What Happened to the Geometry? Examining Spatial and Mathematical Concepts in Computational Toys and Kits for Young Children

Examining Spatial and Mathematical Concepts in Computational Toys and Kits for Young Children

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ABSTRACT

This paper explores the design of computational toys and kits for young children (ages 4-7) as tools for learning integrated mathematical, spatial, and computational thinking concepts. Specifically, we examine how the design features of the toys and kits represent the concepts of rotation on a point and spatial orientation of the agent. We examine toys and kits sold commercially, developed through research, and used in early childhood classrooms. Our findings indicate that the mathematical and spatial concepts are overlooked in some designs. Prior research examined toys for their affordances related to computational thinking, the present study contributes to understanding of how these toys and kits have the potential to foster foundational mathematics and spatial skills. We discuss implications for design of toys and kits as well as recommendations for future research.

CCS CONCEPTS

• **Human-centered computing**; • **Human-centered computing** → Interaction design; Interaction design process and methods; User interface design;

KEYWORDS

Tangible Coding Toys, Turtle Geometry, Turn Angle, Orientation

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1 INTRODUCTION

Contemporary computational toys and kits have historical roots in Logo's Turtle Geometry, which was designed for developing children's mathematical ideas [41, 42]. The Turtle was a virtual point that could be programmed to move using directional commands (FORWARD, BACK) and rotational commands (RIGHT, LEFT). Central to the theoretical underpinnings of Logo's Turtle Geometry were concepts of spatial thinking embedded in mathematical problem solving [42]. Like Turtle Geometry, many contemporary computational toys and kits for children involve navigating a robot or agent in virtual (2-D) or physical (3-D) space.

Although computational toys and kits have been widely used in early childhood settings for over 40 years [6, 48], advances in technology have influenced the designs, affordances, and spread of these toys. Given the pace of innovation, and because children's ideas and interpretations of situations are different from those of adults [18], it is beneficial to evaluate the design features and usability of new toys and kits from a child developmental perspective [6]. For example, recent studies surveyed existing computational toys and kits from the perspective of how the design features support young children's exploration of computational thinking [10, 29, 47, 48]. While these studies are valuable in that they contribute to our understanding of how to use these toys to foster computational thinking (CT) in early childhood, they do not provide information on how the design features represent the math and spatial skills like those that were central to Logo's Turtle Geometry. There is a need for research that examines these toys and kits from a historical perspective that considers the theoretical underpinnings that guided their design [6].

The present study contributes to understanding of how design features of computational toys afford students to engage in spatial mathematical knowledge in meaningful ways. Specifically, we do so by examining how recent computational toys and kits designed and marketed for children, ages 4-7, represent and support their exploration of the concepts of rotation on a point and spatial orientation. We focus on these two concepts for four reasons. First, they were central to the design of Logo. Second, they were concepts children struggled with when using Logo, and third, they are concepts that are foundational to later mathematics learning. Fourth, there is a need for models of integration of mathematics and CT in early childhood and it is important to know how these toys could support this integration [31]. The question that guides our inquiry is: *How do the design features of computational toys and kits represent the concepts of rotation on a point and the orientation of the agent?*

1.1 Defining Spatial Thinking

In the report, *Learning to Think Spatially*, the National Research Council (NRC), defined spatial thinking as a constructive amalgam of three elements: concepts of space, tools of representation, and processes of reasoning. To think spatially involves (1) understanding meanings of space and using its properties for structuring and solving problems, (2) using tools in a variety of ways to represent relationships or communicate ideas about space and spatial relationships, and (3) reasoning about these relationships [21]. Spatial thinking is not a discipline like biology because it is embedded in the practices of STEM domains, such as mathematics [2]. Numerous studies have provided evidence for the link between mathematics learning and spatial thinking [3, 30, 45]. Play with tangible and virtual toys and kits that involve programming agents, like the Turtle in Logo, involves reasoning with the agent's orientations, locations, and navigation in space and operations on those spatial relationships [11]. Spatial thinking is embedded in the mathematical problem-solving children engage in when programming with these tools. Knowing that *not all computational toys and kits are created equal* [6], it is important to explore how design features support children's spatial and mathematics thinking.

1.2 Angles and Rotation on a Point

Angles are a complex idea for children to grasp [7, 34], yet they are critical to mathematics learning [17]. In fact, angles play a role in mathematics every year in primary school. In the early grades, the Common Core State Standards for Mathematics (CCSSM) denote standards for children's learning about angles as an attribute of shapes. For example, in kindergarten children use informal language to analyze and compare shapes by the number of sides and *vertices/ "corners"* (K.G.B.4), and in the first grade, children use angles to distinguish between defining attributes of shapes (1.G.A.1). In grade 2, students "Recognize and draw shapes having specified attributes, such as a *given number of angles* or a given number of equal faces..." (2.G.A.1). In order for children to understand angles, they need to be able to recognize and discriminate angles as critical parts of geometric figures, compare and match angles, and construct and mentally represent the idea of turns, integrating this with angle measure [17]. However, much of this exposure to angles in primary school is through static representations in the context of attributes of shapes. Providing children with early opportunities to explore different representations of angles and contexts for angle measurement is critical for their later mathematical understanding and development [7, 17].

There are three different perspectives of the concept of angle: a static geometric shape represented by the union of two rays and a common endpoint, a dynamic angle as rotation on a point (movement), and the dynamic angle as number or amount of a turn [7, 33]. Rotation on a point or re-orienting an agent's heading is like what Freudenthal described as "turn angle" [33]. Dynamic in nature, turn angles are complex concepts because they require children to pay attention to several aspects at once [33]. For example, children need to understand that a rotation occurs by rotating on a fixed point at a set angle and not translating to an adjacent point. This requires understanding the unit of the angle (i.e., how much one rotation makes the agent turn) and one-to-one correspondence (i.e., that one rotation is one movement). Finally, they need to understand if it is a right rotation or a left rotation (i.e., direction of the movement). Thus, turn angles require children to use both spatial and mathematical knowledge concurrently. This concept of rotation on a point is not intuitive for many children. Young children's exposure to rotations in their daily experience is more akin to the rotation plus movement like a continuous turn of a car or bicycle [44]. Thus, providing dynamic representations of angles in early grades helps children build foundational concepts of angles that they can draw upon and build upon in upper primary [7].

1.3 Spatial Orientation of an Agent

Spatial orientation is knowing how to navigate and move in the world and understanding relationships between different positions in space, from your own position and also from an abstract perspective [17]. While early mathematics standards focus on rotation of objects (e.g., a triangle is still a triangle when it is rotated 30 degrees), spatial orientation in the context of navigation is not typically part of primary mathematics instruction. Spatial orientation of an agent is a difficult concept for children because it requires them to shift their frame of reference from an egocentric perspective to an allocentric perspective [17]. In the context of programming an agent, children need to always consider the agent's heading and not their own. A forward is always a forward no matter what orientation an agent is facing.

1.4 Rotation on a Point and Orientation of the Agent in Logo

In the early design of Logo's Turtle Geometry, the Turtle was represented as a cursor on the computer screen. However, it was meant to be more than that, it was an "object-to-think-with." Papert wrote that "a Turtle is at some place—it, too, has a position—but it also faces some direction–its heading" [42]. Children have their own bodily experiences in the world, they know how to walk around and move (i.e., body geometry), and they were meant to apply their experiences in the world to the properties, actions, and behaviors of the Turtle. The Turtle had two properties: position and heading. Position refers to the location of the Turtle (i.e., its coordinates). Heading refers to the direction the Turtle is facing (e.g., its orientation). FORWARD and BACK commands made the Turtle change position, based on the direction of its heading. RIGHT and LEFT commands made the Turtle change its heading but stay in the same position. Children had to program the direction of the rotation (LEFT or RIGHT) as well as the degree of the rotation. For example, LEFT 90 re-oriented the Turtle's heading 90 degrees to the left. Children had to pay attention to the Turtle's heading when programming movement, rotations and forward and backwards.

1.5 Research on Logo Turtle Geometry

Research on the benefits of Logo for helping children understand rotation on a point and orientation were mixed. While Papert envisioned children taking on a Turtle-centric perspective [42], children often used an ego-centric perspective when programming in Logo. For example, research on Logo found that children had difficulty programming the Turtle when it did not share their orientation [22, 24]; in other words, when the Turtle was facing left, right, or down. This is not surprising given that in early versions of Logo, the graphics of the cursor made it challenging to decipher its heading [26]. In addition, some studies found that children struggled with the concept of angle and angle measurement [19, 20]. However, other studies found that Logo's Turtle Geometry helped children develop turn concepts and measurement [12–15]. In terms of spatial orientation and frames of reference, Geva and Cohen [26] claimed that in order for children to be successful in programming, they needed to be able to distinguish themselves and the Turtle as two distinct frames of reference. Similarly, Fay and Mayer wrote that in order for children to learn Logo, it requires more than just knowing the syntactic rules, they need to be able to discriminate between the actions of turning and moving and the ability to take on a Turtle-centric frame of reference [24].

In response to the mixed findings on children's learning of turn angles and orientation, Clements & Sarama [16], claimed that studies that took an "exposure approach," where researchers assumed that exposure to mathematics concepts alone would help children learn, tended to produce inconclusive findings. Whereas studies on Logo that took a "mediated conceptual framework" approach, tended to find positive results. In these latter studies, teachers mediated instruction and the tasks were focused on how Logo could support children's learning of mathematics. In other words, Logo provided a context for children to construct, test, play, and develop building blocks for later mathematics learning. Thus, while design features of Logo were meant to make some abstract mathematics and spatial concepts more concrete for children, research found that children needed support to do so. These findings are important to the design of contemporary coding toys and kits and have implications for how we use these toys and kits in classroom contexts. It is important to understand how the design features of contemporary toys and kits represent angles and orientation so that educators can better support children's exploration of these mathematical and spatial concepts in a way that is developmentally appropriate.

1.6 Contemporary Perspectives on Spatial Thinking and Mathematics Learning with Toys and Kits

In addition to research on computational thinking, researchers have also focused on the ways that spatial and mathematics concepts emerge as young children play with computational toys and kits. Such concepts include number concepts, measurement, path planning, spatial relations, angles and turns, and mental rotation, [1, 25, 35–39, 46]. For example, Fessakis and colleagues [25] found that children using the Ladybug version of Logo had difficulties with: decoding the symbols used for movement commands and angle and turn concepts, including when the ladybug was not in their orientation. Several studies using the Bee-bot robot explored the relationship between computational thinking and spatial thinking. For example, Diago et al [23] examined the relationship between computational thinking and mental rotation in a study with children in Grade 3. They found students computational thinking improved significantly but did not find any relationship between their computational thinking and mental rotation skills. In a study on preschool children using Bee-bot robot, Palmér [39] , explored the tools potential for fostering math and spatial skills. They found that over a few months, children learned spatial thinking, measurement concepts, counting, and symbol meaning. In another study with the Bee-bot, Angeli and Valenides, [1], found young primary school-aged children learned computational thinking and spatial relations skills. Research on other toys have had similar findings. Welch et al [46] demonstrated how playing with the Cubetto robot helped children understand the concept of dynamic linear units. In a study with children in primary school (1st – 5th), Citta et al [8] explored the relationship between spatial thinking and computational thinking. Children learned about coding and then engaged in embodied and tangible coding games. The authors found that the children's' computational thinking skills were related to mental rotation skills. Shumway et al, [43] is one of the few studies that looked at the mathematical, spatial, and computational thinking skills concurrently. In a study with Botley and Cubetto, they observed kindergarten children using spatial, measurement, and number concepts when solving CT tasks. Given that researchers have already been studying how robot toys and kits foster spatial thinking, mathematics, and computational thinking, there is a need for research on how the design features of these toys represent these concepts.

2 FRAMEWORK FOR EVALUATING THE COMPUTATIONAL TOYS AND KITS

Our analysis of the design features of the toys is guided by Palmer's theory of external representations [40]. Representation plays an important role in the learning and teaching of mathematics [27, 28]. According to Palmer [40], a representation is something that stands for something else; a model of the thing it represents (p. 262). We use Palmer's concept of external representation as a lens to view how the computational toys and kits *represent* rotation on a point and orientation of the agent. Palmer uses "represented world" to describe the thing being represented and the "representing world" as the thing that is doing the representing. He goes on to say that there is a correspondence or mapping between objects in the two worlds in which relations in the represented world are structurally preserved (266-7). Thus, while the representing world only need to represent one aspect or part of the represented world, there needs to be an operational relationship in which information in the represented world is preserved. Palmer wrote that external representations should be described in terms of (1) the represented world, (2) the representing world, (3) what aspects of the represented world are being represented, (4) what aspects of the representing world are doing the modelling and (5) the correspondence between the two worlds. In the present study, *the represented world* is the concept of angles (rotation on a point) and orientation of the agent, and the *representing worlds* are the computational toys and kits. We

Table 1: Computational Toys and Kits for Children Identified in Search

Reference	Toys
Silva et al., (2023)	Bee-Bot, KIBO, ScratchJr
Su, J., Yang, W., & Zhong, Y. (2023)	KIBO, Bee-Bot
Su, Yang, & Li, (2023)	KIBO, Scratch Jr, Bee-Bot, Blue-Bot, Cubetto
Sun & Zhou, (2023)	KIBO & Bee-Bot
Theodoropoulou, Lavidas, & Komis, (2023)	Thymio II, Bee-Bot
Tselegkaridis & Sapounidis, (2022)	Thymio, Bee-Bot, KIBO, Code and Go Mouse, Dash, ScratchJr
Internet Search	Bee-Bot App, Tale Bot Pro, Matatalab
Schools	Botley, Robot Mouse, Bee-bot, ScratchJr, Code.org

examine the operational relationship between the concepts of rotation on a point and orientation of the agent and the representing worlds of the computational toys and kits.

3 METHODS

3.1 Identifying Computational Toys and Kits used in the Present Study

There is a wide range of computational toys and kits that are designed for children. Our goal was to include toys and kits that are sold commercially, developed through research, and used in early childhood classrooms. In addition, there has been a surge of systematic and scoping reviews on robots and computational thinking in early childhood in the past few years. We decided these reviews would be a good place to identify the toys and kits that are popular or widely used.

We identified three criteria for toys and kits to be included in our study: designed for children ages 4-7; a physical robot or a screen-based agent that had a "heading" and could be programmed to move in either 3-D or 2-D space; and available for use as of January 2024. We identified three criteria for exclusion: toys designed for children older than 7, toys that did not involve navigation (e.g., Cubelets), and toys that are no longer available for use (e.g., Kiwi, Cherps).

We used EBSCOhost to search Academic Search Ultimate, Education Full Text (H.W. Wilson), Education Source, and Eric using the terms: (computational toys or robot or computational kits AND early childhood education or preschool or kindergarten AND review or meta-analysis or systematic review NOT social). We restricted the search to 2021 to 2024 because [29, 48] was published in 2020. Given the rate at which technology develops and evolves, we wanted to find the most recent toys and kits.

The search identified 13 articles that were scoping or systematic reviews (see search terms above). After reading abstracts, we dropped six articles because they did not focus on early childhood or computational toys. We read each paper and recorded the robot toys and kits mentioned in the studies. Of the seven articles remaining, only six mentioned toys or kits that fit our inclusion criteria. In addition, we asked teachers what toys and kits children in kindergarten were using in the classroom. Finally, we searched Google and Amazon for robots, coding toys and kits for children ages 4-7. Table 1 presents the results of this inquiry. We identified fifteen toys and kits. We had access to all the toys and kits identified in our search and presented in Table 1 except for Tale Bot Pro and Thymio II. We were able to research enough information about Tale Bot Pro to include it in our analysis, but we could not find enough information to include Thymio II in the present study. Despite our systematic approach, it's important to acknowledge the limitations within our methodology. There is potential bias in our selection of toys and kits, which may influence the findings. One notable constraint involves potential biases in toy selection, which may influence the findings.

After compiling our list of toys, we classified each toy as either tangible, virtual, or hybrid (see Table 2). We used these criteria based on classifications used in previous studies [29, 47, 48]. Tangible kits and toys are "screen free." Children use them without a computer and can touch and manipulate the materials that come with the kit. Virtual toys and kits are used solely on a tablet, smartphone, or computer. There are not any tangible components. Hybrid toys and kits involve a mix of tangible and virtual. We used these criteria as a first step to explore the range of toys and kits.

3.2 Analyzing Computational Toys and Kits

Our analysis included 14 toys and kits. We played with the toys and kits we had available to get a sense of their design features and how children programmed the agent. Three researchers engaged in a systematic examination of the design features of the toys and kits based on Palmer's theory of external representations where the represented world is the concept of angles (rotation on a point) and orientation of the agent, and the representing worlds are each of the computational toys and kits. Initial codes were generated through an inductive approach (playing with toys and kits), with one team member first coding a subset of the toys before coming together in weekly meetings to discuss and refine the codes. Questions were discussed during meetings, including, *How are angles represented? How is orientation represented?* We refined our questions to make sure we were documenting what aspects of the represented world were being represented, what aspects of the representing world were doing the modelling, and the correspondence between the two worlds. Disagreements were resolved through consensus, facilitated by a senior researcher. This iterative process continued until the team agreed that the codebook was complete. We then brought in a fourth researcher, an early childhood mathematics expert, to review the codes and our interpretations of the findings. Our code book with examples is presented in Table 3.

4 FINDINGS

Table 4 presents the toys and kits, the age range, the type of kit, visual representation of the rotations, rotation unit, and unit of

Table 2: Computational Toys and Kits Analyzed in This Paper (N=14)

Tangible	Botley, Botley 2.0, Robot Mouse, Bee-Bot Toy, Cubetto, KIBO, Tale Bot Pro, Matatalab
Virtual	Bee Bot App, ScratchJr, Code.org Pre-Reader Express (2023)
Hybrid	Dash and Dot, Osmo Coding Awbie, Blue-Bot

measurement. Tangible kits have tangible agents and virtual kits have virtual agents. In terms of the hybrid kits, Blue-Bot is a tangible agent and Coding Awbie is a virtual agent that is programmed with tangible codes.

4.1 Arrow Representations of Rotations and Angle Measurement

In 10 of the 14 toys and kits, turn rotations are represented by arrows that often face either left or right and have a curve or bend to them. For example, in KIBO, a rotation is represented by a curved arrow facing either left or right accompanied by text identifying the direction of the turn, "turn left" and "turn right" (Figure 1a). Bee-Bot has arrows on its back, the left and right are slightly curved whereas the forward and back arrows are straight (see Table 4). Cubetto uses color tiles with arrow headings to represent rotation (Figure 1c), and there is a curved aspect to the top left or right tile on the rotation tiles (red and yellow) versus the linear arrows (green and purple). ScratchJr has a rotation code that is represented by an arrow with a rounded end (see Table 4). In most of the toys and kits, rotation arrows look different from the linear movement arrows in that they have a curve to the arrow or tile versus a straight arrow or box/square tile. This means that rotational movements are represented differently than the way the linear movements are represented.

While rotational movement is represented by a curved arrow or tile facing right or left, for two of the kits, Code.org and Coding Awbie, this is not the case. In these kits, all movement in space is represented by a straight arrow, facing up, down, left, or right. The rotation concept is not represented. Instead, direction of linear movement is represented (see Figure 1b and 1d) although the action is actually a rotation. The arrows that point left and right reorient the agent's heading and change their position along with linear movements in that direction.

In 7 of 14 toys and kits, one rotational unit is 90°, cannot be changed, and is represented by an arrow that closely resembles a 90° angle. In ScratchJr, one rotational unit is 30° or 1/12 of a circle (360°) and is represented by a curved arrow without an angular measure. These toys and kits represent rotational movement, but only one kind of rotation. Some toys like Botley 2.0 and KIBO include codes that allow children to use both a 90° angle and one additional option (45° and 360° respectively). For Botley the different measures of rotation are represented by different degrees of angle, without words or numbers. The KIBO's rotation of 360° plus the word "spin" (see Table 4) represents rotation as a spin. Botley and KIBO's additional options broaden the representation of rotational movement in different ways. One commonality across these toys and kits, is that rotation as a turn movement is emphasized in the arrow or tile representation rather than by a precise visual representation of angle measurement. Even the arrows that closely resemble a 90° angle (such as Botley and Robot Mouse) are not precise measurements. This design feature seems to intend the representation of rotational movement over angle measurement.

Dash and Dot and Blue-Bot are different than the other toys and kits in that angle measurement is represented and is represented through numbers. Both can be programmed through virtual apps and have multiple options for entering the degrees of the rotation, which means that both rotational movement and angle measurement are represented by the codes.

4.2 Arrow Representations of Orientation or Direction

Of the 14 agents in the toys and kits, 11 agents have a heading like the Turtle in Logo's Turtle Geometry where a rotation code changes its heading. For example, in the tangible toys like Bee-Bot and Botley, the rotation changes only the heading or orientation and does not include a linear movement. That heading then affects the direction it moves if linear movements are subsequently programmed. A FORWARD is always a FORWARD regardless of the agent's orientation and a rotation changes the orientation but not the agent's position. However, we found that an agent's change in orientation is not always clear in three of the virtual kits: Coding Awbie, Code.org Pre-reader express, and ScratchJr. For example, in Coding Awbie and Code.org's Pre-reader express, orientation is combined with a linear movement. Figure 2 depicts how an agent in Code.org pre-reader express enacts the program ➡ ➡ ⬇ (EAST, EAST, SOUTH). The first picture shows the agent's starting position and orientation. In the second image, the agent has moved one unit to the East. In the third image, the agent has moved two units to the East. The fourth image shows that the South facing arrow programmed the agent to rotate 90° to the Right and move one unit to the South, as evidenced by the agent's face. Children interacting with this kit do not need to program a rotation, rather, a change in orientation and a linear movement both happen with the SOUTH code. The arrow represents the direction of movement, but the code directs the agent to both change direction and move. In this way, the design also represents a "many-actions to one code correspondence". In contrast, the agent in ScratchJr faces the same direction no matter the directional arrow used (UP, DOWN, RIGHT, and LEFT). Those arrows orient the direction of the sprite's movement, but not its orientation. Thus, rotation and orientation are not connected and, specifically, orientation is not represented by the arrows. Figure 3 shows the agent (the cat sprite) enacting the program UP 5, RIGHT 5, DOWN 5.

5 DISCUSSION

The present study extends prior research on the design features of computational toys and kits [29, 47, 48], by examining how design features represent mathematical concepts central to Logo's Turtle

(a) (b) (c) (d)

Figure 1: Representations of Rotation in KIBO, Coding Awbie, Cubetto, and Code.org Pre-Reader Express.

Agent's starting orientation | Agent enacts the East code that is represented by → | Agent enacts the East code that is represented by → | Agent enacts the South code that is represented by ↓

Figure 2: Agent enacting the program E, E, S (East, East, South) in Code.org's Pre-Reader Express the Harvester.

Figure 3: Agent in ScratchJr enacting the program: UP 5, RIGHT 5, DOWN 5.

Geometry. Our findings provide insights into how contemporary computational toys and kits that involve programming an agent or robot, represent rotations and the agent's orientation. After a discussion on the analysis of the design features across the kits examined in the study, we provide implications for design and practice.

5.1 Representations of the Represented World

We surveyed 14 computational toys and kits, a mix of tangible, virtual, and hybrid. Since these toys and kits are made for emerging readers, most of them use arrows to represent angles/rotation and the robot's heading to represent orientation. In terms of the representing world, the arrow shapes and colors are not standard across toys. Bee-Bot and Coding Mouse both have arrows on their backs that children push to program its movement. The representation designates a direction rather than a rotation; however, the movement is a rotation. Being on the back of the robot intuits the robot's heading, for example, if Bee-Bot rotates right, it is easier to "see" which way is forward for the robot no matter one's own orientation in relation to the robot. Cubetto uses colored tiles with a slight arrow shape and curve, which represents direction (arrow) and rotation (curve). While less intuitive and less clearly an arrow, the point and curve seem to represent rotation. Code.org's early levels for children use straight arrows that point East, West, North, and South (the terms the program uses to describe the arrows). ScratchJr uses similar straight arrows that point Left, Right, Up and Down (the terms the program uses to describe the arrows). Osmo has one arrow that a child can reorient to represent the direction the agent should move. These types of arrows represent direction rather than rotation. Given the lack of consistency across toys, it is not surprising that research has found that children struggle with code meaning and what the symbols represent [9, 32]. What is the arrow representing in the represented world – rotational movement (a foundation for angle concepts) and/or direction (learning right

Table 3: Excerpt from Code Book for Analyzing Toys and Kits

	Rotation on a Point	Spatial Orientation	1 to 1 Correspondence	Linear Units
Botley	The agent rotates on a point -90°. Rotations are represented by a curved arrow. Left rotation is yellow and right rotation is blue. There is not a visual indication that it is 90°.	The orientation is represented by robot's heading or the direction it is facing. It moves forward in the direction it is oriented. A Rotate Right arrow makes Botley stay in the same position while reorienting its heading 90° to the right.	Yes. A rotation changes the robots heading but not their position.	Rotation is always 90°
ScratchJr	The sprite rotates on a point 30° to the left or right. It uses clockwise and counterclockwise. You can enter a number for number of rotations. 12= a full rotation. Rotations are represented by a curved arrow facing either left or right.	The sprite's heading is represented by arrows that point Up, Down, Left, and Right. These arrows program the sprite to move in that corresponding direction (Up, Down, Left, and Right arrows). Orientation is equal to direction.	Yes. A rotation changes the robots heading but not their position.	Each rotation is 30° and you can program a number of rotations.

and left)? What does the arrow mean for the robot's orientation in space?

5.2 (Mis) Representations of the Represented World

Our findings identified three ways in which the representations were not structurally preserved in some of the representing worlds. First, in the represented world of early childhood mathematics, a turn rotation is a rotation on a point that changes the agent's heading. Eleven of the toys and kits represented rotation on a point, which emphasizes the angle concept and direction (i.e., not just direction). When a rotation was executed, the tangible agents changed their heading using a measured rotation, but not their position. However, this was not the case for three of the kits that had virtual agents. The virtual agents (such as the Scratch Cat) are two-dimensional representations of the represented world's movements. Hence, the arrows represented the movement of a representation (virtual agent) of the represented world (movement in space in the real-world or three-dimensional world). Hence, in these three kits with virtual agents, the concept of rotation tends to be left out or engaged differently than a three-dimensional agent's movements in space.

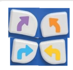
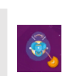
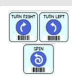
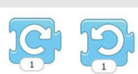
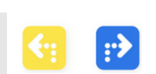
In Coding Awbie and Code.org, rotation on a point was not represented, in other words, there was not a code for children to program a rotation. Both of the programs have movement codes that orient the agent North or Up, Left or West, Right or East, and South or Down. These codes changed the agent's heading by combining heading and direction (as opposed to changing heading via a rotation). Similarly, in ScratchJr, the orientation of the agent was programmed separately from the rotation. ScratchJr has directional codes represented by arrows: UP, DOWN, LEFT, and RIGHT. It also has a rotation code that rotates the sprite in increments of 30°. The sprite can be programmed to move around the screen in linear movements using the UP, DOWN, LEFT, and RIGHT blocks which simply change the direction it moves. In this program, the left direction is a different behavior than rotate left. Thirdly, in Coding Awbie and Code.org, orientation of the agent is represented by one-code-to-multiple-movement correspondence. For example, a right facing arrow makes the agent change their heading and position. In all three of these examples, rotations and orientation of the agent do not follow the same rules as three-dimensional mathematics of the represented world, potentially leading to misunderstandings of how these concepts function in the real world. This is likely due to the constraints of representing movements in two-dimensional space rather than three-dimensional space.

5.3 Implications for Design and Practice

There is a need for models of integrated mathematics and CS lessons for primary school e.g., [31]. Computational toys and kits have potential to help children learn foundational mathematics and spatial skills, however, their potential depend on design features that have an operational relationship with the real world. We identified implications for design of computational toys and kits for young children as well as using these kits in primary classroom settings.

5.3.1 Implications for Design. Here we present four implications for design of future toys and kits. First, designers should consider how the design features foster CT, mathematical knowledge, and spatial thinking in integrated and dynamic ways and leverage these affordances in their designs. Second, representing worlds should be explicit about how they are representing mathematical, spatial thinking, and/or computational thinking concepts and when these concepts are not being portrayed accurately. Any inaccurate representations should be clear to users. Third, children's perceptions and intuitions are different from those of adults [18]. Designers should understand children from a developmental perspective and how children may intuitively interact with these toys. Finally, designers should consider using similar symbols and language for representing movement codes such as curved arrows for left and right rotations and forward and backwards arrows for movement; or at least differentiate a rotation arrow from a forward arrow.

Table 4: Computational Toys and Kits Explored in the Study (n=14)

Tool	Age Range	Kit Type	Representation of Angles (Rotation on a Point)	Rotation Unit	1 to 1 Unit Correspondence
Bee Bot App	3-8	Virtual		90°	1 unit = 1 Button push Linear: 1 Grid Square
Bee Bot Toy	3-8	Tangible		90°	1 unit = 1 Button push Linear: 6"
Blue Bot	3 +	Hybrid		90° 45°	1 unit = 1 Button push Linear: 15 cm
Botley	5-7	Tangible		90°	1 unit = 1 Button push Linear: 8"
Botley 2.0	5-7	Tangible		45° and 90°	1 unit = 1 Button push Linear: 8"
Code.org	5-11	Virtual	Rotations not Represented	90°	1 unit = 1 Code Block Linear: Grid Square
Cubetto	3-6	Tangible		90°	1 unit = 1 Code Block Linear: 6"
Dash and Dot	6-11	Tangible		programmable	1 unit = 1 Code Block Linear: 10 cm
KIBO	4-7	Tangible		90° and 360°	1 unit = 1 Code Block
MatataLab	4-9	Tangible		90°	1 unit= 1 Code Block Linear: 10 cm
Osmo Coding Awbie	5-10	Hybrid	Fixed Rotations not Represented	90°	1 unit = 1 Code Block Linear: Grid Square
Robot Mouse	4+	Tangible		90°	1 unit = 1 Button push Linear: 5"
ScratchJr	5-7	Virtual		30°	1 unit = 1 Code Block Or number on block Linear: Grid Square
Tale Bot Pro	3-5	Tangible		90°	1 unit = 1 Button push Linear: 10cm or 15cm (adjustable setting)

5.3.2 Implications for Practice. Learning in mathematics, more so than other domains, entails a building process in that children build on prior knowledge and experiences [4]. One reason some children struggle with mathematics in primary school is that they have underdeveloped informal knowledge of mathematics [5]. Thus, providing opportunities for children to play with mathematical concepts is important. *But what happens if the mathematical concepts they explore are not correct?* Research already tells us that children have difficulty learning the concepts of angles and rotation (see citations above) and it could be that some of the designers of computational toys and kits thought it would be easier for children to not grapple with angles. However, while angles are complex, they can be taught in the early grades in the context of the concept of rotation on a point. Exposing children to multiple representations of angles is one way to support later learning [7]. As research on Logo Turtle geometry found, children who had support were able to develop an understanding of angles [16]. Thus, computational toys and kits can be used in primary classrooms, but children need support when exploring challenging concepts like angles and orientation. Educators can provide guidance by asking children to consider how the toys and kits are representing rotation/angles and how the toys' movements may be different across toys or from a real-world context. Educators should not assume all computational toys and kits are created equal [6].

6 CONCLUSION

Contemporary computational toys and kits, while rooted in the historical framework of Logo Turtle Geometry, have shifted away from an emphasis on mathematics and spatial thinking towards an emphasis on CT. This is important to recognize because these toys and kits have potential to foster *integrated* mathematics, CT, and spatial skills that are foundational for later mathematics learning. To leverage this potential, it is important to explore the design features of these toys and kits, understand their affordances and limitations, and consider the ways children may make meaning of the represented and representing mathematical worlds these toys and kits embody. By doing so, we can better identify the kinds of supports children need to effectively engage with these tools and build a foundation for later mathematics learning.

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