color you drew out. Have students take turns pulling out one tile at a time and replacing it back into the bag. Explain to students you are conducting a "sampling with replacement" experiment. Each time, record what color was taken from the bag. After 12 times, ask students if they can predict how many of what color tiles are in the bag. Students should be able to predict different combinations that add up to 12 (11 red, 1 yellow; 10 red, 2 yellow, etc.) Have students write their prediction on paper. Continue to draw 12 more tiles, one at a time, from the bag recording the color of tile on the T-chart. Again ask students to predict how many of what color tiles are in the bag. Explain to them that they now have more information than they did after just 12 draws to help them formulate their prediction. Again have students write their new prediction under their first prediction. Draw 12 more tiles, recording them on the board in the T-chart. Lead a discussion about what tiles students think are in the bag now. Have their predictions changed from the first one? Why? Remind students you are conducting an experiment by "sampling with replacement." Have students write a prediction of the contents in the bag by completing the sentence: I think there are ____ yellow tiles and _____ red tiles in the bag because. . .

Curriculum Integration

Math/Science—Read *Probably Pistachio* by Stuart Murphy. This is the story of Jack a young boy who thinks nothing is going his way. Will he get what he wants in his lunch? Probably not! This story centers on probability and Jack's chances of things going his way.

Possible Extensions/Adaptations

Pair up students. Have each pair put 12 tiles of two different colors into a bag, discussing their choices. Have partners trade their bag with another pair. Partners conduct an experiment by sampling with replacement. After 12 draws, have students record their predictions of how many of each color tile are in the bag. Students then draw 12 more times, making another prediction and explaining their thinking in a paragraph. Finally, have students dump the contents of the bag to reveal the correct combination of color tiles.

Assessment Suggestion

Students should keep a log of what happened in their experiment. They can use the T-chart or construct their own way of displaying the information they gathered. After they have completed the assigned number of pulls from the bag and recorded their results, ask students to discuss these questions with their partner.

- How likely is it to pull a yellow tile from the bag when there are 12 yellow and 0 red?
- How likely is it to pull a yellow tile from the bag when there are 11 yellow and 1 red?
- How likely is it to pull a yellow tile from the bag when there are 10 yellow and 2 red?
- How likely is it to pull a yellow tile from the bag when there are 2 yellow and 10 red?
- How likely is it to pull a yellow tile from the bag when there are 1 yellow and 11 red?
- How likely is it to pull a yellow tile from the bag when there are 3 yellow and 9 red?
- How likely is it to pull a yellow tile from the bag when there are 7 yellow and 5 red?

Ask students to write their explanation to the following questions and be prepared to share their responses in a class discussion:

1. What words or numerical display on the chart describe the chance of pulling a yellow tile from the bag when there are 12 yellow tiles and 0 red tiles? Explain how you know this.
2. What words or numerical display describe the chance of pulling a yellow tile from the bag when there are 6 yellow and 6 red tiles? Explain your answer.

Additional Resources

Math by All Means: Probability Grades 3-4 by Marilyn Burns
About Teaching Mathematics: A K-8 Resource, 2nd Edition by Marilyn Burns

Homework & Family Connections

Have students conduct an experiment at home similar to the one done in class. Students gather two different items to put in a paper bag in varying quantities. For example: 4 matchbox cars and 3 fingerboards (toy skateboards). Next, students can sit with a family member and assist them in randomly selecting one item at a time from the bag (without looking at the contents). The student explains how to record their choice on a T-chart. The student (or family member) continues selecting items from the bag, recording it on their chart, and then replacing it. Instruct students to select items and replace them at least 12 times. Students then ask their family member to predict the contents of the bag after considering the results on their T-chart. This activity has many possibilities for length (number of draws from the bag) and options for selecting items to go into the bag. After the family member has completed the activity, ask them to write a short paragraph explaining their thoughts or feelings. Students can share responses with class the following day.

Activity—Spinner Experiment

Standard V

Objectives 1 & 2

Connections

Standard V
Students will use concepts of probability and collect, analyze, and draw conclusions from data.

Objective 1
Formulate and answer questions using statistical methods to analyze data.

Objective 2
Apply basic concepts of probability.

Intended Learning Outcomes
2. Become mathematical problem solvers.
3. Reason mathematically.
4. Communicate mathematically.

Background Information

Part 1 of this lesson introduces children to probability through an experiment in which one outcome is more likely than the others. The experiment provides experience for children to collect and analyze data. The probability of spinning each number provides a context for talking about fractions and percents and engages students in comparing the areas of the regions of a circle. Making their own spinner gives children practice in following directions and helps develop their fine motor skills.

Invitation to Learn

Hold up the sample spinner you made. Tell students they are going to make a spinner like yours. Ask what they notice about its face. Spin the spinner and point out how the indicator line tells what number the spinner lands on. Demonstrate for the children how to make a spinner.

Instructional Procedures

Directions for making a spinner:

1. Cut out the spinner face.
2. Cut the 5-by-8-inch index card in half. Mark a dot near the center of one of the halves. Using a straightedge, draw a line from the dot to one corner of the card.
3. Glue the spinner face to the side of the index card you did not draw on. Cut out the face of the spinner with its new, heavy backing.
4. Bend up the outside part of a paper clip. This part should point straight up when the paper clip is lying flat on your desk.

5. Use the paper clip to poke a hole in the center of the spinner face and through the dot near the center of the index card.
6. Push the bent end of the paper clip through the hole in the index card and use tape to secure the rest of the paper clip to the bottom of the card. Make sure the side of the card with the line is facing up.
7. Put the 1/2-inch length of plastic straw and then the spinner face on the paper clip.
8. Cover the tip of the paper clip with a piece of tape.

Demonstrate for the students how to do the spinner experiment:

1. Write the numerals 1, 2, and 3 at the bottom of the first three columns on the spinner recording sheet.
2. Spin the spinner and record the number it lands of in the lowest square of its column. Point out to the students that they should start writing at the bottom of the columns. Do five or six spins, recording the number each time.
3. Tell students that they will continue spinning and recording until one number reaches the top of its column.
4. Children then cut out the three-column strip and post it on the chalkboard under 1, 2, or 3 heading.

Pose a part of the problem:

Ask, "When you spin the spinner, is any number more likely to come up than any other number? Why do you think so?" Explain: You can keep track of spins on a graph recording sheet that is 3 squares by 12 squares. Demonstrate for the class how to spin and record on the graph. After three or four spins, ask, "What do you think the entire graph will look like when one number reaches the top of the paper?" Ask, "Which number do you think will reach the top of the recording sheet first?" Write your prediction on paper.

Present the problem to be solved:

Explain: Each of you will use your spinner and graph recording sheet to conduct an experiment. When one of the numbers reaches the top, you've completed the experiment. Cut the 3 x 12 recording sheet apart from the graph paper. Then post your 3 x 12 recording sheet on the board under the winning number. (Have students tape their graphs to the board under the heading 1, 2 or 3).

Students should conduct the experiment three times (using their entire graph recording sheet).

Materials

- ❑ Spinner Face A (one per student)
- ❑ 5-by-8-inch index cards, one each student
- ❑ Spinner recording sheet, one to two for each student
- ❑ Paper clips
- ❑ Plastic straws, one 1/2-inch length per student
- ❑ Scissors
- ❑ Tape
- ❑ Sample spinner made ahead of time

Discuss the class results when all students are finished comparing the results to their predictions. Most likely, 3 was the winning number, but there will be instances of 1 or 2 as winner. Ask: how might we find out if 3 actually comes up in half of all the spins?

Possible Extensions/Adaptation

You could have students take their graph recording sheets and cut apart each column. Then cut off any blank squares. Have small groups of students then tape their 1s end to end. Continue doing this with the 2s and 3s. Next combine each group's strips of numbers together to get one long strip of 1s, 2s and 3s. Tape one end to the chalkboard. This will provide a visual for students to see which number actually won. See "Homework & Family Connections" to add to this activity.

Assessment Suggestion

Have children write about what they did, what they had predicted and what the results were. Pose these questions to students who might not know how to begin:

How do the class results compare with your prediction?

How did your individual experiments compare with the class results?

Why do you think mathematicians say that a large sample of data is better for analyzing information than a small sample of data?

Additional Resources

Math by All Means: Probability Grades 3-4 by Marilyn Burns
About Teaching Mathematics: A K-8 Resource, 2nd Edition by Marilyn Burns

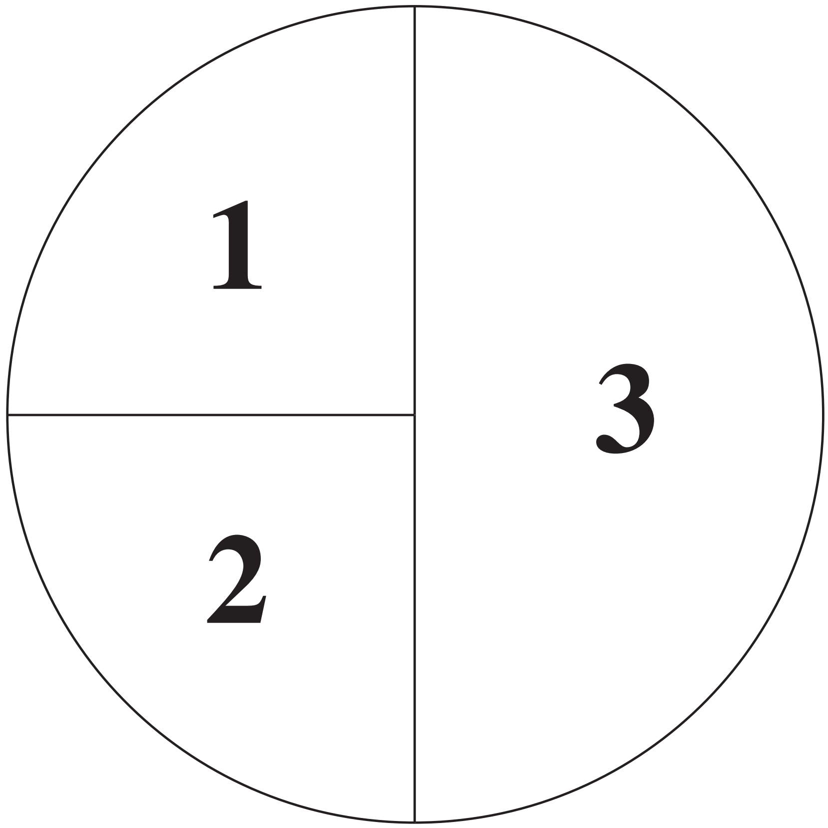
Homework & Family Connections

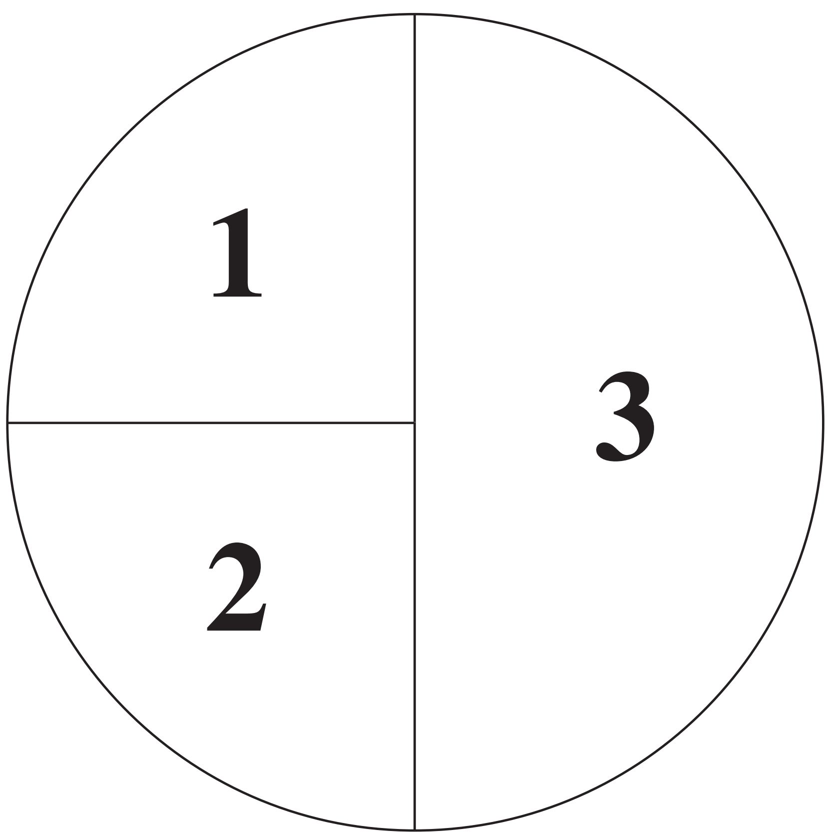
Each student can take their spinner and one graph recording sheet home to conduct the experiment again. Explain to students that they are going to gather more data to add to their first experiment. Ask students why this would be helpful. If students do not suggest it, tell them the mathematical theory of probability says that the more times you spin a spinner, the closer the results will match the theoretical distribution.

The next day, have students again cut apart their numbers, discard blank squares, and add to the class strip of 1s, 2s and 3s. Discuss the results. If there is time, have students actually count the number of 1s, 2s and 3s. This could be accomplished by cutting the strips into groups of ten, then compiling the 10s to form hundreds and so on. Some students may devise their own strategies for counting. Divide the strips among small groups for counting, then gather together for a class total.

Discuss the total numbers finding out if there were any surprises with the additional data. Ask, "Does the mathematical theory of probability (gathering more data) seem to be evident in our experiment? How can you tell? Can you find a way to prove that 3 came up in half of the spins?" Add applicable questions and discussion.

Spinner Faces A





Spinner Recording Sheet

Activity—The Left-Handed Experiment

Standard V

Objective 1

Standard V
Students will use concepts of probability and collect, analyze, and draw conclusions from data.

Objective 1
Formulate and answer questions using statistical methods to analyze data.

Intended Learning Outcomes
1. Demonstrate a positive learning attitude toward mathematics.
4. Communicate mathematically.

Connections

Materials

- ❏ Paper and pencil
- ❏ Graph paper, chart paper, markers, crayons

Background Information

By taking a sample of some people in your school, can you determine approximately how many right-handed people there are for every left-handed person? This experiment has students conduct a sample survey of how many left-handed people there are in your school. Students then conduct a school-wide survey and compare the results of the two surveys.

Invitation to Learn

When students enter the room, have a piece of paper on each student's desk. Any paper will do; used or scratch. Ask students to hold their paper in their hand and do what you do to your paper. Wad up your paper into a ball, tight enough to throw the paper. Then have students stand up, and throw their paper toward the front of the classroom, and try to "hit" the chalkboard/whiteboard. Then have students sit down.

Instructional Procedures

Begin a discussion about throwing. Ask which students used their left hand to throw the paper towards the front of the classroom and which students used their right hands. Discuss how many students write with their left or right hands. Determine as a class how many left-handed and how many right-handed people are in your class. This may need to be clarified by what you use to determine if you are left or right-handed (throwing, writing, eating, etc.). Write the different ways of expressing the results of the class survey. For example: We have 18 right-handed students and 7 left-handed students; 18/25 of the class are right-handed, 7/25 are right-handed; 18 out of 25 students are right-handed and 7 out of 25 are left-handed; we have a ratio of 18:25 right-handed students and 7:25 left-handed students.

Pose a part of the problem:

Tell students you would like to find a way to count the number of people in your school. Ask: How could we find out how many people are in our school? Discuss if you will be counting ALL people, including teachers and staff, or just the students. After you have decided which people you will count for your experiment, tell students you are going to try to find out how many left-handed people there are for every right-handed person. You will then compare the results of your class survey with the results of the school census.

1. Have students devise a way to survey the people in your school. You may want to divide students into teams, then assign each team to survey a particular grade level. As students devise this plan, record important information on the board.
2. Before sending students around to conduct their survey, discuss and decide how you will determine who is right and who is left-handed. Will you do the paper throwing experiment in every class? Or will you just question people by having them raise their hand if they are right, counting the number and doing the same for left-handed people.
3. Be sure to ask permission from teachers ahead of time. Let them know you will be sending students into their classrooms to conduct a quick survey.
4. Conduct the census.

When all people in the school have been surveyed, compile your results. How many right handed people are there for each left-handed person in the entire school? Are the results similar to that of your class survey? Were you satisfied with your sampling procedure (taking a class sample)? If not, how would you improve the way you sampled?

Curriculum Integration

Math/Science—Discuss with students if left-handed vs. right-handed is an inherited trait. Connect this to their study of inherited traits in Science.

Possible Extensions/Adaptation

This procedure can be used by individuals or small groups of students to investigate a variety of topics:

By sampling, determine approximately how many red-, blond-, brown-, and black-haired people there are in your school.

By sampling, decide how many people there are with each eye color.

Use sampling to determine the favorite TV show of students in your school.

Assessment Suggestion

Students will write a report about their right-handed vs. left-handed experiment. Ask students to be sure to include numbers and comparisons in their written report. Students can share their written reports in small groups or with the entire class. You may want students to revise their reports, clarify any unclear details and even illustrating their findings in the form of a graph. Provide graph paper, chart paper, crayons, markers, etc. These can be posted in the classroom or on a favorite bulletin board for all to see.

Additional Resources

About Teaching Mathematics: A K-8 Resource, 2nd Edition by Marilyn Burns

Homework & Family Connections

A possible homework/extension to this activity would be for students to take a survey of at least 10 people from home, their neighborhood, on their bus, etc., to compare with the school wide survey taken.

Students should report their homework results to class the next day. Total the number of left-handed vs. right-handed people outside of the school. Compare these results with the results of the classroom survey and the school wide survey.

Appendix

What was a typical day in math class like when you were in school?

How did it look?	How did it sound?	How did you feel?

Number and Operation

Algebra

Utah Elementary Mathematics Core—Grade ______

Geometry

Measurement

Data Analysis and Probability

ILOs ____

____

Bridging the Gap

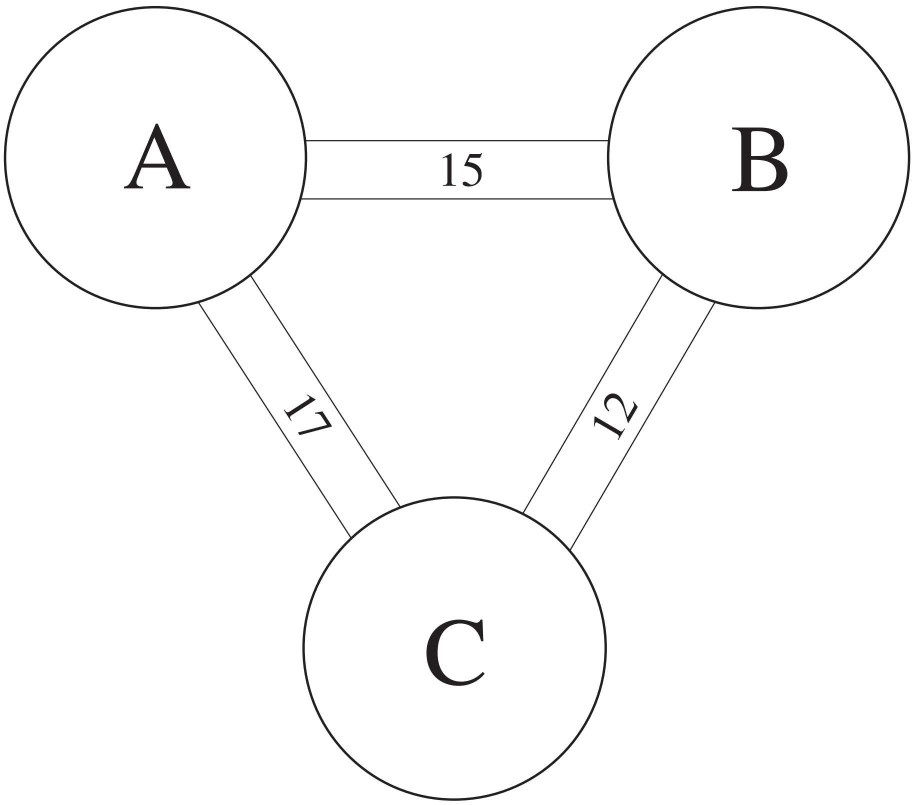
Instructions

How many of you have ever wanted to be part of a construction company that is responsible for building houses? Well today is your lucky day! You are going to be responsible for building houses on three islands. However, you are fortunate to have bridges that connect the islands to one another.

On your blackline master you will notice the architect has written on each bridge, that connects the two islands, the total number of houses on the two islands. The architect has also indicated the total number of houses to be built on the three islands. Your job is to figure out how many houses go on each island. You may use the cm cubes on your table to represent the houses.

Bridging the Gap

Exercise #1



Total Number of Houses = 22

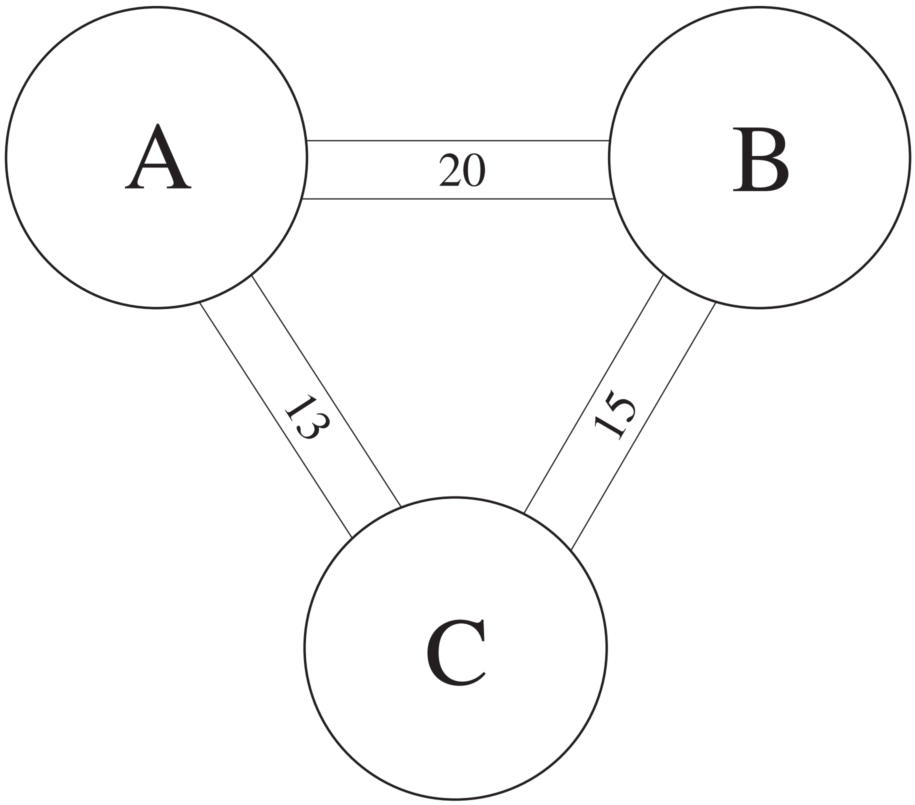
A = ____________

B = ____________

C = ____________

Bridging the Gap

Exercise
#2



Total Number of Houses = 24

A = ____________

B = ____________

C = ____________

Eyewitness

Instructions

Have you ever been the eyewitness for a crime scene? When a crime is committed, what is the role of the eyewitness and the detective? Today you will be working with a partner. One of you will be the eyewitness and the other will be the detective.

1. The detective gives the eyewitness one minute to read the story problem. When time is up, the eyewitness turns the story problem over so he or she can't see it.
2. The detective then asks, "What's this about?"
3. The eyewitness then tells the basic story.
4. The detective then asks the eyewitness to give all the information he or she remembers. The detective asks questions to "learn the story and get the facts."
5. After relating all that can be remembered, the eyewitness is given one more opportunity to have an "instant replay" and read the problem one more time. Again, the detective times the eyewitness for one minute, then turns the problem over again.
6. The detective then asks for any missed details.
7. Following the eyewitness interrogation, the detective writes an equation to represent the case with the help of the witness.
8. If time permits, switch roles and use the second story problem.

Remember: No matter what type of story problem you are solving, you must be a "detective."

Story Problem #1 for Eyewitness:

Jazz Payroll

At approximately 1:00 a.m. on March 25, 2003, the Jazz payroll office was broken into and ransacked. After an investigation all payroll checks were located except the check for John Stockton. The clerk knows that Karl Malone earns 20% more money per game than John Stockton. The clerk computed that Malone earned $120,000 per game. How much money does John Stockton earn per game?

Story Problem #2 for Eyewitness:

Stick-to-it

Johnny bought a box of soccer stickers to use on his project report on soccer. There were two thousand, one hundred sixty stickers in the box, but he could not use some of them. One-ninth of them were stuck so tightly together that he could not detach them from each other. Eighty-three of them were blank. One-fifth of them had no glue. How many were left that he could actually use?

Story Problem #1 for Eyewitness:

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ANSWER: **John Stockton earns $100,000/game**

Story Problem #2 for Eyewitness:

Stick-to-it

Johnny bought a box of soccer stickers to use on his project report on soccer. There were two thousand, one hundred sixty stickers in the box, but he could not use some of them. One-ninth of them were stuck so tightly together that he could not detach them from each other. Eighty-three of them were blank. One-fifth of them had no glue. How many were left that he could actually use?

ANSWER: **1,405 soccer stickers could actually be used**

All in a Name

Instructions

We all know how important our name is. It identifies who we are. It is so important that today we are going to collect data using our names.

1. Quickly walk around the room and record name data for ten participants including yourself. This will give you a chance to meet others in the room and introduce yourself. Have each participant record his or her first and last name in the appropriate columns. Returning to your seat, record the total number of letters for each name in the last column.
2. Using the data collected in your third column, make a quick graph (bar, line, etc.). Describe your data to another participant, in terms of clumps or bunches, gaps or holes, and bumps.
3. Using the same data collected in your third column, find the mean, range, mode, and median. Compare these findings to your graph. Do you see any patterns?
4. Discuss with another participant:
   a. Why/When is it useful to find the mean, range, mode, and median of data?
   b. If more names are added to your data set, predict how your measures and graph will change. Explain why.

Name______________________________

All in a Name

Recording Sheet

First Name	Last Name	Total # of Letters

MEAN:	
RANGE:	
MODE:	
MEDIAN:	

Mean: the sum of all the numbers divided by the number of items you're adding.

Range: the difference between the highest and lowest numbers.

Mode: the number that appears most often (there isn't always a mode, but sometimes there's more than one!).

Median: the middle number.

Name______________________________

All in a Name

Grid

ROPED INTO QUADRILATERALS

Instructions

1. Assign members of your group (table) to do the following:
   a. Cut out the quadrilateral pieces.
   b. Tie the ends of **each** piece of yarn to make three circles.
   c. Cut out the Task Activity Quadrilateral Labels.

2. The object of the activity is to place the quadrilateral pieces appropriately in your circles (yarn) according to the labels. You may need to overlap the circles to form intersections. The number of labels determines the number of circles used.

3. Progress through the labels, placing quadrilateral pieces according to common attributes.

Quadrilateral Labels

Task Activity #1:

At least one right angle	No right angles

Task Activity #2:

No congruent sides	Congruent sides

Task Activity #3:

At least one obtuse angle	At least one acute angle

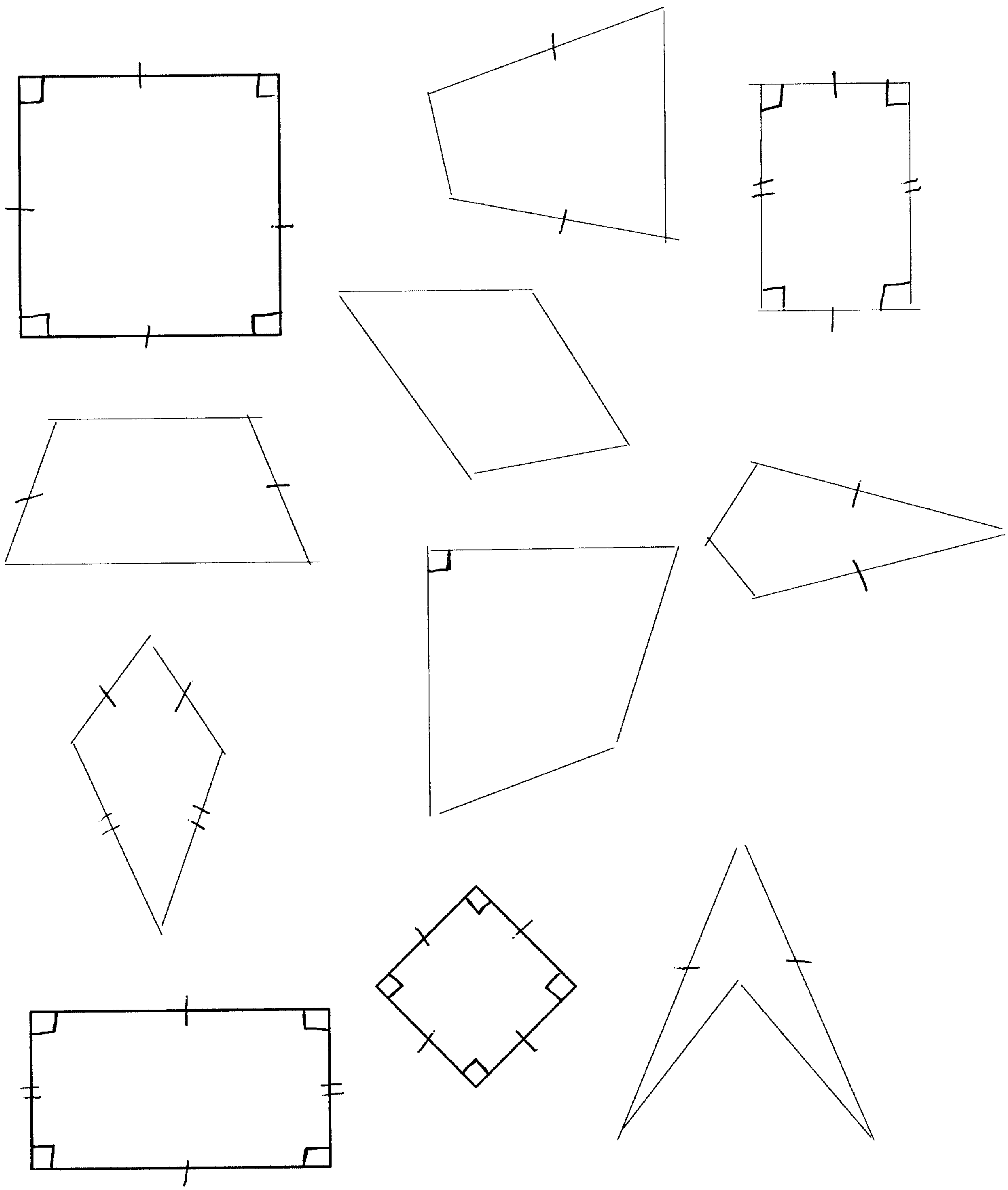
Task Activity #4:

Rectangles	Squares	Rhombi

Task Activity #5:

At least one acute angle	At least one pair of parallel sides

Quadrilaterals



The "Right" Place

Use the 1-6 to complete the equations. Each number can only be used once in an equation.

$$\square \times \square = \square + \square + \square$$

$$(\square - \square) \times \square = \square$$

$$\square \times \square \times \square (\square - \square) = \square\,\square$$

Place the numbers 0-8 to complete the equation.

Hint: The three-digit number is >500.

$$\begin{array}{r} \square\,\square\,\square \\ \times \quad\quad \square \\ \hline \square\,\square\,\square\,\square \end{array}$$

Randomly select 4 of your numbers. Place them in the equation below so that:

1. the product will be a 3-digit number
2. the product is between 750 and 2,500
3. it is an even product
4. the answer is close to 6,000

$$\begin{array}{r} \square\,\square \\ \times\ \square\,\square \\ \hline \end{array}$$

Cut apart:

1	2	3	4	5	6	7	8	9	0

100 Yard Dash

Juan and Maria race for 100 yards. Maria wins by ten yards. Juan demands a second race, but this time Maria spots Juan a handicap – she begins ten yards behind the starting line. Assuming both run the same speed as before, who will win? Why?

Draw a diagram.

Explain your reasoning.

Evaluating and Designing a Rubric

Student Task

Make a graph to illustrate how many hours you spend during a typical school day doing different things. These things might be sleeping, eating, school, homework, playing sports, scouts, dance, playing with friends, playing games, watching TV, etc.

Your task is to collect, organize, and display your data. A chart or table may be helpful because you must in someway show evidence of data organization. You must also choose the best type of graph to represent this data (bar, circle, etc.). Please show any calculations that you make, and write an explanation for why you chose the graph you did and how you made your graph.

Student Rubric

	1 Point	2 Points	3 Points	4 Points
Organization of information	Information about time spent very disorganized	Some data organization evident, but not carried through	Data well-organized	In addition, the data is clearly presented
Graph	Graph chosen is inappropriate to the topic or very poorly executed	Graph chosen is adequate, but execution is poor	Appropriate form of graph and adequate information	In addition, the graph is very accurately and neatly presented
Calculations	Major errors in calculations	A number of errors in calculations	Very few errors in calculations	No calculation errors
Explanation	Explanation very muddled	Explanation difficult to follow	Explanation clear enough to follow	The explanation is clear and displays comprehensive understanding of the relative merits of different types of graphs

A few questions to consider as you evaluate the rubric:

- Is this a “good” rubric for the student task given? Why or why not?
- What changes should be made?
- Are the descriptions well stated and not ambiguous?
- Are there definite differences among the points?

Write any changes you would make on the rubric—use your State Core to see what expectations for student achievement are.

Pizza Party

Ms. Williams' 5th grade class is having a pizza party. They are trying to decide which pizzeria has the cheapest price. The local pizzerias and their prices are listed below.

Pizza Prices

Pizza to Go	$ 8.50	8 slices per pizza
Rosa's Pizza	$10.50	10 slices per pizza
Pizza with Pizazz	$ 6.25	6 slices per pizza

There are 30 students in Ms. Williams' class. Each person (including Ms. Williams) will eat two slices of pizza. All the pizza slices from each pizzeria are the same size. Where should you buy the pizza to get the best deal? Why?

Write a brief description of how you arrived at your decision. Provide your work (any calculations you made), a picture, or a diagram to support your thinking.

SAMPLE TEST SCORE GRID FOR MATH CORE ACADEMY

QUESTION NUMBER	1	2	3	4	5	6	7	8	9	10	**TOTAL**
POINTS POSSIBLE	1	1	1	1	1	1	1	1	1	1	**%**
BILLY	1	1	1	1	1	1	**0**	**0**	1	**0**	70%
SALLY	1	1	1	1	**0**	1	1	**0**	1	**0**	70%
MARK	1	1	1	1	1	1	**0**	**0**	1	**0**	70%
BRETT	1	1	1	1	**0**	**0**	1	**0**	1	**0**	60%
LISA	1	1	1	1	1	1	**0**	**0**	**0**	**0**	60%
JOHN	1	1	1	1	**0**	1	1	**0**	**0**	**0**	60%
SUSAN	1	**0**	1	1	1	1	**0**	**0**	1	**0**	60%
VAN	1	1	1	1	**0**	1	1	**0**	**0**	**0**	60%
YVONNE	1	1	1	**0**	1	1	**0**	**0**	1	**0**	60%
HANS	1	1	1	1	**0**	1	1	**0**	**0**	**0**	60%
FRANCIS	1	1	**0**	1	1	**0**	**0**	1	1	**0**	60%
STEPHANIE	1	1	**0**	1	**0**	1	1	1	**0**	**0**	60%
MILHAUS	1	1	**0**	1	1	**0**	**0**	1	**0**	**0**	50%
WILLIAM	1	1	**0**	1	**0**	**0**	1	1	**0**	**0**	50%
BECKY	1	1	**0**	**0**	1	**0**	**0**	1	**0**	**0**	40%
RALPH	1	1	**0**	**0**	**0**	**0**	1	1	**0**	**0**	40%
RHETT	1	**0**	**0**	**0**	1	**0**	**0**	1	**0**	**0**	30%
SCARLETT	1	**0**	**0**	**0**	**0**	**0**	1	1	**0**	**0**	30%
ISHMAEL	1	**0**	**0**	**0**	1	**0**	**0**	1	**0**	**0**	30%
CHARLES	1	**0**	**0**	**0**	**0**	**0**	1	1	**0**	**0**	30%
QUESTION %	100%	75%	50%	65%	50%	50%	50%	50%	35%	0%	

QUESTION ANALYSIS

1.)

2.)

3.)

4.)

5.)

6.)

7.)

8.)

9.)

10.)

Partial Procuct and Lattice Charts

x			

			x

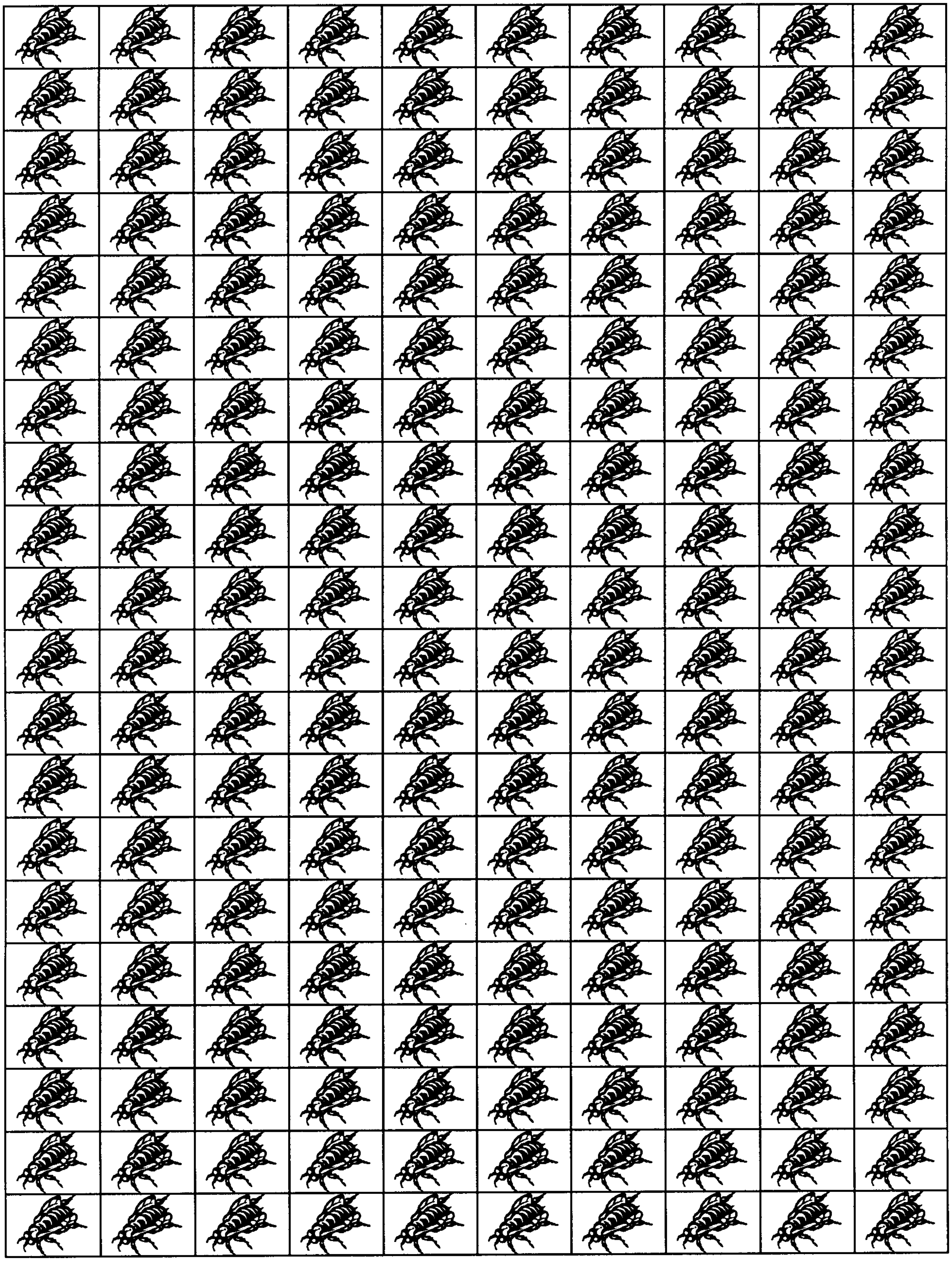
Multiplication Strategy Cards

Partial Product	**Lattice**
Traditional	**Calculator**
Partial Product	**Lattice**
Traditional	**Calculator**

Multiplication Problem Cards

35 x 45	62 x 59	82 x 17
33 x 64	29 x 54	73 x 68
66 x 38	92 x 81	74 x 36
59 x 27	44 x 38	26 x 45
46 x 27	74 x 86	19 x 53
26 x 37	39 x 28	62 x 72

Bug Graph Paper



Remainder of One Riddle (1-25)

Written by: ___________________________________

Solved by: ____________________________________

1. When you divide my number by 1, R =
2. When you divide my number by 2, R =
3. When you divide my number by 3, R =
4. When you divide my number by 4, R =
5. When you divide my number by 5, R =
6. When you divide my number by 6, R =
7. When you divide my number by 7, R =

1 2 3 4 5 6 7 8 9 10

11 12 13 14 15 16 17 18 19 20

21 22 23 24 25

Remainder of One Riddle (1-50)

Written by: ___________________________________

Solved by: ____________________________________

1. When you divide my number by 1, R =
2. When you divide my number by 2, R =
3. When you divide my number by 3, R =
4. When you divide my number by 4, R =
5. When you divide my number by 5, R =
6. When you divide my number by 6, R =
7. When you divide my number by 7, R =

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Mmmm… Fruity Os

Solve these Fruity O riddles.

1. There are six blue. There are twice as many red as blue. There are half as many yellow as blue. How many of each color are there?

   red__________ yellow__________ blue__________

2. There are 5 red. There is one less red than blue. The number of yellow is double the number of blue. How many of each color are there?

   red__________ yellow__________ blue__________

3. There are 19 pieces in all. There are the same number of red and blue and only three yellow. How many of each color are there?

   red__________ yellow__________ blue__________

4. There are three times as many yellow as blue. There are 5 red. There are 25 pieces in all. How many of each color are there?

   red__________ yellow__________ blue__________

Fruity O Fractions

Use common denominators to help you make these Fruity O combinations.

1. Make a yellow, blue, and red combination. The combination should contain 20 Fruity Os:
   1/5 blue, 3/10 yellow, and 1/2 red
   How many of each color is in your mix?

   blue__________ yellow__________ red__________

2. Make a combination using 24 Fruity Os. It should contain:
   1/6 blue, 1/2yellow, and 1/3 red
   How many of each color is in your combination?

   blue__________ yellow__________ red__________

3. Make up your own riddle with 30 Fruity Os in it. Record fractional part of each color.
   __________blue __________yellow __________red
   Exchange problems with a friend and have them fill out the next part. Make sure you know the answer first so you can tell them if they are correct.

   Solved by: ______________________________________

   blue__________ yellow__________ red__________

Fruity O Fractions Part 2

Use common denominators to help you make these Fruity O combinations.

1. Make a yellow, orange, green, and red combination. The combination should contain 24 Fruity Os:

   1/3 yellow 1/3 orange 1/12 green 1/4 red

   How many of each color is in your mix?

   yellow_______ orange_______ green_______ red_______

2. Make a combination using 30 Fruity Os. It should contain:

   1/6 blue, 1/5yellow, 1/3 green, ___?___red

   How many of each color is in your combination?

   blue_______ yellow_______ green_______ red_______

3. Make a combination with 40 Fruity Os in it. It should contain:

   2/5 blue 1/4 red 3/20 orange

   1/20 green 1/20 yellow 1/10purple

   How many of each color is in your combination?

   blue_______ red_______ orange_______

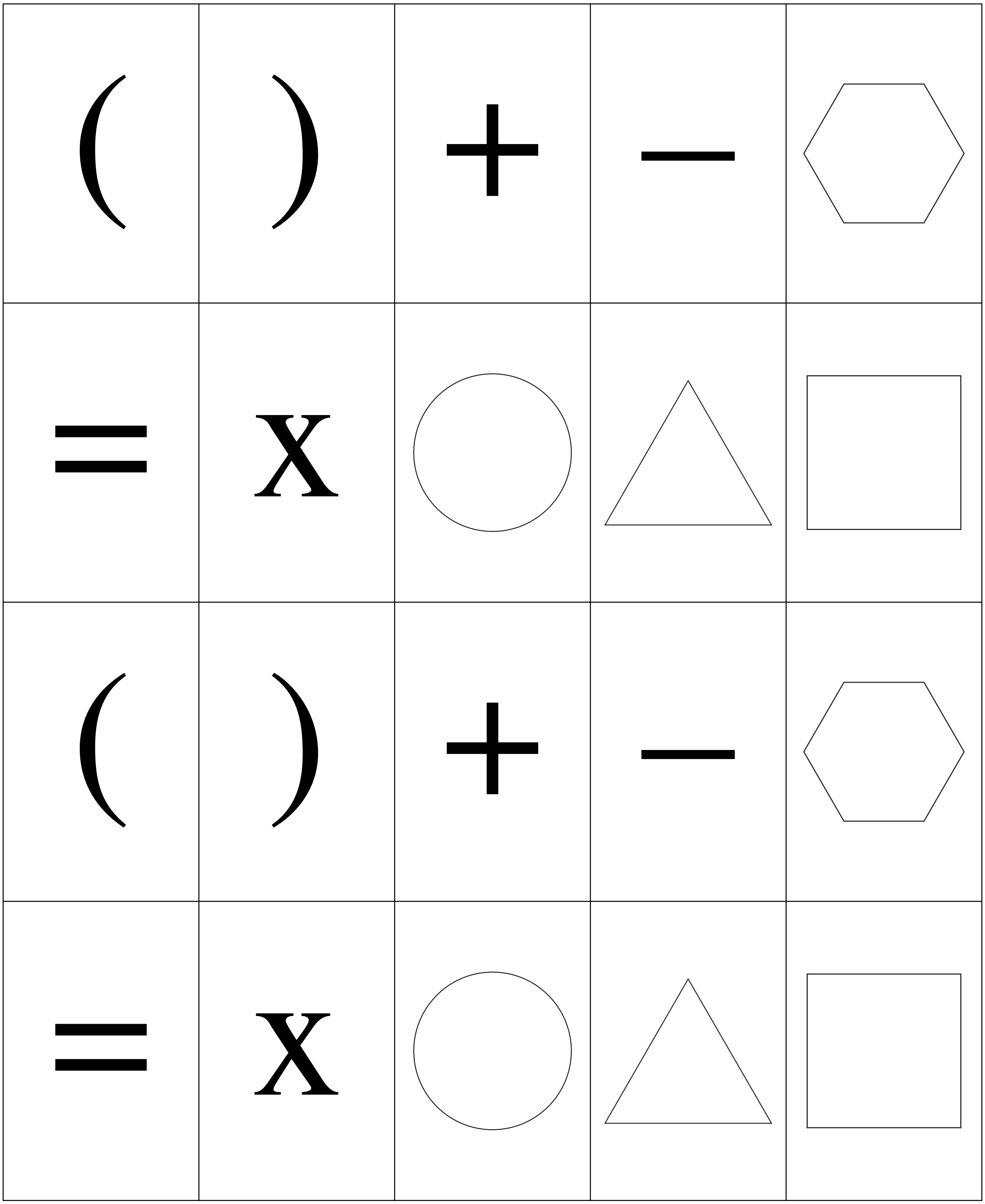
   green_______ yellow_______ purple_______

Comparing Fractions War Game Cards

6/8	7/9	4/9	1/6
2/4	3/3	1/8	5/7
1/4	7/10	2/8	8/10
3/7	8/12	4/10	2/6

1/3	3/8	2/3	4/5
5/6	7/12	3/5	4/7
5/12	1/2	4/8	3/9
3/4	9/12	2/5	4/6

Symbol Cards



Game Board for Equation Game

Game Pieces for Equation Game

0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4
0_1	1_1	2_1	3_1	4_1	5_1	6_2	7_5	8_2	9_4

10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	$-_1$	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	$-_1$	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	$-_1$	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	$-_1$	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	$-_1$	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	X_2	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	X_2	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	X_2	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	X_2	X_2	$-_1$
10_1	11_1	12_4	$*_2$	$\bullet_2$	$+_1$	$+_1$	X_2	X_2	$-_1$

13_{3}	15_{3}	17_{4}	19_{4}	21_{2}	23_{3}	25_{3}	27_{4}	29_{4}	2
13_{3}	15_{3}	17_{4}	19_{4}	21_{2}	23_{3}	25_{3}	27_{4}	29_{4}	2
13_{3}	15_{3}	17_{4}	19_{4}	21_{2}	23_{3}	25_{3}	27_{4}	29_{4}	2
13_{3}	15_{3}	17_{4}	19_{4}	21_{2}	23_{3}	25_{3}	27_{4}	29_{4}	2
13_{3}	15_{3}	17_{4}	19_{4}	21_{2}	23_{3}	25_{3}	27_{4}	29_{4}	2
14_{2}	16_{2}	18_{3}	20_{2}	22_{2}	24_{3}	26_{3}	28_{3}	30_{2}	2
14_{2}	16_{2}	18_{3}	20_{2}	22_{2}	24_{3}	26_{3}	28_{3}	30_{2}	2
14_{2}	16_{2}	18_{3}	20_{2}	22_{2}	24_{3}	26_{3}	28_{3}	30_{2}	2
14_{2}	16_{2}	18_{3}	20_{2}	22_{2}	24_{3}	26_{3}	28_{3}	30_{2}	2
14_{2}	16_{2}	18_{3}	20_{2}	22_{2}	24_{3}	26_{3}	28_{3}	30_{2}	2

= 0	= 0	= 0	= 0	= 0	= 0	(0	) 0	(0	) 0
= 0	= 0	= 0	= 0	= 0	= 0	(0	) 0	(0	) 0
= 0	= 0	= 0	= 0	= 0	= 0	(0	) 0	(0	) 0
= 0	= 0	= 0	= 0	(0	) 0	(0	) 0	(0	) 0
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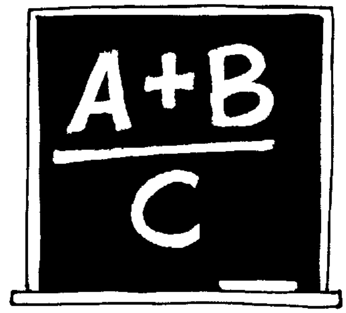
= 0	= 0	= 0	= 0	(0	) 0	(0	) 0	(0	) 0
= 0	= 0	= 0	= 0	(0	) 0	(0	) 0	(0	) 0
= 0	= 0	= 0	= 0	(0	) 0	(0	) 0	(0	) 0



Find the Missing Addend

* First write the equation using one or more variables, then solve the problem.

1. Rose had some marbles. Betty had 22 marbles. Together they had 50. How many marbles did Rose have?

2. Tommy made $12 mowing lawns which he added to his savings. He now has $38. How much did Tommy have in his savings before?

3. There were 54 children at a party. Some more children came to the party and then there were 99 children. How many more children came?

4. Dana bought a cd on sale for $8.50. She saved $4.75. How much was the cd at regular price?

5. Alan bought a new helmet for $7.00 and also bought a pair of knee-pads. He had $20.00 to begin with and came home with $3.25. How much were the knee-pads?

6. Jennifer invited some friends to a party. Twice as many people showed up than what she invited. Not including her, there was a total of 24 people at her party. How many friends did she invite?

7. For dinner, Matt bought some pizzas and cut each one into 8 slices. He ate three slices while waiting for everyone else to come to the table and then there were 21 slices left. How many pizzas did he buy?

8. My secret number is 24 more than Mary’s. Her number is 31. What is my number?

9. Blake’s secret number is half of Michael’s. If Michael’s secret number is 128, what is Blake’s?

10. Caitlin’s secret number is 9. Haley’s secret number is 7 more than 4 times Caitlin’s. What is Haley’s number?

11. Rebecca has ten less than the sum of Tessa’s number and Emily’s number. If Emily’s number is 42 and Tessa’s number is 28, what is Rebecca’s number?

12. If Ken’s secret number is twice John’s plus four more; and John’s secret number is 78, what is Ken’s number?

Equation Mysteries

* Solve each equation to find the missing addend/variable. Make sure to use the variable in your answer (e.g., “n = 4”).

1. $\square + 12 = 20$ ______________________________
2. $36 - \bigcirc = 16$ ______________________________
3. $29 + \triangle = 42$ ______________________________
4. $8 * n = 40$ ______________________________
5. $(20 - n) + 2 = 17$ ______________________________
6. $3(2) + n = 10$ ______________________________
7. $24/n = 8$ ______________________________
8. $(x \bullet 12) = 60$ ______________________________
9. $(3 \bullet 7) + y = 32$ ______________________________
10. $\dfrac{2 + 8}{x} = 2$ ______________________________

* The following problems involved simultaneous equations and use two variables. Have fun and good luck!

11. $\bigcirc + \square = 8$
    $\bigcirc - \square = 4$ ______________________________

12. $\bigcirc + \square = 10$
    $\bigcirc - \square = 4$ ______________________________

13. $\bigcirc + \square = 12$
    $\bigcirc - \square = 6$ ______________________________

14. $\bigcirc + \square = 11$
    $\bigcirc - \square = 5$ ______________________________

15. $\bigcirc + \square = 18$
    $\bigcirc - \square = 0$ ______________________________

Math Station Log

Date	Station Name	Description of Activity	Finished	Score

In/Out Functions Cards

In/Out Functions Card #1

In	Out
1	100
2	200
3	300
4	400
5	500
6	600

1. What's the rule?
2. What are the next three numbers?
3. What is the equation?

In/Out Functions Card #2

In	Out
1	4
2	8
3	12
4	16
5	20
6	24

1. What's the rule?
2. What are the next three numbers?
3. What is the equation?

In/Out Functions Card #3

In	Out
1	1
2	4
3	9
4	16
5	25
6	36

1. What's the rule?
2. What are the next three numbers?
3. What is the equation?

In/Out Functions Card #5

In	Out
1	5
2	6
3	7
4	8
5	9
6	10

1. What's the rule?
2. What are the next three numbers?
3. What is the equation?

In/Out Functions Card #4

In	Out
1	0
2	1
3	2
4	3
5	4
6	5

1. What's the rule?
2. What are the next three numbers?
3. What is the equation?

In/Out Functions Card #6

In	Out
5	15
8	25
10	30
15	45
20	60
25	75

1. **What's the rule?**
2. **What are the three more examples?**
3. **What is the equation?**

In/Out Functions Card #7

In	Out
1	3
2	5
3	7
4	9
5	11
6	13

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #8

In	Out
18	9
16	8
14	7
12	6
10	5
8	4

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #10

In	Out
20	4
25	5
30	6
35	7
40	8
45	9

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #9

In	Out
1	1.5
2	2.5
3	3.5
4	4.5
5	5.5
6	6.5

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #11

In	Out
1	7
2	16
3	25
4	34
5	43
6	52

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #12

In	Out
1	4
2	9
3	14
4	19
5	24
6	29

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #13

In	Out
1	4
2	7
3	10
4	13
5	16
6	19

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #15

In	Out
8	7
12	9
20	13
24	15
30	18
100	53

1. **What's the rule?**
2. **What are the three more examples?**
3. **What is the equation?**

In/Out Functions Card #14

In	Out
0	2
1	7
2	12
3	17
4	22
5	27
6	32

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #16

In	Out
1.5	3.0
2.4	4.8
3.0	6.0
4.75	9.50
5.1	10.2
7.3	14.6

1. **What's the rule?**
2. **What are the three more examples?**
3. **What is the equation?**

In/Out Functions Card #17

In	Out
1	1/2
2	1
3	1 1/2
4	2
5	2 1/2
6	3

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #18

In	Out
1	6
2	9
3	12
4	15
5	18
6	21

1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

In/Out Functions Card #20

In	Out
3	33
0	-3
10	117
4	45
2	21
5	57

1. **What's the rule?**
2. **What are the three more examples?**
3. **What is the equation?**

In/Out Functions Card #19

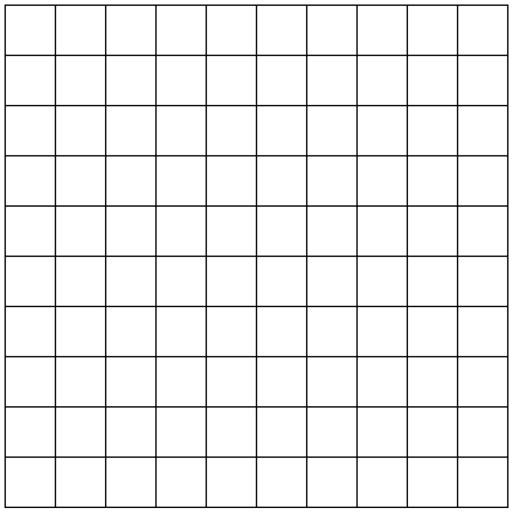
In	Out
1	6
2	8
3	10
4	12
5	14
6	16

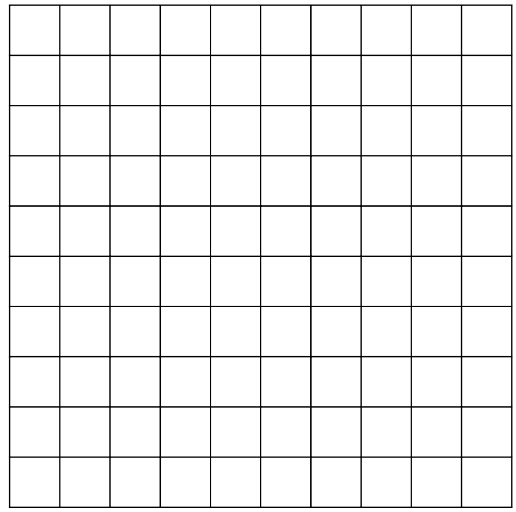
1. **What's the rule?**
2. **What are the next three numbers?**
3. **What is the equation?**

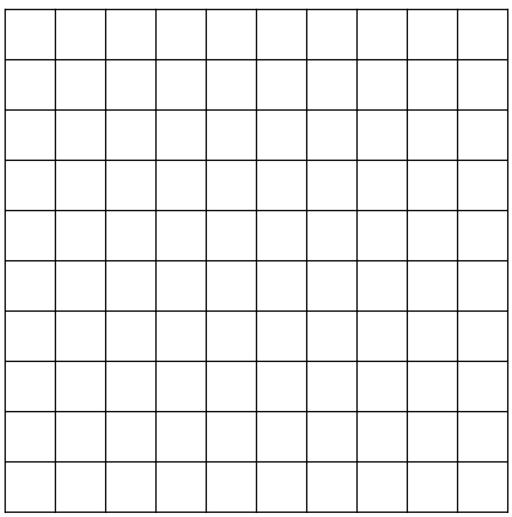
Decimal Sequencing Cards

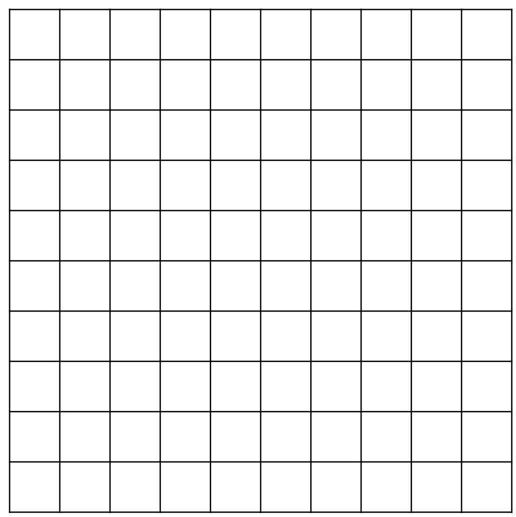
0.01	0.03	0.2
0.02	0.07	0.4
0.04	0.11	0.6
0.08	0.15	0.8
0.16	0.19	1.0
0.32	0.23	1.2
0.64	0.27	1.4
1.28	0.31	1.6

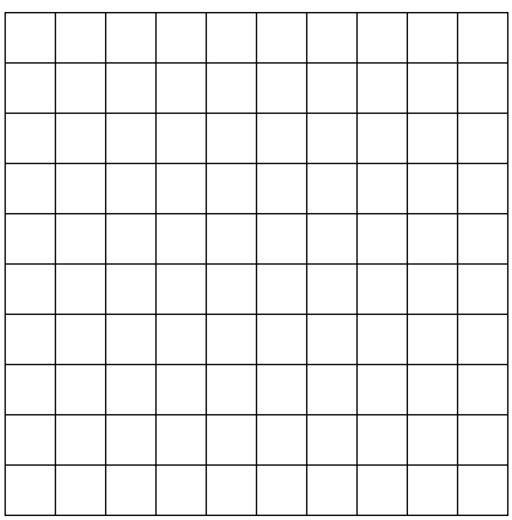
Hundreds Squares Worksheet

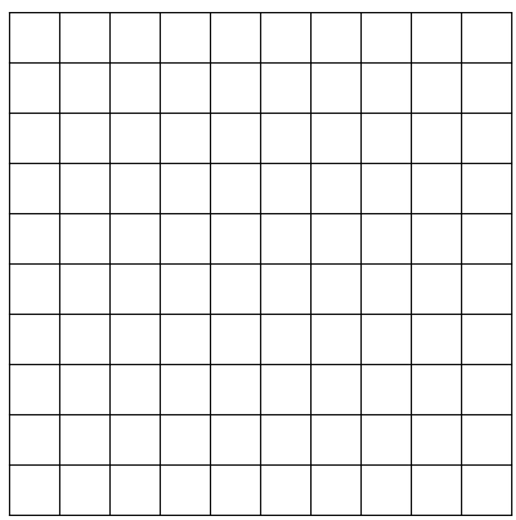










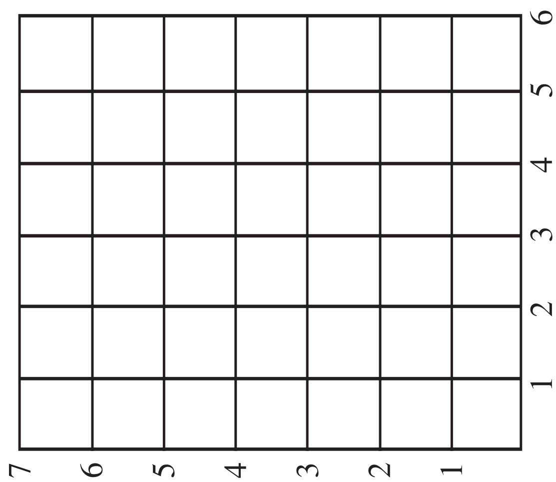


Pass It On Cards

Angle	**Vertex**	**Acute Angle**
Pyramid	**Obtuse Angle**	**Edge**
Face	**Polygon**	**Equilateral**
Ray	**Triangle**	**Isosceles**
Right Angle	**Scalene**	**Prism**

The plane figure that serves as one side of a solid figure. The faces of a cube are square.	A triangle that has exactly two congruent sides.	A three-dimensional figure with two parallel and congruent faces that are polygons. The rest of the faces are parallelograms.
An angle that measures exactly 90°.	A polygon with three sides.	Two rays sharing a common endpoint.
The point at which two line segments, lines or rays meet to form an angle.	An angle that measures less than 90°.	A triangle that has no congruent sides.
A polyhedron whose base is a polygon, and whose other faces are triangles that share a common vertex.	A closed plane figure made by line segments.	An angle that measures greater than 90° but less than 180°.
A part of a line that has one endpoint and goes on forever in one direction.	The line segment where two faces of a solid figure meet.	A triangle with all sides and angles equal.

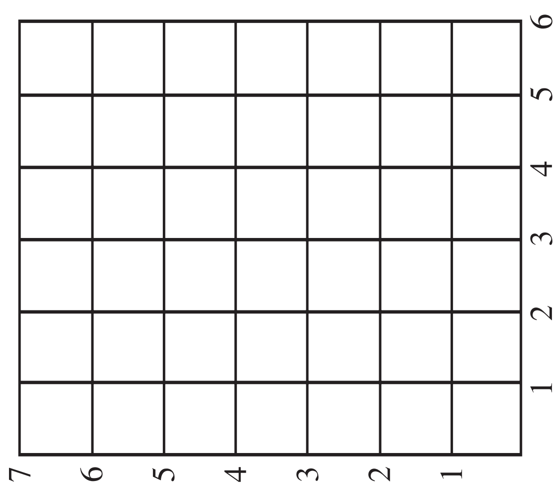
Match My Masterpiece



Coordinates for letter #1

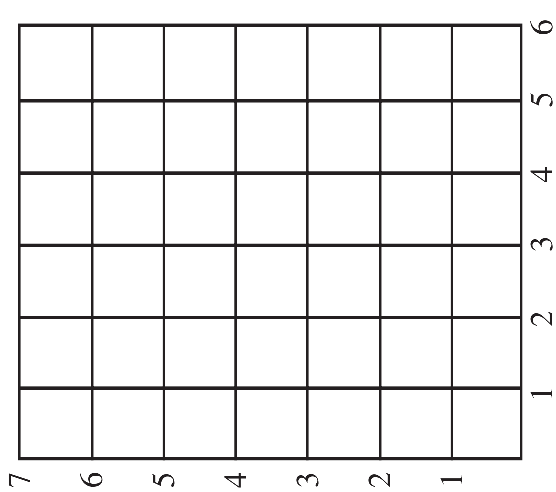
Letter formed: ______

(Cut along this line)



Coordinates for letter #1

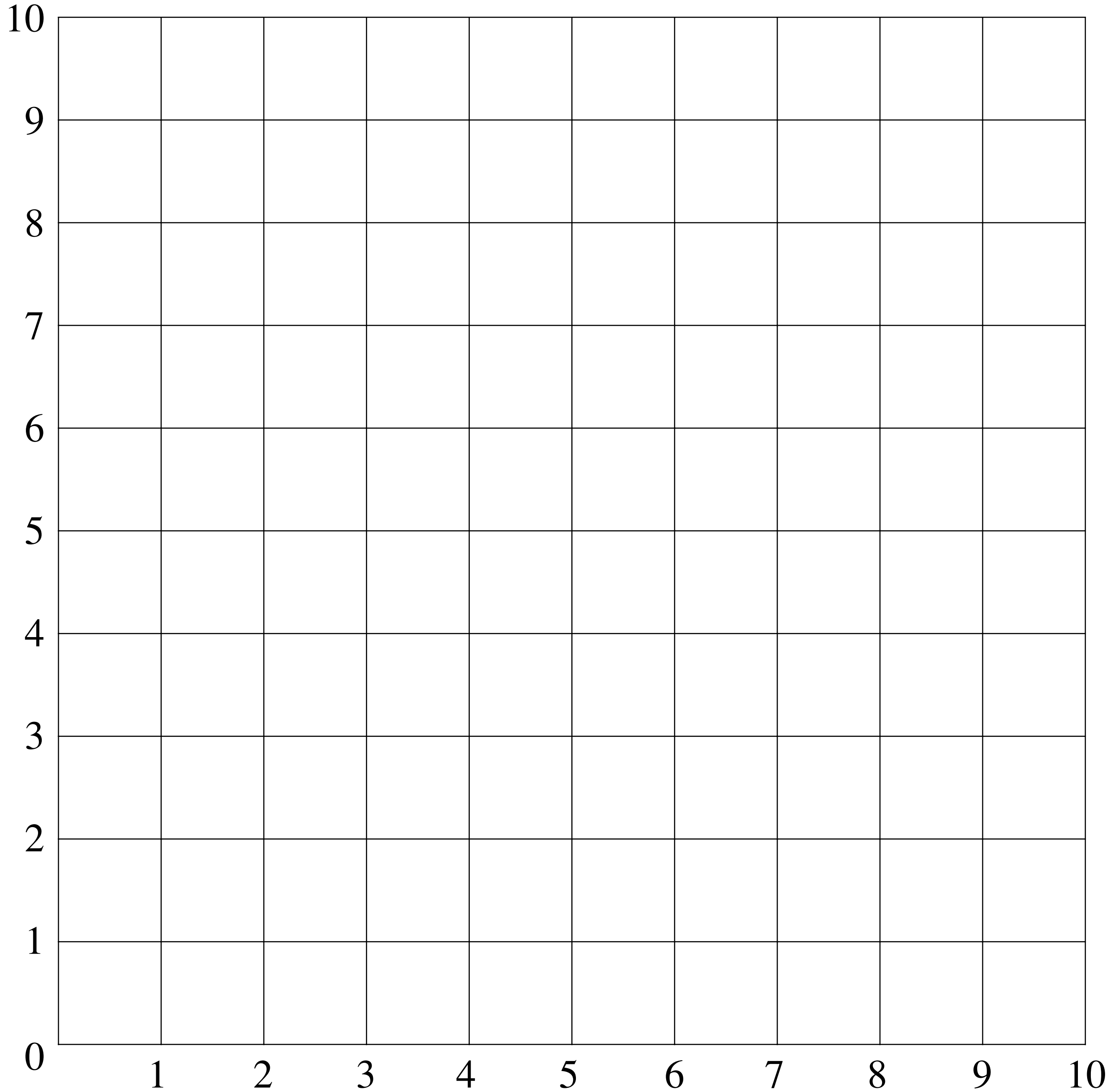
Letter formed: ______



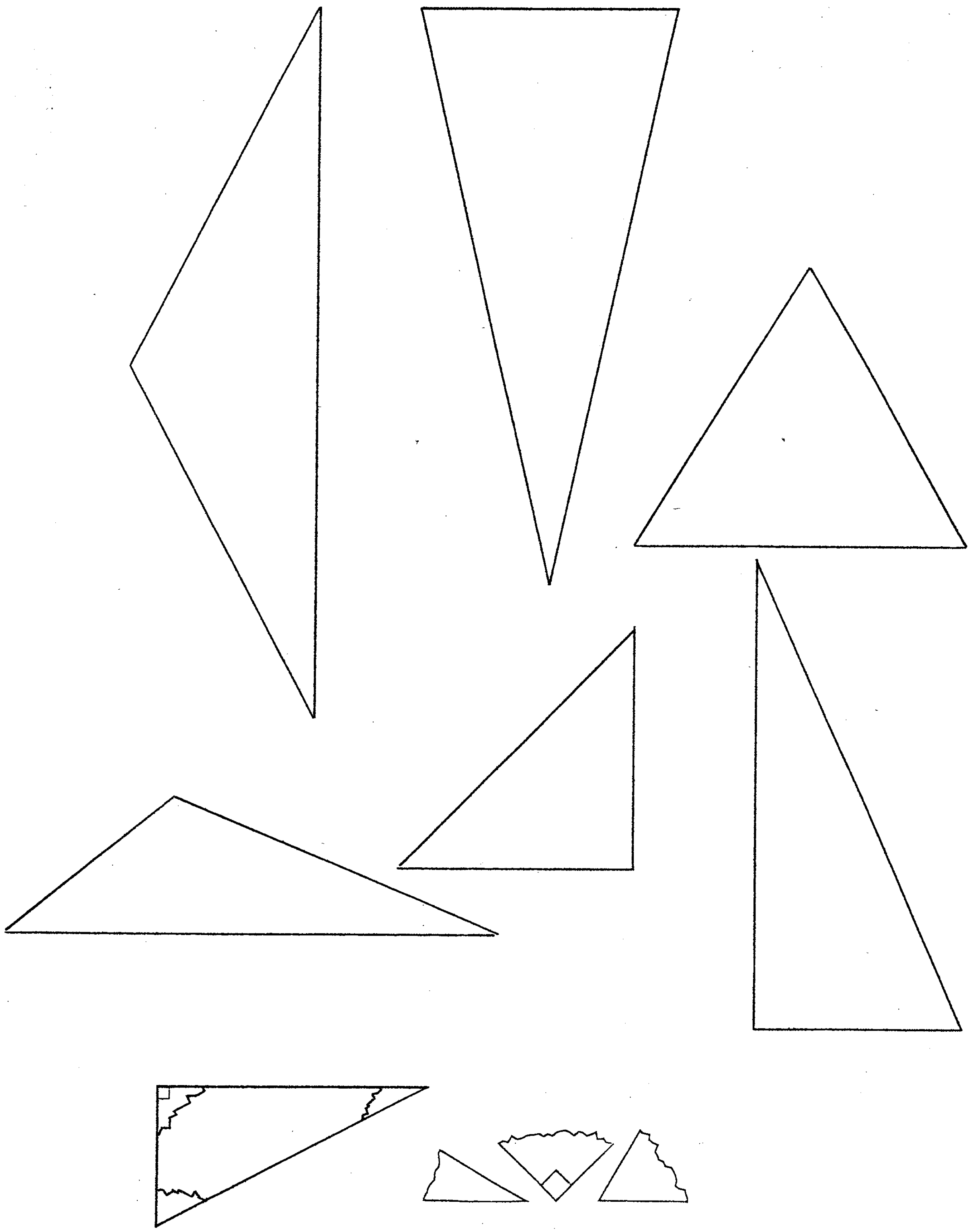
Coordinates for letter #1

Letter formed: ______

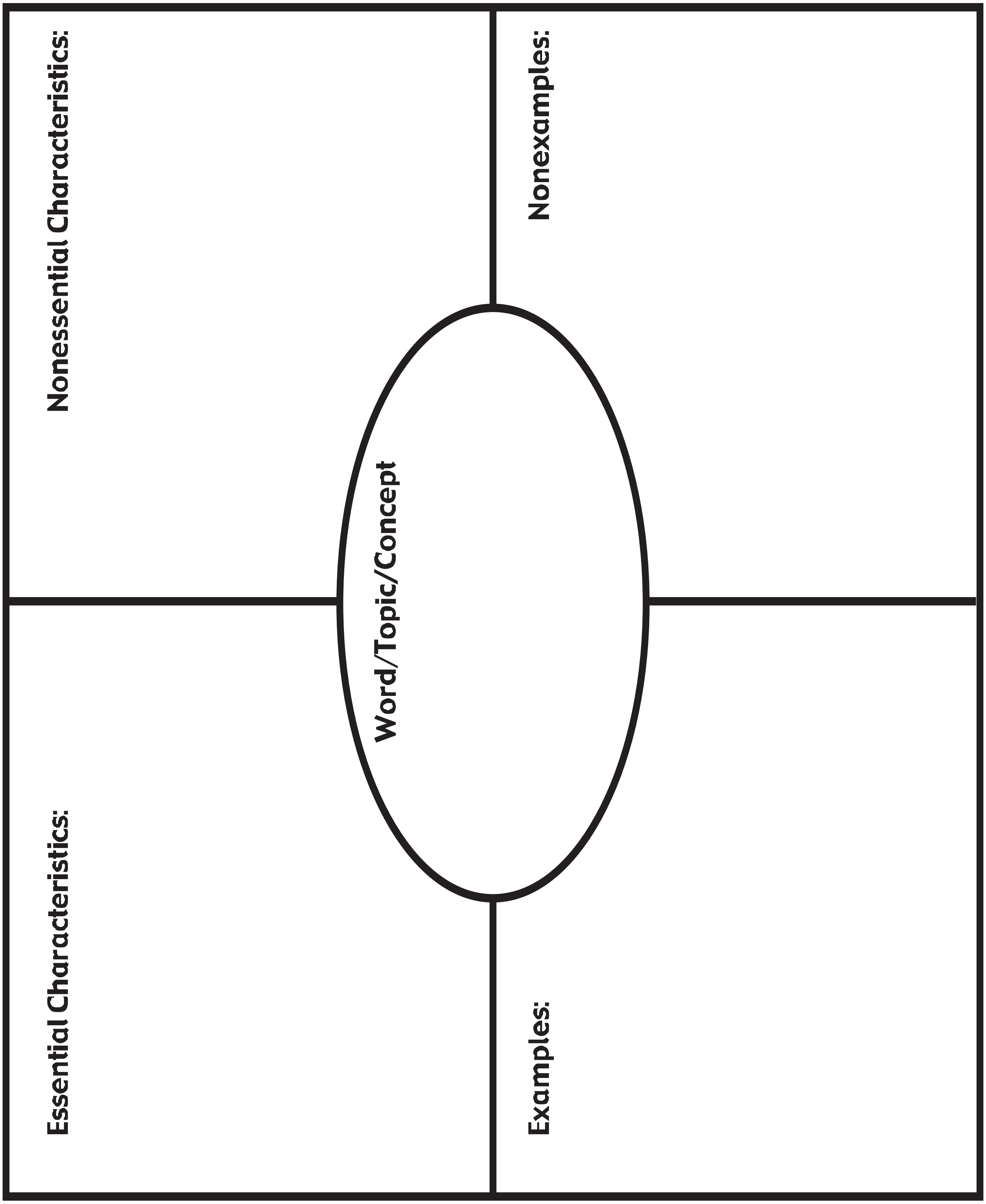
10 x 10 Grid Paper



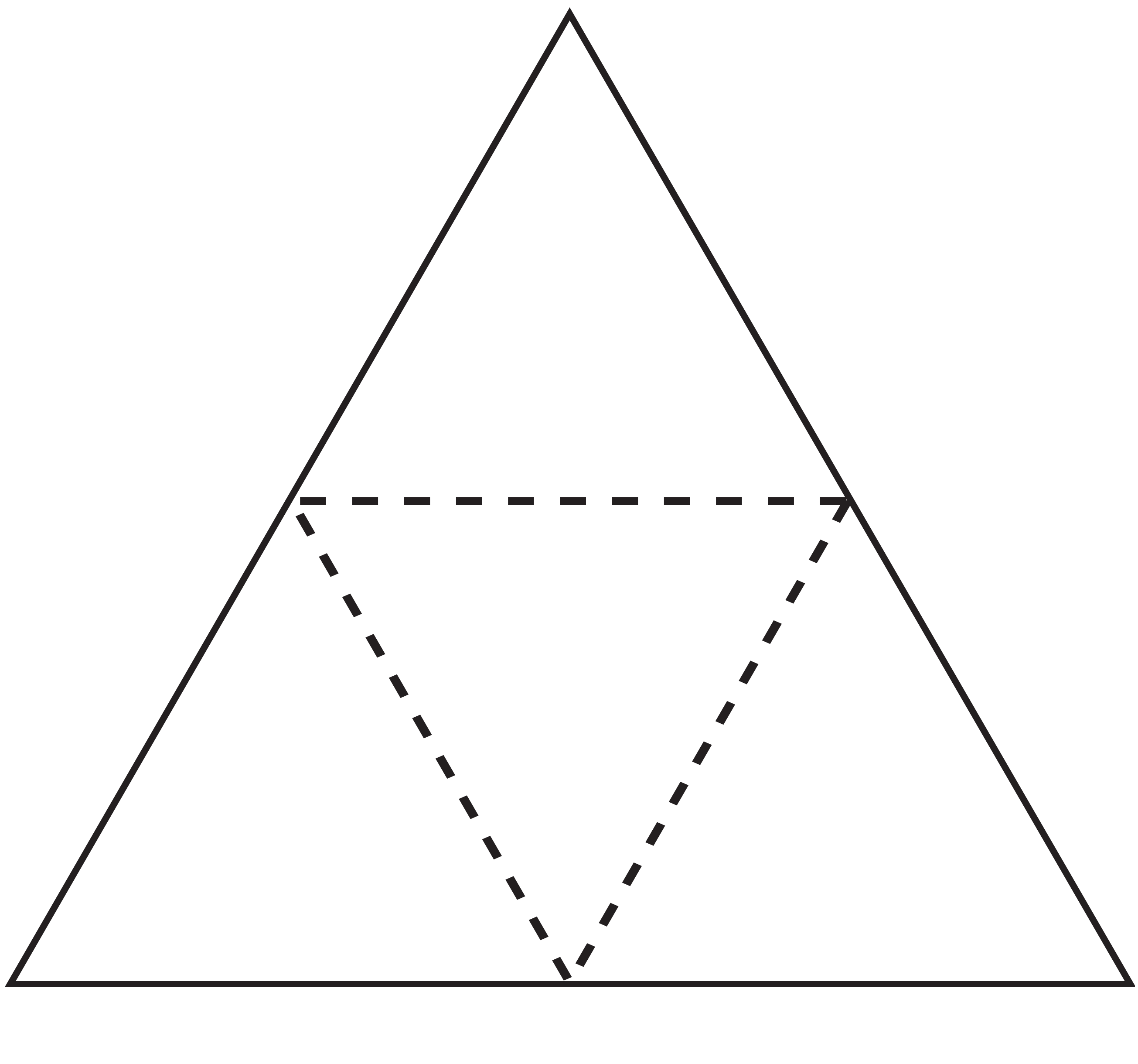
Triangle Patterns

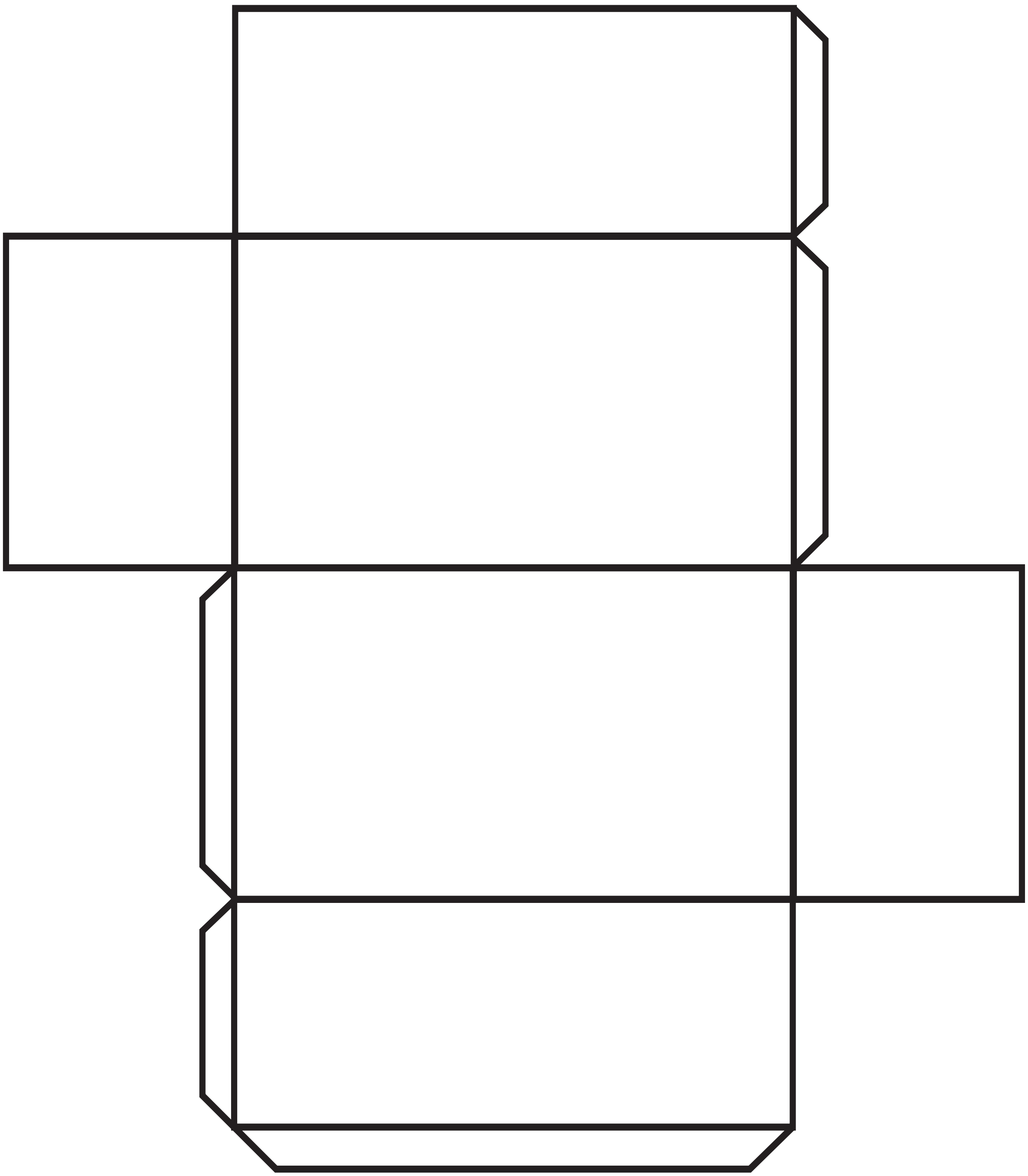


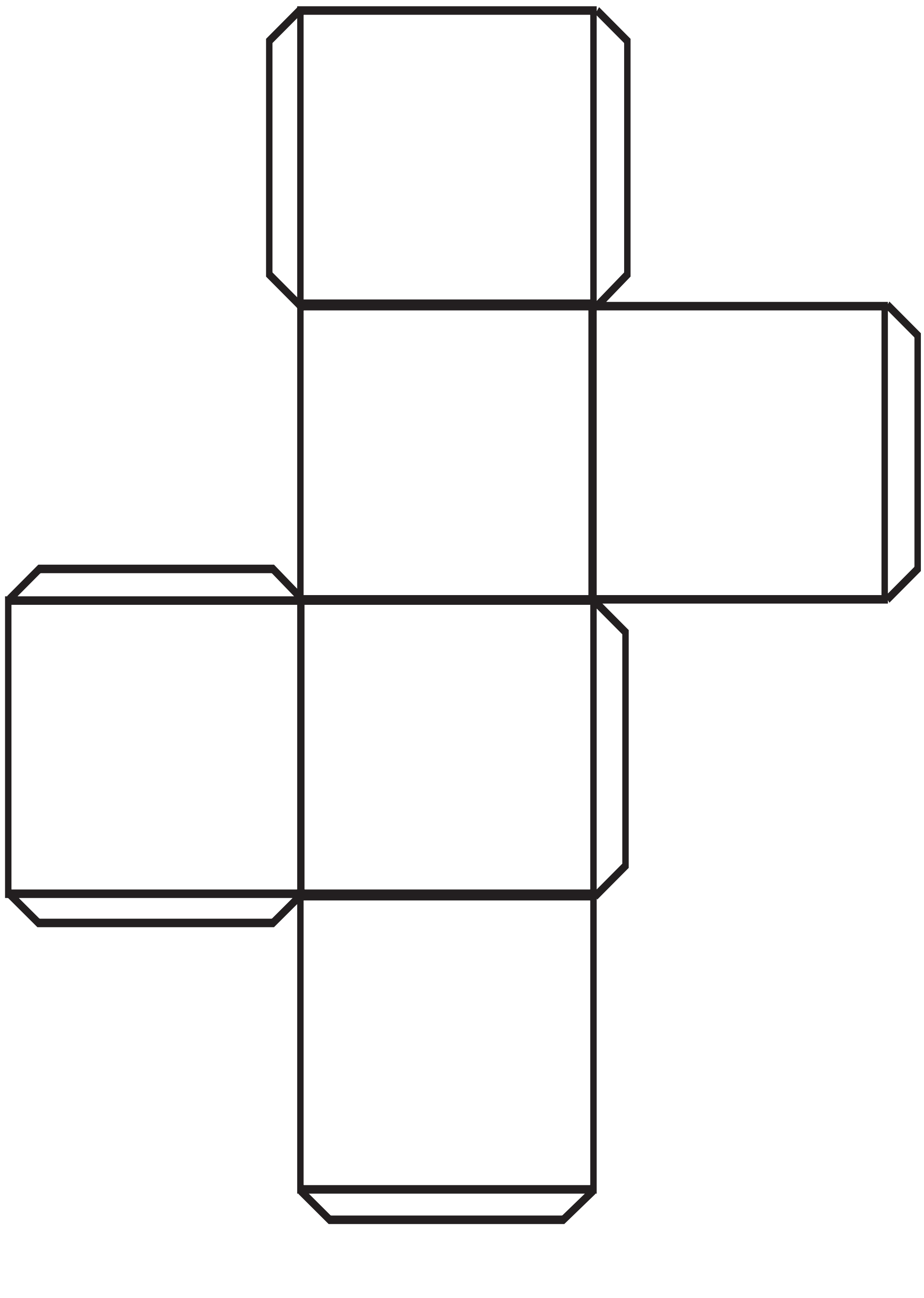
Frayer Model



Figures for Prisms and Pyramids







Name______________________________

Measuring in Feet and Inches

Measure each item to the nearest 1/8 inch.

#	Item Name	Estimate		Exact Measurement	
		feet ' and inches "	inches " only	feet ' and inches "	inches " only
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Wheat Thins Measure Up!

Estimate how many wheat thins it would take to put around the edge of your math book, then see how many it takes. Do the same with other objects in the class. Record your answers and watch for possible patterns.

Object	Estimate	Actual
1. Math Book		
2.		
3.		
4.		
5.		

Write patterns you observe: ______________________________

Estimate how many wheat thins it would take to cover your math book when placed side-by-side, then see how many it takes. Do the same with other objects in the class. Record your answers and watch for possible patterns.

Object	Estimate	Actual
1. Math Book		
2.		
3.		
4.		
5.		

Write patterns you observe: ______________________________

Estimate the perimeter of your math book in inches, then find the actual measurement. Estimate the area of your math book to the nearest inch, then find the actual measurement. Do the same with other objects in the class. Record your answers and watch for possible patterns.

Object	Perimeter Estimate	Perimeter	Area Estimate	Area
1. Math Book				
2.				
3.				
4.				
5.				

Write patterns you observe: ______________________________

Name______________________________

← ÷ King Henry Metrics x →

King	Henry	Danced	Merrily	Down	Center	Main
kilo	hecto	deka	liter meter gram	deci	centi	milli
Ex: 0.1	**1**	**10**	**100**	**1,000**	**10,000**	**100,000**
1.						**0.1**
2.				**728**		
3.		**79**				
4.					**300**	
5. **91**						
6.		**0.92**				
7.	**51**					
8.						**21**
9.				**0.839**		
10.			**0.467**			
11.	**0.3**					
12. **1.806**						
13.				**0.9**		
14.			**21**			
15.						**0.76**
16.					**0.4630**	
17. **25**						
18.		**1.324**				
19.						**28**
20.				**750**		

Name______________________________

Candy Measuring

Measure each object and record the correct length on the appropriate chart.

Nonstandard

Measure with your index finger.

Objects	Peanut	Cereal	Pencil	Cracker	Foot	Pretzel	Wrist	Pixie Stix	Jolly Rancher

Customary

Measure to the nearest 1/8 inch.

Objects	Peanut	Cereal	Pencil	Cracker	Foot	Pretzel	Wrist	Pixie Stix	Jolly Rancher

Metric

Measure to the nearest centimeter.

Objects	Peanut	Cereal	Pencil	Cracker	Foot	Pretzel	Wrist	Pixie Stix	Jolly Rancher

Name_____________________________

Customary Measuring

1. ______ 1/4 cups = 1 cup
2. ______ 1/2 cups = 1 cup
3. ______ 1 cups = 1 pint
4. ______ 1 cups = 1 quart
5. ______ 1/4 cups = 1/2 cup
6. ______ 1/2 cups = 3 cups
7. ______ quarts = 1 half-gallon
8. ______ quarts = 1 gallon
9. ______ cups = 1 gallon
10. ______ 1/2 cups = 2 cups
11. ______ 1/3 cups = 1 cup
12. ______ pints = 1 quart
13. ______ 1/3 cups = 2 cups
14. ______ 1/4 cups = 2 cups
15. ______ pints = 2 quarts
16. ______ pints = 1/2 gallon
17. ______ pints = 1 gallon
18. ______ 1/2 cups = 1 gallon

Measuring Spoons

19. ______ teaspoons = 1 Tablespoon
20. ______ 1/2 teaspoons = 1 teaspoon
21. ______ 1/4 teaspoons = 1 teaspoon
22. ______ 1/8 teaspoons = 1 teaspoon
23. ______ 1/3 teaspoons = 2 teaspoons
24. ______ 1/3 teaspooons = 1 teaspoon
25. ______ 1/4 teaspoons = 2 teaspoons
26. ______ 1/8 teaspoons = 2 teaspoons

Equal, Greater Than, or Less Than

27. 1 gallon ◯ 4 quarts
28. 3 cups ◯ 1 pint
29. 2 quarts ◯ 8 pints
30. 4 pints ◯ 1 cup
31. 2 cups ◯ 1 quart
32. 3 pints ◯ 1 quart
33. 8 cups ◯ 1 gallon
34. 2 quarts ◯ 10 cups

Name______________________________

Metric Measuring

Measure the following:

1. ______ milliLiters = 1 Liter
2. ______ milliLiters = 2 Liters
3. ______ milliLiters = 4 Liters
4. ______ deciLiters = 1 Liter
5. ______ deciLiters = 2 Liters
6. ______ deciLiters = 3 Liters
7. ______ centiLiters = 1 Liter
8. ______ centiLiters = 2 Liters
9. ______ centiLiters = 3 Liters
10. ______ Liters = 20 deciLiters
11. ______ deciLiters = 100 milliLiters
12. ______ Liters = 2,000 milliLiters

Extend your thinking:

1. ______ Liters = 1 dekaLilter
2. ______ Liters = 1 hectoLilters
3. ______ Liters = 1 kiloLilters
4. ______ deciLiters = 1 hectoLiter
5. ______ deciLiters = 1 kiloLiter
6. ______ hectoLiters = 5 Liters
7. ______ Liters = 2 dekaLiters
8. ______ Liters = 2 hectoLiters
9. ______ Liters = 2 kiloLiters
10. ______ deciLiters = 1 kiloLiters
11. ______ milliLiters = 2 centiLiters
12. ______ deciLiters = 2 hectoLiters

Concentration Cards

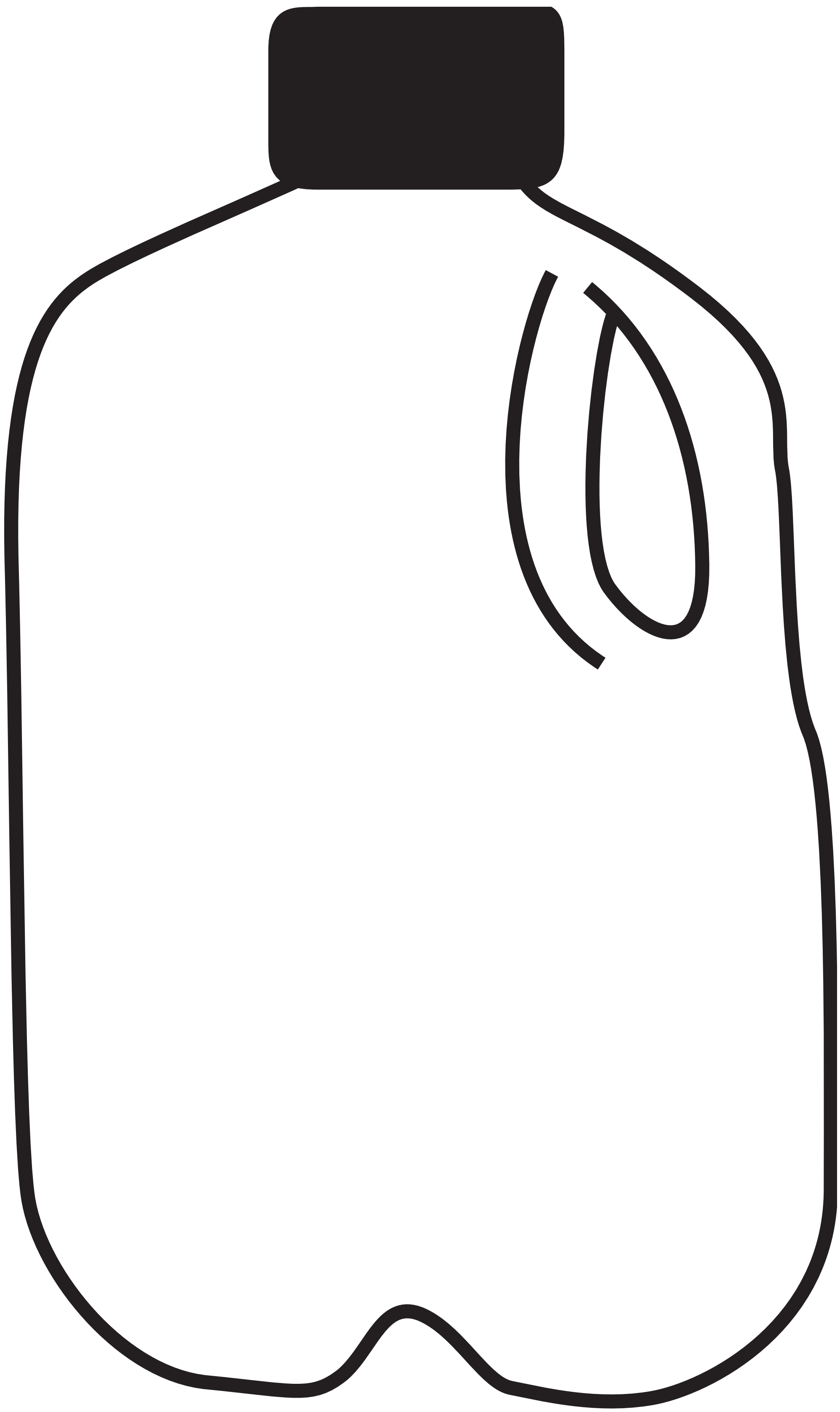
3 m	**4.25 cm**	**8,000 mm**
300 cm	**.425 dm**	**800 cm**

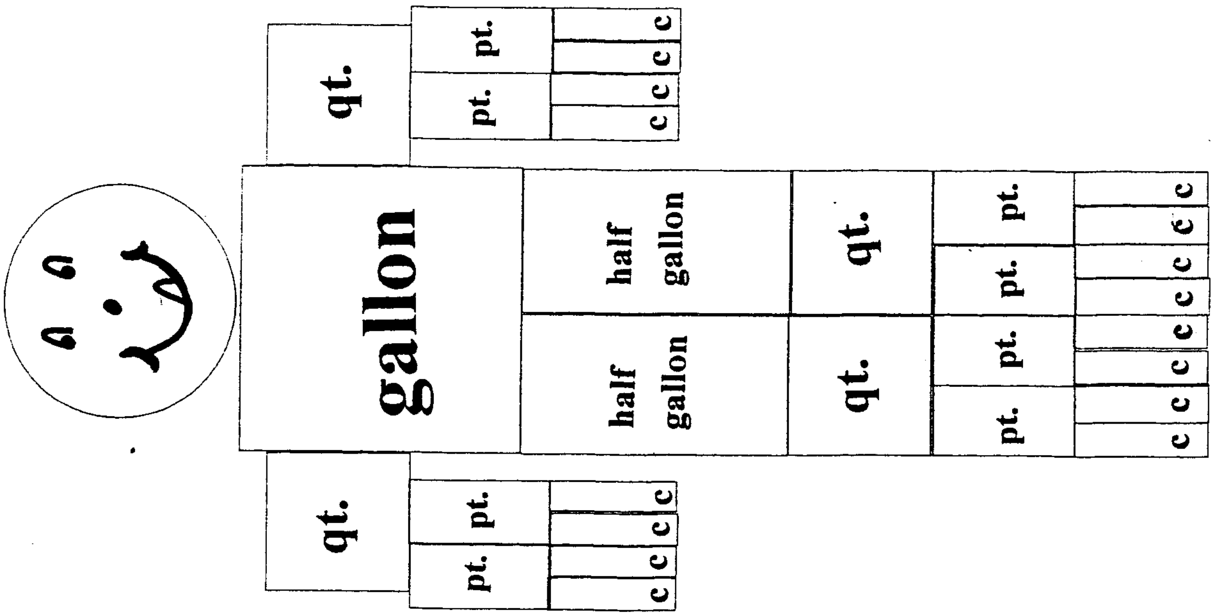
4.43 km	**.25 hm**	**78 dam**
4,430 m	**25 m**	**780 m**

9.9 km	**.52 m**	**52 cm**
99 hm	**5.2 dm**	**520 mm**

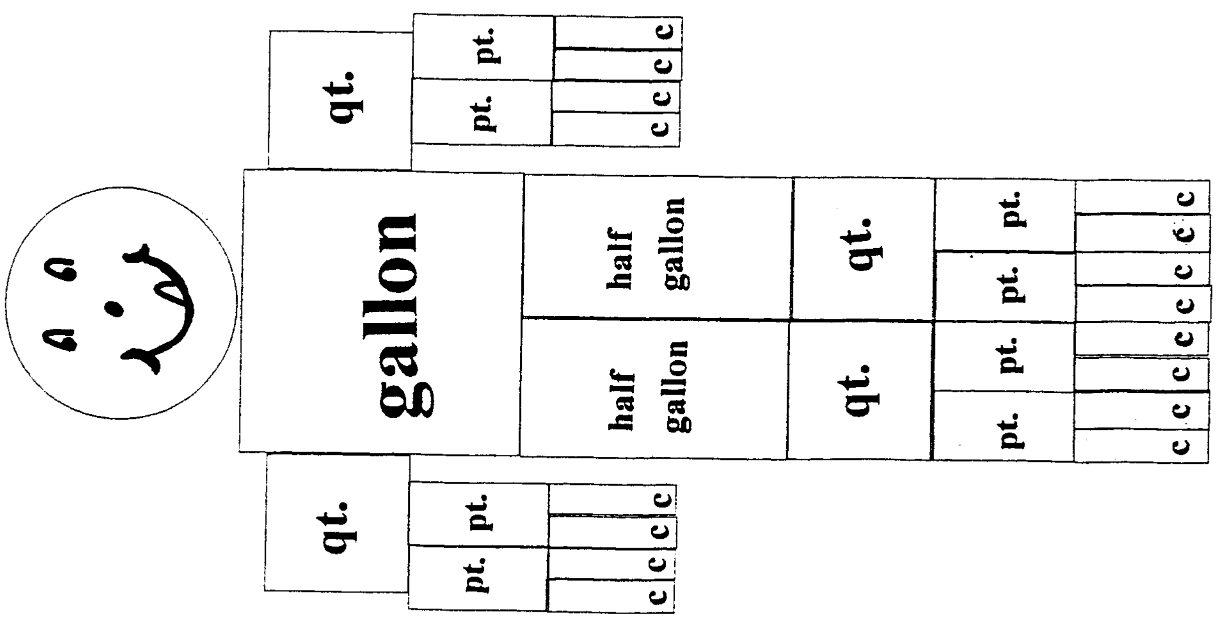
7.8 hm	**46 hm**	**8.8 dm**
7,800 dm	**460 dam**	**88 cm**

CAPICITY





Gallon-t Robo



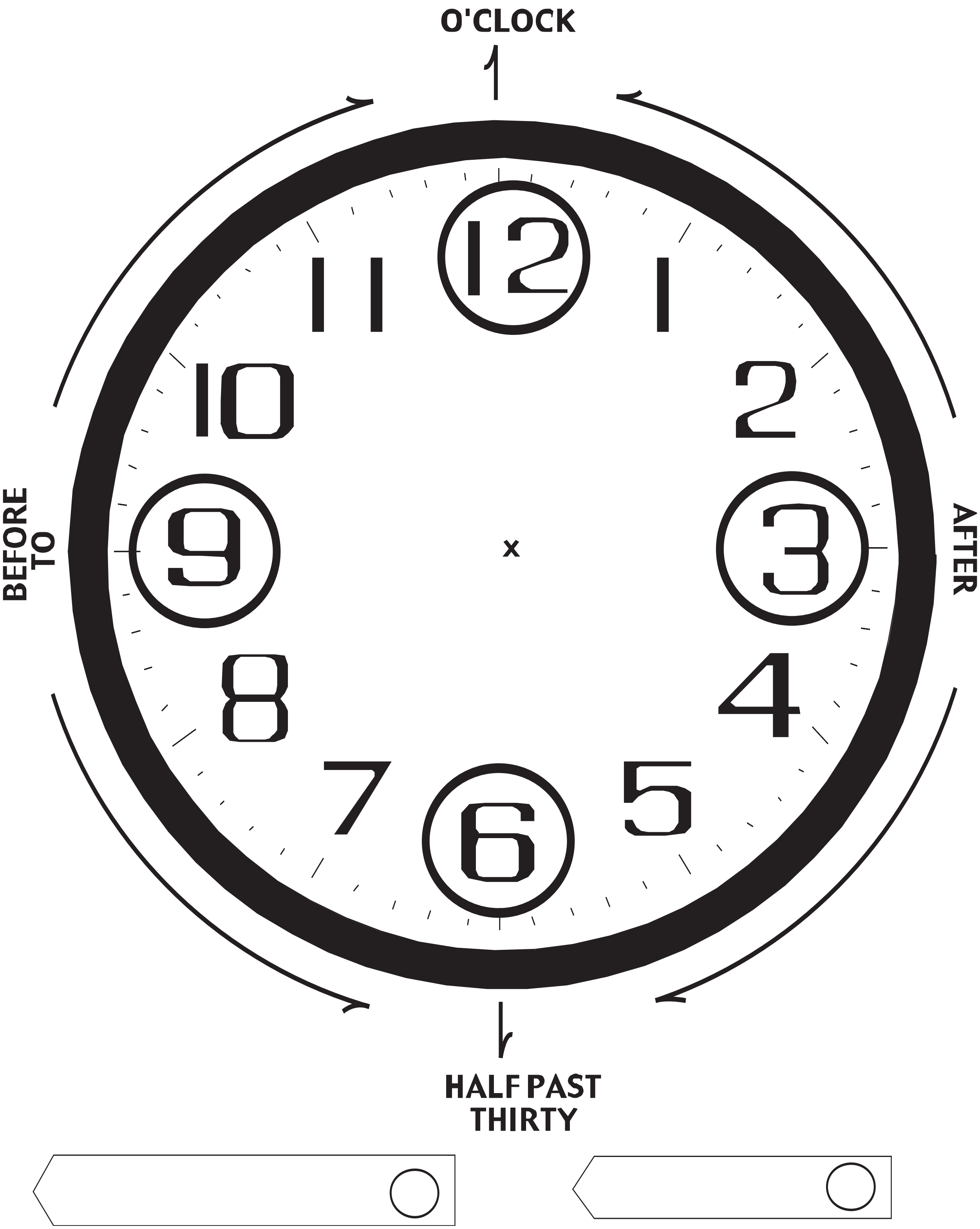
Gallon-t Robo

Name______________________________

Gallon-t Robo Combinations

Sketch your different Gallon-t Robo combinations in the squares below:

Clock Pattern



2-12 Number Line

12

11

10

9

8

7

6

5

4

3

2

Name______________________________

Dice Sums

Roll 2 dice. Mark an X under the sum. Do this until one sum reaches the Finish Line.

2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

Finish Line

Recored the sum that finished first on the class chart.

Dodecahedron Race

Player #1_________________________

I predict ______ will be rolled more than any other number.

Player #2_________________________

I predict ______ will be rolled more than any other number.

1										
2										
3										
4										
5										
6										
7										
8										
9										
10										
11										
12										

Probability Questions

1. What are the chances that 2 will win the next race you play? Why? ________________

2. What are the chances that 6 will win the next race you play? Why? ________________

3. Write at least two different ways of expressing the probability of rolling a 6. (Hint: as a fraction and as a ratio) ________________

4. Write at least two different ways of expressing the probability of rolling a 12. ________

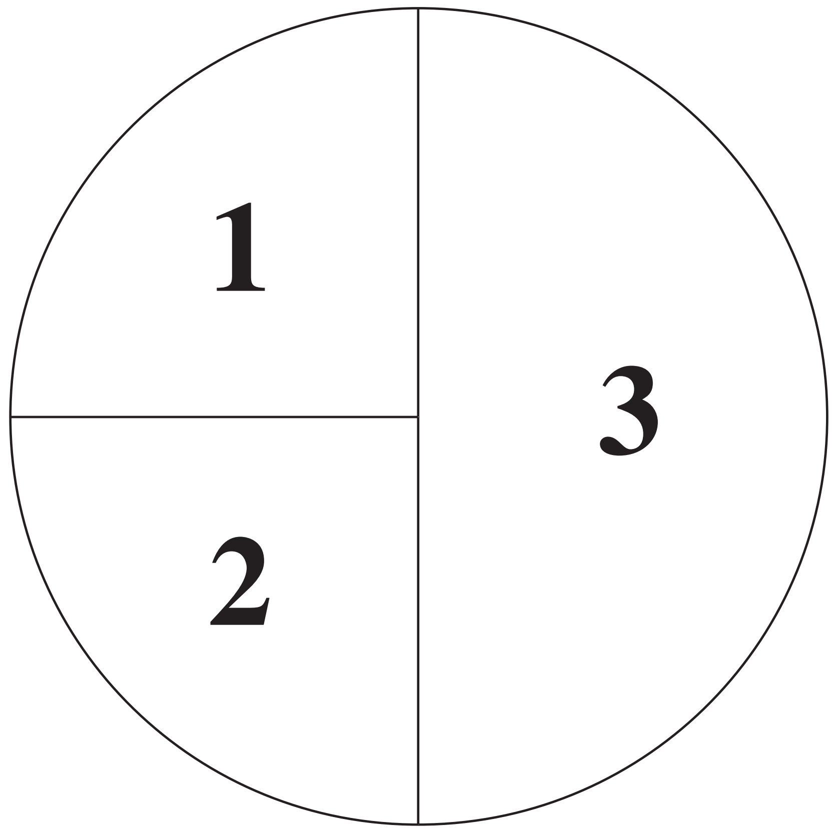
5. What is the probability of rolling a number less than 6? Explain ________________

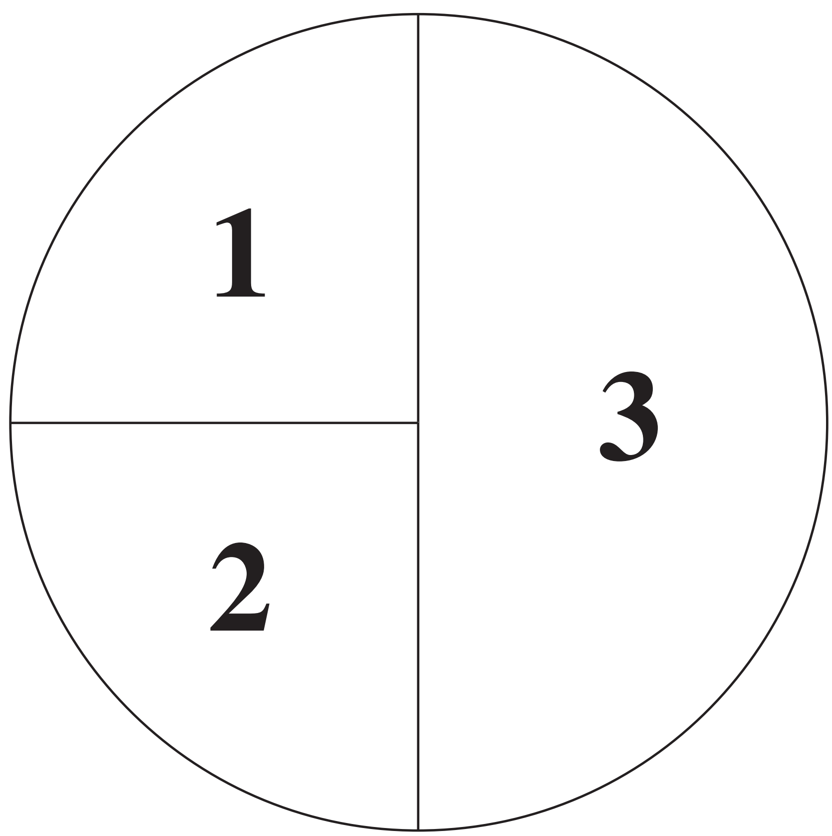
6. What is the probability of rolling a number more than 3? Explain. ________________

BONUS:

Write three different ways to express the chances of rolling an 11. ________________

Spinner Faces A

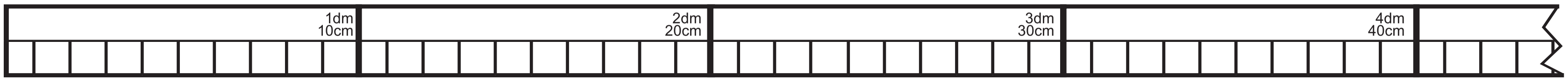




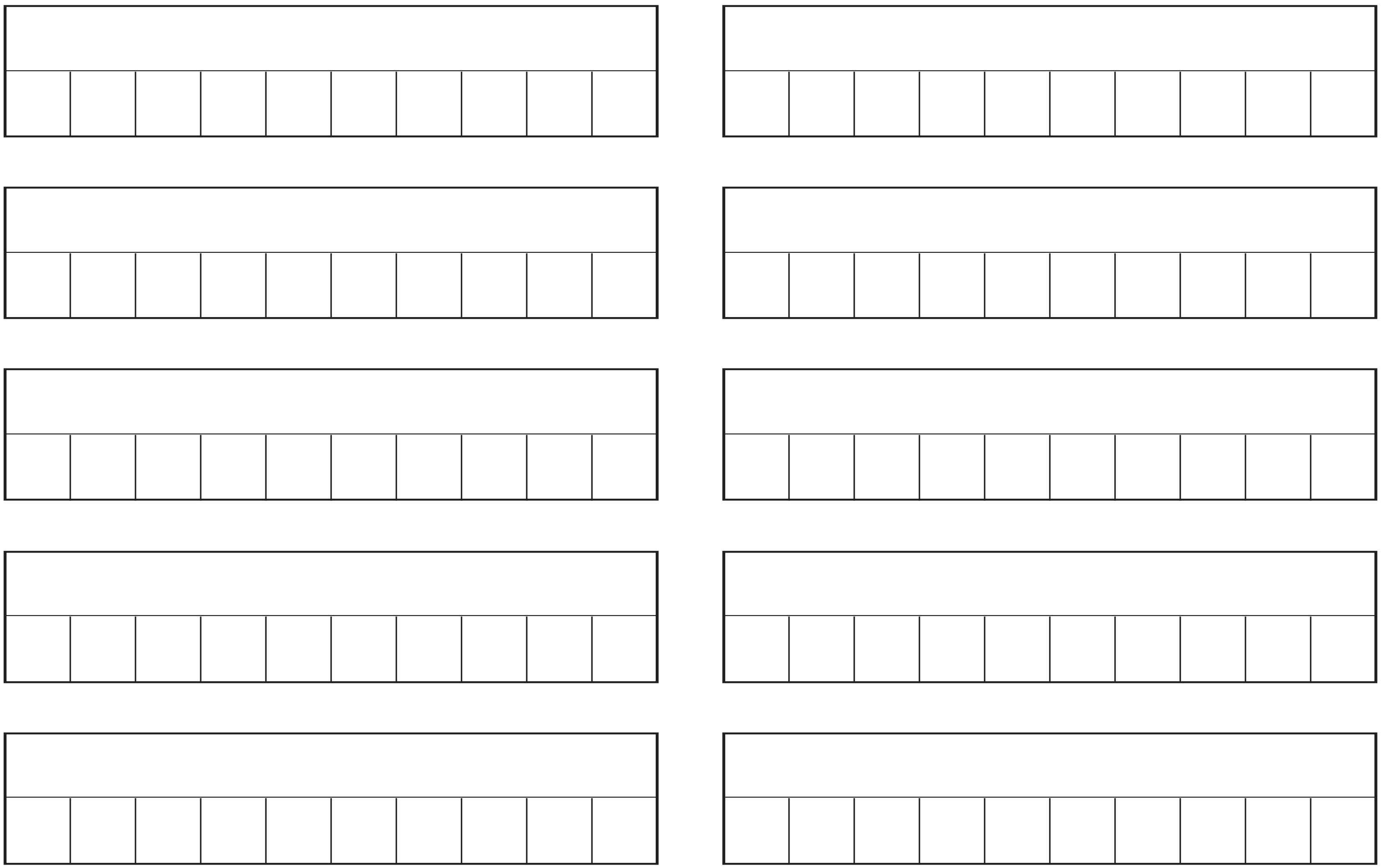
Spinner Recording Sheet

Metric Tape Measure 1

Cut each rectangle down the center of the bold lines. Tape rectangles together to create a meter tape. Label the meter tape as shown in the example.

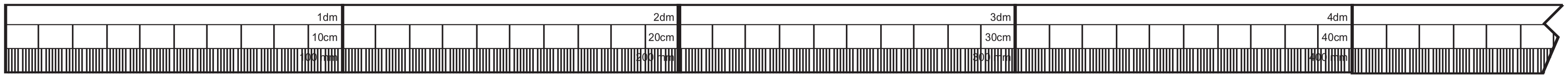


Adapted from Gayle Cloke

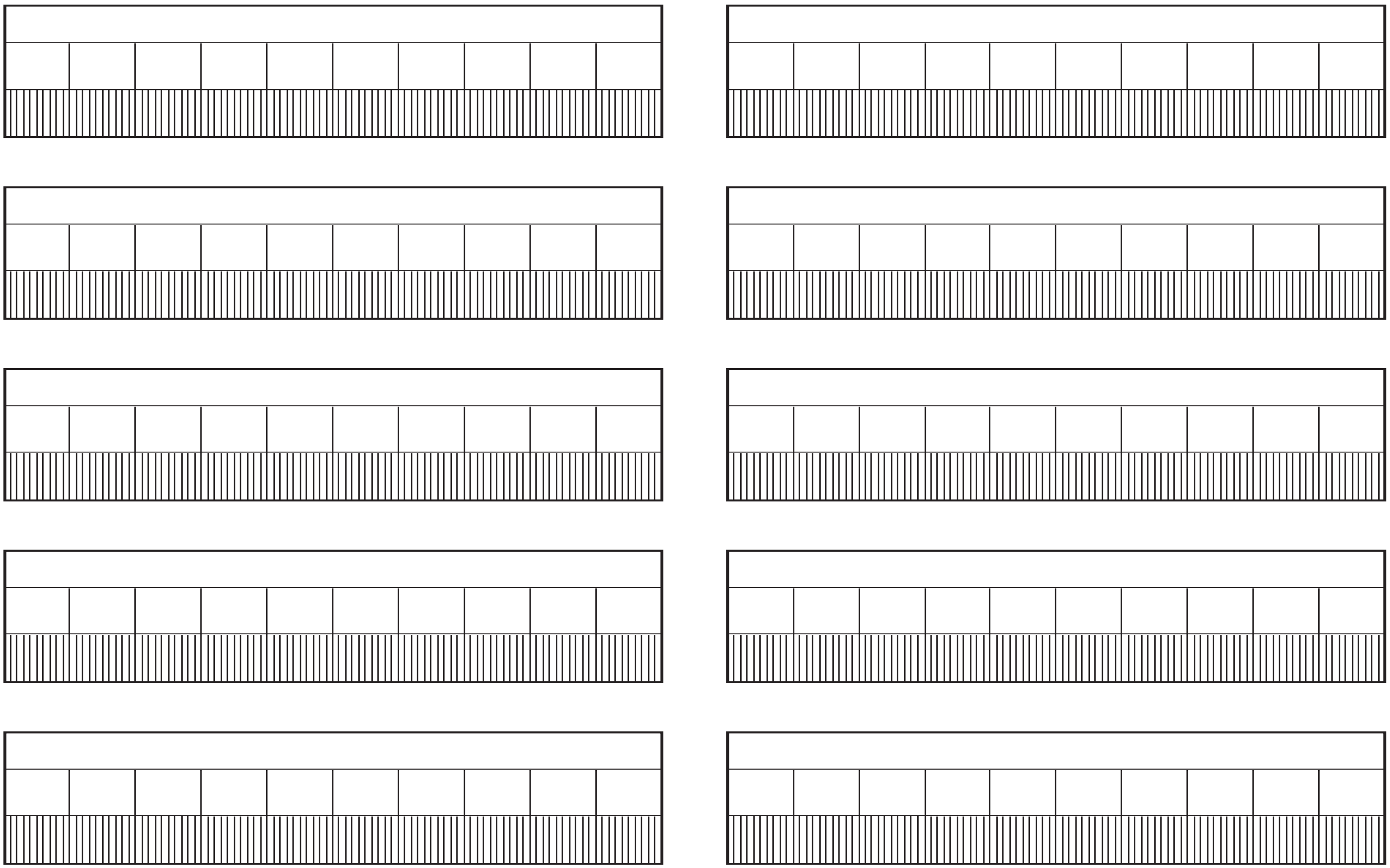


Metric Tape Measure 2

Cut each rectangle down the center of the bold lines. Tape rectangles together to create a meter tape. Label the meter tape as shown in the example.



Adapted from Gayle Cloke



Academy Notes

Academy Notes

Professional Development Activity
Evaluation Form 2002-03
Utah State Office of Education

Course Title ______Elementary CORE Academy______________________________

Facilitator ____

Dates _____________________________ to _________________________________

Location ____

		N/A	Strongly disagree	Disagree	Agree	Strongly Agree
1.	Inservice aligned with the Utah Core Curriculum.	0	1	2	3	4
2.	Useful assessment practices related to subject were presented.	0	1	2	3	4
3.	Time allocated for this professional development was appropriate to meet my learning needs. If your answer was “strongly disagree” or “disagree”, please check one of the following: ___More time needed ___Less time needed	0	1	2	3	4
4.	Inservice was well organized.	0	1	2	3	4
5.	Facilitator(s) and presenter(s) clearly stated objectives of professional development.	0	1	2	3	4
6.	Presenter(s) had adequate knowledge of subject matter.	0	1	2	3	4
7.	Professional development provided information relevant to my classroom.	0	1	2	3	4
8.	Accommodations and facilities promoted learning.	0	1	2	3	4
9.	I will recommend this professional development experience to other teachers.	0	1	2	3	4

10. Rate the use and effectiveness of each mode of instruction in this professional development.

	Not Used	Used Occasionally	Used Often	Not Used Effectively	Used Effectively
a) Lecture	0	1	2	0	1
b) Hands-on	0	1	2	0	1
c) Cooperative Groups	0	1	2	0	1
d) Discussion	0	1	2	0	1
e) Technology	0	1	2	0	1
f) Field Trips	0	1	2	0	1

11. How do you plan to implement the information from this professional development into your classroom instruction?

12. In what way will this professional development provide long-term benefit to the quality of your instruction?

13. What suggestions do you have for improving this professional development?

If you wish to send additional comments regarding this professional development experience, or to indicate other professional activities you would like to take in the future, please email or phone:

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1. Check the category that best describes your school position:
_____Teacher _____School and/or District Administrator / Supervisor
_____Grade Level
_____Pre-service teacher candidate _____Other (Specify:______________________)

2. Gender: _____Male _____Female

3. How many students, total, did you teach during the previous school year? ___________
How many students do you expect to teach in the coming school year? ___________

4. Are you teaching at a Title I School?
❏ Yes I am ❏ No I am not ❏ I don't know

School ______________________________ District __________________________